자료구조

Chap 8. Search

2018년 1학기

컴퓨터과학과 민경하

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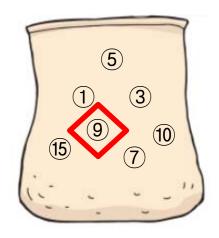
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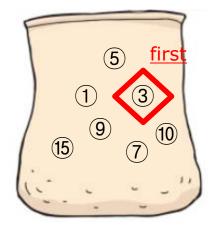
- 7.0 Introduction
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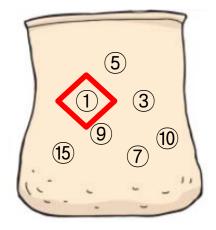
- 3 types of search
 - Find an arbitrary element in a given set
 - Find the first/last element in a give set
 - Find the top (maximum/minimum) element in a given set



Find arbitrary



Find first/last



Find top

Traversal

- Find all the elements reachable from an element in a given set
- Used in tree and graph
 - Depth first search = depth first traversal
 - Breadth first search = breadth first traversal
 - Inorder search = inorder traversal
 - Preorder search = preorder traversal
 - Postorder search = postorder traversal

• All about search (1)

search	operation	Data structure	algorithm	Tim	e comple	exity
Scarcii	орстастот	Data Structure	algoritiiii	search	insert	delete
		Unsorted array	Linear search	O(n)	0(1)	
			Linear search	O(n)		O(n)
		Sorted array	Binary search	O(log n)	O(n)	O(n)
Arbitrary	Find x		Interpolation search	O(log n)		
		Binary search tree	Search	O(log n) / O(n)	O(log n) / O(n)	O(log n) / O(n)
		Hash	Hashing	0(1)	O(1)	O(1)
		Unsorted array	Find max/min	O(n)	0(1)	O(n)
Тор	Find max/min	Sorted array	Find max/min	O(1)	O(n)	O(n)
	max/min	Неар	Find max/min	0(1)	O(log n)	O(log n)
	Find first/	Stack	Рор			
Arrival	Arrival last	Queue	DeleteQ	O(1)	O(1)	O(1)

• All about search (2)

Туре	Data structure	Type of	Algorithm	Tim	e comple	exity	
Турс	Data Structure	search	Algorithm	search	insert	delete	
	Unsorted array	Arbitrary	Linear search	O(n)	O(1)		
	Olisoited array	Тор	Find max/min	O(n)	0(1)		
			Linear search	O(n)		O(n)	
Linaan	Sorted array	Arbitrary	Binary search	O(log n)	O(n)	O(II)	
Linear	Sorted array		Interpolation search	O(log n)	0(11)		
		Тор	Find max/min	0(1)			
	Stack		Рор	0(1)	0(1)	0(1)	
	Queue	Arrival	Push	0(1)	O(1)	O(1)	
Hiearchical	Binary search tree	Arbitrary	Search	O(log n) / O(n)	O(log n) / O(n)	O(log n) / O(n)	
	Неар	Тор	Find max/min	0(1)	O(log n)	O(log n)	
Hash	Hash	Arbitrary	Hashing	0(1)	0(1)	0(1)	

- Linear data structure
 - data structure
 - A data structure whose elements are mapped by indices

index	1	2	3	4	5	6	7	8	9	10	11	12	
element	20	42	55	62	78	92	112	132	140	145	150	170	

- List
- Implementation
 - Consecutive VS separate
 - Array VS linked list
 - Index VS pointer

(1) Linear search

- Definition:
 - Search an element by visiting the elements from the first one to the last one

Next element to visit: (i+1)

index	1	2	3	4	5	6	7	8	9	10	11	12
element	20	42	55	62	78	92	112	132	140	145	150	170

- Working on unsorted list
- -O(n)

(2) Binary search

- Definition:
 - Search an element by visiting the element in the center index (mid)

$$mid = \frac{(high + low)}{2} = low + \frac{(high - low)}{2}$$

	index	1	2	3	4	5	6	7	8	9	10	11	12
	element	20	42	55	62	78	92	112	132	140	145	150	170
		Î											
low						mid						high	

low: 1 mid =
$$1 + (12-1)/2 = 6.5 \approx 6$$

high: 12

(2) Binary search

- Example:
 - Search 150 from List[1..12]

index	1	2	3	4	5	6	7	8	9	10	11	12
element	20	42	55	62	78	92	112	132	140	145	150	170



- Step 1. Search 150 from List[1..12]
 - mid = 1 + (12-1)/2 = 6.5 = 6
 - Compare List[6] & 150
 - If 150 > List[6] → Search 150 from List[7..12]
 - Else → Search 150 from List[1..5]

(2) Binary search

- Example:
 - Search 150 from List[1..12]

index	1	2	3	4	5	6	7	8	9	10	11	12
element	20	42	55	62	78	92	112	132	140	145	150	170



- Step 2. Search 150 from List[7..12]
 - mid = 7 + (12-7)/2 = 9.5 = 9
 - Compare List[9] & 150
 - If $150 > \text{List}[9] \rightarrow \text{Search } 150 \text{ from } \text{List}[10..12]$
 - Else → Search 150 from List[7..8]

- Binary search
 - Example:
 - Search 150 from List[1..12]

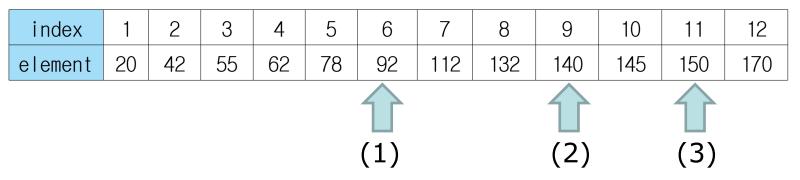
index	1	2	3	4	5	6	7	8	9	10	11	12
element	20	42	55	62	78	92	112	132	140	145	150	170



- Step 3. Search 150 from List[10..12]
 - mid = 10 + (12-10)/2 = 11
 - Compare List[11] & 150
 - If 150 > List[11] → Search 150 from List[12]
 - Else → Search 150 from List[10]
 - If List[11] == 150 → Bingo!!

(2) Binary search

- Example:
 - Search 150 from List[1..12]



– Problem of binary search?

(2) Binary search

- Example: Search 150 from List[1..12]

index	1	2	3	4	5	6	7	8	9	10	11	12
element	20	42	55	62	78	92	112	132	140	145	150	170
						(1)			(2)		(3)	

- Problem of binary search?
 - 150 (key to find) is closer to 170 (high) than to 20 (low) (|150 170| < |150 20|)
 - Searching from high is more efficient than searching from mid or low

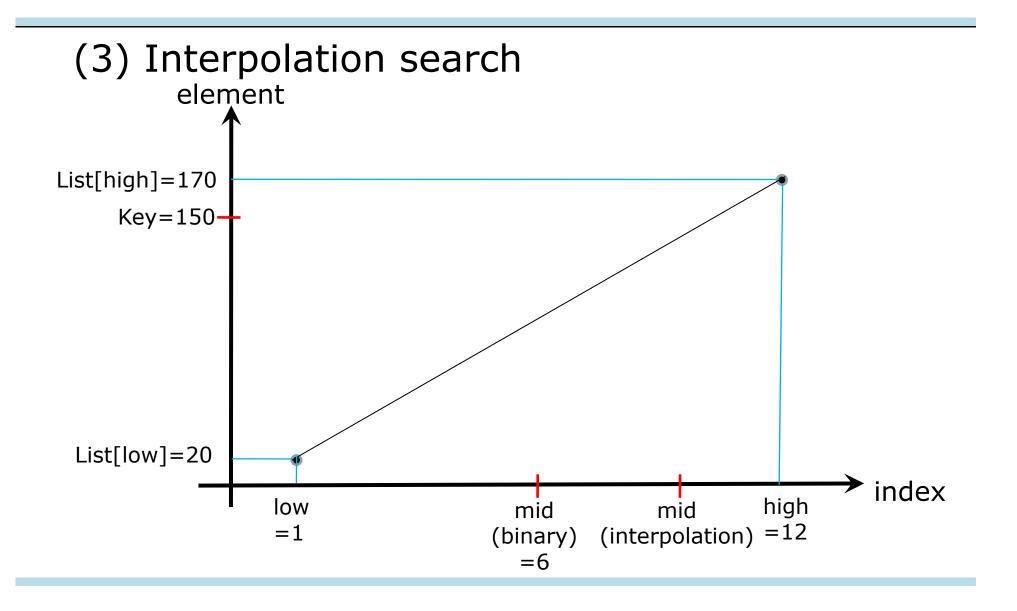
(3) Interpolation search

 Search an element by visiting the element in the center index (mid)

$$mid = low + \frac{(Key - List[low])}{(List[high] - List[low])}(high - low)$$

In binary search,

$$mid = low + \frac{1}{2}(high - low)$$



(3) Interpolation search

Example: Search 150 from List[1..12]

index	1	2	3	4	5	6	7	8	9	10	11	12
element	20	42	55	62	78	92	112	132	140	145	150	170



$$mid = low + \frac{(Key - List[low])}{(List[high] - List[low])}(high - low)$$

- Step 1. Search 150 from List[1..12]
 - mid = 1 + ((150-20)/(170-20))*(12-1) = 10.5 = 10
 - Compare List[10] & 150
 - If 150 > List[10] → Search 150 from List[11..12]
 - Else → Search 150 from List[1..9]

(3) Interpolation search

Example: Search 150 from List[1..12]

index	1	2	3	4	5	6	7	8	9	10	11	12
element	20	42	55	62	78	92	112	132	140	145	150	170

$$mid = low + \frac{(Key - List[low])}{(List[high] - List[low])}(high - low)$$

- Step 2. Search 150 from List[11..12]
 - mid = 11 + ((150 150)/(170 150))*(12 11) = 11
 - Compare List[11] & 150
 - If 150 > List[11] → Search 150 from List[12]
 - Else → None
 - If 150 == List[11] → Bingo!!

Summary

	Linear search	Binary search	Interpolation search		
Next one to visit	i + 1	$low + \frac{1}{2}(high - low)$	low + $\frac{(\text{Key} - \text{List[low]})}{\left(\text{List[high]} - \text{List[low]}\right)}$ (high – low)		
Time complexity	O(n)	O(log n)	Improved O(log n)		
On sorted list?	No	Yes	Yes		
Depends on implementation?	No	Yes	Yes		

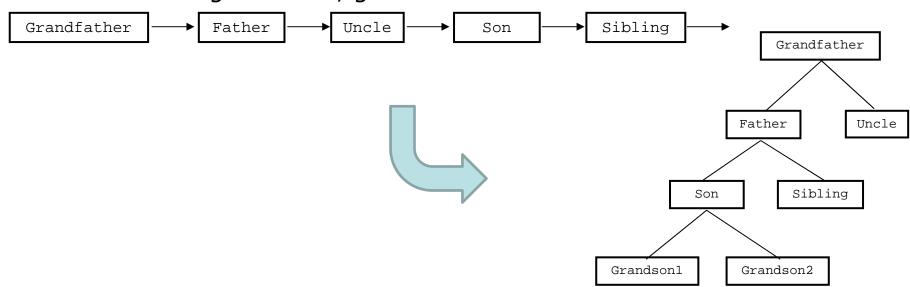
- Hierarchical data structure
 - Binary search tree

- Heap

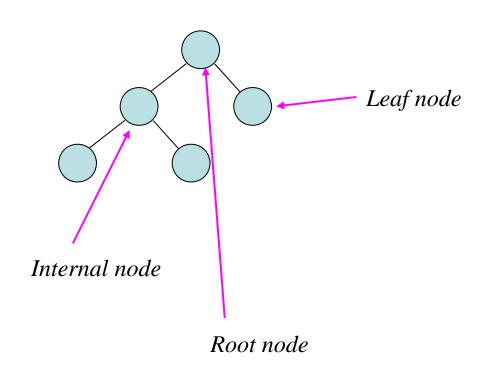
- AVL tree

- B+ tree

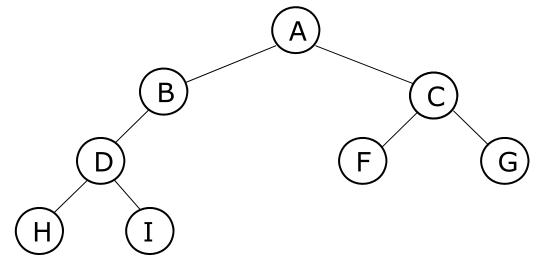
- Hierarchical data structure
 - Limitations of linear data structure?
 - Representation of "family record"
 - grandfather
 - father, uncle
 - son, sibling
 - grandson1, grandson2



- Hierarchical data structure
 - Tree

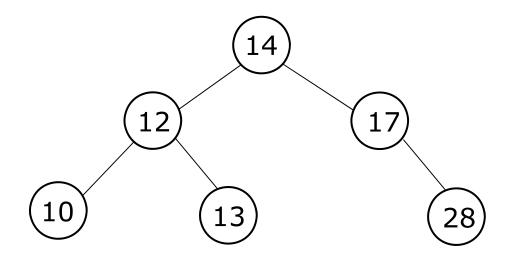


- Hierarchical data structure
 - Binary tree
 - A tree whose nodes have at most two child nodes



- Tree traversal algorithm: O(n)
 - Pre-order search
 - In-order search
 - Post-order search

- Binary search tree (BST)
 - A binary tree (may be empty)
 - Satisfies the following properties
 - Each node has exactly one key, which is distinct
 - The keys in the left subtree < the key in root
 - The keys in the right subtree > the key in root
 - The left and right subtrees are also binary search tree



- Binary search tree (BST)
 - Search algorithm

```
element search ( BST root, KEY key )
{
    if ( !root )
        return NULL;
    if ( key == root->key )
        return root->data;
    if ( key < root->key )
        return search ( root->lchild, key );
    else
        return search ( root->rchild, key );
}
```

```
12 17 17 10 13 22 2
```

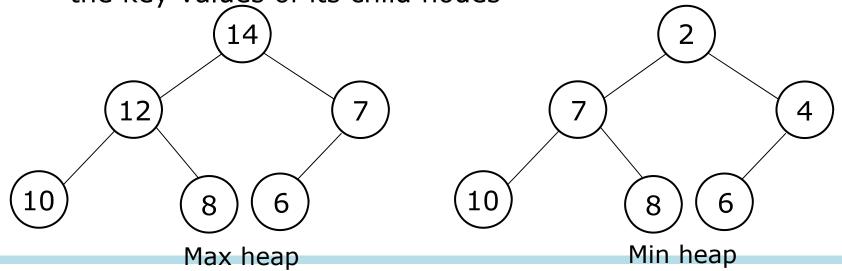
- Binary search tree (BST)
 - Time complexity?
 - O (log₂ n)
 - Is it better than binary search on list?
 - Why binary search tree is better than sorted list?

```
element search ( BST root, KEY key )
{
    if ( !root )
        return NULL;
    if ( key == root->key )
        return root->data;
    if ( key < root->key )
        return search ( root->lchild, key );
    else
        return search ( root->rchild, key );
}
```

Heap

- Priority queue
 - The element to be deleted is the one with the highest (or lowest) priority
- A complete binary tree
- Max heap (Min heap)

 The key value in each node is no smaller (greater) than the key values of its child nodes

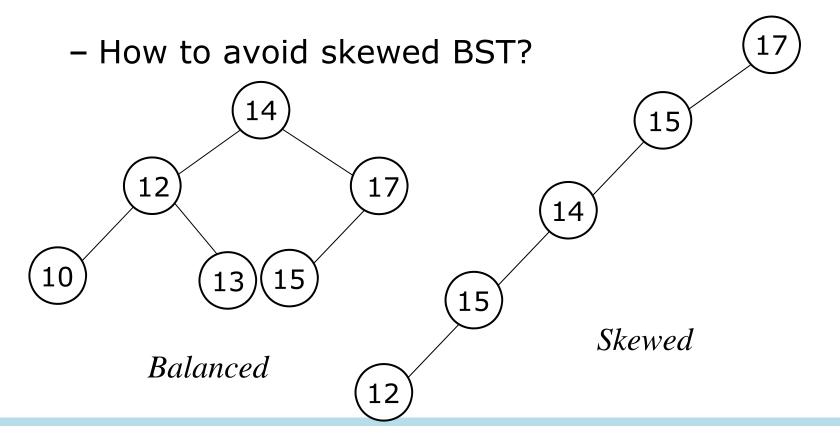


Comparison of the data structures

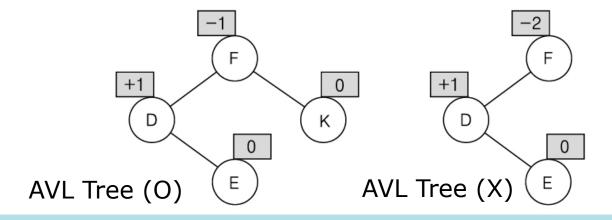
	Sorted list	BST	Heap
What is it?	A list whose elements are sorted (array VS linked list)	A binary tree	A complete binary tree
Arbitrary	Sorted array: O (log n) Linked list: O (n)	O (log n)/O(n)	-
Тор	Sorted array: O (1) Linked list: O(1)	O (log n)/O(n)	0 (1)
Insert/Delete	Array: O (n) Linked list: O (n)	O (log n)/O(n)	O (log n)

- Balanced Binary Search Tree (BBST)
 - Self-balancing search tree
 - Height-balanced search tree
 - a search tree that attempts to keep its height as small as possible at all times, automatically
 - (1) AVL tree
 - (2) Red-black tree
 - (3) 2-3 tree
 - (4) B+ tree

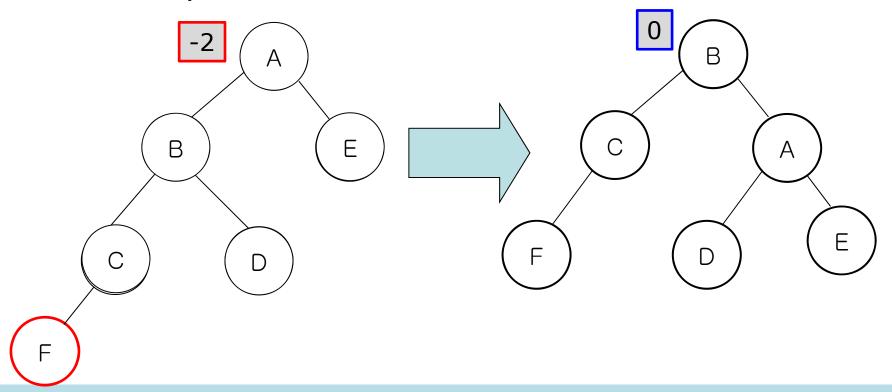
- Weakness of binary search tree
 - On a skewed BST, all operations take O (n)



- G. M. Adelson-Velskii & E. M. Landis
- the heights of the two child subtrees of any node differ by at most one
- Balance factor
 - = Height of right subtree Height of left subtree



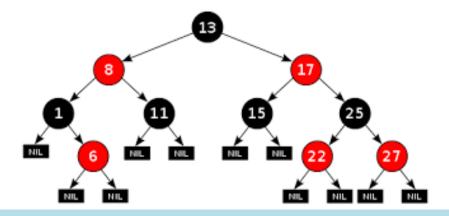
- After insertion/deletion, balance can be broken
- Using balancing operations, the tree is modified to keep the balance



- Balancing operations
 - Single left rotate
 - Single right rotate
 - Double left rotate
 - Double right rotate

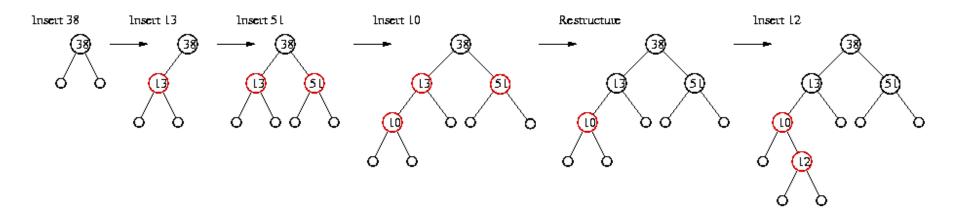
(2) Red-black tree

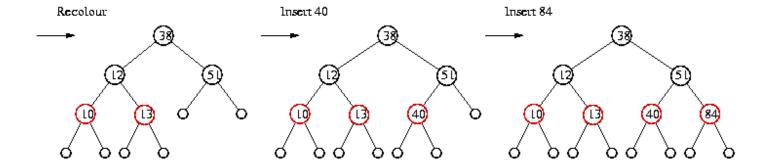
- L. J. Guibas and R. Sedgewick (1978)
- Five properties
 - Each node is either red or black
 - The root is black
 - All leaf nodes (NIL node) are black
 - If a node is red, then its both child nodes are black
 - Every path from a given node to all of its descendent
 NIL nodes contain the same number of black nodes



(2) Red-black tree

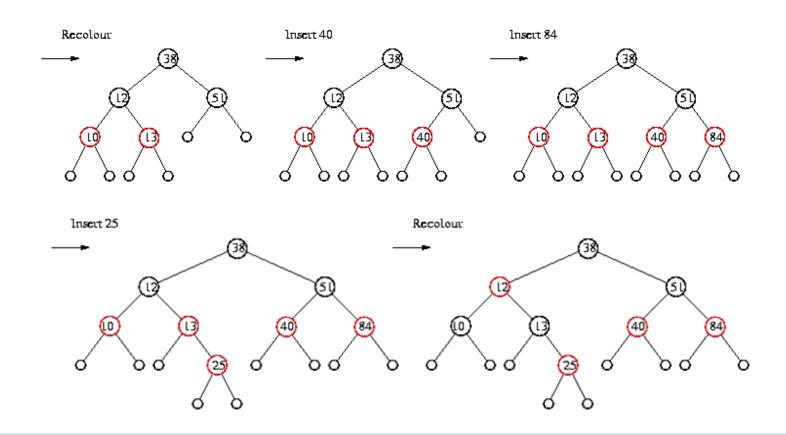
Insertions: 38, 13, 51, 10, 12, 40, 84, 25



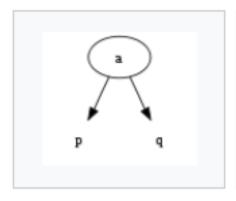


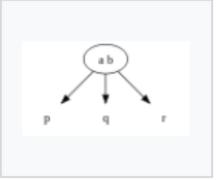
(2) Red-black tree

Inscitions: 38, 13, 51, 10, 12, 40, 84, 25



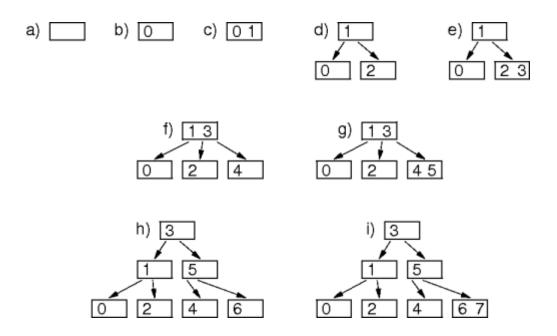
- (3) 2-3 tree
 - J. Hopcroft (1970)
 - A search tree whose nodes are either 2-node or 3-node.
 - 2-node
 - One key value with two child nodes
 - 3-node
 - Two key values with three child nodes





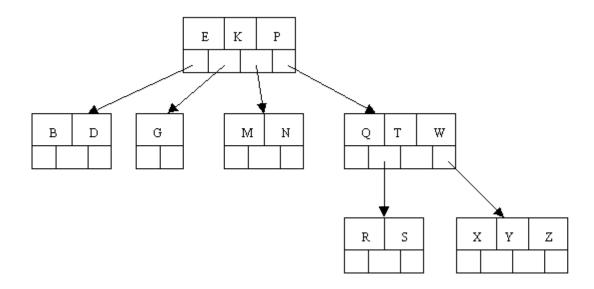
(3) 2-3 tree

- Insert 0, 1, 2, 3, 4, 5, 6, 7



(4) B+ tree

- A multi-way tree
 - Each node stores
 - k keys
 - (k+1) subtrees
- Time complexity: O (log_k n)



7.3 Search on graph

Travesal

 Find a vertex of a graph that contains the key element

- Depth-first traversal
- Breadth-first traversal

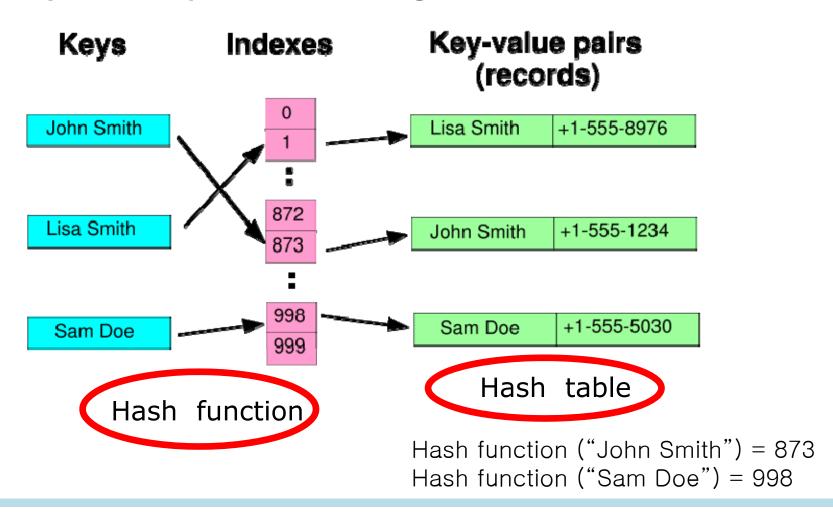
- Why hashing?
 - 이진 탐색
 - O(log n)의 탐색 시간
 - 자료의 크기에 따라서 검색 시간이 결정됨
 - 자료의 크기에 상관없이 실시간에 탐색이 수행되어야 하는 경우가 있음
 - O(1)의 탐색 시간을 보장하는 탐색 알고리즘

Definition

"모든 키의 레코드를 산술 연산에 의해 한 번에 바로 접근할 수 있는 기법"

- Hash function
- Hash index
- Hash table
- Collision & collision resolution

Key concept of hashing



- Hash function (1)
 - ① 자릿수 선택 (digit selection)
 - 키의 값 중에서 일부 자릿수만 골라내서 인덱스를 생성하는 함수
 - 예
 - 13자리 주민등록번호 중 홀수 자릿수만 선택

```
h(8812152051218) = 8112528
```

• 충돌이 발생하는 경우는?

$$h(8812152051218) = 8112528$$

$$h(8711142152238) = 8112528$$

- Hash function (2)
 - ② 자릿수 접기 (digit folding)
 - 키의 각각의 자릿수를 더해서 인덱스를 생성하는 함수
 - 예 h(8812152051218) = 8 + 8 + 1 + ... + 8 = 44
 - 충돌이 발생하는 경우는?

- Hash function (3)
 - ③ 모듈로 연산 (modulo function)
 - 키를 해쉬 테이블의 크기로 나눈 나머지를 인덱스로 생성하는 함수
 - h(KEY) = KEY mod TableSize
 - 충돌이 발생하는 경우는? h(8812152051218) = 8812152051218%100 = 18

h(8713142152218) = 8713142152218%100 = 18

- 충돌 (collision)
 - 서로 다른 키를 가진 레코드가 해쉬 함수에 의해서 동 일한 인덱스로 대응되는 현상
 - 예
 - Hash function:

```
H(K) = K \mod m, where m = 17
```

- For $K_1 = 18 \& K_2 = 171$, - $H(K_1) = 18 \mod 17 = 1$ - $H(K_2) = 171 \mod 17 = 1$
- Collision !!!

• 충돌

- 예

0	1	2	3	4	5	6	7	8	9	
	18									
	18									

- 충돌 해소
 - 충돌이 발생한 레코드를 다른 주소에 저장하는 기법
 - 열린 어드레싱 (open addressing)
 - 충돌이 발생한 레코드를 저장할 다른 주소를 찾는 방법
 - 레코드의 주소 (address)가 변경될 수 있기 때문에 열린 (open) 어드레싱이라고 함
 - 선형 탐사 (linear probing)
 - 제곱 탐사 (quadratic probing)
 - 이중 해시 (double hash)
 - 닫힌 어드레싱 (closed addressing)
 - 충돌이 발생한 레코드를 동일한 주소에 저장하는 방법
 - 주소가 변경되지 않으므로 닫힌 (closed) 어드레싱이라고 함
 - 버켓 (bucket)
 - 별도 체인 (separate chain)

- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 충돌이 일어나면 그 다음 빈 곳에 원소를 저장함
 - T[h(KEY)]가 점유되어 있을 때,
 - -T[h(KEY) + 1]
 - -T[h(KEY) + 2]
 - ...
 - 배열의 끝을 만나면 다시 처음으로 되돌아와서 거기서부터 빈자리를 찾음
 - T[n-1]
 - -T[0]

- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 예: h(x) = x % 31

0	1	2	3	4	5	6	7	8	9	
			65							
										•
			65							

- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 예: h(x) = x % 31
 - 65를 삽입 → 65%31 = 3
 - 34를 삽입 → 34%31 = 3

0	1	2	3	4	5	6	7	8	9	
			65							
			34							

- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 예: h(x) = x % 31
 - 65를 삽입 → 65%31 = 3
 - 34를 삽입 → 34%31 = 3

0	1	2	3	4	5	6	7	8	9	
			65	34						
				<u> →</u>	-					•
			 34							

- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 예: h(x) = x % 31
 - 65를 삽입 → 65%31 = 3
 - 34를 삽입 → 34%31 = 3
 - 127을 삽입 → 127%31 = 3

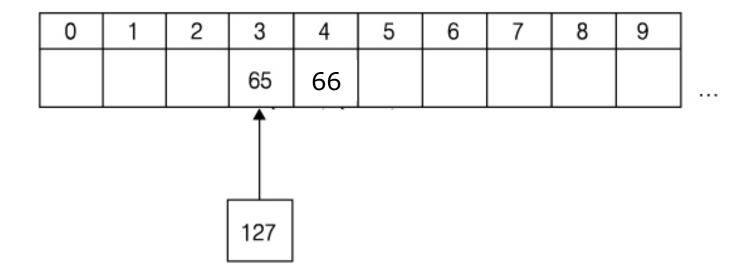
_	<u> </u>	·/宣 (<u>" " "</u>	<u> </u>	<u> </u>	$\perp = \zeta$	<u> </u>				
	0	1	2	3	4	5	6	7	8	9	
				65	34						
				127		-					

- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 예: h(x) = x % 31
 - 65를 삽입 → 65%31 = 3
 - 34를 삽입 → 34%31 = 3 → T[3+1] = T[4]
 - 127을 삽입 → 127%31 = 3 → T[3+2] = T[5]
 0 1 2 3 4 5 6 7 8 9
 65 34 127 ...

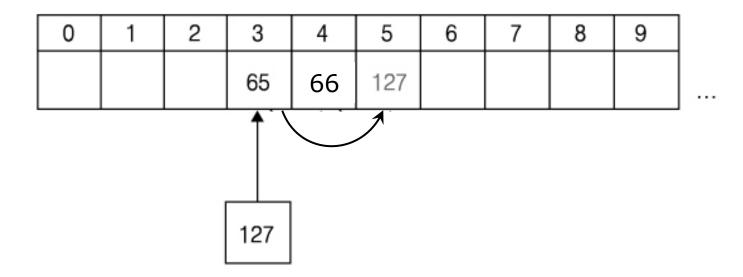
- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 태그 (tag)
 - 선형 탐사에서 다음 원소를 가리키는 포인터

0	1	2	3	4	5	6	7	8	9	
			65	34	127					
				X						J

- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 태그 (tag)의 필요성 (1)
 - 다음 자리 (h(key)+1)에 이미 다른 원소가 존재할 때



- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 태그 (tag)의 필요성 (1)
 - 다음 자리 (h(key)+1)에 이미 다른 원소가 존재할 때
 - 다음 원소의 위치를 가리킴



- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 태그 (tag)의 필요성 (2)
 - 중간의 원소를 삭제할 때

0	1	2	3	4	5	6	7	8	9			
			65		127							

- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 태그 (tag)의 필요성 (2)
 - 중간의 원소를 삭제할 때
 - 태그를 수정함

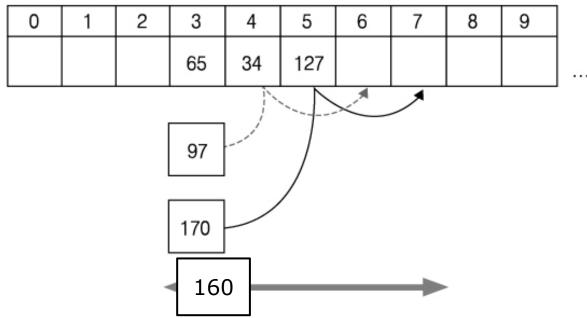
0	1	2	3	4	5	6	7	8	9	
			65		127					
					1					

- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 단점: 클러스터링 (clustering)
 - 레코드가 분산되지 않고 군집되는 현상

0	1	2	3	4	5	6	7	8	9	
			65	34	127					

...

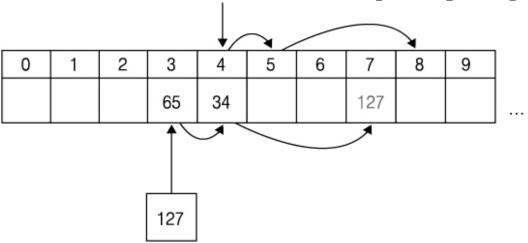
- 열린 어드레싱 (1)
 - ① 선형 탐사 (linear probing)
 - 단점: 클러스터링 (clustering)
 - 예: 97을 삽입: 97%31 = 4
 - 예: 160을 삽입: 160%31 = 5



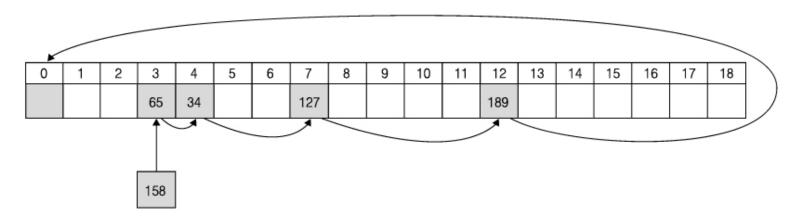
- 열린 어드레싱 (2)
 - ② 제곱 탐사 (quadratic probing)
 - 충돌이 일어날 때 바로 그 뒤에 넣지 않고 조금 간격을 두고 삽입
 - T[h(KEY)]가 점유되어 있을 때,
 - -T[h(KEY) + 1]
 - $-T[h(KEY) + 2^2]$

- ...

- 열린 어드레싱 (2)
 - ② 제곱 탐사 (quadratic probing)
 - 충돌이 일어날 때 바로 그 뒤에 넣지 않고 조금 간격을 두고 삽입
 - 예: h(x) = x % 31
 - 65를 삽입 → 65%31 = 3
 - 34를 삽입 → 34%31 = 3 → T[3+1] = T[4]
 - 127을 삽입 → 127%31 = 3 → T[3+2²] = T[7]



- 열린 어드레싱 (2)
 - ② 제곱 탐사 (quadratic probing)
 - 클러스터링을 피할 수 있는가?
 - 연속적인 배열을 피할 수 있음
 - 동일한 키를 가진 레코드는 동일한 자리에 삽입됨
 - 예: 158의 경우, 65, 34, 127, 189 다음의 자리에 삽입
 - 궁극적으로는 클러스터링을 피할 수 없음
 - → 2차 클러스터링



- 열린 어드레싱 (3)
 - ③ 이중 해쉬 (double hash)
 - 제곱탐사의 단점인 2차 클러스터를 방지
 - 두 개의 해쉬 함수 h₁, h₂를 사용
 - $-h_1$ 은 주어진 키로부터 인덱스를 계산하는 해쉬 함수
 - h₂는 충돌 시 탐색할 인덱스의 간격(Step Size)을 의미

- 열린 어드레싱 (3)
 - ③ 이중 해쉬 (double hash)
 - 예

$$- h_1 = KEY \% 13$$

$$-h_2 = 1 + KEY \% 11$$

• 14를 삽입 (KEY = 14)

$$- h_1 = 14 \% 13 = 1 → Collision$$

0	1	2	3	4	5	6	7	8	9	10	11	12
	79			69	98		72		15		50	

- 열린 어드레싱 (3)
 - ③ 이중 해쉬 (double hash)
 - 여

$$- h_1 = KEY \% 13$$

$$-h_2 = 1 + KEY \% 11$$

• 14를 삽입 (KEY = 14)

$$- h_1 = 14 \% 13 = 1$$
 → Collision

0	1	2	3	4	5	6	7	8	9	10	11	12
	79			69	98		72		15		50	
	1				1							
	14											

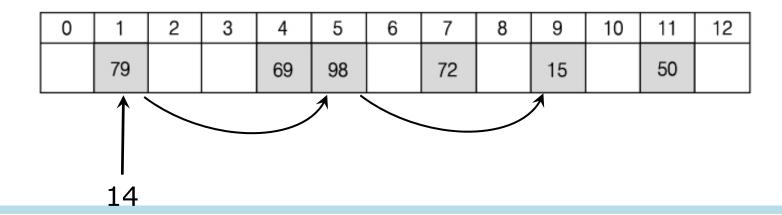
- 열린 어드레싱 (3)
 - ③ 이중 해쉬 (double hash)
 - 例

$$- h_1 = KEY \% 13$$

$$-h_2 = 1 + KEY \% 11$$

• 14를 삽입 (KEY = 14)

$$- h_1 = 14 \% 13 = 1$$
 → Collision

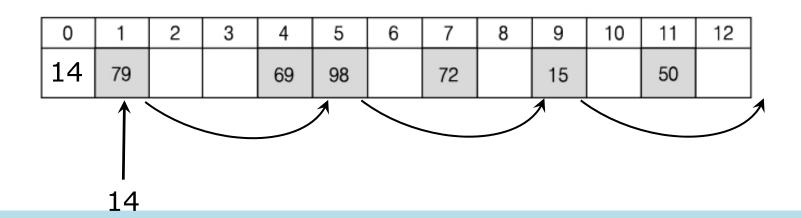


- 열린 어드레싱 (3)
 - ③ 이중 해쉬 (double hash)
 - 例

$$- h_1 = KEY \% 13$$

$$-h_2 = 1 + KEY \% 11$$

- 14를 삽입 (KEY = 14)
 - $h_1 = 14 \% 13 = 1$ → Collision
 - h₂ = 1 + 14% 11 = 4 → 4칸씩 건너뛰면서 삽입



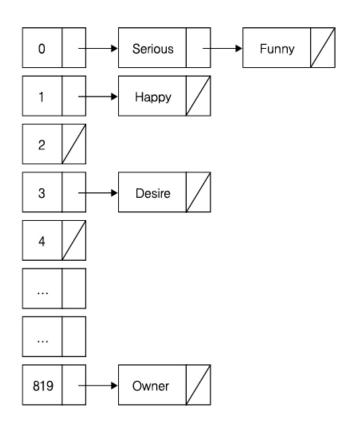
- 닫힌 어드레싱 (1)
 - ① 버켓 (bucket)
 - 해쉬 테이블의 각 원소들이 다시 여러 개의 원소로 이루어짐
 - 2차원 배열
 - 충돌되는 레코드를 하나의 인덱스에 둠
 - 사용되지 않는 배열 공간이 낭비됨

		3			4	
65	34	127	189			

• • •

- 닫힌 어드레싱 (2)
 - ② 별도 체인 (separate chain)
 - 해쉬 테이블의 각 원소들가 연결 리스트를 가리키는 헤드
 - 충돌이 일어날 때마다 해당 레코드를 연결 리스트의 첫 위치 에 삽입
 - 동적 구조(Dynamic Structure)

- 닫힌 어드레싱 (2)
 - ② 별도 체인 (separate chain)



키	해시 값
Funny	0
Нарру	1
Serious	0
Owner	819
Desire	3

7.5 Heuristic search

- Graph search algorithm
 - Find a path from a source node to a target node by estimating the best route
 - Depends on the heuristic
 - Find the best path according to the heuristic
 - After a failure, it searches an alternative
 - An evaluation function quantifies the heuristic to find the best path
- A* search
 - $f^{(n)} = g^{(n)} + h^{(n)}$
- Game programming

Contents

- 1. Introduction
- 2. Analysis
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- 5. Queue
- 6. Sorting
- 7. Tree
- 8. Search
- 9. Graph