Review 4-2

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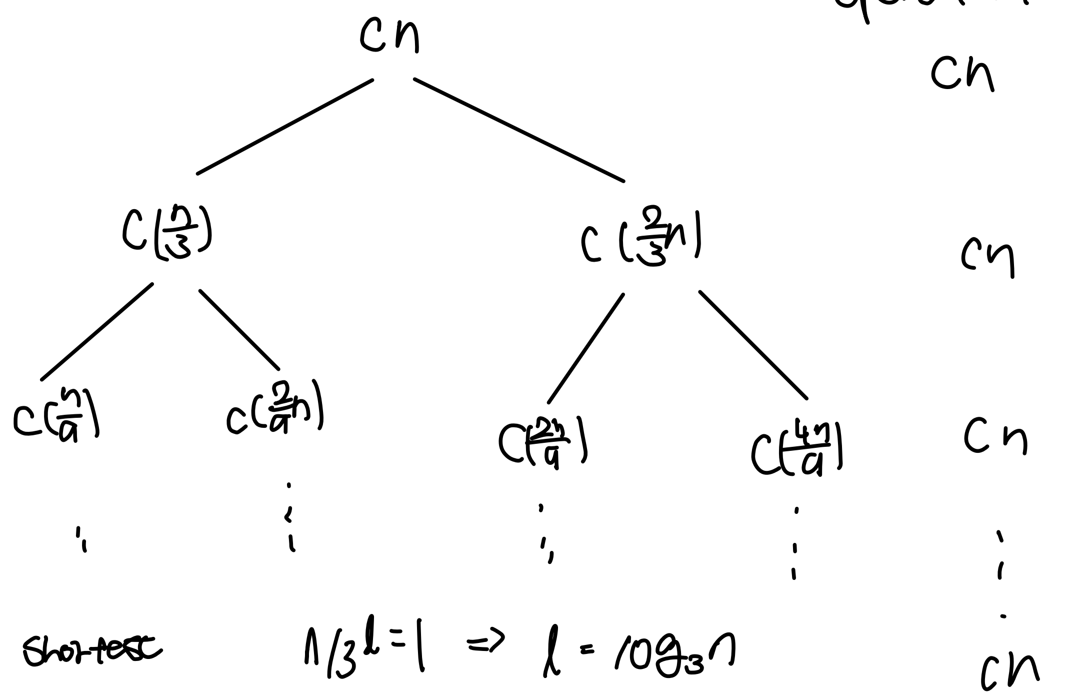
1. Guess the solution to the recurrence $T(n) = T(n/3) + T(2n/3) + cn$, where c is a constant, is $\Theta(n \lg n)$ (both $O(n \lg n)$ and $\Omega(n \lg n)$) by appealing to a recursion tree.

For $\Omega(n\log n)$, it is obvious because of $T(n) \geq cn$

For $O(n\log n)$

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + O(n)$$

Each level's operation



for shortest $\quad n/3^l = 1 \Rightarrow l = \log_3 n$

for longest $\quad \left(\frac{3}{2}\right)^l = n \Rightarrow l = \log_{\frac{3}{2}} n$

For Big-O notation, choose $\log_{\frac{3}{2}} n$

Whole operation $\quad cn \times \log_{\frac{3}{2}} n = cn\log_{\frac{3}{2}} n \Rightarrow O(n\log n)$

$$T(n) = T\left(\tfrac{2}{3}n\right) + T\left(\tfrac{n}{3}\right) + cn \leq d\left(\tfrac{n}{3}\right)\log\left(\tfrac{n}{3}\right) + d\left(\tfrac{2}{3}n\right)\log\left(\tfrac{2}{3}n\right) + cn$$

$$= dn\log n + d\left(-\tfrac{n}{3}\log 3 + \tfrac{2}{3}n\log\tfrac{2}{3}\right) + cn$$

$$= dn\log n + d\left(-\tfrac{n}{3}\log 3 + \tfrac{2}{3}n\log 2 - \tfrac{2}{3}n\log 3\right) + cn$$

$$= \cancel{dn\log n} + dn\left(-\log 3 + \tfrac{2}{3}\right) + cn \leq \cancel{dn\log n}$$

$$\Rightarrow d \geq \frac{c}{\log 3 - \frac{2}{3}}$$