

Review 3

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1. Write O if an entry is true or X otherwise.

| | $O(n \lg n)$ | $\Omega(n \lg n)$ | $\Theta(n \lg n)$ |
|-------------|--------------|-------------------|-------------------|
| $\lg n$ | o | X | X |
| n | o | X | X |
| $n \lg n$ | O | O | O |
| $n \lg^2 n$ | X | o | X |
| n^2 | X | o | X |

2. Show $3n + 1 = O(n^2)$ by the definition of O .

$$3n + 1 \leq Cn^2$$

$$\text{when } C = 4, n_0 = 1$$

$$\rightarrow 3n + 1 \leq Cn^2 \quad (n \geq 1)$$

$$\therefore 3n + 1 = O(n^2)$$

3. Write asymptotic notations that satisfy each relation and explain why.

(1) Transitivity

ex> O is transitive because $f(n) = O(g(n))$ and $g(n) = O(h(n))$ implies $f(n) = O(h(n))$.

Ω is transitive because $f(n) = \Omega(g(n))$ and $g(n) = \Omega(h(n))$ implies $f(n) = \Omega(h(n))$

Θ is transitive because $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n))$ implies $f(n) = \Theta(h(n))$

o is transitive because $f(n) = o(g(n))$ and $g(n) = o(h(n))$ implies $f(n) = o(h(n))$

ω is transitive because $f(n) = \omega(g(n))$ and $g(n) = \omega(h(n))$ implies $f(n) = \omega(h(n))$

(2) Reflexivity

Θ is reflexive because $f(n) = \Theta(f(n))$

o is reflexive because $f(n) = o(f(n))$

Ω is reflexive because $f(n) = \Omega(f(n))$

(3) Symmetry

Θ is symmetric because $f(n) = \Theta(g(n))$ if and only if $g(n) = \Theta(f(n))$

(4) Transpose symmetry

O and Ω is transpose symmetric because $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$

o and ω is transpose symmetric because $f(n) = o(g(n))$ if and only if $g(n) = \omega(f(n))$