Review 4-1

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1. Show that the solution of $T(n) = 2T(\lfloor n/2 \rfloor) + n$ is $O(n \lg n)$ by the substitution method. (Show the inductive step only.)

Inductive step

assumption $T(m) = O(m \log m)$

$T(m) \le c m \log m$

$T(n) = 2T(\lfloor \frac{n}{2} \rfloor) + n \le 2T(\frac{n}{2}) + n$

$= 2 \cdot (c \frac{n}{2} \log \frac{n}{2}) + n$

$= cn \log \frac{n}{2} + n$

$= cn(\log n - \log 2) + n$

$= cn \log n - cn \log 2 + n$

$\le cn \log n - n(c-1)$

WHEN $c \ge 1$

$\le cn \log n$

Boundary conditions

If $n_0 = 1$

$T(1) = 1 > c \log 1 = 0$

If $n_0 = 2$

$T(2) = 2T(1) + 2 = 4$

$T(2) \le 2c$

$\Rightarrow c \ge 2$

Check $T(3)$

$T(3) = 2T(1) + 3 = 5$

$T(3) \le 6 \log 3$

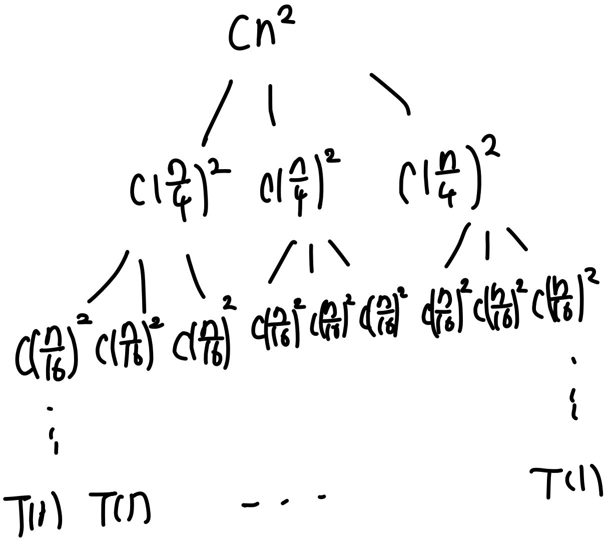
$T(4) = 2T(2) + 4$

$T(5) = 2T(2) + 5$

$T(n) \rightarrow$ covered by inductive step

2. Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 3T(\lfloor n/4 \rfloor) + \theta(n^2)$.

Assumption $T(n) = \theta(n^2) \Rightarrow T(n) = cn^2$



max level $= \log_4 n + 1$

number of nodes at level i: 3^i

Number of each step's computation at level i

$$= c\left(\frac{3}{16}\right)^i n^2$$

$$T(n) = \sum_{i=0}^{\log_4 n - 1} c\left(\frac{3}{16}\right)^i n^2 + \theta(n^{\log_4 3})$$

$$\le \sum_{i=0}^{\infty} c\left(\frac{3}{16}\right)^i n^2 + \theta(n^{\log_4 3})$$

$$= \frac{1}{1 - \frac{3}{16}} cn^2 + \theta(n^{\log_4 3})$$

$$= \frac{16}{3} cn^2 + \theta(n^{\log_4 3}) = \theta(n^2)$$

Derived guess: $T(n) = \theta(n^2)$