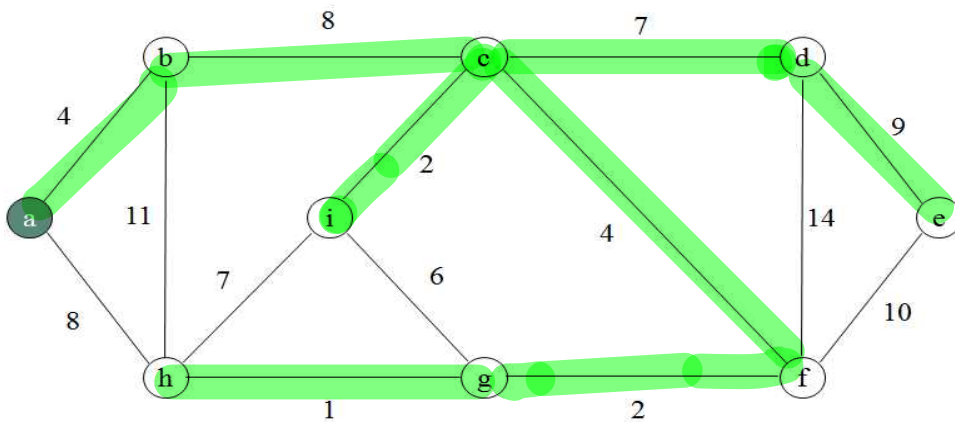


# Review 14

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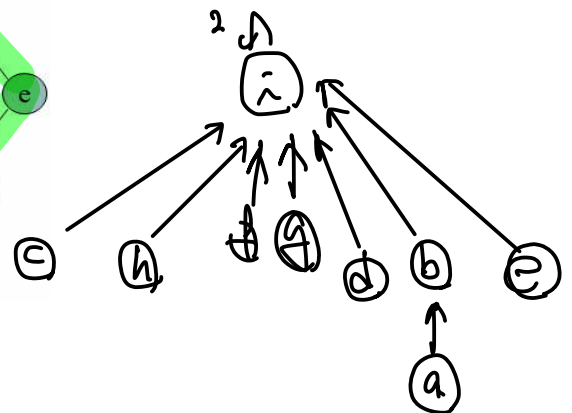
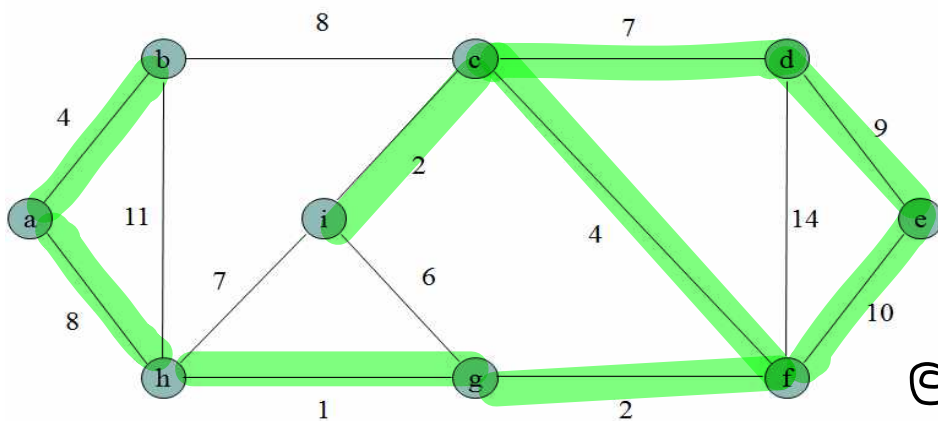
1. Show the minimum spanning trees of Prim's algorithm and Kruskal's algorithm for the following graph.

(1) Prim's algorithm: If there are 2 or more candidate vertices, select the alphabetically earliest one.



a b c i f g h d e

(2) Kruskal's algorithm: If there are 2 or more candidate edges, select the edge whose endpoint is the alphabetically earliest vertex.



2. What are the time complexities of the Prim's algorithm and the Kruskal's algorithm?

(1) Prim's algorithm (Use Min-Heap as the priority queue Q.)

MST-PRIM( $G, w, r$ )

```

1  for each  $u \in G.V$ 
2       $u.key = \infty$ 
3       $u.\pi = \text{NIL}$ 
4   $r.key = 0$ 
5   $Q = G.V$ 
6  while  $Q \neq \emptyset$ 
7       $u = \text{EXTRACT-MIN}(Q)$ 
8      for each  $v \in G.Adj[u]$ 
9          if  $v \in Q$  and  $w(u, v) < v.key$ 
10              $v.\pi = u$ 
11              $v.key = w(u, v)$ 

```

Handwritten notes for Prim's algorithm:

- Lines 1-3:  $\Theta(W)$
- Line 4:  $\Theta(V \log V + E \log V)$
- Line 5:  $E \geq V-1 \rightarrow \theta$ ,  $V = \theta(E) \rightarrow \boxed{\theta(E \log V)}$
- Line 7:  $\rightarrow \theta(V \log V)$
- Line 11: ~~decrease-key~~  $\rightarrow \theta(E \log V)$

(2) Kruskal's algorithm

MST-KRUSKAL( $G, w$ )

```

1   $A = \emptyset$ 
2  for each vertex  $v \in G.V$ 
3      MAKE-SET( $v$ )
4  sort the edges of  $G.E$  into nondecreasing order by weight  $w$ 
5  for each edge  $(u, v) \in G.E$ , taken in nondecreasing order by weight
6      if FIND-SET( $u$ )  $\neq$  FIND-SET( $v$ )
7           $A = A \cup \{(u, v)\}$ 
8      UNION( $u, v$ )
9  return  $A$ 

```

Handwritten notes for Kruskal's algorithm:

- Line 3:  $n = n + 1 + n \rightarrow O(V+E)$
- Line 4:  $\rightarrow O(m \alpha(n)) = O((V+E) \alpha(n))$
- Line 5:  $E \geq V-1 \rightarrow \boxed{O(E \alpha(n))}$
- Line 6:  $\rightarrow \boxed{O(E \log E)}$
- Line 7:  $O(E \log E + E \alpha(n))$
- Line 8:  $E \leq V^2 \rightarrow O(V^2 \log V + V)$
- Line 9:  $\sim O(E \log V)$