Problem_4

Suppose you are given the following information:

- Assume no-arbitrage
- It is currently time 0
- A policyholder has an initial investment of 200,000 and receives direct participation rate p_{dp} on their investment
- The continuous (flat) interest rate is r = 2%
- The time horizon is T = 1 year
- S_t denotes the stock price at time t
- One unit of at-the-money European option A has a payoff of $max[\frac{S_1}{S_0} - 1, 0]$
- One unit of option A costs \$0.04 for purchase today

If the insurer replicates the guarantee by purchasing a zero-coupon bond and call options, and assuming no-arbitrage, determine the dollar position they should invest in:

(a) Zero coupon bonds

$$200,000e^{-.02*1} = 196,039.74$$

(b) Risk budget spent on options

$$200,000 - 196,039.74 = 3,960.26$$

(c) Determine p_{dp}

$$V_0 * p_{dp} = 1 - e^{-.02}$$

$$.04 * p_{dp} = 1 - e^{-.02}$$

$$p_{dp} = \frac{1 - e^{-.02}}{.04} = 49.5\%$$

(d) State the cost of options f and g (according to the notation set up in (QFIQ-132)

- The cost of option f is .04
- The cost of option g is $200,000 * .04 = 8,000$

(e) Suppose the stock return over the holding period is 6%. Show all cash flows to the insurer at maturity.

The payoff from the zero coupon bond is:

$$200,000$$

The payoff of the option is:

$$6\% * 200,000 * 49.5\% = 5,940$$

The policyholder receives:

$$200,000 + 200,000 * 49.5\% * 6\% = 205,940$$

The net cash flow to the insurer is 0.

(f) State the three drawbacks of the VolTarget mechanism.

1. Only works well in specific markets. The VolTarget mechanism works well in falling markets with high volatility or rising markets with low volatility. For atypical market conditions outside of these two it can be an ineffective strategy.

2. The rule-based nature of the strategy can lead to significant losses in nonstandard market environments like a falling market with low volatility.
3. May not be sufficient to solely define portfolio management decisions and should be combined with other asset allocation strategies.