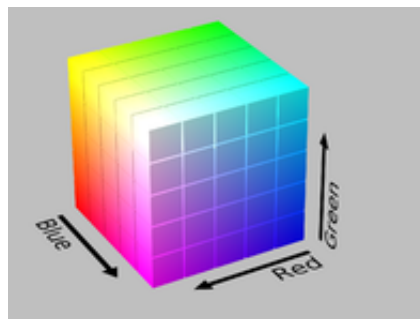


Math behind colorspace conversions, RGB-HSL

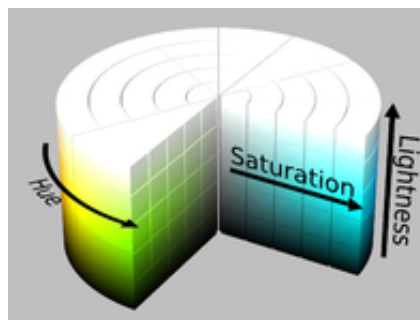
by 2013/05/08

In this article I will try to explain the math behind converting RGB values to HSL and back.

RGB is a way to describe a color in a cube, where Red, Green and Blue are on the different axis.



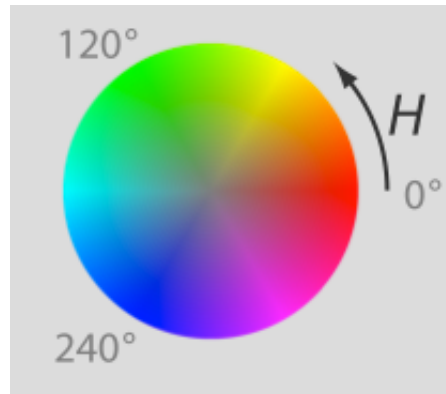
HSL is another way to describe a color. Here we use Hue (the angle on the color wheel), Saturation (the amount of color/chroma) and Lightness (how bright the color is).



RGB is used in many cases but sometimes it's a bit difficult to know what color you are describing just by looking at the numbers. R= 24, G= 98, B=118. It's a Deep Cyan color.



In HSL it's described like $H=193^\circ$ $S=67\%$ $L=28\%$. This tells us it's quite saturated and dark. When we look at the color wheel we see that 193° is very close to cyan.



So how is this conversion done?

RGB – HSL

1

Convert the RGB values to the range 0-1, this can be done by dividing the value by 255 for 8-bit color depth

$$R = 24 / 255 = 0.09$$

$$G = 98 / 255 = 0.38$$

$$B = 118 / 255 = 0.46$$

2

Find the minimum and maximum values of R, G and B.

min = 0.09 (the R value)

max = 0.46 (the B value)

3

Now calculate the Luminance value by adding the max and min values and divide by 2.

$$L = (0.09 + 0.46)/2 = 0.275 \text{ which rounded up is equal to } \mathbf{28\%}$$

4

The next step is to find the Saturation.

If the min and max value are the same, it means that there is no saturation. If all RGB values are equal you have a shade of grey. Depending on how bright it's somewhere between black and white. If there is no Saturation, we don't need to calculate the Hue. So we set it to 0 degrees.

But in our case min and max are not equal which means there is Saturation.

5

Now we know that there is Saturation we need to do check the level of the Luminance in order to select the correct formula.

If Luminance is less or equal to 0.5, then Saturation = $(\text{max} - \text{min}) / (\text{max} + \text{min})$

If Luminance is bigger then 0.5. then Saturation = $(\text{max} - \text{min}) / (2.0 - \text{max} - \text{min})$

In our case Luminance is smaller then 0.5, so we use the first formula.

$$S = (0.46 - 0.09) / (0.46 + 0.09) = 0.37 / 0.55 = 0.672 \text{ which is rounded down equal to } \mathbf{67\%}$$

6

Two done, one to go. We still need to calculate the Hue.

The Hue formula is depending on what RGB color channel is the max value. The three different formulas are:

If Red is max, then Hue = $(G - B) / (\text{max} - \text{min})$

If Green is max, then Hue = $2.0 + (B - R) / (\text{max} - \text{min})$

If Blue is max, then Hue = $4.0 + (R-G)/(max-min)$

The Hue value you get needs to be multiplied by 60 to convert it to degrees on the color circle

If Hue becomes negative you need to add 360 to, because a circle has 360 degrees.

Blue is our max, we have to use the third formula.

$$H = 4.0 + (0.09 - 0.38) / (0.46 - 0.09) = 4.0 + (-0.29)/0.37 = 3.21$$

Now convert it to degrees

$$H = 3.21 \times 60 = 192.9 \text{ which rounded up is } \mathbf{193 \text{ degrees}}$$

This is how simple it is to calculate HSL from RGB, what about the other way around?

HSL – RGB

1

If there is no Saturation it means that it's a shade of grey. So in that case we just need to convert the Luminance and set R,G and B to that level.

For example $H = 0$, $S = 0$ and $L = 40\%$, we get $0.4 * 255 = 102$, so $R = 102$, $G = 102$ and $B = 102$

In our case $S = 67\%$ which means we have saturation. In that case we need to go ahead.

2

We need to create some temporary variables. The variables are used to store temporary values which makes the formulas easier to read.

There are two formulas to choose from in the first step.

If Luminance is smaller than 0.5 (50%) then $temporary_1 = Luminance \times (1.0 + Saturation)$

If Luminance is equal or larger than 0.5 (50%) then $temporary_1 =$

Luminance + Saturation – Luminance x Saturation

Our Luminance is 28%, so we use the first formula.

$$\text{temporary_1} = 0.28 \times (1.0 + 0.67) = 0.28 \times 1.67 = \mathbf{0.4676}$$

3

We need one more temporary variable, temporary_2

$$\text{temporary_2} = 2 \times \text{Luminance} - \text{temporary_1}$$

$$\text{temporary_2} = 2 \times 0.28 - 0.4676 = \mathbf{0.0924}$$

4

The next step is to convert the 360 degrees in a circle to 1 by dividing the angle by 360.

$$\text{Hue} = 193/360 = \mathbf{0.536}$$

5

And now we need another temporary variable for each color channel, temporary_R, temporary_G and temporary_B.

$$\text{temporary_R} = \text{Hue} + 0.333 = 0.536 + 0.333 = \mathbf{0.869}$$

$$\text{temporary_G} = \text{Hue} = \mathbf{0.536}$$

$$\text{temporary_B} = \text{Hue} - 0.333 = 0.536 - 0.333 = \mathbf{0.203}$$

All values need to be between 0 and 1. In our case all the values are between 0 and 1

If you get a negative value you need to add 1 to it.

If you get a value above 1 you need to subtract 1 from it.

6

Now we need to do up to 3 tests to select the correct formula for each color channel. Let's start with Red.

test 1 – If $6 \times \text{temporary_R}$ is smaller then 1, $\text{Red} = \text{temporary_2} + (\text{temporary_1} - \text{temporary_2}) \times 6 \times \text{temporary_R}$

In the case the first test is larger then 1 check the following

test 2 – If $2 \times \text{temporary_R}$ is smaller then 1, $\text{Red} = \text{temporary_1}$
 In the case the second test also is larger then 1 do the following

test 3 – If $3 \times \text{temporary_R}$ is smaller then 2, $\text{Red} = \text{temporary_2} + (\text{temporary_1} - \text{temporary_2}) \times (0.666 - \text{temporary_R}) \times 6$

In the case the third test also is larger then 2 you do the following

$\text{Red} = \text{temporary_2}$

Ok lets do it for our Red value

$6 \times \text{temporary_R} = 6 \times 0.869 = 5.214$, so it's larger then 1, we need to do
 test 2

$2 \times \text{temporary_R} = 2 \times 0.869 = 1.738$, it's also larger then 1, we need to do
 test 3

$3 \times \text{temporary_R} = 3 \times 0.869 = 2.607$, it's larger then 2, so we go for the
 last formula

Red = $\text{temporary_2} = 0.0924$ which rounded down is **0.09**, which is a
 number we recognize from the RGB to HSL conversion

Now let's do Green

$6 \times \text{temporary_G} = 6 \times 0.536 = 3.216$, to large

$2 \times 0.536 = 1.072$, also to large

$3 \times 0.536 = 1.608$, which is below 2, so

$\text{Green} = \text{temporary_2} + (\text{temporary_1} - \text{temporary_2}) \times (0.666 - \text{temporary_G}) \times 6$

Green = $0.0924 + (0.4676 - 0.0924) \times (0.666 - 0.536) \times 6 = 0.0924 + 0.3752 \times 0.13 \times 6 = 0.3850$ which rounded down is equal to **0.38**, another
 number we recognize.

And finally Blue

$6 \times \text{temporary_B} = 6 \times 0.203 = 1,218$, to large

$2 \times 0.203 = 0.406$, below one, so we go with this formula

Blue = temporary_1

Blue = 0.4676, which rounded down is **0.46**, and a another number we know.

7

Convert them to 8-bit by multiply them with 255 and we are done

$R = 0.0924 \times 255 = 23,56$ round up to 24

$G = 0.3850 \times 255 = 98.17$ round down to 98

$B = 0.4676 \times 255 = 119,23$ round down to 119, this is not 118 because we rounded the numbers when converting from RGB to HSL. It shows that when you do conversions you need to be as accurate as possible and not round the numbers.

Happy converting

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