Repport of the application of FreeFem++ to heat transfer by conduction in two dimensional systems ⋆

Kori Ditmeyer-Moreau [a]- MNI 2

[a] *University of Strasbourg, France*

Abstract

The aim of this homework exercise is to determine the energy transferred (in 2D) from heating elements embedded in a large heating plate with isothermal surfaces at $T = 310K$.

Key words: FreeFem++; Heat transfer; 2D.

1 Introduction

The physical properties of the plate are :

- thermal conductivity $k = 0.5\ Wm^{-1}/K^{-1}$;
- specific heat capacity $c_p = 920\ Jkg^{-1}/K^{-1}$;
- volumetric mass $\rho = 2400\ kgm^{-3}$.

The initial condition at $t = 0s$ is a constant plate temperature $T = 310K$. The heating element is at $T = 400K$.

2 Selection of the domain

In order to simplify the resolution, we had to take into account the symmetry and the conduction problems. As we simulate an infinite plate and that there is a symmetry along the X and Y axes, we can take only the domain that is shown in the figure 1.

Once we have define the domain we have meshed and compute the project with FreeFem++. The mesh size that we have choose is indicate for each side in parenthesis with the appropriate side: a(45), b(90), c(60), d(75), e(80). The result of the meshing is shown in the figure 1.

The calculus parameters that I have taken are the following:

- calculus time of $t_s = 10^6\ s$;

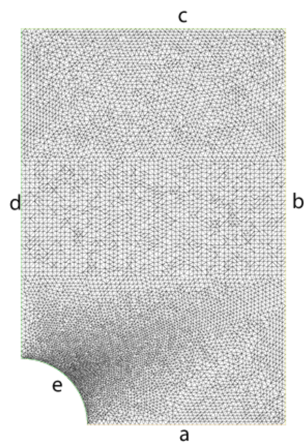


Fig. 1. Domain that will be simulated

- time step $d_t = 6000\ s$;

As we have a symmetry on the (b) side, we impose for the calculation a no flux condition. Hence,at this side, $V_0 = 0$.

3 Time for achieving the steady state

To find the time were the steady state is achieved, we have to introduce a convergence criterion $\varepsilon = 10^{-5}$ and to compute an error value in our code. In this case to calculate the error value, we have choose a point distant enough from the heating source (e). The error value is the difference of the Temperature values at this point between two iterations. If this difference decrease and go below the convergence criterion, we consider that the

⋆ See Mr Shaeffer Exercice statement

Email address: `kditmeye@engees.eu` (Kori Ditmeyer-Moreau).

value doesn't change significantly and that the problem has reached the steady state.

As we can see from the figure 2,the steady state obtained for the selected criterion of $\varepsilon = 10^{-5}$ is after $t = t_{stat} = 4.296 * 10^6\ s$, which is the equivalent as $t_{stat} = 49\ days\ and\ 16\ hours$ or after 716 iterations.

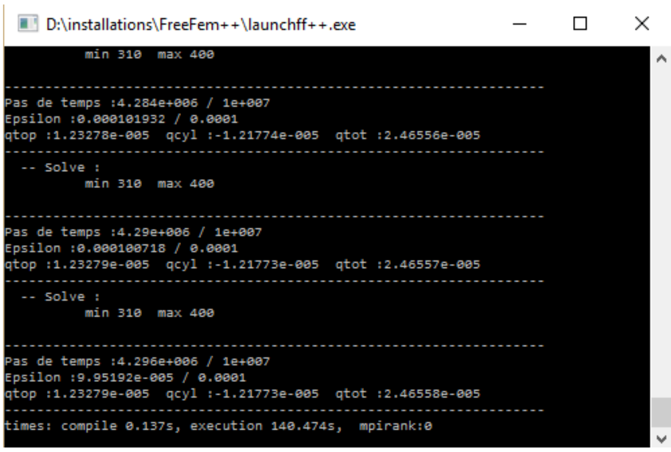


Fig. 2. Calculus Stop at steady state in FreeFem++

The t_{stat} obtained is not unique, hence it depends on the precision that we want for the value to be constant. Here, we have choose $\varepsilon = 10^{-5}$ because in the literature, it's the most common use criteria for knowing that we achieved the steady state. At this calculated steady state, the values of the temperature are as shown in the figure 3.

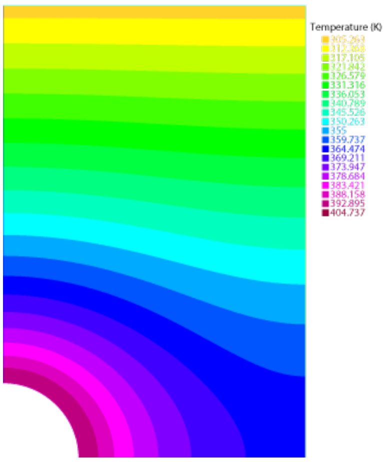


Fig. 3. Temperature Values at steady state (K)

4 Energy transferred by steady-state conduction

To determine the energy produce by the e side and the energy that exits by c side , we have to compute the heat flux conduction, generally expressed by the 'Fourrier rate equation' (1):

$$\vec{q} = -kA\nabla T\vec{n} \tag{1}$$

Where A is the area normal to the heat flow (m^2) and $\vec{q}$ is the heat transfer rate vector (W).

To implement this equation into our FreeFem++ code, we have to decompose our equation into basic variational formulation. Hence the equation (1) becomes for the side:

$$c : q_{top} = \int_c (-k) * dx(T) * N_x + (-k) * dy(T) * N_y \tag{2}$$

$$e : q_{cyl} = \int_e (-k) * dx(T) * N_x + (-k) * dy(T) * N_y \tag{3}$$

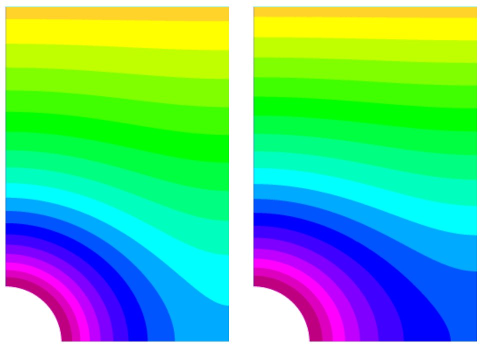
At the steady state the values of q_{top} and q_{cyl} are respectively $1.23 * 10^{-5}$ and $-1.22 * 10^{-5}$ Watts. Here the value of q_{cyl} is negative because this element loses heat and the q_{top} get back this heat.

We can also have used these values to compute the error function and so to determine the steady state. Furthermore, this method is more accurate because we don't take the value of only one point like in the §3 but the take a value on a hole surface.

The total energy transferred from the $T = 0$ of the simulation to the steady-state is $q_{tot} = 2.47 * 10^{-5}$ Watts (see figure 2) .

5 Temperature field for the plate at different times

According to the demands of the exercise 2 of M.Schaeffer we should have have calculated the corresponding temperature fields for $t = 0.33 * t_{stat} = 1.418 * 10^6\ s$ and $t = 0.66 * t_{stat} = 2.835 * 10^6\ s$, however as we have chosen a high convergence criterion at $t = 0.66 * t_{stat}$, we observe almost no change the with the steady state.Hence, we have choose to compute the value at $t = 0.10 * t_{stat}$. We can see the results in the figure below:



(a) 0.2*T (b) 0.33*T

Fig. 4. Comparison of temperature iso-surfaces at different time period

6 Influence of the mesh grid on numerical results

We have computed the calculation for several definition of grid meshes and different types of grid mesh generation. Indeed, we can define by ourself the number of nodes or we can choose with an adapt mesh function that the software by itself calculate the best number of nodes for the calculation.

6.1 Influence of mesh refinement

Hence, first, we have ourselves define the number of nodes in the mesh and have compute the model with a high number of nodes (curve (1) in the figure 6) and an other with a more fewer number of nodes (curve (2) in the figure 6). The computational time is $t = 141.709\ s$ with a high number of nodes and decrease to $t = 35.39\ s$ with a lower number of nodes.

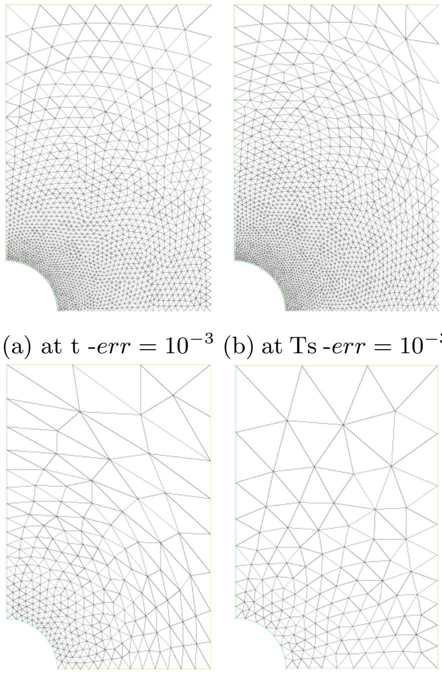
6.2 Influence of automatically mesh refinement

Second, we have used an automatic adaptation mesh method. Here in FreeFem++, we use the keyword "adaptmesh", in order the mesh to be adapted at each time step with respect to a given finite element. In the adapt mesh, we can choose a precision mesh error. We have choose $err = 10^{-2}$ and 10^{-3}. The figures of the different mesh adptation are represent in the figures 5. We can see, in these figures that the mesh change with time and with the err value that we choose.

With the automatically adapted mesh method with $err = 10^{-2}$ (curve (4) in the figure 6), the computation time to reach the steady state is $t = 82.86\ s$ whereas with an $err = 10^{-3}$ (curve (3) in the figure 6) the computation time rise to $t = 162.06\ s$.

6.3 Conclusion

In the figure 6, we have compared the values obtained for the q (the heat transfer) at the top of the domain (c) for the different mesh refinements and for different adaptmesh err values. We can see that there is a difference at the first iterations of the computation between an adaptmesh and a non-adaptmesh. However, all the curves converges to the same value and at reach the steady state, nearly at the same time. Hence, he main difference in the meshes refinement and method being the computational time.



(a) at t -$err = 10^{-3}$ (b) at Ts -$err = 10^{-3}$

(c) at t -$err = 10^{-2}$ (d) at Ts -$err = 10^{-2}$

Fig. 5. Comparison adaptmesh function for different time period and different err values

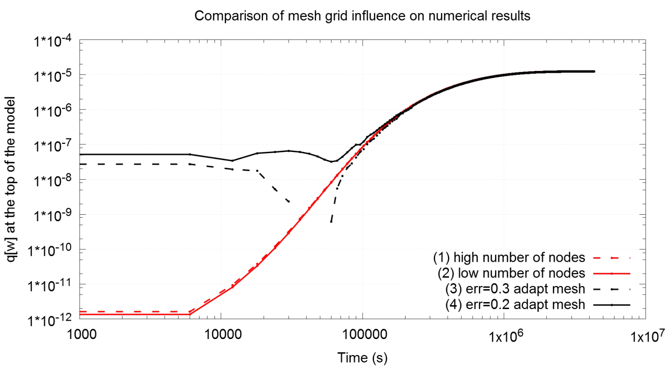


Fig. 6. Comparison of mesh grid influence on numerical results

References

[Shaeffer, 2015] (2015) *Applied Computational Engineering for Heat and mass transfer.* university of Strasbourg.