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Problem 5

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

approx: $\sin\left(\frac{1}{n}\right) \approx \frac{1}{n}$ $\cos\left(\frac{1}{n}\right) \approx 1 - \frac{1}{2n^2}$ $\Rightarrow$ error $\sin\left(\frac{1}{n}\right)$

$$\sin\left(\frac{1}{n}\right) = \frac{1}{n} - \frac{1/n^3}{6} + \frac{1/n^5}{5!} - \frac{1/n^7}{7!} + \cdots$$

$$\underbrace{\sin\left(\frac{1}{n}\right) - \frac{1}{n}}_{\text{error}} = -\frac{1}{6n^3} + \frac{1}{5!n^5} - \frac{1}{7!n^7} + \cdots$$

$$\boxed{\text{error}_{\sin}\left(\frac{1}{n}\right) = O(n^3)}$$

$\Rightarrow \text{error}_{\cos}\left(\frac{1}{n}\right)$

$$\cos\left(\frac{1}{n}\right) = 1 - \frac{1/n^2}{2} + \frac{1/n^4}{4!} - \frac{1/n^6}{6!} + \cdots$$

$$\underbrace{\cos\left(\frac{1}{n}\right) - \left(1 - \frac{1}{2n^2}\right)}_{\text{error}} = \frac{1}{4!n^4} - \frac{1}{6!n^6} + \cdots$$

$$\boxed{\text{error}_{\cos}\left(\frac{1}{n}\right) = O(n^4)}$$