Bubble Sort

```
bool swap;
    do {
        swap = false;
        for(int i=0; i < n-1; i++) {
            if(a[i] > a[i+1]) {
                int temp = a[i];
                a[i] = a[i+1];
                a[i+1] = temp;
                temp = true;
            }
        }
    } while (swap);
```

$$O_b + \sum_{i=0}^{n-1} \left(O_i + \sum_{i=0}^{n-2}(O_i + PO_s) \right) \overset{(\text{let } O_i + PO_s = O_{is})}{=} O_b + \sum_{i=0}^{n-1} \left(O_i + (n-1)O_{is} \right)$$

$$= O_b + \left(O_i + O_{is}n - O_{is} \right)(n-1-0+1)$$

$$= O_b + \left(O_i + O_{is}n - O_{is} \right)n \qquad = O_b + \left(O_i - O_{is} \right)n + O_{is}\,n^2$$

$$= C_0 + C_1 n + C_2 n^2 = f(n) \qquad \Rightarrow \boxed{f(n) \text{ is } O(n^2)}$$

Selection Sort

```
int  alx;  aih;
 for (int pos=0; pos < n-1; pos++){          O(n)
    aih = a[pos];  indx =pos;
    for (int  i = pos +1 ; i < n ; i++){
       if (a[i] < aih) {
          aih = a[i];              O(1)
          indx =i ;
          }
       }
       a.[alx] = a[pos];
       a [pos] = aih;
       }
```

$$(\text{let } O_i + PO_s = O_{is})$$

$$O_b + \sum_{k=0}^{n-2} \left(O_k + \sum_{i=k+1}^{n-2} (O_i + PO_s)\right) = O_b + \sum_{k=0}^{n-2} \left(O_k + O_{is}(n-k-1)\right)$$

$$= O_b + \sum_{k=0}^{n-2} \left(O_k + O_{is}(n-1) - k O_{is}\right)$$

$$= O_b + (n-1)\left(O_k + O_{is}(n-1)\right) - O_{is} \sum_{k=0}^{n-2} k$$

$$= O_b + (n-1) O_k + (n-1)^2 O_{is} - \frac{(n-2)(n-1)}{2} O_{is}$$

$$= \left(\tfrac{3}{2} O_{is}\right) n^2 + \left(-\tfrac{7}{2} O_{is} + O_k\right) n + \left(2 O_{is} - O_k + O_b\right)$$

$$= C_2 n^2 + C_1 n + C_0 = f(n) \implies \boxed{f(n) = O(n^2)}$$