Example Homework: Computer Science

CS 123

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Problem 1. Consider the given code segment. *(Omitted here.)*

(a) When $f(k)$ is called, how many times is it recursively called in total? Include the initial call and give the answer in terms of k.

$f(k)$ calls $f(k-1)$ recursively until the argument equals 0. Thus the number of times $f(k)$ is called is $T(k) = T(k-1) + 1, T(0) = 1$.

$$\begin{aligned} T(k) &= T(k-1) + 1 \\ &= T(k-2) + 1 + 1 \\ &= T(k-3) + 1 + 1 + 1 \\ &= T(k-n) + n \\ &= T(0) + k \qquad\qquad \text{when } n = k \\ T(k) &= k + 1 \end{aligned}$$

(b) What does $f(k)$ return? Give your answer in terms of k.

Based on the code, we can define $f(k)$ mathematically as $f(k) = f(k-1) + 3k + 2, f(0) = 2$.

$$\begin{aligned} f(k) &= f(k-1) + 3k + 2 \\ &= (f(k-2) + 3(k-1) + 2) + 3k + 2 \\ &= ((f(k-3) + 3(k-2) + 2) + 3(k-1) + 2) + 3k + 2 \\ &= f(k-3) + 3(k-2) + 2 + 3(k-1) + 2 + 3k + 2 \\ &= f(k-n) + 3 \sum_{i=k-n+1}^{k} i + n(2) \end{aligned}$$

Now let $n = k$.

$$\begin{aligned} f(k) &= f(0) + 3 \sum_{i=1}^{k} i + 2k \\ &= 2 + 3 \cdot \frac{k(k+1)}{2} + 2k \\ f(k) &= \frac{3k^2 + 7k + 4}{2} \end{aligned}$$

(c) How many times is f called in total? Give the answer in terms of n.

$f(j)$ is called once from each iteration of the loop. The loop runs from $j = 1$ to $j = n$. Thus the number of times f is called is the following sum, where $T(n)$ represents the number of times f is called when invoked as $f(n)$. We know from part **(a)** that $T(n) = n + 1$.

$$\sum_{j=1}^{n} T(j) = \sum_{j=1}^{n} (j+1) = \sum_{j=1}^{n} j + \sum_{j=1}^{n} 1 = \frac{n(n+1)}{2} + n = \frac{n^2 + 3n}{2}$$

Problem 2. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magnam aliquam quaerat.

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```
procedure EXPONENTIATE(n):
1  if n = 0 then
2  |    return 182
3  sub ← EXPONENTIATE(n − 1)
4  return MULMOD(sub, sub)
```

(b) Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor.

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We will now prove that $E(n) = 182^{(2^n)} \bmod 2000$ using induction. Let $P(n)$ be the statement $E(n) = 182^{(2^n)} \bmod 2000$.

Basis step:

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Inductive step:

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Conclusion:

Since $P(0)$ is true and $P(n-1) \to P(n)$, by induction on n, $P(n)$ is true for all $n \in \mathbb{N}^+ \cup \{0\}$.

Problem 3. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magnam aliquam quaerat voluptatem. Ut enim aeque doleamus animo, cum corpore dolemus, fieri tamen permagna accessio potest, si aliquod aeternum et infinitum impendere malum nobis opinemur. Quod idem licet transferre in voluptatem, ut postea variari voluptas distinguique possit, augeri amplificarique non possit. At etiam Athenis, ut e patre audiebam facete et urbane Stoicos.

Proof by cases. Below, a node in a graph represents a TA and an edge between two nodes represents a handshake between the two TAs.

Case 1: Everyone shook hands

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Case 2: Not everyone shook hands

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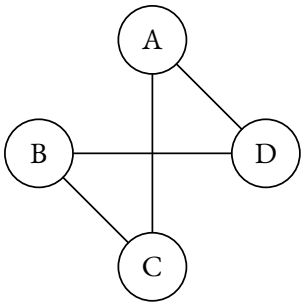

Figure 1: A graph which demonstrates that having two disjoint pairs of TAs who have not shaken hands contradicts UTA Bob's observation and thus is impossible. $\{A, B\}$ and $\{C, D\}$ are the pairs of TAs who have not shaken hands.

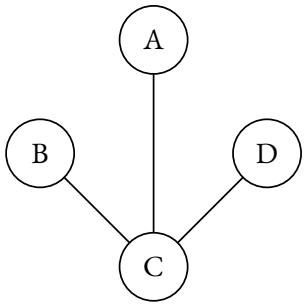

Figure 2: A graph which demonstrates that having two pairs of TAs who have not shaken hands is possible as long as one TA is in both pairs. $\{A, B\}$ and $\{A, D\}$ are the pairs of TAs who have not shaken hands.

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magnam aliquam quaerat. Figure 2 demonstrates that this still satisfies UTA Bob's observation, so it can exist.

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Problem 4. For the following graph questions, either give an example graph or prove that there are none. For parts **(d)** and **(e)**, note that sequence of indegree and outdegree corresponds to the same vertices.

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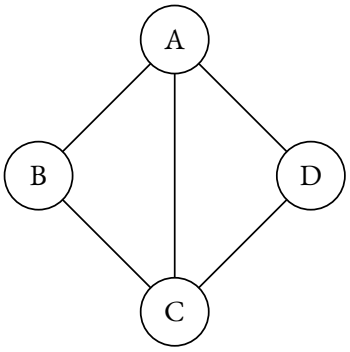

Figure 3: Graph for problem 4.a.

(b) Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor.

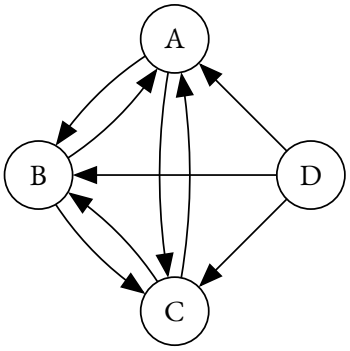

Figure 4: Graph for problem 4.b.

(c) Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore.

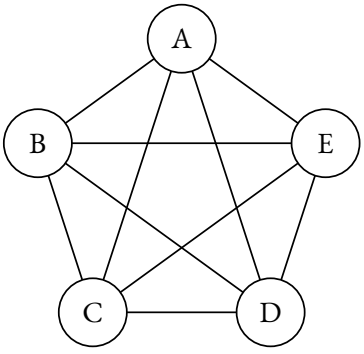

Figure 5: Graph for problem 4.c.