Almost local metrics

The problem at hand is the following: we wish to construct a notion of distance between two points in $B_e(S^1, \mathbb{R}^2)$ by defining a metric, such that the distance between two points is the length of geodesics between the points. As we have seen, the distance induced by the L^2-metric vanishes on $B_e(S^1, \mathbb{R}^2)$, so we seek to define metrics, which do not vanish. One type of such metrics is *almost local metrics*, which, given $f \in B_e(S^1, \mathbb{R}^2)$, are metrics of the form

$$G_f^\Phi(h,k) = \int_M \Phi(\mathrm{Vol}(f), H_f, K_f)\bar{g}(h,k)\mathrm{vol}(f^*\bar{g}),$$

where $\Phi : \mathbb{R}^3 \to \mathbb{R}_{>0}$ is smooth, $\mathrm{Vol}(f) = \int_M \mathrm{vol}(f^*\bar{g})$ is the total volume of $f(M)$, H_f is the mean curvature of f and K_f is the Gauss curvature of f. Both H_f and K_f are local invariant properties with respect to the Riemanninan metric, g, and so Φ is often chosen to only depend on one of the two curvatures.