Phys 3820, Fall 2010
Problem Set #1, Hint-o-licious Hints

1. *Griffiths*, **6.2** (a) The exact energies of the unperturbed system are $E_n^0 = (n+\frac{1}{2})\hbar\omega$ where $\omega = \sqrt{\frac{k}{m}}$. We can the exact answer for the new system by replacing k by $k(1+\epsilon)$. Use a Taylor expansion to get E_n as a series in ϵ.

b) With

$$H' = \tfrac{1}{2}\epsilon k x^2$$

the E_n^1 are given by

$$E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle = \tfrac{1}{2}\epsilon k \langle n | x^2 | n \rangle$$

Use *operator methods* to evaluate this matrix element... it would be very to use the explicit wave functions. Write x^2 as

$$x^2 = \left(\frac{\hbar^2}{2m\omega} \right) (a_+^2 + a_- a_+ + a_+ a_- + a_-^2)$$

and use the action of the operators on a state $|n\rangle$ and orthogonality of the states to get

$$\langle n | x^2 | n \rangle = \left(\frac{\hbar^2}{2m\omega} \right) (2n+1)$$

and use this to show that E_n^1 just gives the first–order term in the *series* for E_n found in (a).

2. *Griffiths*, **6.3** The ground state of the unperturbed system has the symmetric wave function

$$\psi(x_1, x_2) = \frac{2}{a}\psi(x_1)\psi(x_2)$$

with energy

$$E_{\rm gs} = 2E_1 = \frac{\pi^2\hbar^2}{ma^2}$$

The first excited state has the wave function

$$\psi(x_1, x_2) = \frac{1}{\sqrt{2}}(\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1))$$

with energy

$$E_{\rm 1exc} = E_1 + E_2 = \frac{5\pi^2\hbar^2}{2ma^2}$$

The results for the first–order energy corrections I get are

$$E_{\rm gs}^1 = -\frac{3V_0}{2} \qquad E_{\rm 1exc}^1 = -2V_0$$

3. *Griffiths*, **6.5** Use operator methods to show

$$\langle m | x | n \rangle = \sqrt{\frac{\hbar}{2m\omega}} \left(\sqrt{n+1}\, \delta_{m,n+1} + \sqrt{n}\, \delta_{m,n-1} \right)$$

4. *Griffiths*, **6.8** The perturbation

$$H' = a^3 V_0 \delta\left(x - \frac{a}{4}\right)\delta\left(y - \frac{a}{2}\right)\delta\left(z - \frac{3a}{4}\right)$$

gives non-zero corrections to the energy of the ground and first excited state.

You should be able to write down the ground-state wave function and energy for this system. You should find

$$E^1_{\text{gs}} = 2V_0$$

The first excited state is triply degenerate. Say that the first of these states has the wave function

$$\psi_{1\text{ex},I} = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{\pi y}{a}\right)\sin\left(\frac{\pi z}{a}\right)$$

In this scheme calculate

$$W_{ij} = \langle \psi^0_i | H' | \psi^0_j \rangle$$

For the matrix W you ought to get

$$\begin{pmatrix} 4V_0 & 0 & -4V_0 \\ 0 & 0 & 0 \\ -4V_0 & 0 & 4V_0 \end{pmatrix}$$

and then get the eigenvalues of this to get the possible shifts in energ for the first excited states.

5. *Griffiths*, **6.11** Fairly easy algebra.

6. *Griffiths*, **6.14** The relativistic perturbation to the Hamiltonian is

$$H' = -\frac{p^4}{8m^3c^2}$$

so we want to calculate

$$E^1_{n,\text{rel}} = \left\langle n \left| -\frac{p^4}{8m^3c^2} \right| n \right\rangle$$

Use

$$p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-) \qquad \Longrightarrow \qquad p^2 = -\frac{\hbar m\omega}{2}(a_+^2 + a_-^2 - a_+a_- - a_-a_+)$$

in

$$E^1_{n,\text{rel}} = -\frac{1}{8m^3c^2}\langle p^2\psi^0_n | p^2\psi^0_n \rangle$$

and use orthogonality of the HO wave functions. Get

$$E^1_{n,\text{rel}} = -\frac{3\hbar^2\omega^2}{32mc^2}(2n^2 + 2n + 1)$$

When this is used in the formula for E^2_n there are only two terms in the sum and you get

$$E^2_n = -\frac{(qE)^2}{2m\omega^2}$$

Once again the perturbed potential gives a problem which actually does have and exact solution. By completing the square you can *show* that the new potential is

$$V(x) = \tfrac{1}{2}kx^2 - qEx = \frac{k}{2}\left(x - \frac{qE}{k}\right)^2 - \frac{(qE)^2}{2k}$$

But with a change of variable to

$$x' \equiv x - \frac{qE}{k} = x - \frac{qE}{m\omega^2}$$

the Schrödinger equation is that of a harmonic oscillator *with an added constant potential term*. And you know the energy eigenvalues for that immediately.