

2D Motion.

constant acceleration

x | y

$$v_x = v_{0x} + a_x t$$

$$v_y = v_{0y} + a_y t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$x = x_0 + \frac{1}{2}(v_{0x} + v_x)t$$

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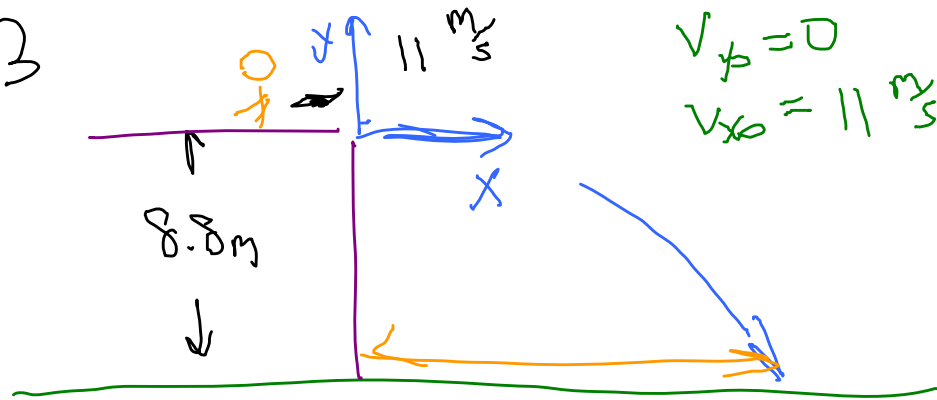
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Free-fall

$$a_x = 0$$

$$a_y = -9.8 \frac{\text{m}}{\text{s}^2} = -g$$

3.33



$$v_y = 0$$

$$v_{x0} = 11 \frac{\text{m}}{\text{s}}$$



- a) How long in flight?
- b) How far move horiz.

a) How long in flight?

When does $y = -8.8\text{m}$?

$$a_y = -g$$

$$y = \cancel{v_{y0}t} + \frac{1}{2} a_y t^2$$

$$= \frac{1}{2} (-9.8 \frac{\text{m}}{\text{s}^2}) t^2 = -8.8\text{m}$$

$$t = 1.34\text{s}$$

b) Horiz distance?

What is x at that time?

$$x = \cancel{x_0} + v_{x0}t + \frac{1}{2}a_x t^2$$

$11 \frac{m}{s}$

$1.34 s$

$$= (11 \frac{m}{s})(1.34 s) = 14.7 m$$

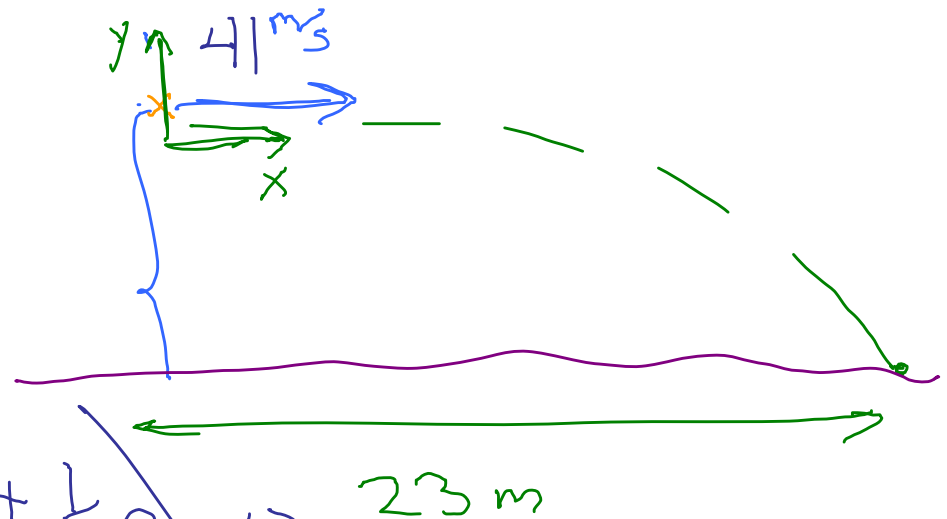
3.34 An arrow fired horizontally at $41 \frac{m}{s}$ travels horizontally 23 m, from what height was it shot?

Time in flight.
At impact

$$X = 23 \text{ m} = \cancel{x_0} + v_{x0}t + \frac{1}{2}a_x t^2$$

$$= (41 \frac{m}{s})t$$

$$t = \frac{23}{41} \text{ s} = 0.561$$



What is y at this time?

$$y = -\frac{1}{2}gt^2 = -1.54 \text{ m}$$

↑ $h = 1.54 \text{ m}$

New Problem

Projectile Problem

Find R , H , T , etc...

$$a_x = 0 \quad a_y = -9.8 \frac{m}{s^2}$$

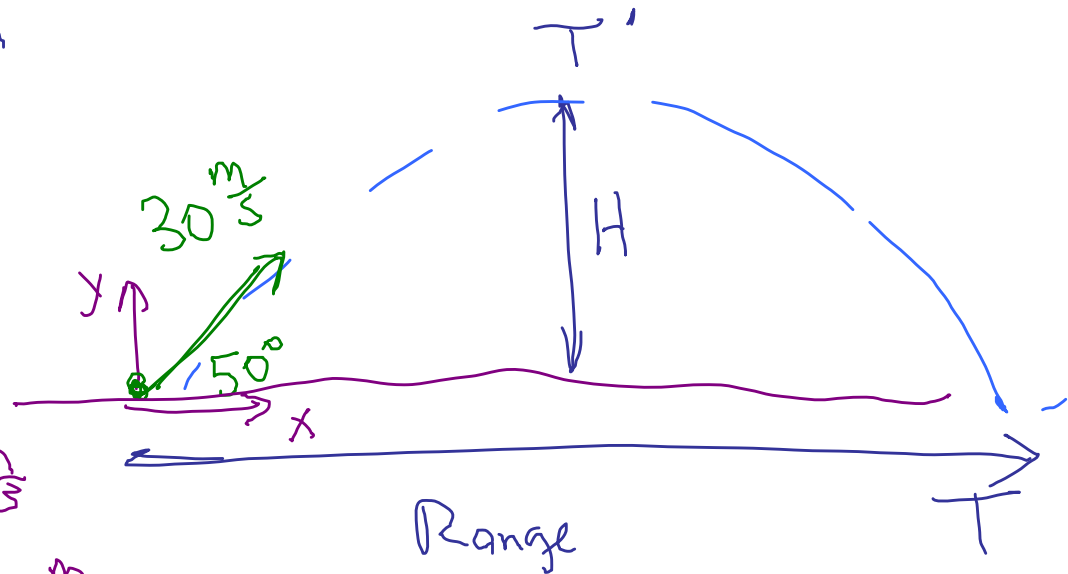
$$V_{x0} = 30 \frac{m}{s} \cos 50^\circ = 19.28 \frac{m}{s}$$

$$V_{y0} = 30 \frac{m}{s} \sin 50^\circ = 22.98 \frac{m}{s}$$

$$y = \cancel{y_0} + V_{y0}t - \frac{1}{2}gt^2$$
$$= (22.98 \frac{m}{s})t - \frac{1}{2}gt^2$$


$$x = \cancel{x_0} + V_{x0}t$$
$$= (19.28 \frac{m}{s})t$$

a) How long was it "in air",
What is t when $y = 0$?



$$y = 22.98 t - \frac{1}{2} 9.8 t^2 = 0$$

$$t (22.98 - 4.9 t) = 0$$



$$4.9 t = 22.98$$

$$t = \frac{22.98 \frac{\text{m}}{\text{s}}}{4.9 \frac{\text{m}}{\text{s}^2}} = \underline{\underline{4.7 \text{ s}}}$$

What is Range?

What is x at this time?

$$x = (19.28 \frac{\text{m}}{\text{s}}) t = \underline{\underline{90.4 \text{ m}}}$$

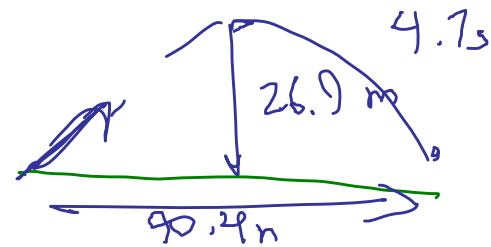
When did ball get to max h?

When does $v_y = 0$?

$$v_y = v_{y0} + a_y t = (22.98 \frac{m}{s}) - (9.8 \frac{m}{s^2})t$$

$$= 0$$

$$t = \frac{22.98}{9.8} = 2.34 s$$

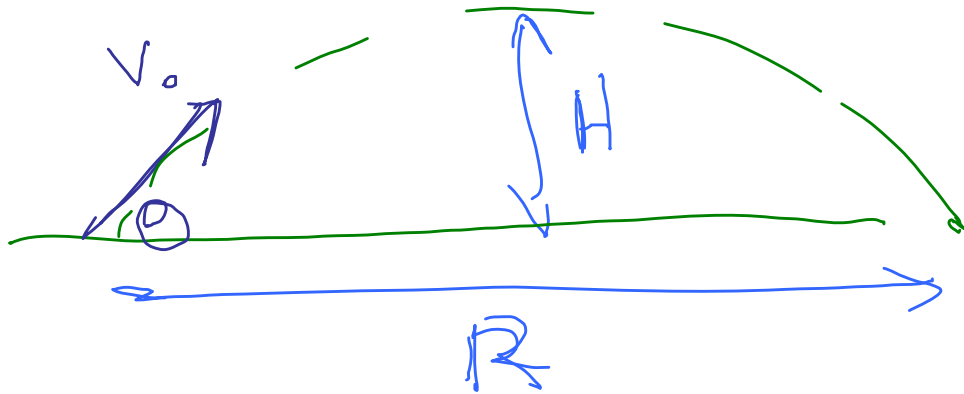


What is max?

$$y = \cancel{x} + (22.98 \frac{m}{s})t - \frac{1}{2}gt^2$$

What is y at this time?

$$= 26.9 m$$



$$V_{x0} = V_0 \cos \theta$$

$$a_x = 0$$

$$V_{y0} = V_0 \sin \theta$$

$$a_y = -g$$

How long in flight? When does $y = 0$?

$$y = \underbrace{(V_0 \sin \theta)}_{V_p} t - \frac{1}{2} g t^2 = 0 = t \left(V_0 \sin \theta - \frac{g}{2} t \right)$$

Two answers.

~~$t = 0$~~

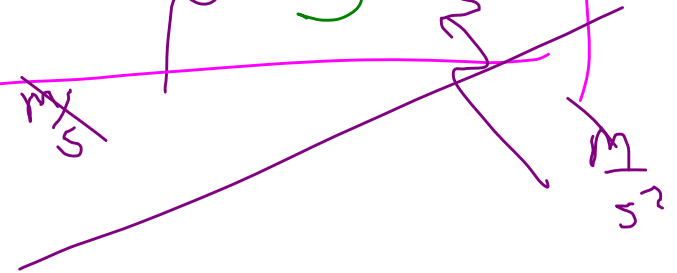
$$V_0 \sin \theta = \frac{gt}{2} \quad \text{or} \quad t = \frac{2V_0 \sin \theta}{g}$$

What is x at this time?

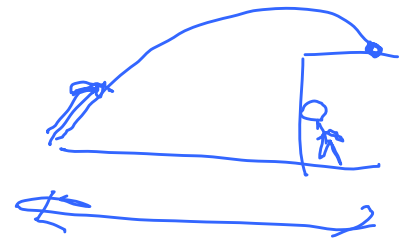
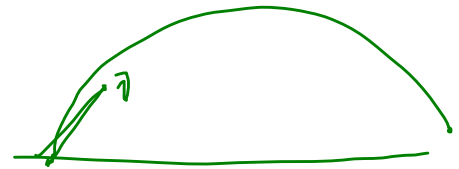
$$x = (V_0 \cos \theta) t$$

$$= (V_0 \cos \theta) \frac{2V_0 \sin \theta}{g}$$

$$= \frac{2V_0^2 \sin \theta \cos \theta}{g} = R$$



$= S$



$$R = \frac{2v_0^2 \sin \theta \cos \theta}{g}$$

$$= \frac{v_0^2 \sin 2\theta}{g}$$

Keep v_0 constant

For what θ is R biggest?

$$2\theta = 90^\circ$$

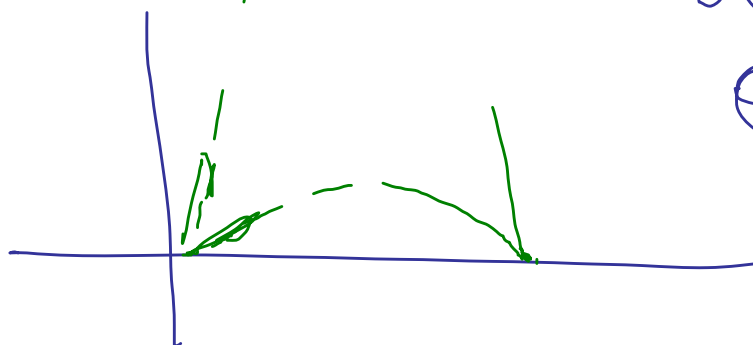
$$\theta = 45^\circ$$

$$m = \frac{1}{2} \times \frac{1}{2}$$

$$16, 800$$

$$28, 700$$

$$38, 600$$



Time to get to max height:

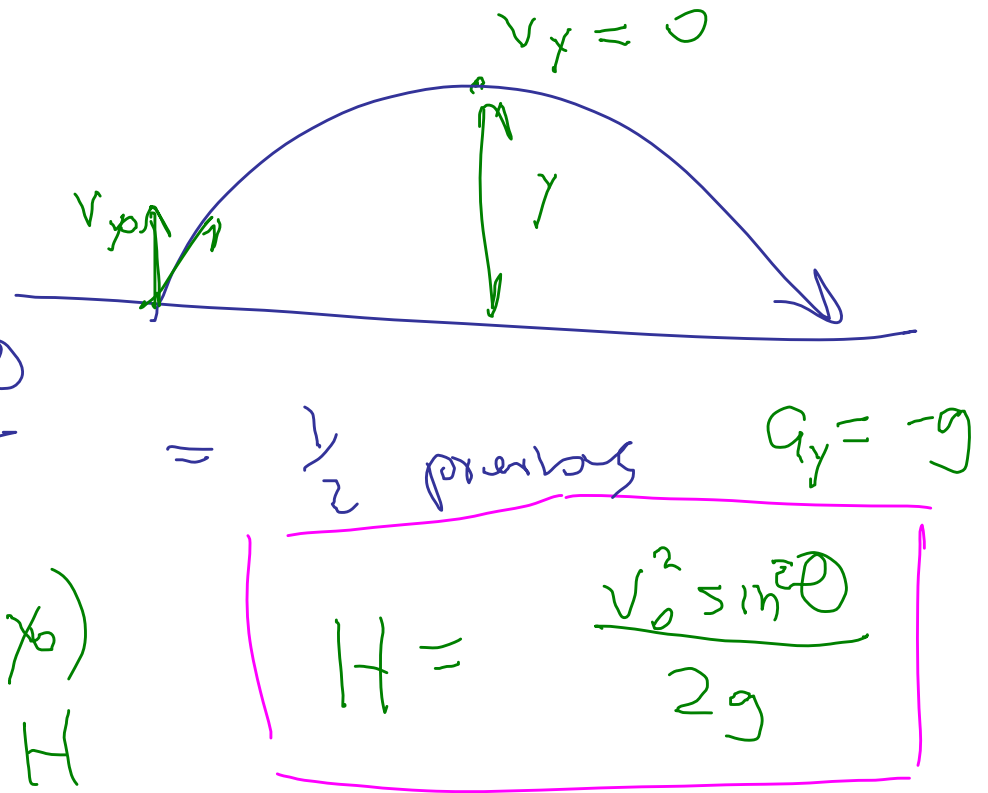
$$V_y = 0$$

$$V_y = v_{y0} - g t$$

$$t = \frac{v_{y0}}{g} = \frac{v_0 \sin \theta}{g}$$

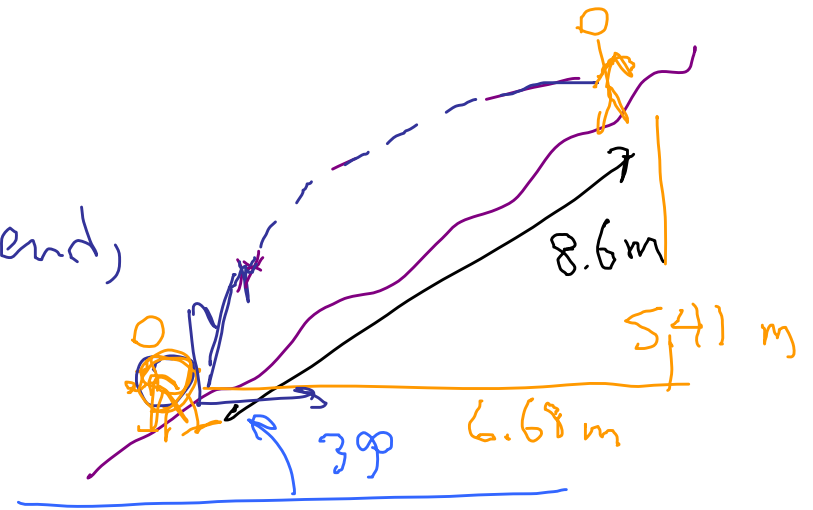
$$V_f^2 = v_{y0}^2 + 2a(y - y_0)$$

$$0 = (v_0^2 \sin^2 \theta) - 2gH$$



3.62 Toss protein bar

Det. initial vel. so that
when it gets to thin friend,
moving horiz'lly.



$$a_y = -g$$

$$v_x$$

$$v_y = 0$$

$$\Delta y = 5.41 \text{ m}$$

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$$

Annotations:
 - v_{y0} is marked with a question mark and an arrow pointing to the initial velocity vector.
 - a_y is marked with an arrow pointing to $-9.8 \frac{\text{m}}{\text{s}^2}$.
 - $y - y_0$ is marked with an arrow pointing to 5.41 .

$$v_{y0} = 10.3 \frac{\text{m}}{\text{s}}$$

$$v_y = v_{y0} - g t$$

Annotations:
 - t is circled.
 - $t = 1.05 \text{ s}$ is written next to the equation.
 - 10.3 is marked with an arrow pointing to the initial velocity vector.

X eqn:

$$X = v_{x0} t$$

$$6.60 \text{ m} = v_{x0} (1.0 \text{ s})$$

$$\longrightarrow 6.36 \frac{\text{m}}{\text{s}} = v_{x0}$$

$$t_{\text{up}} = \frac{10.3}{6.36}$$

$$\theta = 58.3^\circ$$

