Phys 4610, Fall 2007
Problem Set #3

1. In this problem (excercise, project, whatever) we want to get a numerical solution to the Laplace equation with arbitrary boundary conditions for a rectangular geometry (in two dimensions).

We will use the fact that the cells in a spreadsheet can be programmed to be some function of the values of the surrounding cells. Using the Copy and Paste features we can easily "paint" regions of the spreadsheet with a selected numerical value or formula.

To start, choose a rectangular chunk of the spreadsheet, about 12 cells across and 30 cells up and down (so that all of it can be seen on the screen). On the top row put the value 5.0 in each cell, to represent 5.0 V. Fill the bottom row with 5's as well. Fill the first and last columns with 0's. It doesn't matter much what values the corner cells have.

Now if your region starts in cell `A1` then in cell `B2` you will want to enter the following:

```
+0.25*(B1+A2+C2+B3)
```

which tells cell `B2` to always be the average of its nearest neighbors; this gives a local solution to the Laplace equation, as we discussed in class. Then copy this cell and paint the entire interior region with this formula.

Now, Excel will probably complain if you attempt this, due to the problem of self-referencing discussed in class. You can tell it not to do the Paste just yet and go to the sequence of menus

$$\text{Tools} \qquad \Longrightarrow \qquad \text{Options} \qquad \Longrightarrow \qquad \text{Calculation}$$

whereupon you can change the number of iterations that will be done on the sheet to something big enough so that we can be sure things will converge: Try 100 at first. This is no sweat for any modern PC. When you turn on the Iterations feature (and paint the interior region with the formula, if you still need to do this) the interior region will calculate and recalculate to make each cell be the average of its neighbors. After 100 iterations it surely will have converged to all the accuracy we could want.

a) *Pick any cell in the interior and verify with a calculator that is indeed the average of its neighbors to high accuracy.*

Now make a surface plot of $V(x, y)$. Select the *entire* potential region (including the boundary) and hit the graphing button. Select a surface plot. Make some choices so as to get a clear view of the surface.

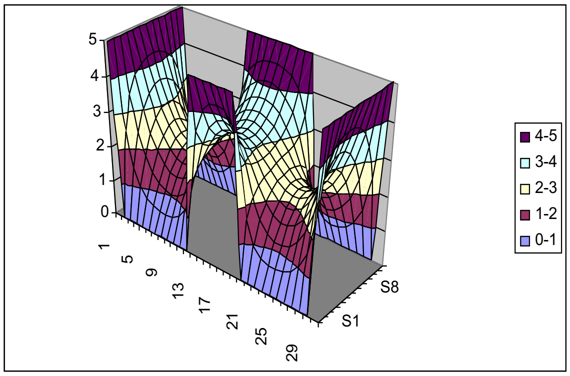
b) *Copy the surface plot and print it out. (You can combine all your plots together in one Word document; that would be nicer.)*

Now try to do something more interesting with the boundary values and the shape of the 2D region. Change the values of V on the boundary and possibly try a different–shaped region. You could try a corner, a cross...

In each case note that the Laplace equation does what it can to make the solution as "boring" as possible!

c) *Make three other choices for the shape of the region and the boundary values; shoot for something interesting! Get the plots for $V(x,y)$ and print them out. Explain what you did in each case.*

Here's a keen one I got by keeping two sides at 5 volts and then keeping just the middle regions of the other two sides at 5 volts (and otherwise at zero). It looks great, doesn't it? Especially when I embed it in a LaTeX document like this? Come on, admit it, you *know* it does.



2. *Griffiths*, **2.32**

3. *Griffiths*, **2.39**

4. *Griffiths*, **3.7**

5. *Griffiths*, **3.12**

6. *Griffiths*, **3.13**

7. *Griffiths*, **3.14**