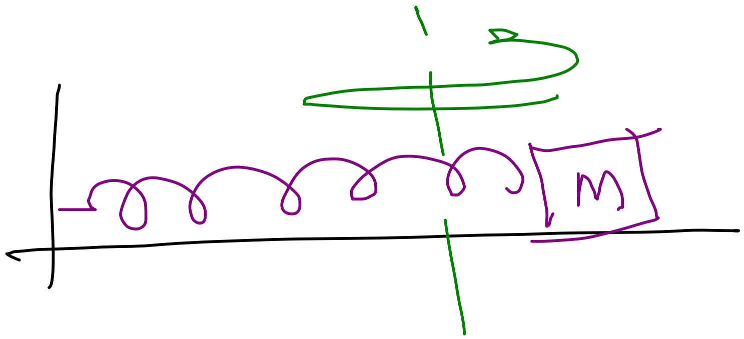
Phys 2110-5 11/16/12

Note Title 11/16/2012

~~Osc~~illa~~tors~~



Linear restoring force

$$= -\omega^2 x$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

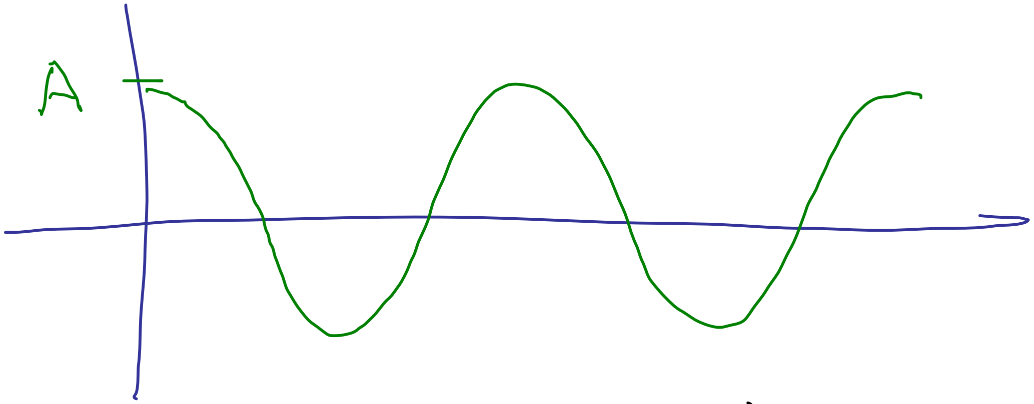
$x(t)$

$$x(t) = A\cos(\omega t + \phi)$$

$\phi = 0$ for now

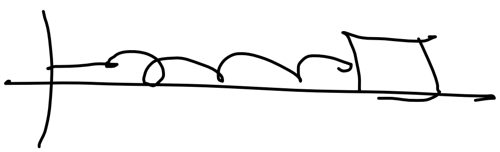
$$= A\cos(\omega t)$$

ω = ang. freq $= 2\pi f$ $\quad f = 1/T$



$A\cos(\omega t)$

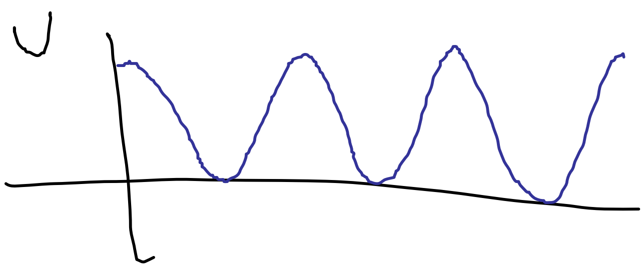
$x_{max} = A \quad v_{max} = \omega A \quad a_{max} = \omega^2 A$

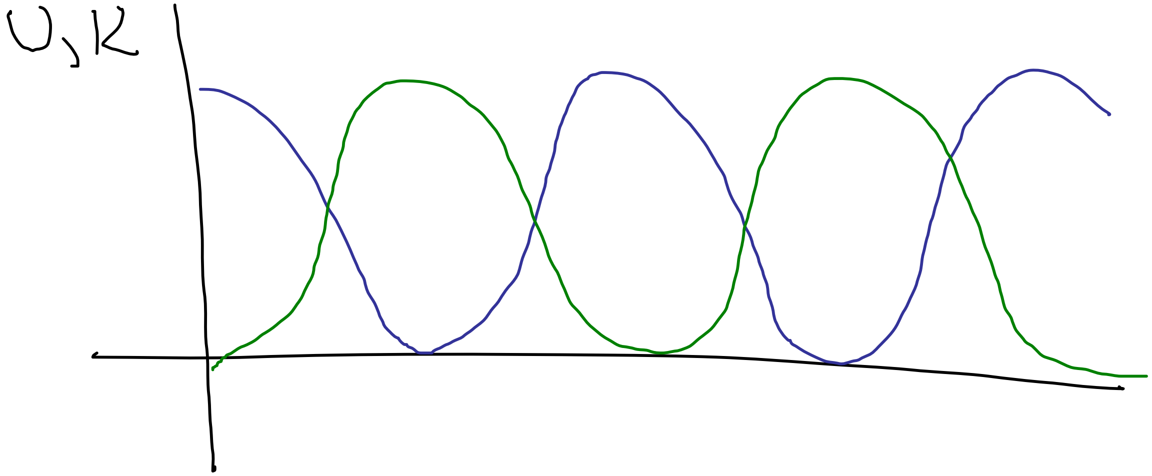


$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t)$$

$$v = dx/dt = -\omega A\sin(\omega t)$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t)$$





$U + K = \text{const}$

$$U = \frac{1}{2}kA^2\cos^2(\omega t) + \frac{1}{2}\underbrace{m\omega^2}_{=k} A^2\sin^2(\omega t)$$

$\omega^2 = k/m$ sub, m cancels

$$= \frac{1}{2}kA^2\cos^2(\omega t) + \frac{1}{2}kA^2\sin^2(\omega t)$$

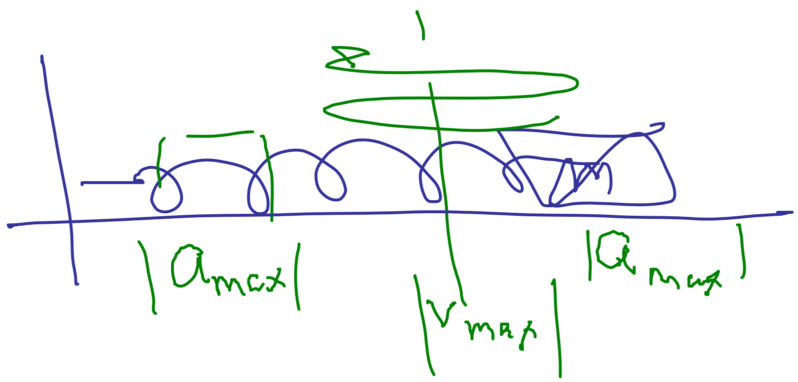
$$= \frac{1}{2}kA^2\left(\cos^2(\omega t) + \sin^2(\omega t)\right) = \frac{1}{2}kA^2$$

= energy in spring

$$?. = \frac{1}{2}mv_{max}^2 = \frac{1}{2}\underbrace{m\omega^2}_{=k}A^2 = \frac{1}{2}kA^2$$

13.25 A 50g mass is attached to spring & undergoes SH Motion max accel = $15 \frac{m}{s^2}$ max speed is $3.5 \frac{m}{s}$ Find

a) Ang. frequency
b) Spring const
c) Amplitude.



$$a_{max} = \omega^2 A = 15 \frac{m}{s^2}$$

$$V_{max} = \omega A = 3.5 \frac{m}{s}$$

$$\frac{a_{max}}{V_{max}} = \frac{\omega^2 A}{\omega A} = \omega = \frac{15 \frac{m}{s^2}}{3.5 \frac{m}{s}}$$

$$\boxed{= 4.29 \frac{rad}{s}}$$

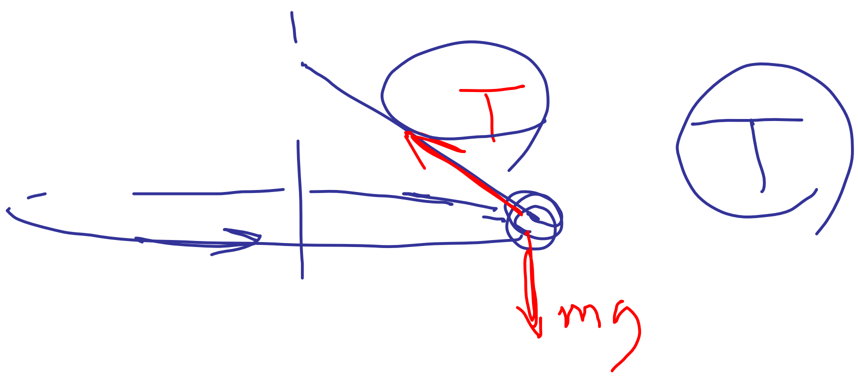
$$f = \omega/2\pi = 0.682 \, \frac{osc}{s}$$

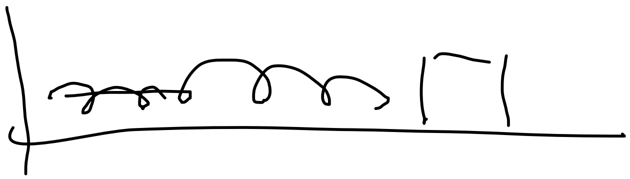
$$Hz$$

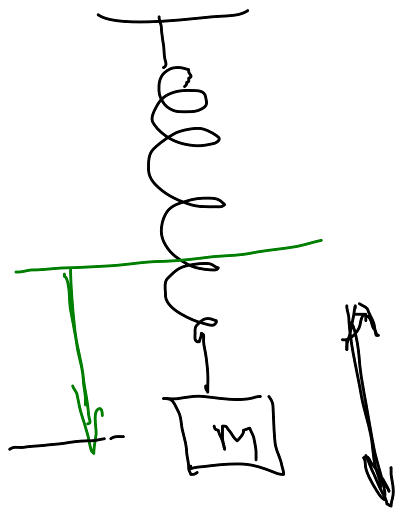
$$V_{max} = \omega A \qquad A = \frac{V_{max}}{\omega} = \frac{3.5 \, \frac{m}{\cancel{s}}}{4.29 \, \frac{rad}{\cancel{s}}} = 0.817 \, m$$

$$\omega = \sqrt{\frac{k}{m}} \qquad \omega^2 = k/m \qquad k = m\omega^2 = 0.920 \, \frac{kg}{s^2}$$

$$= 0.920 \, N/m$$







What is ω for this system

$$\omega = \sqrt{k/m}$$

Same formula Assuming $m_{spring} = 0$

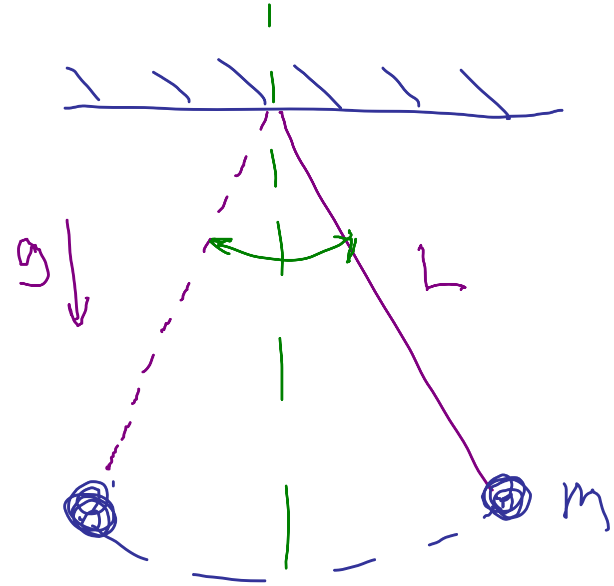
To correct for spring's mass

$$\omega = \sqrt{\frac{k}{m + m_{spring}/3}}$$

Pendulum

Goes back & forth.

$\Rightarrow T \quad (\omega, \to f, \to T)$

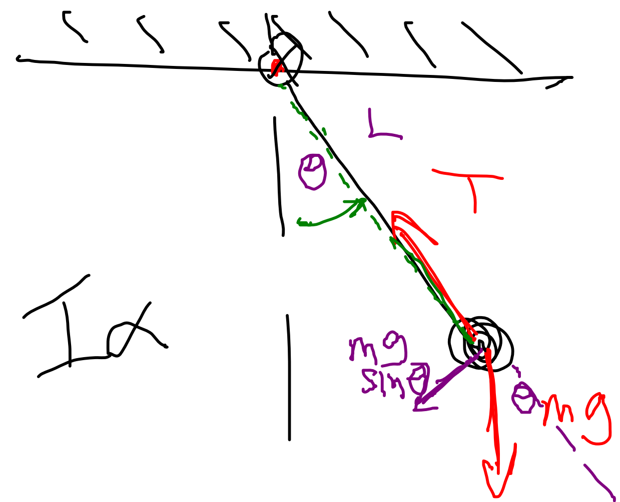


Treat as rotating system

Torque about axis

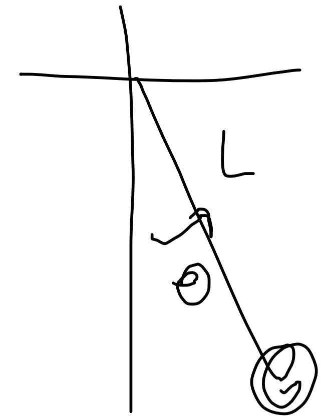
T no torque

$\tau = \underline{\quad - \quad} L\, mg \sin\theta = I\alpha$



$$\tau = -mgL\sin\theta = I\alpha$$

$$= mL^2\alpha = mL^2\,\frac{d^2\theta}{dt^2}$$



$$\frac{d^2\theta}{dt^2} = \frac{-\cancel{mgL}}{\cancel{mL^2}}\sin\theta = -\frac{g}{L}\sin\theta$$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Cheat:

θ is in radians

$\sin\theta \approx \theta$

as long as θ is small.

θ		$\sin\theta$
0.1 rad	5.73°	0.09983
0.2 rad	11.45°	0.19866

$\theta \approx \sin\theta$

$\theta = 45^\circ = \frac{\pi}{4}$ rad

$\sin\theta = 0.707$

$\frac{\pi}{4} = 0.785$

Promise that θ is always small.

I didn't know that!

Tay

$\sin x = x - \frac{x^3}{3!} + \ldots$

$\sin\theta \to \theta$

Substitute

If x small $\sin x \approx x$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta}$$

Approximate

$$d^2x/dt^2 = -w^2 x$$

$$x = A\cos(wt)$$

Just from the DE thing in front of the coordinate x, θ

$= w^2$

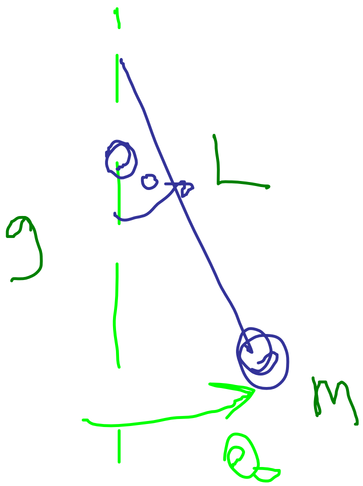
$$\theta(t) = \theta_0 \cos(wt)$$

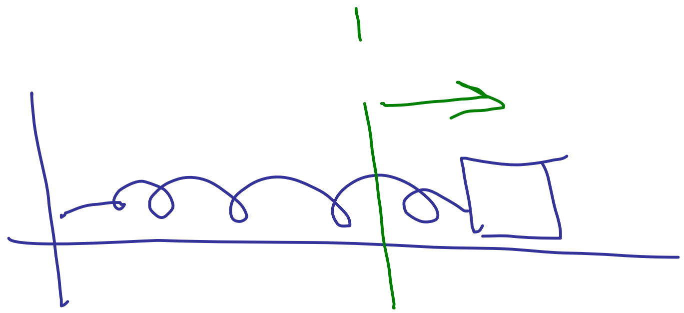
$$w^2 = g/L \qquad \boxed{w = \sqrt{g/L}}$$

$$\omega = \sqrt{g/L} \qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{g/L}$$

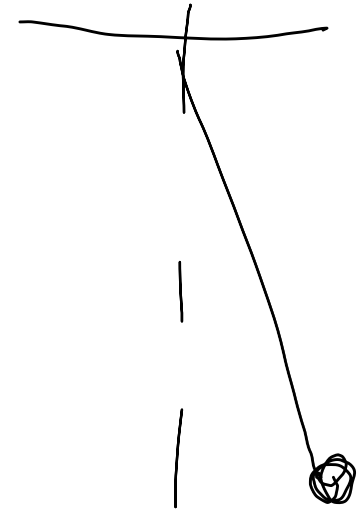
$$T = \frac{1}{f} = 2\pi\sqrt{L/g} \qquad \sqrt{\frac{\cancel{m}}{\cancel{m}/s^2}} = s$$

Note! Doesn't depend on mass (pt mass at end)





Find period of pendulum of length 1m.

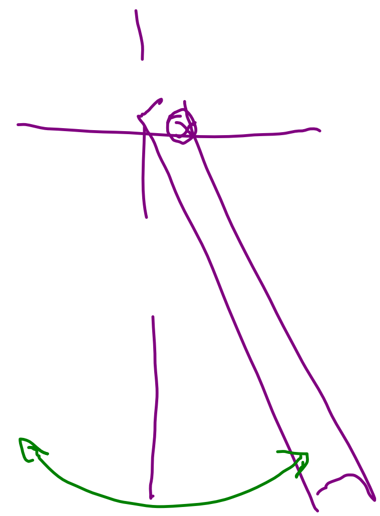


$$T = 2\pi\sqrt{\frac{1.0\,m}{9.8\,m/s^2}} \doteq 2.01\,s$$

Plug in stuff on HW

Suppose we swing 1 m stick about end.

"Physical pendulum"



Next time...