

**Phys 2920, Spring 2010**  
**Problem Set #2**

1. If

$$\mathbf{a} = 5.62\mathbf{i} - 3.58\mathbf{j} + 9.41\mathbf{k} \quad \text{and} \quad \mathbf{b} = -8.92\mathbf{i} + 6.77\mathbf{j} + 2.11\mathbf{k}$$

find  $\mathbf{a} \cdot \mathbf{b}$  and  $\mathbf{a} \times \mathbf{b}$  using Maple or another computer-math system. (You can just include a printout of the result, or explain what you typed and what it gave.)

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2. a) Show that the set of vectors

$$\mathbf{e}'_1 = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k}) \quad \mathbf{e}'_2 = \frac{1}{\sqrt{2}}(\mathbf{j} - \mathbf{k}) \quad \mathbf{e}'_3 = \frac{1}{\sqrt{6}}(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

is orthonormal and thus can serve as a basis for 3D vectors.

b) Express the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  in terms of the vectors  $\mathbf{e}'_1$ ,  $\mathbf{e}'_2$  and  $\mathbf{e}'_3$ .

c) If we denote our usual unit vectors by

$$\mathbf{e}_1 = \mathbf{i} \quad \mathbf{e}_2 = \mathbf{j} \quad \mathbf{e}_3 = \mathbf{k}$$

Express your answer in (b) as a matrix/vector relation of the form

$$\mathbf{e}_i = \sum_{j=1}^3 A_{ij} \mathbf{e}'_j$$

That is, find the matrix  $A$  here.

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As an example of an abstract vector space (with an inner product), and a suitable basis, we will consider functions defined on a certain finite interval in  $x$ . In such a vector space, “adding the vectors” amounts to just adding the functions, with scalar multiplication being just simple multiplication. For each, the basis is a set of functions indexed by  $n$ ,  $f_n(x)$ .

The inner product of two “vectors” is defined as the *integral of the product* of the two functions. (The integral is taken over the appropriate interval in  $x$ .)

3. Consider the set of functions

$$f_n(x) \equiv \sqrt{2} \sin(n\pi x) \quad n = 1, 2, 3 \dots \quad \text{on the interval } 0 \leq x \leq 1$$

That is, the basis of this vector space is

$$\hat{\mathbf{e}}_n = f_n(x) = \sqrt{2} \sin(n\pi x)$$

a) Show that these basis “vectors” are orthonormal. As stated, the definition of the inner product here is

$$\langle m | n \rangle = \int_0^1 f_m(x) f_n(x) dx$$

b) Any smooth function which is zero at both  $x = 0$  and  $x = 1$  can be expressed as a vector in this space. Express the function

$$f(x) = 2 \sin^3(\pi x)$$

as a linear combination of the basis functions (vectors). You just need to get the coefficients and at very least you can find them from

$$c_n = \langle n | f \rangle$$

4. A second example of a set of functions which can be treated as basis vectors can be gotten from the *Legendre polynomials*, a set of polynomials indexed by  $n = 0, 1, \dots$ . They are defined on the interval  $-1 \leq x \leq 1$ . The first few are given by:

$$P_0(x) = 1 \quad P_1(x) = x \quad P_2(x) = \frac{1}{2}(3x^2 - 1) \quad P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3) \quad P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$$

a) Show that the first three functions  $P_n(x)$  are mutually *orthogonal*. (For the inner product, do the integral from  $x = -1$  to  $x = 1$ .)

b) The polynomials as given do *not* have magnitude 1 so the set is not *orthonormal*. For the first three, divide each function (vector) by its magnitude to get *unit* basis vectors. (You can call these  $\tilde{P}_n(x)$ , maybe.)

c) Express the polynomial  $1 - x^2$  in terms of the unit basis functions  $\tilde{P}_n(x)$ .

5. For

$$\mathbf{x} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} 2 & -4 & 7 \\ -3 & 0 & 6 \\ 1 & 8 & -5 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 3 & -1 \\ 0 & 2 & -1 \\ -3 & 0 & 4 \end{pmatrix}$$

find:

a)  $\mathbf{Ax}$

b)  $\mathbf{A}^2$

c)  $\mathbf{AB}$

c)  $\mathbf{BA}$

c)  $\mathbf{A}^{-1}$ . (Use a calculator or computer.)