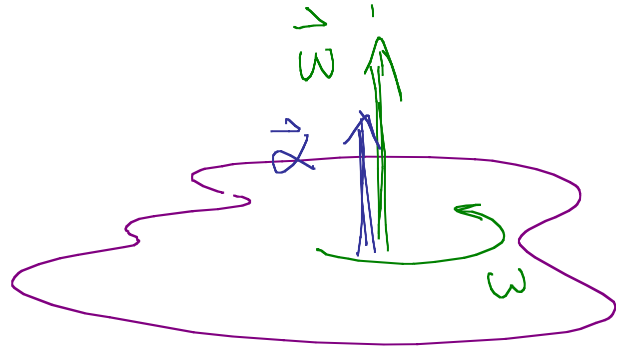
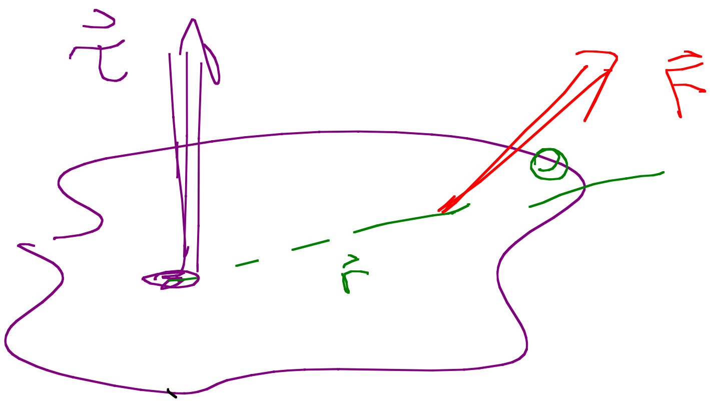
Phys 2110-3 11/12/10

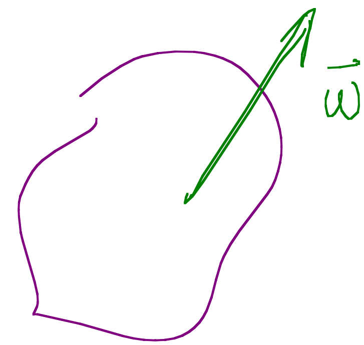
Note Title 11/12/2010

Chap 11



$\vec{\alpha}$ comes from torque.

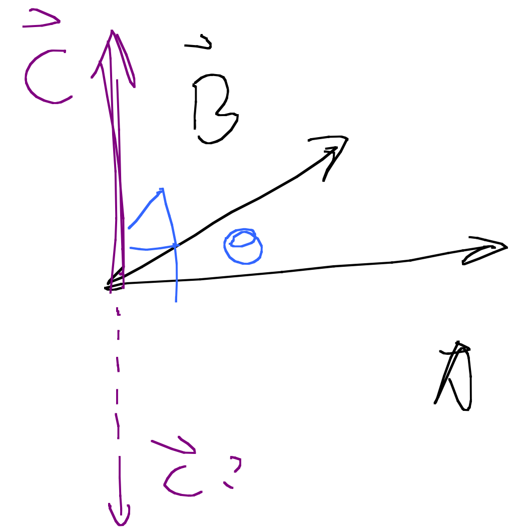




Torque is a cross-product of $\vec{r}$ and $\vec{F}$

Dot product $\vec{A}\cdot\vec{B} = AB\cos\theta = A_xB_x + \cdots$

$\vec{A} \times \vec{B}$ gives a vector



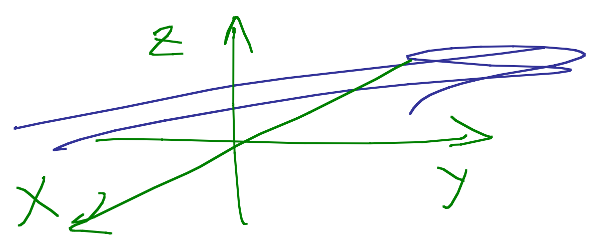
$\vec{A} \times \vec{B} = \vec{C}$

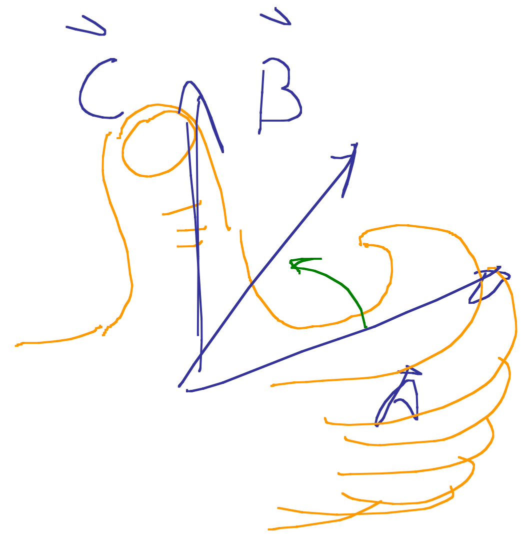
$\vec{C}$ points perp to plane containing $\vec{A}$ & $\vec{B}$

$C = AB|\sin\theta|$

This almost specifies $\vec{C}$

Right-hand rule





In coordinates

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

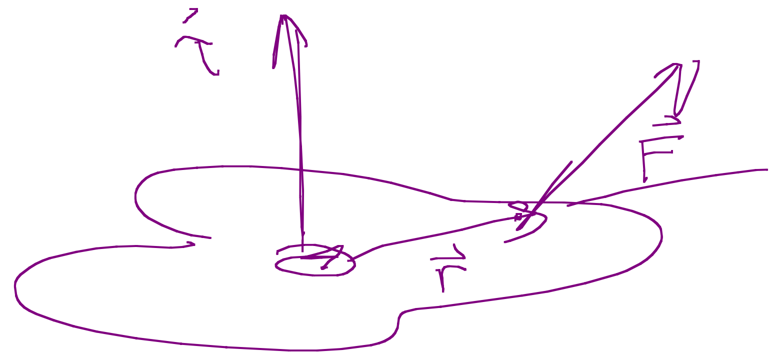
$$\vec{B} = B_x \hat{i} + \dots$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

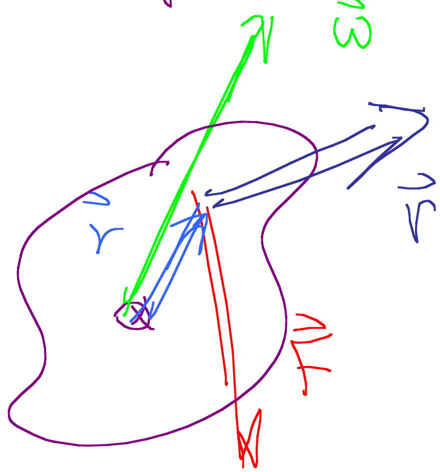
$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$



More complicated



$$\vec{r} \times \vec{F} = \vec{\tau}$$

Rotations are complicated

Momentum

$$\vec{p} = m\vec{v}$$

$$L = I\omega$$

Angular momentum

Scalar Vector

Units? $L = I\omega \longrightarrow kg\ m^2\ \frac{1}{s}$

Units? $L \longrightarrow \frac{kg\ m^2}{s}$

Same units $h = J \cdot s$

$p \cdot x$ $\frac{kg\ m}{s} \cdot m$

action

Vector $L = I\omega$

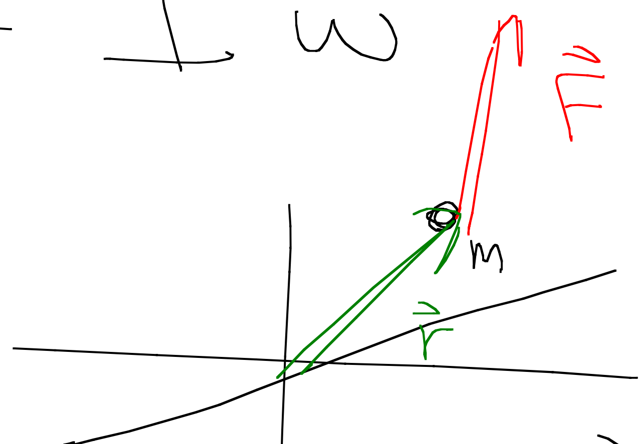
$\vec{L} = I\vec{\omega}$

$\vec{L}$

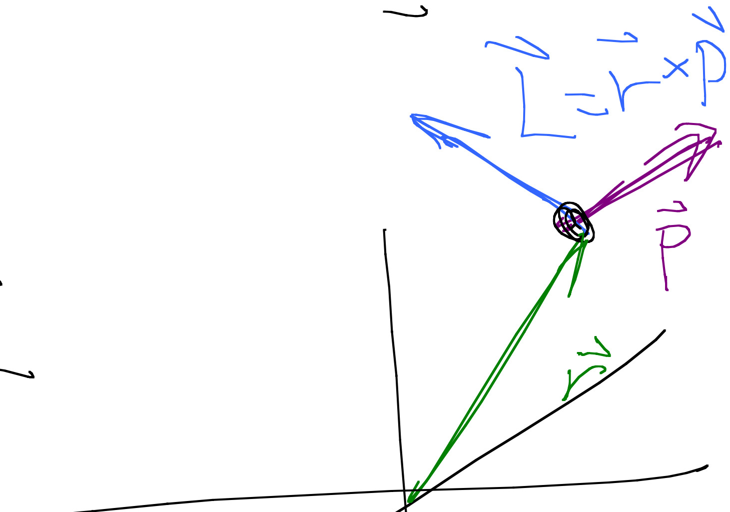
Not nec. parallel to ω

$\vec{L} = I\vec{\omega}$

Ref def'^s



$\tau = \vec{r} \times \vec{F}$

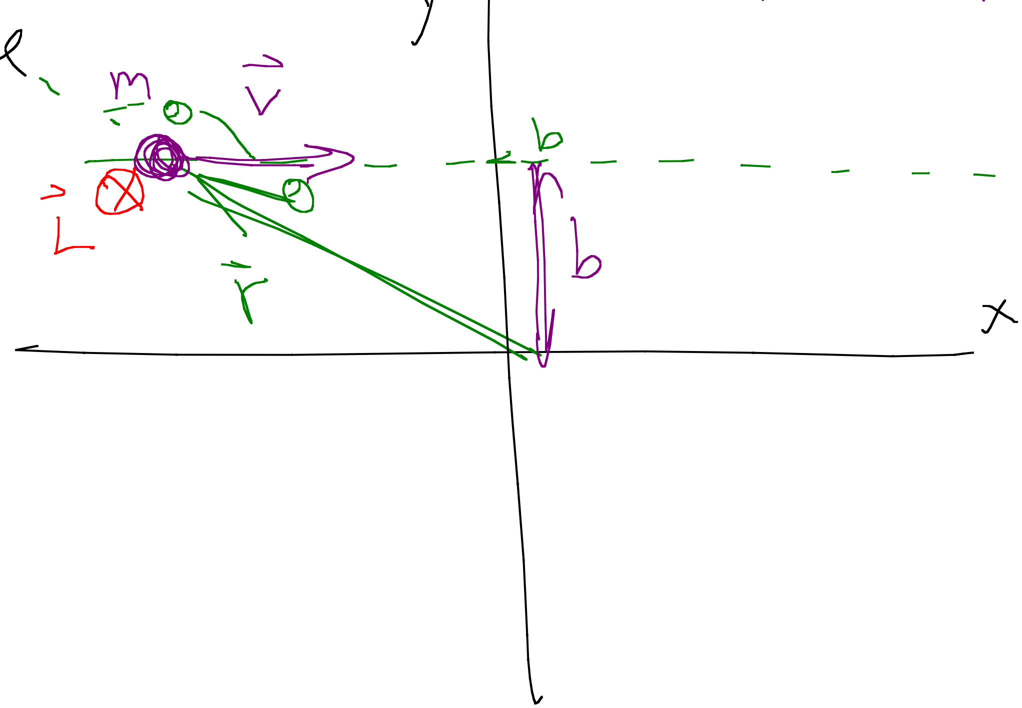


$\vec{L} = \vec{r} \times \vec{p}$

Simple example

$\vec{p} = m\vec{v}$

What is ang. mom. of this blob?



$$\vec{L} = \vec{r} \times \vec{p}$$

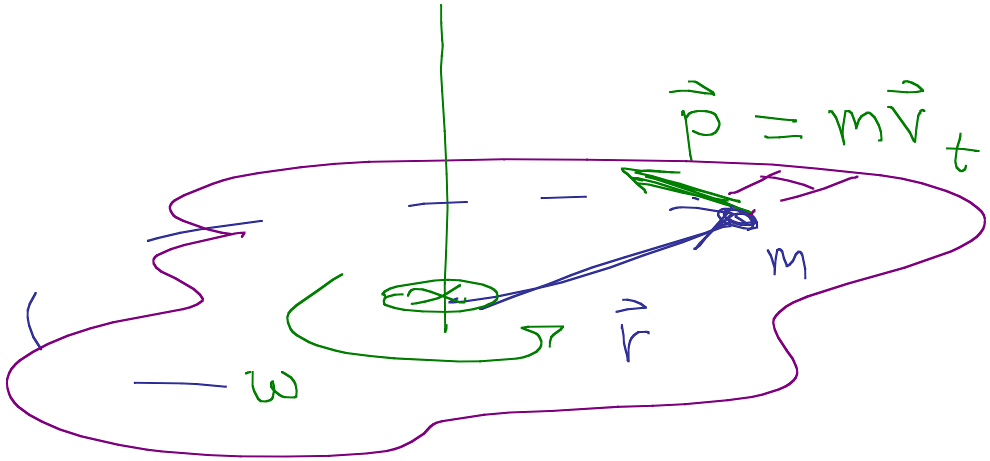
Points into page

$$|\vec{L}| = r\underbrace{p}_{mv}\sin\theta = rmv\sin\theta$$

$$= \vec{L}$$

$$= mvb(-\hat{k})$$

In



$$|\vec{L}| = r m v_t$$

$$= r m r \omega$$

$$= \underbrace{(mr^2)}_{I_i} \omega$$

Whole object:

Add up mass bits

$$\vec{L}_{Total} = I \omega \hat{k}$$

$$\vec{L} = mr^2 \omega \hat{k}$$

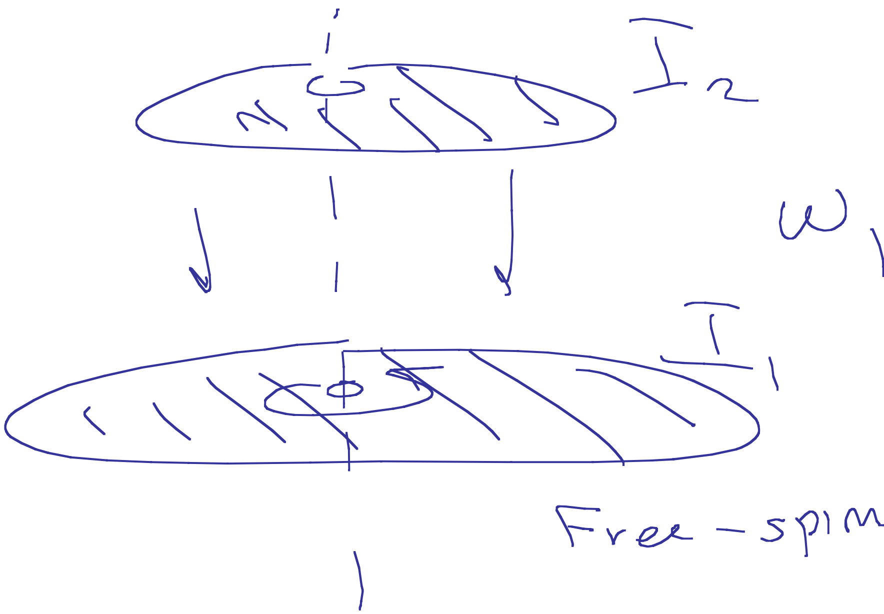
What is it good for?

$\vec{F}_{ext} = d\vec{p}/dt$ Can show $\vec{\tau}_{ext} = d\vec{L}/dt$

No external forces, total momentum stayed constant

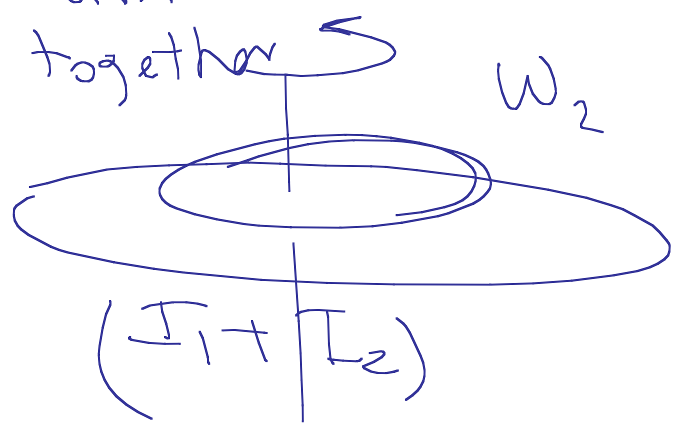
No external torques total ang mom of system.

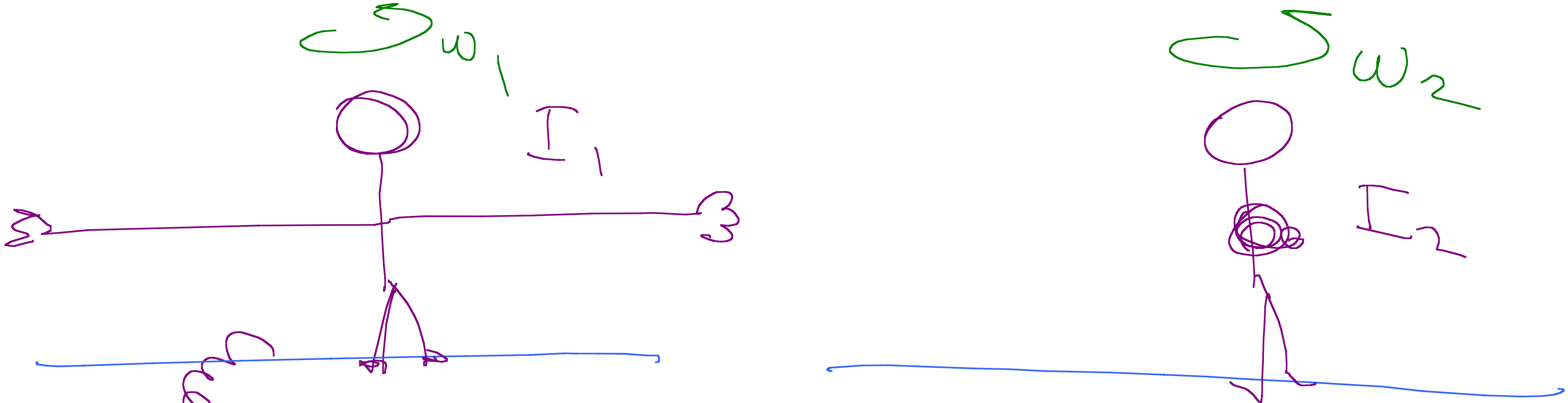
$I_1\omega_1 = (I_1 + I_2)\omega$



Free-spinning disk

Turn together





Ang mom conser

$$I_1 \omega_1 = I_2 \omega_2$$

$$\omega_2 = \frac{I_1}{I_2}\,\omega_1$$

Energy <u>not</u> conser