

Phys 3810, Spring 2011
Problem Set #1, Hint-o-licious Hints

1. Griffiths, 1.3 (a) Change the integration variable to $z = x - a$. Use the Gaussian integrals from the back of the book or elsewhere. (b) Make the same change of integration variable (you have no choice) but note now that because of the factor of x in front, the integral picks up extra terms.

2. Griffiths, 1.7 Start with

$$\langle p \rangle = -i\hbar \int \Psi^* \frac{\partial \Psi}{\partial x} dx$$

and evaluate $d\langle p \rangle/dt$ by moving the derivative inside as a partial time derivative. Use the product rule, note that $\partial/\partial t$ commutes with $\partial/\partial x$ and use the Schrödinger equation and its complex conjugate to substitute. Use integration by parts to see that all the terms cancel except for the one with $\frac{\partial V}{\partial x}$.

3. Griffiths, 1.14 Start with

$$P_{ab} = \int_a^b \psi^*(x, t) \psi(x, t) dx$$

and operate on this with $\frac{d}{dt}$ which means you can move the $\frac{\partial}{\partial t}$ inside. Use the product rule; show that the complex conjugate of the Schrödinger equation can be written

$$\frac{\partial \psi^*}{\partial t} = -\frac{i\hbar}{2m} \frac{\partial^2 \psi^*}{\partial x^2} + \frac{i}{\hbar} V \psi^*$$

(assuming V is real!). Substitute and then show how the whole integrand can be written as a derivative $\frac{\partial}{\partial x}$ of something and then basically you can remove the integral sign!

Use the definition of $J(x, t)$ and arrive at

$$\frac{dP_{ab}}{dt} = J(a, t) - J(b, t)$$

6. Griffiths, 1.18 Once in a while we need to plug some numbers into formulae... the results can teach us something, as they do here.

In (a) set d equal to the De Broglie wavelength λ , we can solve for the temperature T which makes them the same. For electrons with a “separation” equal to the lattice spacing, one finds that the temperature is *far* above room temperature (unless you live inside the Sun) so that electrons *must* be treated as a quantum system. Of course, they are a *many-body* quantum system.

You’ll find that sodium atoms at the same spacing must be treated with quantum mechanics when the temperature is very low, about 3 K.

(b) Take the atomic spacing in a gas to be the cube root of the volume per atom:

$$d = \left(\frac{V}{N} \right)^{1/3}$$

and you can get V/N from the ideal gas law (assuming it applies). Equating this d to the De Broglie wavelength you can derive the bound on the temperature given in the problem. For Helium at 1 atm the result is also about 3 K. For H_2 in outer space at the given intermolecular spacing the “quantum temperature” is *extremely* low, so we can treat it classically. Show all this.

4. Griffiths, 2.4 Nothing new here, just some integrals to work out. Be sure that your answers are plausible. For $\langle p \rangle$ you can use

$$\langle p \rangle = \frac{d\langle x \rangle}{dt}$$

The answer for $\langle x^2 \rangle$ is

$$\langle x^2 \rangle = a^2 \left[\frac{1}{3} - \frac{1}{2n^2\pi^2} \right]$$

The answer for $\langle p^2 \rangle$ is

$$\langle p^2 \rangle = \left(\frac{n\pi\hbar}{a} \right)^2$$

The “uncertainty product” $\sigma_x\sigma_p$ ought to be bigger than $\hbar/2$ for all states.