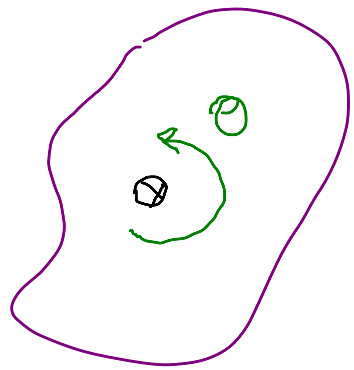
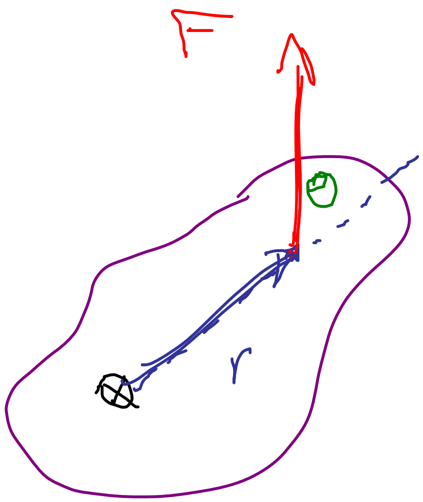
Phys 2110-4 3/23/12

Note Title 3/23/2012

Rotations



θ, ω, α



$$\tau = rF\sin\theta$$

Scalar

+/−

Units N·m

$$\tau_{\substack{net \\ ext}} = \left(\sum_i m_i r_i^2\right)\alpha$$

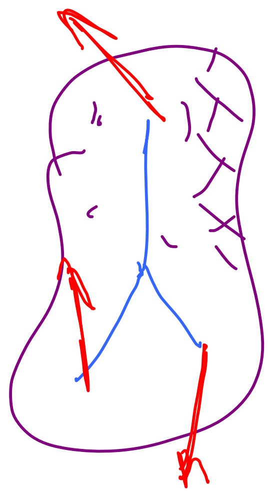
N's second law for rotation

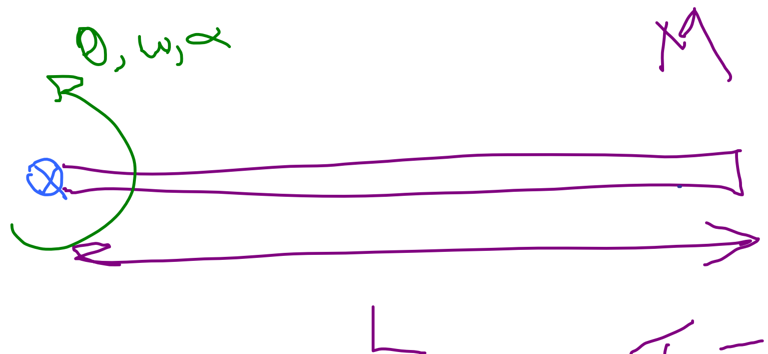
$$= I\alpha$$

Moment of inertia

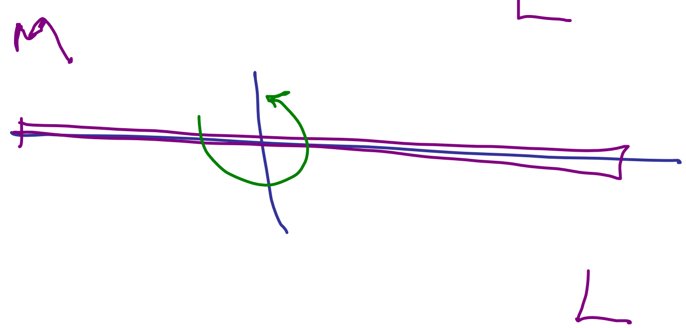
scalar

$kg \cdot m^2$

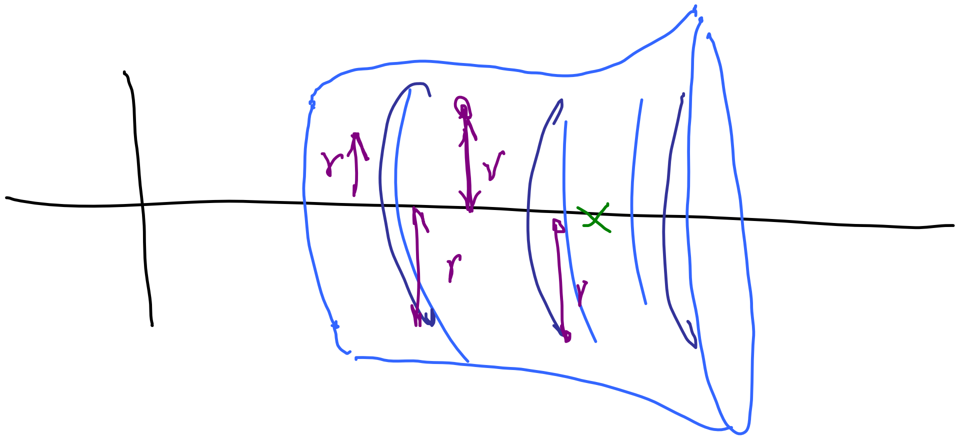




$$I_{\substack{stick \\ end}} = \frac{1}{3}ML^2$$

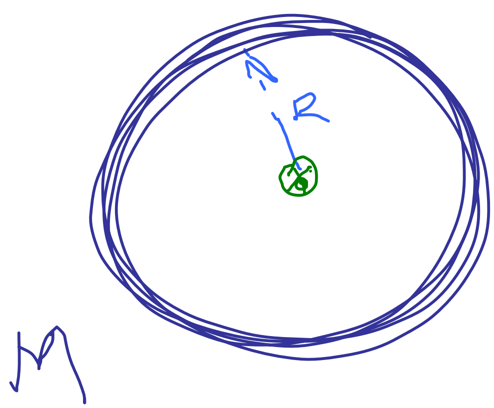


$$I_{\substack{stick \\ middle}} = \frac{1}{12}ML^2$$



$$I = \int r^2 \underbrace{\boxed{\rho \, dV}}_{dm}$$

Simple Shapes

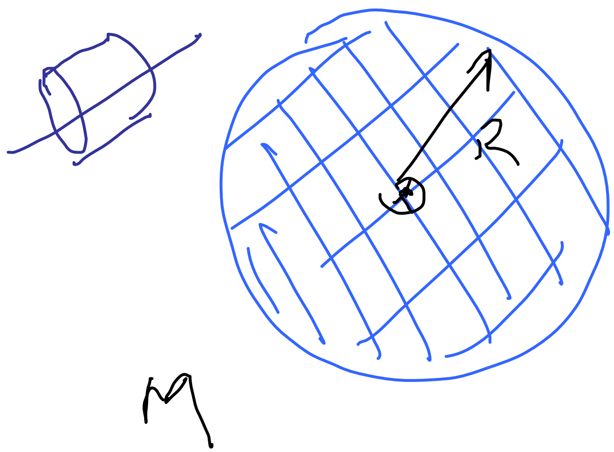


Ring of radius R

Hoop of radius R

⋮

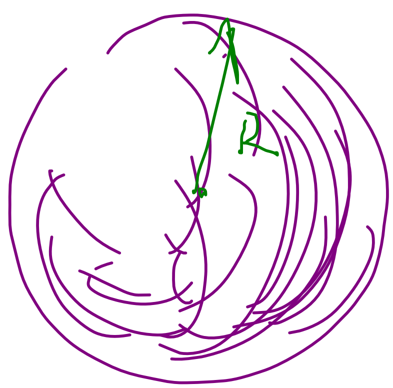
$$I = MR^2$$



Disk : Mass M, radius R

$$I_{disk,\ cyl.} = \frac{1}{2} M R^2$$

Uniform.



Solid (uniform) sphere

Mass M, radius R

$$I_{sol.\,sph} = \frac{2}{5} M R^2$$

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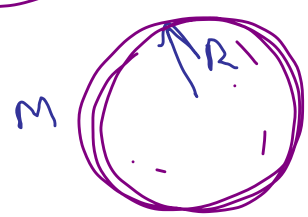
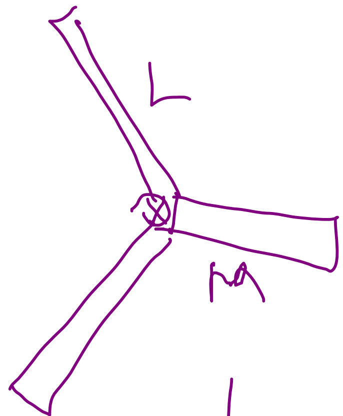


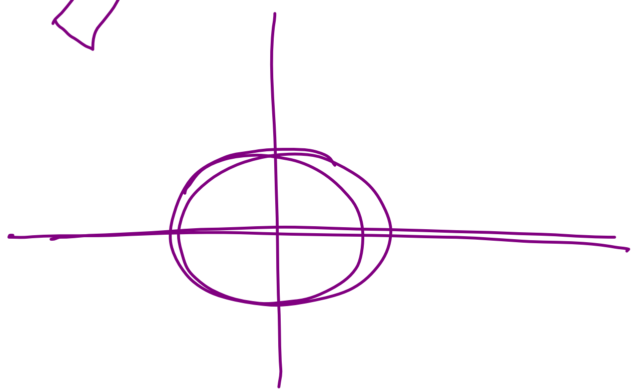
Sph. shell.

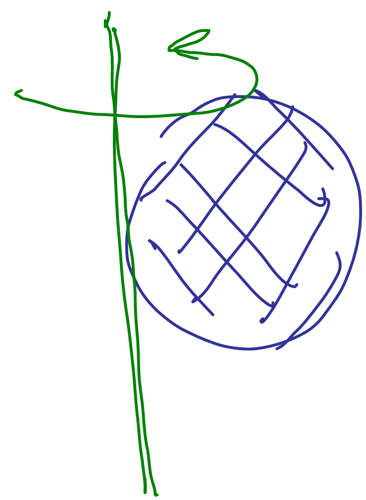
$$I_{sph\ shell} = \frac{2}{3} M R^2$$



$. \ 3\left(\frac{1}{3}ML^2\right) \ldots$ —

Moments of inertia just add together...



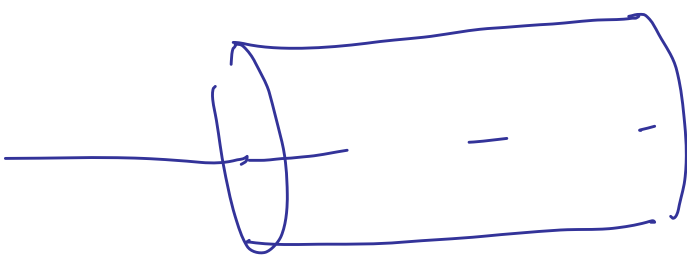


I depends on axis

$$\tau = I\alpha \qquad F = ma$$

Newton's 2nd Law for rotations

10.26 The shaft connecting a power plant's turbine ... solid cylinder of mass 6.8 Mg Diameter 85 cm. Find rot'l inertia



$r = 42.5\ \text{cm}$

$M = 6.8 \times 10^3\ \text{kg}$

$$I = \frac{1}{2}Mr^2 = 6.14 \times 10^2\ \text{kg m}^2$$

10.32 108 g Frisbee 24 cm in diam.

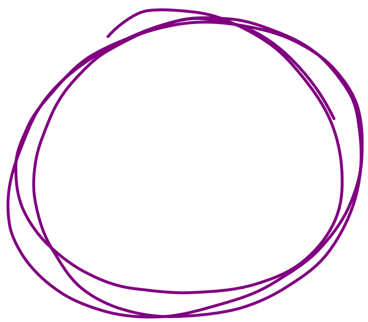
Treat frisbee as
1/2 mass in rim
1/2 mass is in flat disk

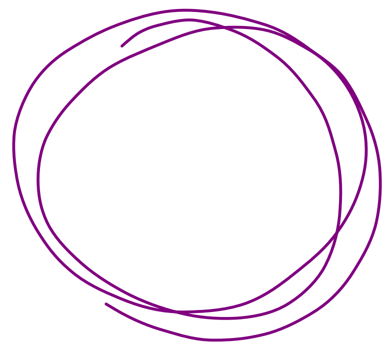
a) what is rot'l inertia?

$$I_{disk} + I_{rim} = \frac{1}{2}\left(\frac{M}{2}\right)R^2 + \left(\frac{M}{2}\right)R^2$$

$$= \frac{3}{4} M R^2 \qquad I = 1.17 \times 10^{-3}\ kg\ m^2$$

b) With a quarter-turn flick of wrist student makes it rotate a 550 rpm what torque did student apply?





$\omega_0 = 0$

$\omega = 550\ \text{rpm}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)$

$\frac{1}{4}$ turn $= \pi/2 = \theta$

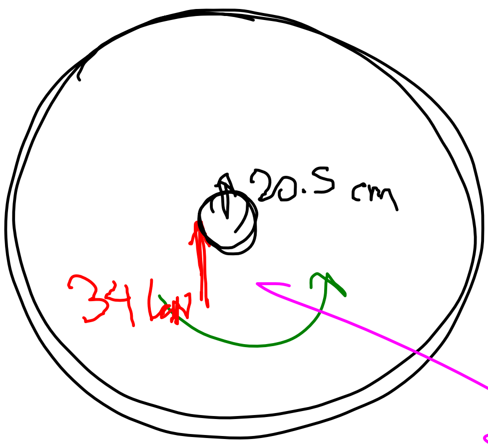
$\omega^2 = \omega_0^2 + 2\alpha\theta \implies \alpha = 1056\ \text{rad}/\text{s}^2$

$$\tau = I\alpha = (1.17\times10^{-3}\ \mathrm{kg\,m^2})\cdot\left(1056\ \frac{\mathrm{rad}}{\mathrm{s^2}}\right)$$

$$= 1.24\ \mathrm{N\cdot m}$$

$$\frac{\mathrm{kg\,m^2}}{\mathrm{s^2}} = \mathrm{N\cdot m}$$

10.33 At MIT Magnet Laboratory energy stored in flywheel mass 7.7×10^4 kg radius 2.4 m. Flywheel is on shaft 41 cm. Fric torque of 34 kN acts tangentially on shaft; how long to stop from 360 rpm rot'n rate?



$M,\ R = 2.4\,m$

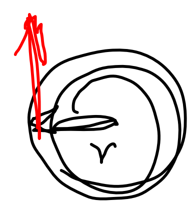
$\omega_0 = 360\ rpm$

Torque, τ

$\tau = Fr$

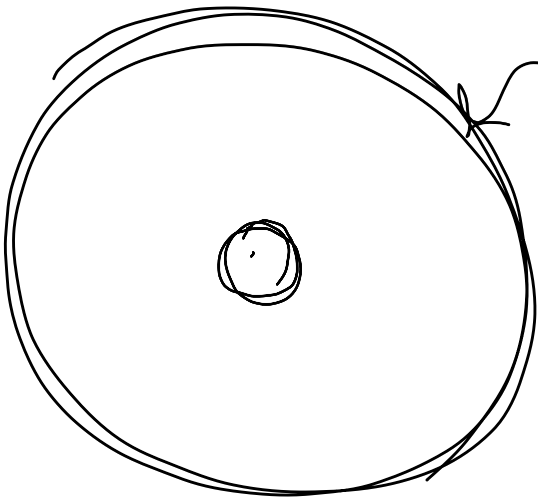
$= (34 \times 10^3 N)(20.5 \times 10^{-2} m)$

$=$



$\tau = I\alpha$

Then $\alpha = \Delta\omega / \Delta t$



$M,\ R$ $\quad \frac{1}{2}MR^2 = I$

Approx I_{shaft} is small

$$I = \frac{1}{2}(7.7\times10^4\,kg)(2.4\,m)^2$$

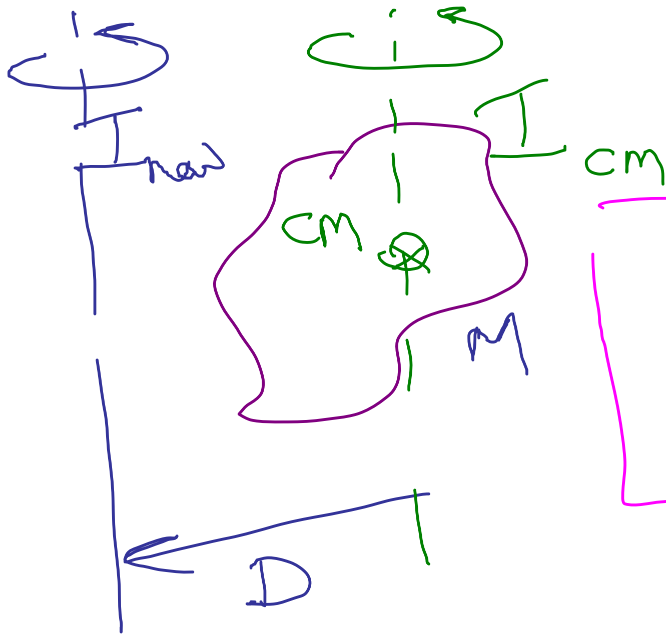
$= \ldots$

$$\tau = I\alpha \implies \alpha \quad \text{(neg number)}$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$t = 20.0$ min

Parallel Axis Thm



Parallel axis

$$\boxed{I_{new} = I_{cm} + MD^2}$$