Phys 3810, Spring 2012
Problem Set #2, Hint-o-licious Hints

1. *Griffiths*, **2.37** Find A from normalization and find the c_n's from (2.37). Then $\Psi(x,t)$ is given by (2.36).

A clever trick on part (a) is to write $\Psi(x,0)$ in terms of the stationary states (which are proportional to $\sin(n\pi x/a)$) before squaring it. You can look up in a book how $\sin^3 w$ is related to $\sin(3w)$. Then when you square and integrate you can use orthonormality of the stationary states.

Get

$$A = \frac{4}{\sqrt{5a}}$$

In finding $\langle x \rangle$ you'll need to evaluate the integral

$$\int_0^a x \sin(\pi x/a) \sin(3\pi x/a)\, dx$$

which you might not find in a table of integrals. You can use a trig identity which makes a product of trig functions into a sum and then it becomes a couple of integrals of x times a single trig function, and that *can* be found.

2. *Griffiths*, **2.10** Use (2.66) and you can use the result for $\psi_1(x)$ from 2.47. You will find:

$$\psi_2(x) = \frac{1}{\sqrt{2}}\, a_+ \psi_1(x) = \frac{1}{\sqrt{2}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \left(\frac{2m\omega}{\hbar} x^2 - 1\right) e^{-\frac{m\omega}{2\hbar}x^2}$$

3. *Griffiths*, **2.11** This one can be a little tedious; it will be OK if you just do the $\psi_0(x)$ state *or* the $\psi_1(x)$ state. For ψ_0, the nonzero answers are

$$\langle x^2 \rangle = \frac{\hbar}{2m\omega} \qquad \langle p^2 \rangle = \frac{\hbar m\omega}{2}$$

and for ψ_1 the nonzero answers are

$$\langle x^2 \rangle = \frac{3\hbar}{2m\omega} \qquad \langle p^2 \rangle = \frac{3\hbar m\omega}{2}$$

For part (c), recall how T and V are related to p^2 and x^2, respectively. Then use the results from part (a).

4. *Griffiths*, **2.15** The classical turning point for the ground state is $a = \sqrt{\frac{\hbar}{m\omega}}$. The probability we want is

$$P = \int_{|x|>a} |\psi_0(x)|^2\, dx$$

I get $P = 0.157299$ (probability to be where, classically, it shouldn't be). But whatever probability you get, make sure it's less than 1.

5. *Griffiths*, **2.19** Use the definition of J from Problem 1.14 and be careful with the complex conjugates. You get an answer which makes sense, as it is proportional to the classical velocity.

6. *Griffiths*, **2.21** (a) Show that $A = \sqrt{a}$. (b) Find $\phi(k)$ with its definition; it may help to use

$$e^{-ikx} = \cos kx - i \sin kx$$

and the fact that cos is an even function and sin is odd. You should get

$$\phi(k) = \sqrt{\frac{2}{\pi}} \frac{a^{3/2}}{(a^2 + k^2)}$$

(c) Use this $\phi(k)$ to write out $\Psi(x, t)$ as an integral. (d) examine the behavior of $\psi(k)$ and $\Psi(x, 0)$ for large and small a.