Phys 2920, Spring 2012
Problem Set #8

1. Go back to Problem 5 on the last set and find the divergence of that vector field, $\nabla \cdot \mathbf{a}$. Then for the cylindrical volume of that problem, evaluate $\int_V (\nabla \cdot \mathbf{a})\, dV$.

Did you get what you expected?

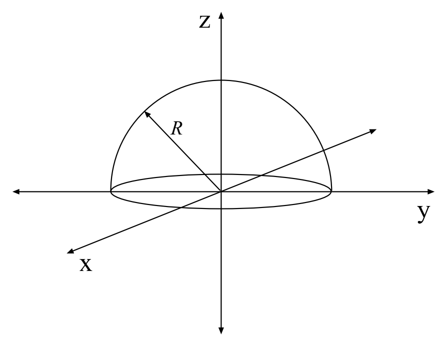
2. *(VA 6.55)* If S is any closed surface enclosing a volume V and $\mathbf{A} = ax\,\hat{\mathbf{i}} + by\,\hat{\mathbf{j}} + cz\,\hat{\mathbf{k}}$, prove that

$$\int\int_S \mathbf{A} \cdot \mathbf{n}\, dS = (a + b + c)V$$

3. Check the divergence theorem for the function

$$\mathbf{v} = r^2 \cos\theta\, \hat{\mathbf{e}}_r + r^2 \cos\phi\, \hat{\mathbf{e}}_\theta - r^2 \cos\theta \sin\phi\, \hat{\mathbf{e}}_\phi$$

using as your volume the upper half of the sphere of radius R centered on the origin.



4. *(VA 6.63)* Verify Stokes' theorem for $\mathbf{A} = (y - z + 2)\,\hat{\mathbf{i}} + (yz + 4)\,\hat{\mathbf{j}} - xz\,\hat{\mathbf{k}}$, where S is the surface of the cube $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 2$, $z = 2$ above the xy plane.

5. Check Stokes' theorem using the function $\mathbf{v} = 2y\,\hat{\mathbf{i}} + 3x\,\hat{\mathbf{j}}$ where the path is the unit circle in the xy plane. (This is the same thing as checking "Green's theorem in a plane" for this case.

6. Evaluate:

a) $\int_0^1 \cos x\, \delta(x - \frac{\pi}{4})\, dx$

b) $\int_0^4 (3x^2 - 2x - 1)(\delta(x - 2) + \delta(x - 5))\, dx$

c) $\int_V (5\mathbf{r}^2 - 2\mathbf{r} \cdot \mathbf{c} - 7)\, \delta^3(\mathbf{r} - 2\,\hat{\mathbf{k}})\, dV$ where $\mathbf{c} = 3\,\hat{\mathbf{i}} - 5\,\hat{\mathbf{k}}$ and V is the sphere of radius 3 centered at the origin.

7. *(CV 1.53 g))* Evaluate, in simple $x + iy$ form,

$$\frac{(2 + i)(3 - 2i)(1 + 2i)}{(1 - i)^2}$$

8. *(CV 1.54 b, j))* If $z_1 = 1 - i$, $z_2 = -2 + 4i$, $z_3 = \sqrt{3} - 2i$, find

(a) $\left| \dfrac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right|$ (b) $\mathrm{Im}\,\{z_1 z_2 / z_3\}$