

2D Motion

$$x(t), y(t)$$

$$v_x, v_y \quad a_x, a_y$$

Constant Accel

a_x, a_y both constant

$$v_x = v_{0x} + a_x t$$

$$v_y = v_{0y} + a_y t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

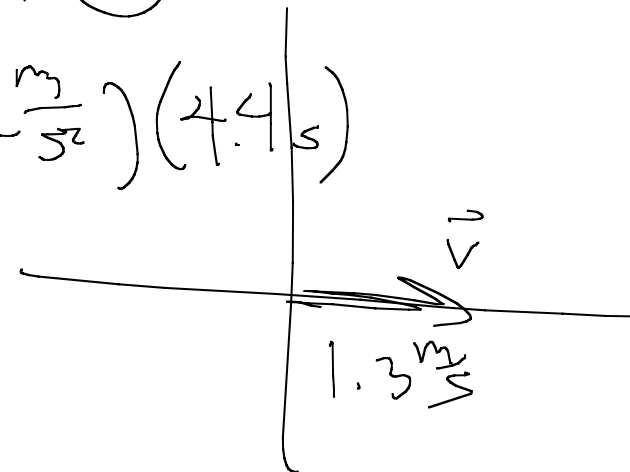
$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

3.32 An object is moving in the x dir at $1.3 \frac{m}{s}$; it is subj'd to accel of $\vec{a} = 0.52 \hat{y} \frac{m}{s^2}$. What is velocity vector after 4.4 s of acceleration?

$$v_x = v_{0x} + a_x t = 1.3 \frac{m}{s} + 0$$

$$v_y = v_{0y} + a_y t = 0 + (0.52 \frac{m}{s^2})(4.4 s)$$

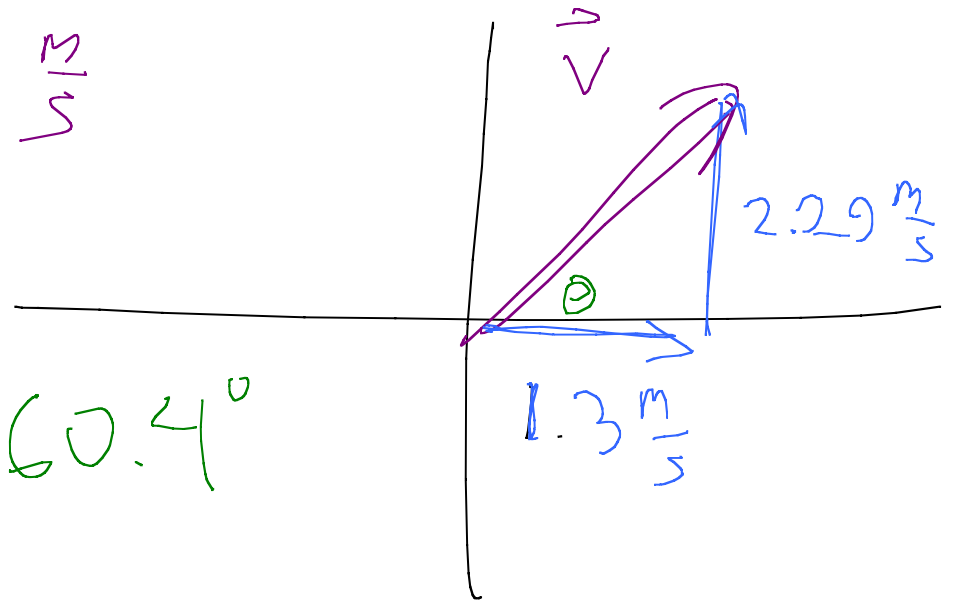
$$v_x = 1.3 \frac{m}{s} \quad v_y = 2.29 \frac{m}{s}$$



$$V = \sqrt{(1.3)^2 + (2.29)^2} \frac{\text{m}}{\text{s}}$$

$$= 2.63 \frac{\text{m}}{\text{s}}$$

$$\theta = \tan^{-1} \left(\frac{2.29}{1.3} \right) = 60.4^\circ$$



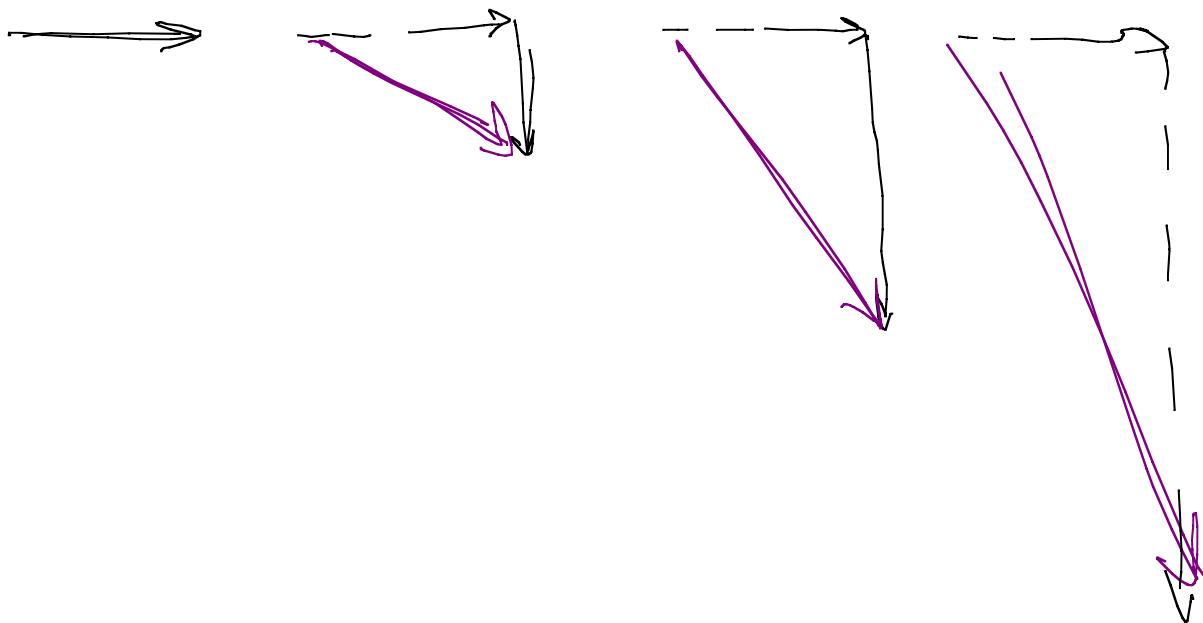
Free-fall

Ignore air!

a_x, a_y

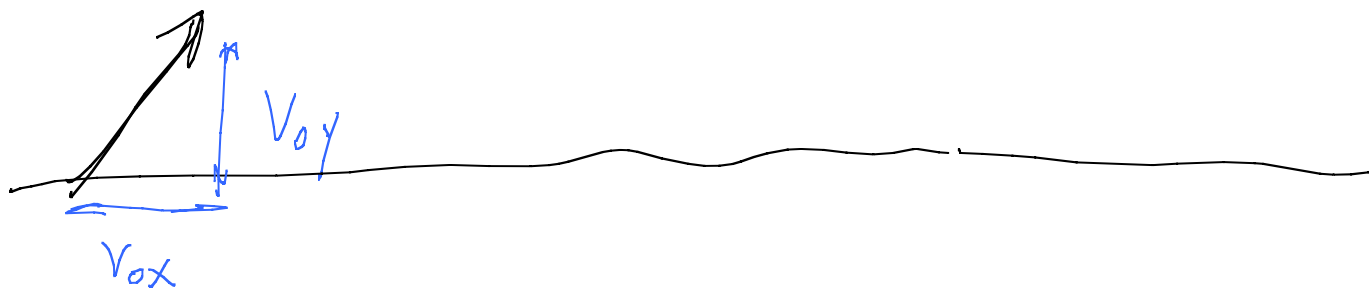
$$a_y = -9.8 \frac{\text{m}}{\text{s}^2}$$

$$a_x = 0$$



x coordinate
changes regularly
 $\sim t$

y coord. changes
 $\sim t^2$

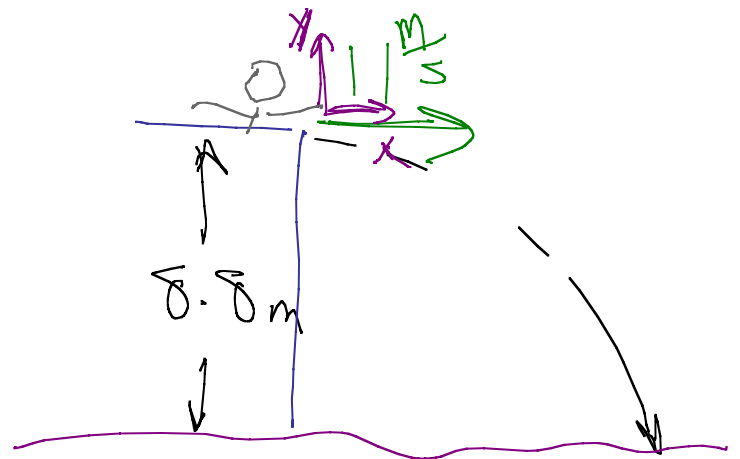


3.40 A carpenter tosses a shingle off
8.8 m - high roof giving it an
initial horiz. velocity of $11 \frac{\text{m}}{\text{s}}$.

a) How long does it take shingle to reach
ground?

b) How far shingle move horizontally
in this time?

$$\begin{aligned}x &= x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\&= 0 + 11 \frac{\text{m}}{\text{s}} t + 0 \\&= (11 \frac{\text{m}}{\text{s}}) t\end{aligned}$$



$$\begin{aligned} y &= y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \\ &= 0 + 0 + \frac{1}{2}(-9.8 \frac{\text{m}}{\text{s}^2}) t^2 \\ &= -\frac{1}{2}(9.8 \frac{\text{m}}{\text{s}^2}) t^2 \end{aligned}$$

a) t when $y = -8.8 \text{ m}$

$$-8.8 \text{ m} = -\frac{1}{2}(9.8 \frac{\text{m}}{\text{s}^2}) t^2$$

$$t = 1.34 \text{ s}$$

b) $x = (11 \frac{\text{m}}{\text{s}}) t = (11 \frac{\text{m}}{\text{s}})(1.34 \text{ s}) = 14.7 \text{ m}$

3.41 An arrow fired horizontally at $41 \frac{\text{m}}{\text{s}}$ travels 23 m horizontally before it hits ground.
From what height fired?

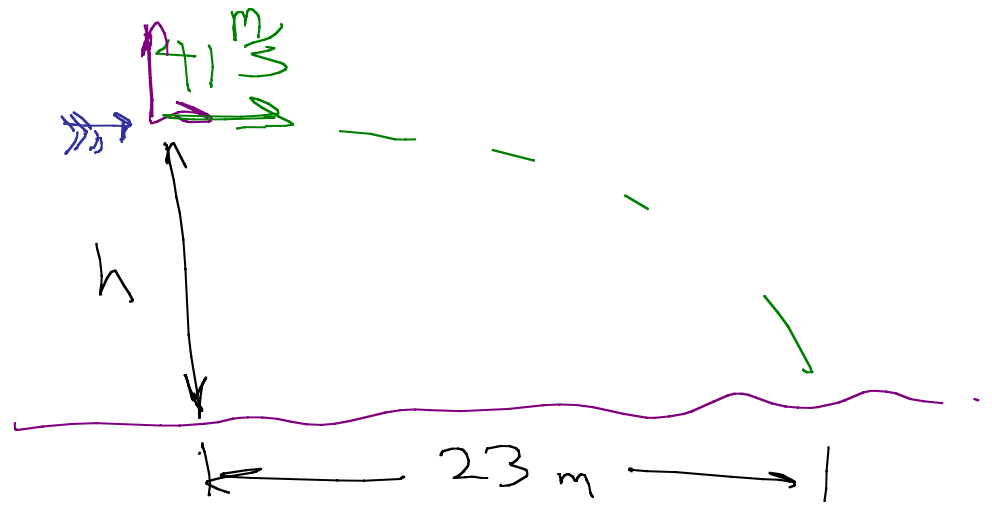
$$x = (41 \frac{\text{m}}{\text{s}})t$$

$$y = -\frac{1}{2}gt^2$$

Find time in flight

$$x = 23 \text{ m} = (41 \frac{\text{m}}{\text{s}})t$$

$$t = 0.561 \text{ s}$$



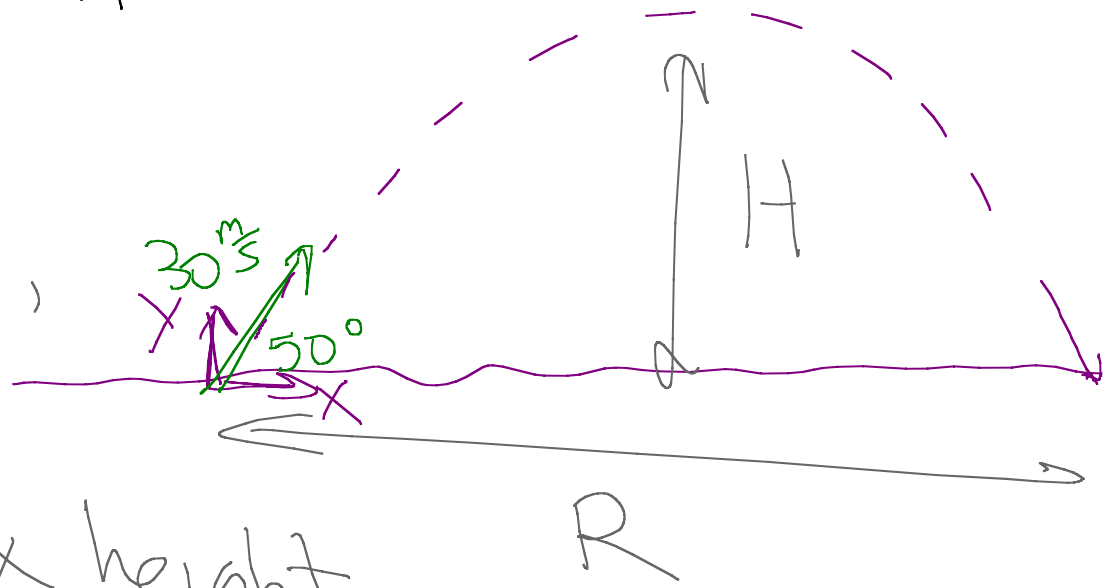
At this time

$$y = -\frac{1}{2}(9.8 \frac{\text{m}}{\text{s}^2})(0.561 \text{ s})^2$$
$$= -1.54 \text{ m}$$

Fired from height of 1.54 m

"Projectile problem"

Projectile fired
from ground level,
as shown



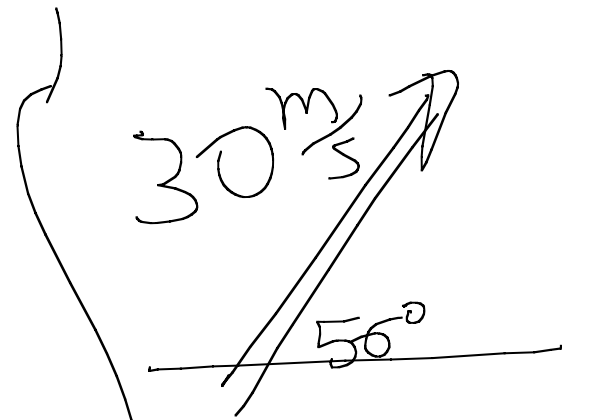
find \rightarrow Range, max height
 H , time in flight

$$\begin{aligned}
 X &= X_0 + V_{0x}t + \frac{1}{2}a_xt^2 \\
 &= 0 + (19.3 \frac{\text{m}}{\text{s}})t + 0 \\
 &= (19.3 \frac{\text{m}}{\text{s}})t
 \end{aligned}$$

$$V_x = V_{0x} = 19.3 \frac{\text{m}}{\text{s}}$$

$$y = y_0 + V_{0y}t + \frac{1}{2}a_yt^2$$

$$\begin{aligned}
 &= 0 + (22.98 \frac{\text{m}}{\text{s}})t + \frac{1}{2}(-9.8 \frac{\text{m}}{\text{s}^2})t^2
 \end{aligned}$$



A diagram showing a velocity vector of magnitude $30 \frac{\text{m}}{\text{s}}$ at an angle of 56° to the horizontal. A vertical line is drawn from the tip of the vector to the horizontal axis, forming a right triangle.

$$\begin{aligned}
 V_{0x} &= 30 \frac{\text{m}}{\text{s}} \cos 56^\circ \\
 &= 19.28 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

$$\begin{aligned}
 V_{0y} &= 30 \frac{\text{m}}{\text{s}} \sin 56^\circ \\
 &= 22.98 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

Range = Value of y when it hits

When does $y = 0$?

$$y = (22.98 \frac{\text{m}}{\text{s}})t - \frac{1}{2}(9.8 \frac{\text{m}}{\text{s}^2})t^2$$

$$t \left[(22.98 \frac{\text{m}}{\text{s}}) - \frac{1}{2}(9.8 \frac{\text{m}}{\text{s}^2})t \right] = 0$$

$$t = 0 \quad \text{or} \quad t = \frac{2(22.98 \frac{\text{m}}{\text{s}})}{(9.8 \frac{\text{m}}{\text{s}^2})} = 4.75$$

What is x at that time?

$$x = (19.28 \frac{\text{m}}{\text{s}})(4.75) \\ = 90.4 \text{ m}$$

$$R = 90.4 \text{ m}$$

What is max ht?

How long to get to max ht?

$$\text{Max } h \uparrow : v_y = 0$$

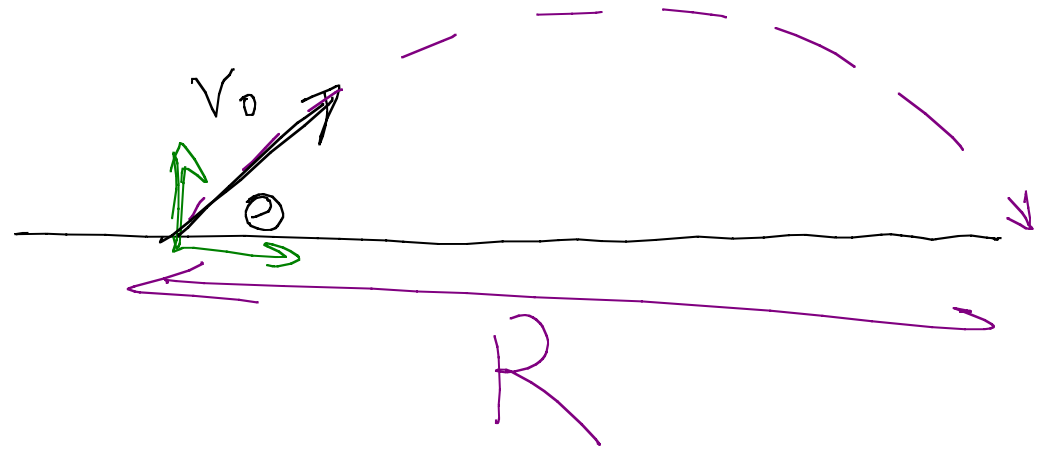
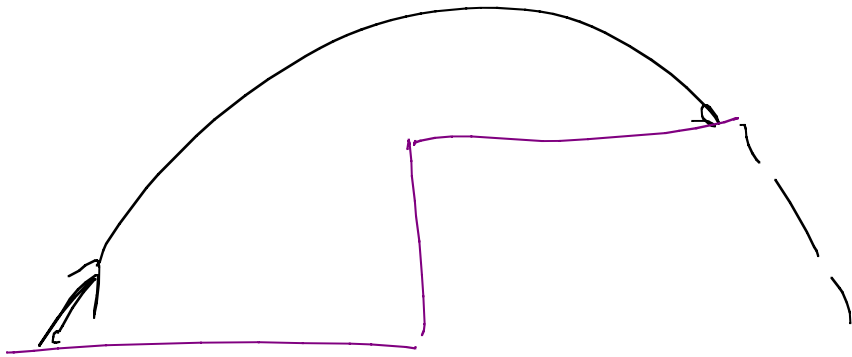
$$v_y = v_{oy} + a_y t$$

$$= (22.98 \frac{\text{m}}{\text{s}}) + (-9.8 \frac{\text{m}}{\text{s}^2}) t$$
$$= 0$$

$$t = 2.34 \text{ s}$$

(Half as long
to get to max
h \uparrow)

$$y = (22.98 \frac{\text{m}}{\text{s}})(2.34 \text{ s}) - \frac{1}{2}(9.8 \frac{\text{m}}{\text{s}^2})(2.34 \text{ s})^2$$
$$= 26.9 \text{ m} = H$$



$$x = 0 + (v_0 \cos \theta)t + 0$$