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Dec. 13, 2011

Phys 4900, Fall 2011
Quiz #3

1. What trick (well, a very reasonable assumption) did we use to find the threshold energies for the production of particles in various collisions?

We assumed that at the threshold condition, all of the product particles would be at rest in the center-of-momentum frame.

2. Explain what is meant by the following terms which refer to groups:

a) The *closure* property.

The product of any two elements of the group gives *another* member of the group.

b) The (possible) *non-commutative* property.

The order in which the product is written (i.e. the symmetry operations are performed) matters; there are elements R_i and R_j for which $R_i R_j \neq R_j R_i$.

c) A *representation* for the members of a group

This is a set of matrices associated with the respective members of the group which obey the same "multiplication table" as the associated group members.

3. Give spins of the

a) electron $\frac{1}{2}$ **b)** proton $\frac{1}{2}$ **c)** neutron $\frac{1}{2}$
d) photon 1 **e)** pion 0 **f)** Δ^+ $\frac{3}{2}$
g) quark $\frac{1}{2}$ **h)** neutrino $\frac{1}{2}$

4. Consider a quantum state where we know the individual angular momenta of two particles with $j_1 = \frac{3}{2}$ and $j_2 = 1$; they are

$$|\tfrac{3}{2} \;\; -\tfrac{1}{2}\rangle \qquad \text{and} \qquad |1 \;\; +1\rangle \; .$$

Write this as a linear combination of states of total angular momentum. (There will be three terms.)

Using the Clebsch--Gordan tables (properly) we find

$$|\tfrac{3}{2} \;\; -\tfrac{1}{2}\rangle \, |1 \;\; +1\rangle = \sqrt{\tfrac{3}{10}} |\tfrac{3}{2} \;\; -\tfrac{1}{2}\rangle \, |1 \;\; +1\rangle - \sqrt{\tfrac{8}{15}} |\tfrac{3}{2} \;\; -\tfrac{1}{2}\rangle \, |1 \;\; +1\rangle + \sqrt{\tfrac{1}{6}} |\tfrac{3}{2} \;\; -\tfrac{1}{2}\rangle \, |1 \;\; +1\rangle$$

5. State the basic assumption that was behind our *predictions* for the relations between cross sections for $\pi-$nucleon scattering. (Hint: It is a statement about isospin...)

The *most basic* assumption was that the fundamental laws for the particle interactions are invariant if we perform a rotation of all the isospins in their (abstract) vector space. It implies that isospin is conserved.

6. a) What is the difference between a vector and a pseudovector? When all three axes are reversed, the components of a vector will change sign; those of a pseudovector will not.

b) The magnetic field is a pseudovector; how do you know that?

It is formed from a cross-product via the Biot-Savart law (where we have a croos-product of two *vectors*). When the components of both vectors change sign, those of the cross-product does not, so the magnetic field is a pseudovector.

c) In spite of the answer to (b), the *force* on a charged particle moving in a magnetic field is a vector. How do you know that?

The magnetic force is given by

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

so that with the reversal of axes the sign of the components of $\mathbf{v}$ will change and so will those of the corss product here and thus so will the force $\mathbf{F}$. So the force is a vector.

7. In the famous Wu experiment which led to the "downfall of parity", what was measured?

The beta-decay of cobalt nuclei which had been polarized so that their spins were all along the same direction. The direction of motion of the product electrons was measured to see if there was a preferred direciton with respect to the spin direction. There was.

8. What particle(s) were involved in the first experiments showing the non-conservation of CP symmetry?

The K^0 and $\overline{K}^0$ particles.

9. What do physicists mean by "Deep Inelastic Scattering" ?

This is the scattering of electrons (or other leptons) from protons (or any nucleus) at sufficiently high energies (see next problem) so that the details of nucleon substructure are resolved. The experimental procedure whereby we can find what the quarks inside the nucleon are doing.

10. Repeat the rough (order-of-magnitude) argument as to what minimum electron energy is needed for experiments with scattered electrons to resolve the structure of the nucleon.

The structure of the nucleon can be resolved if the wavelength of the scattered particles is less than the nucleon radius, which is roughly 10^{-15} m. Thus, with

$$\lambda = 1\ \mathrm{fm} = 1 \times 10^{-15}\ \mathrm{m}$$

we find the energy of the (relativistic) electron to be

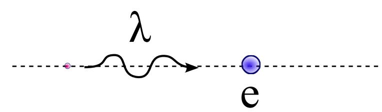
$$E \approx pc = \frac{h}{\lambda}c = \frac{(6.626 \times 10^{-34}\ \mathrm{J \cdot s})}{(1 \times 10^{-15}\ \mathrm{m})}(3.00 \times 10^{8}\ \tfrac{\mathrm{m}}{\mathrm{s}}) = 2 \times 10^{-10}\ \mathrm{J}$$

which in dog-years is

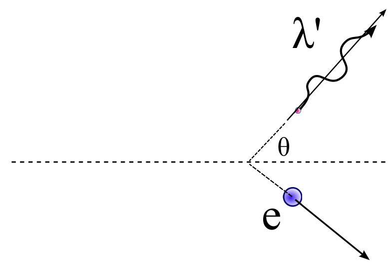
$$E = 2 \times 10^{-10}\ \mathrm{J}\frac{(1.6 \times 10^{-19}\ \mathrm{eV})}{(1\ \mathrm{J})} = 1.24 \times 10^{9}\ \mathrm{eV} = 1.2\ \mathrm{GeV}$$

from which we conclude that at electron energies *beginning at* 1 GeV we can *start* to make sense of the structure of the proton.

11. Really easy problem. I swear, I can't believe how easy this is. This is so easy, I mean, like, I swear, my dog could do it.



For Compton scattering, a photon of wavelength λ collides elastically with a charged particle of mass m (you can take it to be an electron). The photon scatters at angle θ; we want an expression for the outgoing wavelength λ' in terms of θ, λ and m.



I don't need you to re-do this homework problem here, but just show how the solution is *set up* and get it to the point where "the rest is algebra".

If the recoiling electron has 3-momentum $\mathbf{p}$ and the angle at which the electron recoils is phi then since the energy of a photon is $E = h\nu = \frac{hc}{\lambda}$ then energy conservation gives

$$\frac{hc}{\lambda} + mc^2 = \frac{hc}{\lambda'} + \sqrt{\mathbf{p}^2c^2 + m^2c^4}\ . \tag{1}$$

The momentum of a photon is $E/c = h/\lambda$, so conservation of x momentum gives

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + p\cos\phi \tag{2}$$

and conservation of y momentum gives

$$\frac{h}{\lambda'}\sin\theta = p\sin\phi \tag{3}$$

where we mean $p = |\mathbf{p}|$. These are three equations for three unknowns λ', p and ϕ (where λ and θ are given). They tell us in the math department that it should be possible to get an expression for λ' if we are clever enough.

We can eliminate ϕ by isolating $\sin\phi$ and $\cos\phi$, squaring the respective equations and adding

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$$c = 2.998 \times 10^{8}\ \tfrac{\mathrm{m}}{\mathrm{s}} \qquad h = 6.626 \times 10^{-34}\ \mathrm{J\cdot s} \qquad \hbar = 1.0546 \times 10^{-34}\ \mathrm{J\cdot s}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \beta = \frac{v}{c} \qquad c = 3.00 \times 10^{8}\ \tfrac{\mathrm{m}}{\mathrm{s}} \qquad x' = \gamma(x - vt) \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + (v_{AB} v_{BC}/c^2)} \qquad t_{\frac{1}{2}} = (\ln 2)\tau \qquad p = (E/c, \mathbf{p})$$

$$E = \gamma m c^2 \qquad \mathbf{p} = \gamma m \mathbf{v} \qquad E = \sqrt{\mathbf{p}^2 c^2 + m^2 c^4} \qquad \lambda = \frac{h}{p} \qquad E_\gamma = \hbar\omega = h\nu$$

$$|j_1\ m_1\rangle\ |j_2\ m_2\rangle = \sum_{j=|j_1-j_2|}^{(j_1+j_2)} C^{j\,j_1\,j_2}_{m\,m_1\,m_2}\ |j\ m\rangle \qquad |j\ m\rangle = \sum_{j_1,\,j_2} C^{j\,j_1\,j_2}_{m\,m_1\,m_2}\ |j_1\ m_1\rangle\ |j_2\ m_2\rangle$$