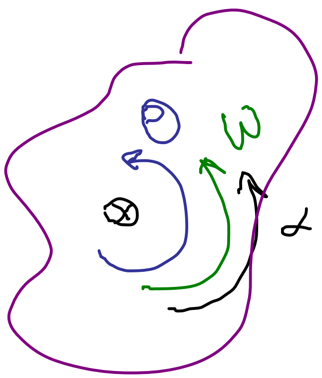
Phys 2110 - 4 3/19/12

Note Title 3/19/2012

Rotations Chap 10, Chap 11



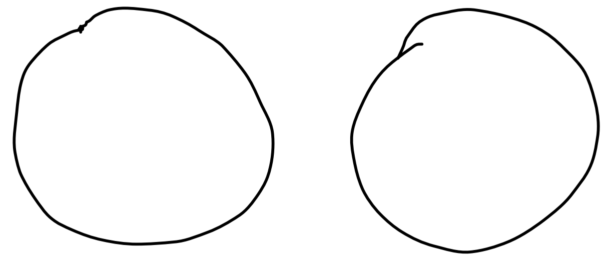
Constant α :

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

10.18 a) How long?



$\omega_0 = 0$ $\qquad$ $\omega = 3600\text{ rpm}$

$= 377 \frac{rad}{s}$

$\alpha = 0.52 \frac{rad}{s^2}$

$$\alpha = \frac{\Delta\omega}{t}$$

$$t = \frac{\Delta\omega}{\alpha} = \frac{377 \frac{rad}{s}}{0.52 \frac{rad}{s^2}}$$

$$= 750\,s \cong 12\,min$$

b) How many revolutions?

$$\theta = \frac{1}{2}(\omega + \omega_0)\,t \qquad \theta = \frac{1}{2}\alpha t^2$$

Get same answer

$$\theta = \underline{1.37 \times 10^5\,rev} = \underline{2.17 \times 10^4\,rad}$$

10.19 Merry-go-round starts rest
accel's $0.010 \, rad/s^2$ for 14 s.

a) How many rev's during that time?

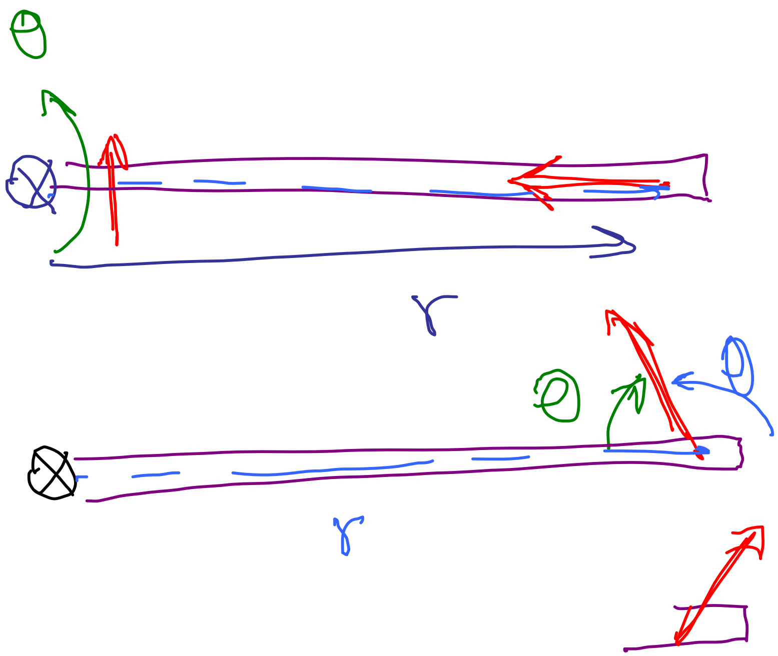
b) Average angular speed?

a) $\theta = \frac{1}{2}\alpha t^2 + \cancel{\omega_0 t} = 0.98 \, rad$

$= 0.156 \, rev$

Kinematics $\longrightarrow$ Dynamics

Why do thing rotate (Forces) but . . .



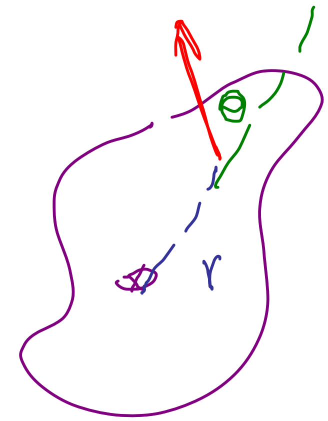
Push at large r.

F, r, $\sin\theta$

$$\tau = rF\sin\theta$$

torque

Torque $rF\sin\theta$



Plus or minus

ccw : $+$ cw : $-$

For now, it's scalar :

Units?
Units?

$[\tau] = m \cdot N$
$= N \cdot m$

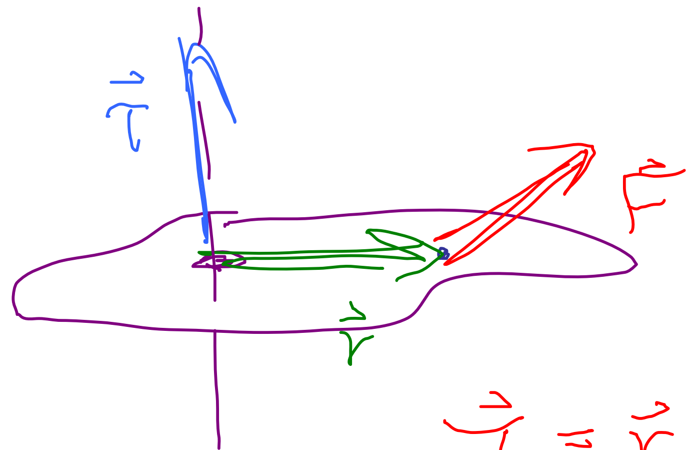
$1\ J = N \cdot m$

Energy J

Torque N·m

Eng) ft·lb lb·ft

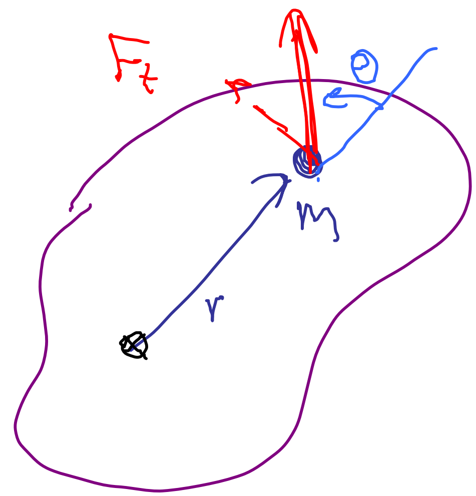
Later: Vector



$$\vec{\tau} = \vec{r} \times \vec{F}$$

Newton's 2nd Law for Rotations

Heuristic derivation



$$m a_t = F_t = F \sin\theta$$

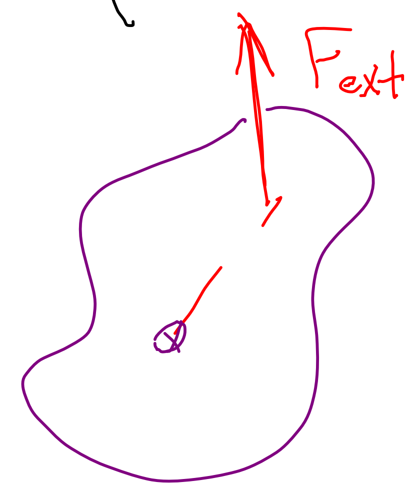
$$m r \alpha = F \sin\theta$$

Mult by r

$$mr^2\alpha = rF\sin\theta = \tau$$

$$\tau = (mr^2)\alpha$$

Add up over all particles



$$\tau_{net} = \sum_i \left(m_i r_i^2 \alpha\right)$$

$$\tau_{net} = \left[\sum_i m_i r_i^2\right]\alpha$$

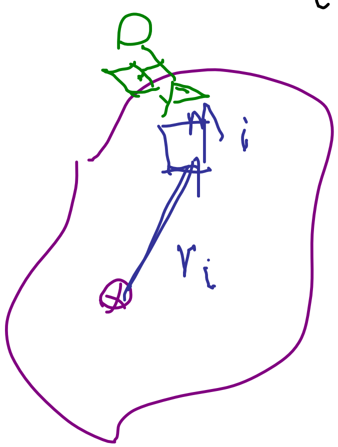
$$F = ma$$

$$\tau \qquad I \qquad \alpha$$

$$I = \sum_i m_i r_i^2$$

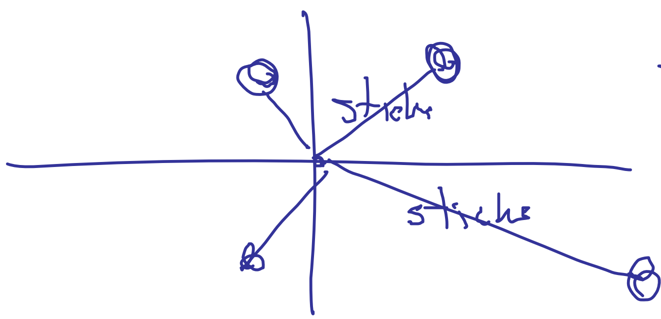
Moment on inertia

Rotational inertia



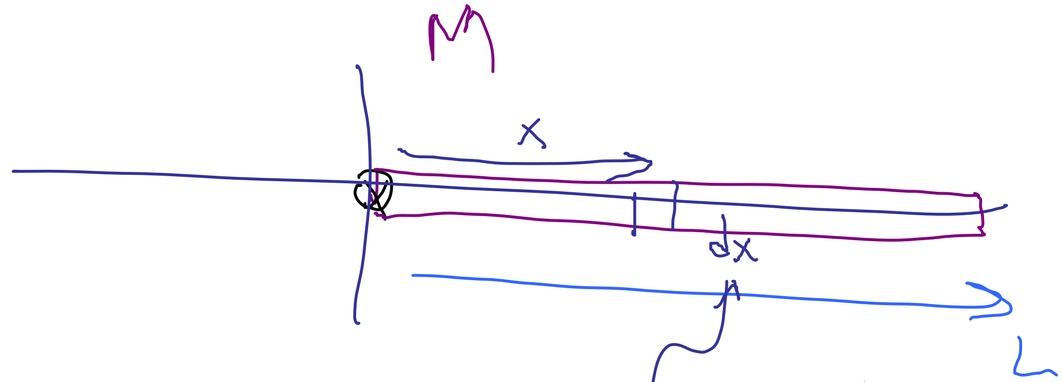
Scalar (Matrix!)

What is it? Units: $kg \cdot m^2$



$\sum m r^2$

Example



M, L

$$I = \sum m r^2$$

$$\sum x^2 \cdot \frac{M\,dx}{L}$$

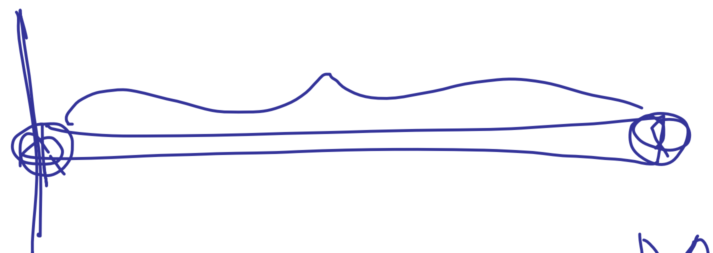
$$\frac{dx}{L} M$$

$$x : 0 \to L$$

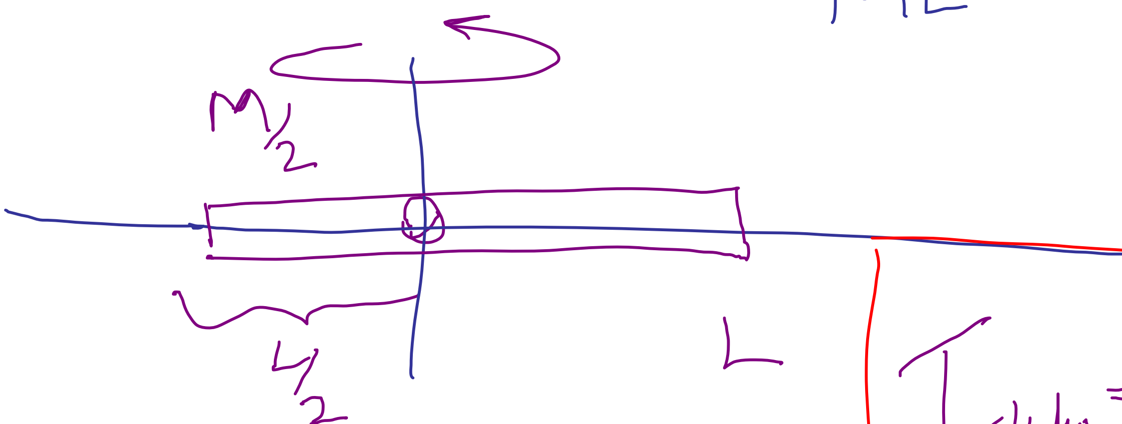
$$I = \int_0^L \frac{M}{L} x^2\,dx = \frac{M}{L}\int_0^L x^2\,dx$$

$$\frac{M}{L}\frac{x^3}{3}\Big|_0^L = \frac{1}{2}\frac{M}{L}L^3 = \boxed{\frac{1}{3}ML^2}$$

$I_{\text{stick end}}$

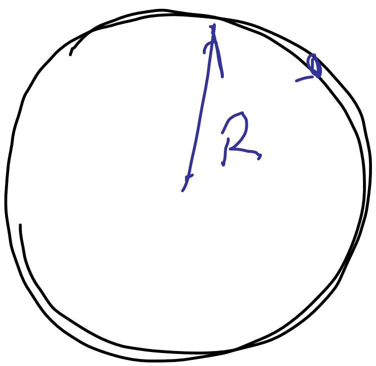


ML^2



$$I = \frac{1}{3}\frac{M}{2}\left(\frac{L}{2}\right)^2 + \frac{1}{3}\frac{M}{2}\left(\frac{L}{2}\right)^2$$

$$\boxed{I_{\text{sticky middle}} = \frac{1}{12}ML^2}$$



Hoop

$$I_{hoop} = MR^2$$