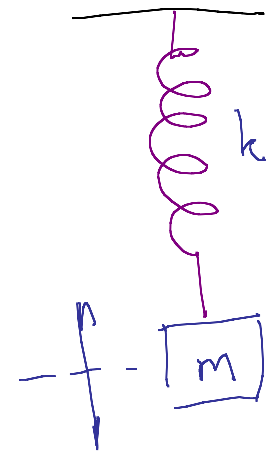
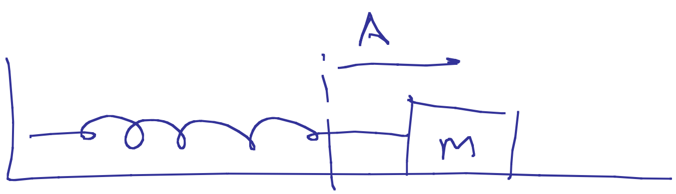
Note Title 11/19/2010

Phys 2110-3 11/19/10

Mass-spring system

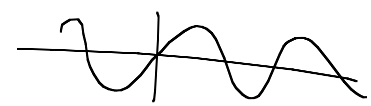
$$\omega = \sqrt{k/m} \qquad f = \frac{\omega}{2\pi} \qquad T = 1/f$$





Same period for any A (!?)

$$x(t) = A\cos(\omega t + \phi)$$

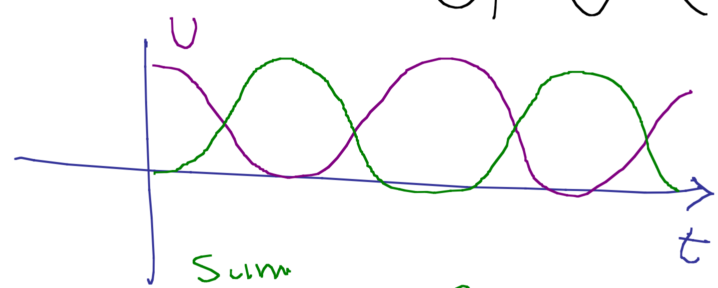


Energy Conservation:

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t)$$

$\phi = 0$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}kA^2\sin^2(\omega t)$$



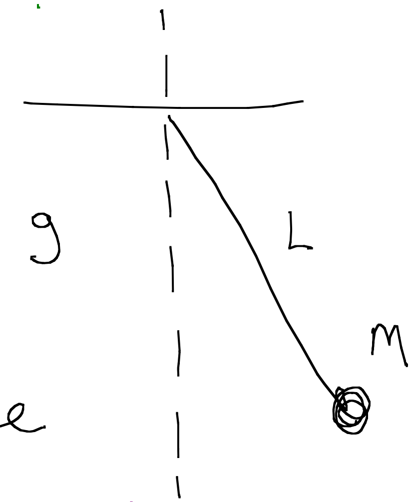
Sum $= \frac{1}{2}kA^2$

$$\frac{d^2x}{dt^2} = -\underbrace{\frac{k}{m}}_{\omega^2} x \longrightarrow \omega = \sqrt{\frac{k}{m}}$$

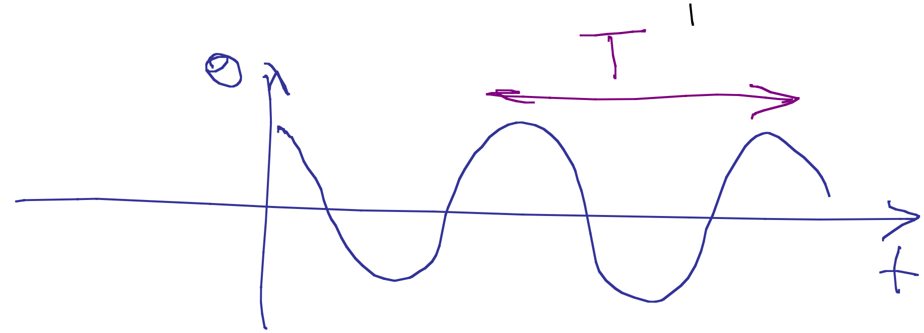
$$x = A\cos(\omega t)$$

Sqrt of thing in front of x is ω!

Simple pendulum

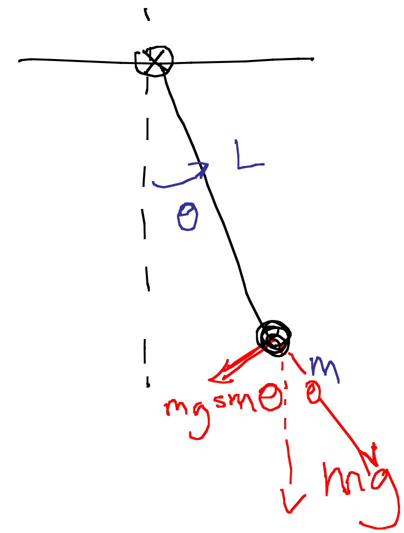


Point mass m on end of string of length L. Pull back to some angle Θ_0, let it go.



Find period.

Do it with rotations.



$$\tau = -(mg\sin\theta)\,L$$
$$= -mgL\sin\theta$$
$$= I\alpha = (mL^2)\frac{d^2\theta}{dt^2}$$

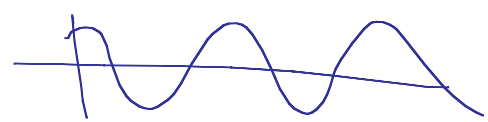
$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

$$d^2x/dt^2 = -\underbrace{\frac{k}{m}}_{\omega^2}x$$

Problem: Don't have θ on rhs.

Cheat:

θ is in radians

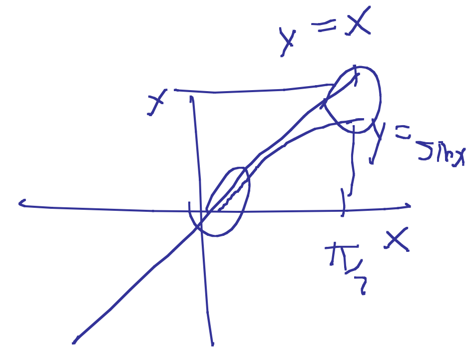


$\sin\theta \approx \theta$ when θ is small.

x, deg	x	sin x
5.73°	0.1	0.09983
11.4°	0.2	0.19866
0.573°	0.01	0.0099998.

Promise (heh heh.)

$\theta_{deg} < 10°$



Did you know this?

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots$$

Useful when $x \ll 1$ $\sin x \approx x$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta = -\frac{g}{L}\theta$$

$\frac{g}{L} \to \omega$

$$\omega = \sqrt{g/L} \qquad f = \omega/2\pi = \frac{1}{2\pi}\sqrt{g/L}$$

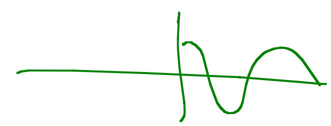
$$T = 1/f = 2\pi\sqrt{L/g}$$

Period of simple pend. 1 m long.

$$T = 2\pi\sqrt{\frac{(1.0\ m)}{(9.8\ \frac{m}{s^2})}} = 2.01\ s$$

Recall: Approx. $\longrightarrow$ $\theta(t) = \theta_0 \cos(\omega t)$

Depends on L, g $\qquad T \propto \sqrt{L}$



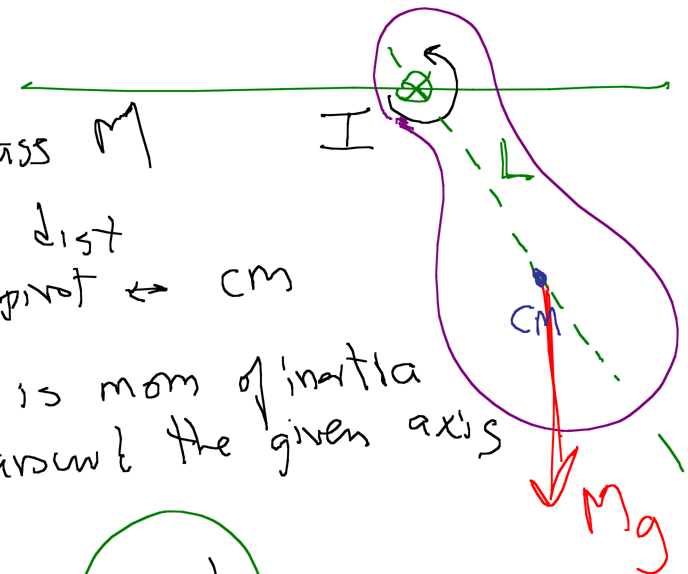
<u>Not</u> on m and not on θ_0 as long as it's small.

More general pendulum:

Mass M

L dist pivot $\leftrightarrow$ cm

I is mom of inertia around the given axis



$$\tau = -mgL\sin\theta$$

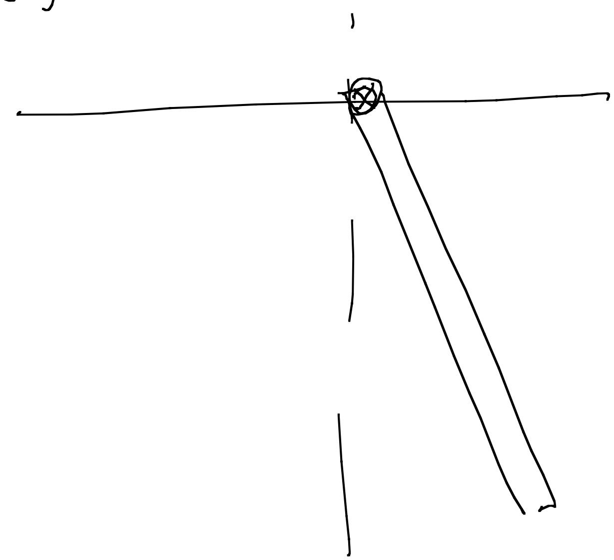
$$= I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{mgL}{I}\sin\theta \simeq -\frac{mgL}{I}\theta$$

$\frac{mgL}{I} \rightarrow \omega^2$

$$\omega = \sqrt{\frac{mgL}{I}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgL}}$$

Make pendulum out of meter stick; axis is at one end.



$$\ell = 1\,\text{m}$$

$$L = \ell/2 = 0.50\,\text{m}$$

$$I = \tfrac{1}{3} M \ell^2$$

Then

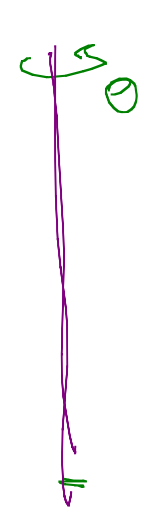
$$T = 2\pi\sqrt{\frac{I}{MgL}} = 2\pi\sqrt{\frac{\frac{1}{3}M\ell^2}{Mg\,\ell/2}}$$

$$= 2\pi\sqrt{\tfrac{2}{3}\,\ell/g}$$

"physical pendulum"

Another kind of pendulum

Torsion fibers



Fiber gives resisting torque.

$$\tau = -K\theta$$

torsional constant, $\frac{N \cdot m}{rad.}$

Torsional pendulum

$$\tau = -K\theta$$

$$= I\, d^2\theta/dt^2$$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{K}{I}\theta}$$

ω^2

T, f fall out

I