Phys 4620, Spring 2006
Exam #3

1. In the problem set we found the (classical) rate of energy loss of the electron in a hydrogen atom. It was too damn big!

In the Phys 2020 magnetic fields lab, we accelerate electrons through a potential difference of about 150 V. Then they are made to go in a circular path by a magnetic field which is perpendicular to the plane of their motion. The radius of the path is about 3.0 cm.

a) What is the speed of the electrons as they move on the circular path? Do we need to worry about relativity in studying their motion? If not, why not?

b) What is the magnitude of the acceleration of the electrons?

c) Make the low–velocity approximation made in the text and find the total power radiated by the electrons.

d) At this rate of energy loss, how long would it take the electrons to lose 10% of their kinetic energy? Is this number bigger than a 2020 lab period?

2. In the lab reference frame, particle A moves in the $+x$ direction with speed $\frac{1}{2}c$ and particle B moves in the $+x$ direction with speed $\frac{3}{4}c$.

What is the speed of particle B in the reference frame of particle A?

3. What is the (relativistic) kinetic energy of a proton which has a speed of $0.9c$?

4. A proton with a kinetic energy of 1.0 GeV is incident on a stationary proton (in the lab frame).

a) Find the momentum of the incident proton in the lab frame. (Use $m_pc^2 = 938.27$ MeV.)

b) Using the invariance of the square of the total momentum 4–vector, find the energy of (either) proton in the center-of-momentum frame.

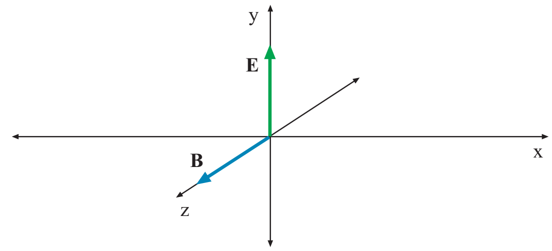
c) Find the speed of (either) proton in the center-of-mementum frame.

d) If this collision produces a particle X, i.e.

$$\mathrm{p} + \mathrm{p} = \mathrm{p} + \mathrm{p} + \mathrm{X}$$

what is the largest value possible for the mass of X? (Consider conservation of energy–momentum in CM frame...)

5. In the “lab” frame there is a uniform E field of magnitude $1.0 \times 10^7\ \frac{\mathrm{N}}{\mathrm{C}}$ in the $+y$ direction and a B field of magnitude 0.100 T in the $+z$ direction.



a) What are the values (magnitudes and directions) of the E and B fields in a refrence frame which moves in the $+x$ direction with a speed of $5.0 \times 10^7\ \frac{\mathrm{m}}{\mathrm{s}}$?

b) Is there a reference frame in which there is *no* electric field? Specify this reference frame and find the value of the magnetic field in that frame.

c) Is there a reference frame in which there is no magnetic field? If so, specify this frame and find the value of the E field in that frame.

6. What does it mean to say that a mathematical object is a 4-tensor? Specifically, what property does the object $t^{\mu\nu}$ have to have?

7. If we let $\mu = 3$ in the relativistic “inhomogeneous” Maxwell equation, we have

$$\frac{\partial F^{3\nu}}{\partial x^{\nu}} = J^3$$

Show how this is the same as one of the Maxwell equations written in our old vector notion.

8. If we let $\mu = 0$ and $\nu = 1$ in the definition of the $\mu\nu$ element of the EM field tensor we get

$$F^{01} = \frac{\partial A^1}{\partial x_0} - \frac{\partial A^0}{\partial x_1}$$

Show that this agrees with the relations we previously had between the fields and the potentials.

Useful Equations

$$\int_{\mathbf{a}}^{\mathbf{b}} (\nabla T)\cdot d\mathbf{l} = T(\mathbf{b}) - T(\mathbf{a}) \qquad \int_{\mathcal{V}} (\nabla\cdot\mathbf{v})d\tau = \oint_{\mathcal{S}} \mathbf{v}\cdot d\mathbf{a} \qquad \int_{\mathcal{S}} (\nabla\times\mathbf{v})\cdot d\mathbf{a} = \oint_{\mathcal{P}} \mathbf{v}\cdot d\mathbf{l}$$

Spherical:

$$d\mathbf{l} = dl_r\,\hat{\mathbf{r}} + dl_\theta\,\hat{\boldsymbol{\theta}} + dl_\phi\,\hat{\boldsymbol{\phi}} \qquad d\tau = r^2\ \sin\theta\, dr\, d\theta\, d\phi$$

Gradient:

$$\nabla T = \frac{\partial T}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}} \tag{1}$$

Divergence:

$$\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} \tag{2}$$

Curl:

$$\nabla\times\mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}} \tag{3}$$

Laplacian:

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2} \tag{4}$$

Cylindrical:

$$d\mathbf{l} = dl_s\,\hat{\mathbf{s}} + dl_\phi\,\hat{\boldsymbol{\phi}} + dl_z\,\hat{\mathbf{z}} \qquad d\tau = s\, ds\, d\phi\, dz$$

Gradient:

$$\nabla T = \frac{\partial T}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial T}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}} \tag{5}$$

Divergence:

$$\nabla\cdot\mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z} \tag{6}$$

Curl:

$$\nabla\times\mathbf{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}} \tag{7}$$

Laplacian:

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial\phi^2} + \frac{\partial^2 T}{\partial z^2} \tag{8}$$

More Math

Gradients:

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$$

Divergences:

$$\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$

Product Rules:

(1) $\nabla\cdot(\nabla T)$ (Divergence of curl)

$$\nabla\cdot(\nabla T) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \nabla^2 T$$

(2) $\nabla\times(\nabla T)$ (Curl of gradient)

$$\nabla\times(\nabla T) = 0$$

(3) $\nabla(\nabla\cdot\mathbf{v})$ (Gradient of divergence)

Nothing interesting about this; does not occur often.

(4) $\nabla\cdot(\nabla\times\mathbf{v})$

$$\nabla\cdot(\nabla\times\mathbf{v}) = 0$$

(5) $\nabla\times(\nabla\times\mathbf{v})$ (Curl of a curl)

$$\nabla\times(\nabla\times\mathbf{v}) = \nabla(\nabla\cdot\mathbf{v}) - \nabla^2\mathbf{v}$$

Physics:

$$\nabla\cdot\mathbf{E} = \frac{1}{\epsilon_0}\rho \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

$$c = 2.998\times10^8\ \tfrac{\mathrm{m}}{\mathrm{s}} \qquad \epsilon_0 = 8.854\times10^{-12}\ \mathrm{C^2/N{\cdot}m^2} \qquad \mu_0 = 4\pi\times10^{-7}\ \mathrm{N/A^2} \qquad e = 1.602\times10^{-19}\ \mathrm{C}$$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad \nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t} \qquad \mathcal{E} = -\frac{d\Phi}{dt}$$

$$\mathbf{E} = -\nabla V - \frac{\partial\mathbf{A}}{\partial t} \qquad \mathbf{A}' = \mathbf{A} + \nabla\lambda \qquad V' = v - \frac{\partial\lambda}{\partial t}$$

$$\text{Coulomb}: \quad \nabla\cdot\mathbf{A} = 0 \qquad \text{Lorentz}: \quad \nabla\cdot\mathbf{A} = -\mu_0\epsilon_0\frac{\partial V}{\partial t}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r}', t_r)}{\mathscr{r}}\,d\tau' \qquad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}', t_r)}{\mathscr{r}}\,d\tau'$$

$$V(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\frac{qc}{\mathscr{r}c - \boldsymbol{\mathscr{r}} \cdot \mathbf{v}} \qquad \mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\frac{qc\mathbf{v}}{(\mathscr{r}c - \boldsymbol{\mathscr{r}} \cdot \mathbf{v})} = \frac{\mathbf{v}}{c^2}V(\mathbf{r},t)$$

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0}\frac{\mathscr{r}}{(\boldsymbol{\mathscr{r}} \cdot \mathbf{u})^3}[(c^2 - v^2)\mathbf{u} + \boldsymbol{\mathscr{r}} \times (\mathbf{u} \times \mathbf{a})] \qquad \mathbf{B}(\mathbf{r},t) = \frac{1}{c}\boldsymbol{\mathscr{r}} \times \mathbf{E}(\mathbf{r},t)$$

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0}\frac{1 - v^2/c^2}{\left(1 - v^2\sin^2\theta/c^2\right)^{3/2}}\frac{\hat{\mathbf{R}}}{R^2} \qquad \mathbf{B} = \frac{1}{c}(\hat{\boldsymbol{\mathscr{r}}} \times \mathbf{E}) = \frac{1}{c^2}(\mathbf{v} \times \mathbf{E})$$

$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \Delta\bar{t} = \sqrt{1 - v^2/c^2}\,\Delta t \qquad \Delta\bar{x} = \frac{1}{\sqrt{1 - v^2/c^2}}\Delta x$$

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + (v_{AB}v_{BC}/c^2)} \qquad \bar{t} = \gamma\left(t - \frac{v}{c^2}x\right) \qquad \bar{x} = \gamma(x - vt) \qquad \bar{y} = y \qquad \bar{z} = z$$

$$\bar{x}^\mu = \sum_{\nu=0}^{3}(\Lambda^\mu_\nu)x^\nu \qquad \Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\eta^\mu = \gamma(c, v_x, v_y, v_z) \qquad \mathbf{p} = \frac{m\mathbf{u}}{\sqrt{1 - u^2/c^2}} \qquad p^\mu = (E/c, p_x, p_y, p_z) \qquad E = \gamma mc^2$$

$$p^\mu p_\mu = -m^2c^2 \qquad E^2 = p^2c^2 + m^2c^4$$

$$K^\mu = \frac{dp^\mu}{d\tau} \qquad J^\mu = (c\rho, J_x, J_y, J_z) \qquad A^\mu = (V/c, A^x, A^y, A^z)$$

$$\begin{aligned} \bar{E}_x &= E_x \qquad \bar{E}_y = \gamma(E_y - vB_z) \qquad \bar{E}_z = \gamma(E_z + vB_y) \\ \bar{B}_x &= B_x \qquad \bar{B}_y = \gamma(B_y + \frac{v}{c^2}E_z) \qquad \bar{B}_z = \gamma(B_z - \frac{v}{c^2}E_y) \end{aligned}$$

$$F^{\mu\nu} = \left\{ \begin{array}{cccc} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{array} \right\} \qquad F^{\mu\nu} = \frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A^\mu}{\partial x_\nu}$$

Invariants: $\mathbf{E} \cdot \mathbf{B}\,,\qquad (E^2 - c^2B^2)$

$$\frac{\partial J^\mu}{\partial x^\mu} = 0 \qquad \frac{\partial F^{\mu\nu}}{\partial x^v} = \mu_0 J^\mu \qquad \frac{\partial G^{\mu\nu}}{\partial x^\nu} = 0 \qquad K^\mu = q\eta_\nu F^{\mu\nu}$$