Phys 2920, Spring 2011
Problem Set #9

1. Use a computer program which does "3D graphics", make a plot of the function

$$f(x, y) = 5\sin(2x)\sin(5y)$$

for the range $0 \le x \le 2\pi$, $0 \le y \le 2\pi$.

Hint: In Maple, you can learn how the procedure `plot3d` *is used. You may also want to change the option* `numpoints` *or* `grid` *to get a better picture.*

2. Using a computer program produce (any way you can) a plot of the function

$$f(\rho, \phi) = 5\sin(\rho^2/3)\cos(\phi)e^{-\rho^2/10}$$

for the range $-5 \le x \le 5$, $-5 \le y \le 5$

3. Find $\cos(-2 + 5i)$, using both your calculator and demonstrating the result using only a real-variable ($10) calculator.

4. Evaluate $\cos^{-1}(10)$; get the decimal value from your calculator and show how this value comes about using the definition of $\cos^{-1}(z)$ and real-variable math.

5. *(CV 3.44)* Prove that $\frac{d}{dz}(z^2 z^*)$ does not exist anywhere.
(If it were analytic, the C–R equations would have to be satisfied...)

6. *(CV 3.47)* Verify that the real and imaginary parts of the following functions satisfy the Cauchy–Riemann equations and thus deduce the analyticity of each function:

$$(a) \quad f(z) = z^2 + 5iz + 3 - i\,, \qquad (b) \quad f(z) = ze^{-z}\,, \qquad (c) \quad f(z) = \sin 2z$$

In other words, use $z = x + iy$, find $u(x, y)$ and $v(x, y)$ and test the C–R equations.

7. *(CV 3.78)* Evaluate

$$(a) \quad \lim_{z\to 2i} \frac{z^2 + 4}{2z^2 + (3 - 4i)z - 6i} \qquad (b) \quad \lim_{z\to e^{\pi i/3}} (z - e^{\pi i/3}) \left(\frac{z}{z^3 + 1}\right) \qquad (c) \quad \lim_{z\to i} \frac{z^2 - 2iz - 1}{z^4 + 2z^2 + 1}$$

(A little practice with l'Hopital's rule and derivatives.)