Phys 2920, Spring 2009 Problem Set #7

1. (VA 5.28) If
$$\mathbf{R}(t) = (3t^2 - t)\mathbf{i} + (2 - 6t)\mathbf{j} - 4t\mathbf{k}$$
, find (a) $\int \mathbf{R}(t) dt$, and (b) $\int_2^4 \mathbf{R}(t) dt$

2. (*VA* 5.34) Evaluate
$$\int_2^3 \mathbf{A} \cdot \frac{d\mathbf{A}}{dt} dt$$
 if $\mathbf{A}(2) = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{A}(3) = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.

3. (VA 5.38) If $\mathbf{F} = (5xy - 6x^2)\mathbf{i} + (2y - 4x)\mathbf{j}$, evaluate $\int_C \mathbf{A} \cdot d\mathbf{r}$ along the curve C in the xy plane, $y = x^3$ from the point (1,1) to (2,8).

4. (VA 5.37) If $\mathbf{A} = (2y+3)\mathbf{i} + xz\mathbf{j} + (yz-x)\mathbf{k}$, evaluate $\int_C \mathbf{A} \cdot d\mathbf{r}$ along the following paths C:

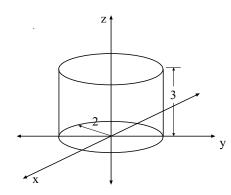
(a)
$$x = 2t^2$$
, $y = t$, $z = t^3$ from $t = 0$ to $t = 1$.

- (b) the straight lines from (0,0,0) to (0,0,1), then to (0,1,1), and then to (2,1,1).
- (c) the straight lines joining (0,0,0) and (2,1,1).

5. Evaluate the surface integral $\oint_S \mathbf{a} \cdot d\mathbf{S}$, where the vector field is given (in cylindrical coordinates) by

$$\mathbf{a} = \rho^2 \cos^2 \phi \, \hat{\mathbf{e}}_{\rho} + \rho \sin \phi \, \hat{\mathbf{e}}_{\phi} + \rho z^3 \, \hat{\mathbf{e}}_z$$

and the closed surface is a circular cylinder of radius 2 whose axis is the z axis; it has height 3 and extends from z=0 to z=3.



6. Find the moment of inertia (about the z axis) of the "ice cream cone" volume which was used in another example in class and which is shown here. (It is a sector of a solid sphere of radius R, out to an angle $\theta = \pi/6$ out from the z axis)

Assume its mass density ρ_{mass} is uniform. Express the answer in terms of the total mass M of the object.

