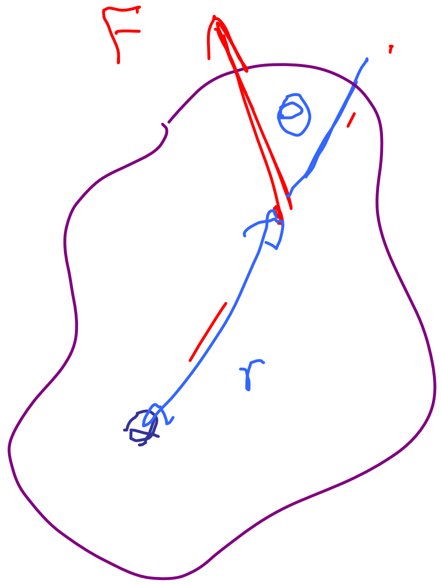
Rotational Dynamics

$F_{x_{net}} = m a_x \Longrightarrow \boxed{\tau_{net} = I\alpha}$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega = \omega_0 + \alpha t$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$



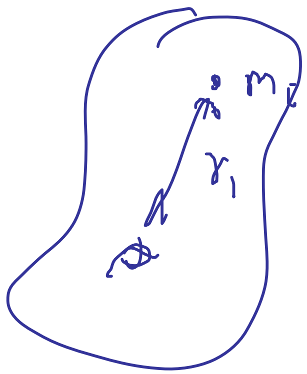
$$|\tau| = rF\sin\theta$$

you put in sign

↺ positive

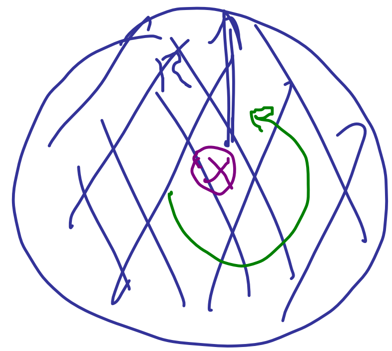
↻ negative

$$I = \sum_i m_i r_i^2 \quad (\text{kg m}^2)$$



Particular cases:

$$I_{\substack{\text{stick}\\ \text{end}}} = \frac{1}{3} M L^2$$



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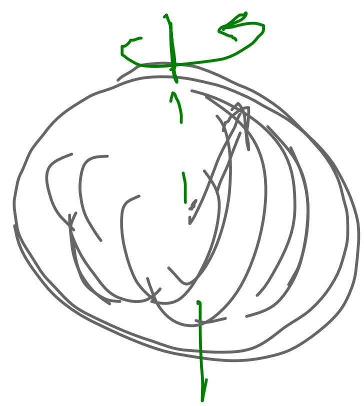
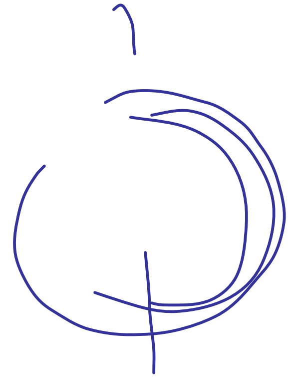
$$I_{\substack{\text{cyl}\\ \text{middle}}} = \frac{1}{2} M R^2$$



$$I_{\substack{\text{sphere}\\ \text{solid}}} = \frac{2}{5} M R^2$$

$$I_{\text{sphere hollow}} = \frac{2}{3} M R^2$$



10.33 Huge flywheel, solid cylinder

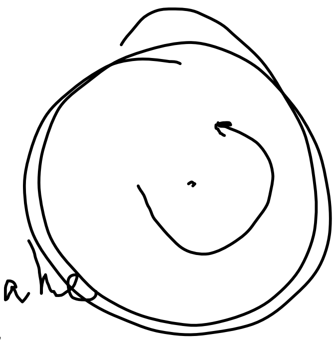
7.7×10^4 kg, radius 2.4 m

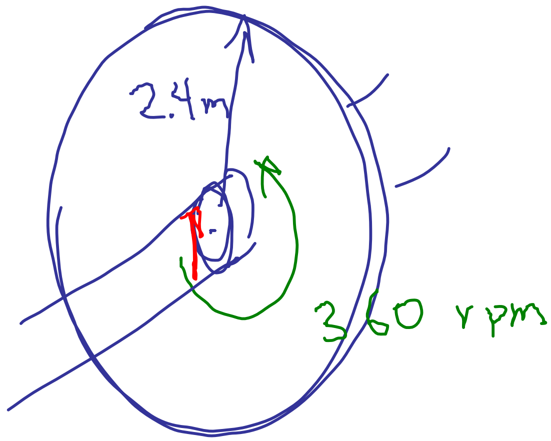
Shaft is 20.5 cm radius

Fric force of 34 kN acts

tang'ly on shaft, how long it take

flywheel to stop from usual rate 360 rpm





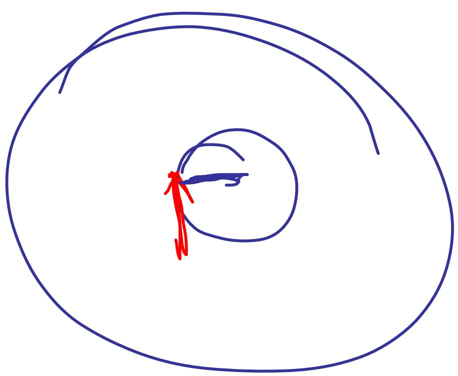
$$I = \frac{1}{2} M R^2$$

$$= \frac{1}{2}(7.7\times10^4\ kg)(2.4\ m)^2$$

$$= 2.22\times10^5\ kg\ m^2$$

$$\tau = -Fr(1) = -(34\times10^3\ N)(0.205\ m)$$

$$= -6.97\times10^3\ N\cdot m$$



$$= I\alpha$$

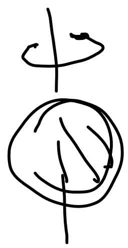
$$\alpha = \tau/I = -3.2\times10^1\ \frac{rad}{s^2}$$

$$\omega_0 = 360 \frac{rev}{min} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 37.7 \frac{rad}{s}$$

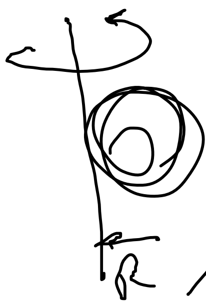
$$\omega = 0 = \omega_0 + \alpha t \quad \text{etc.}$$

$$\Longrightarrow \quad t = 20 \text{ min!}$$

Example



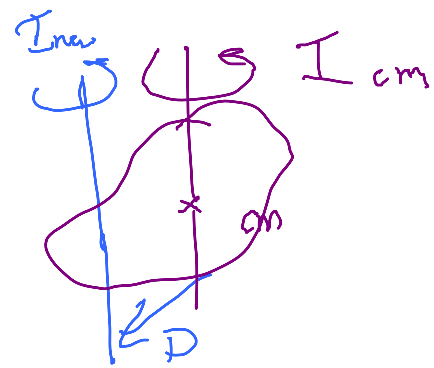
$I = \frac{2}{5} M R^2$



$$I = \frac{2}{5} M R^2 + M R^2$$
$$= \frac{7}{5} M R^2$$

Parallel Axis Thm

$$I_{new} = I_{cm} + M D^2$$



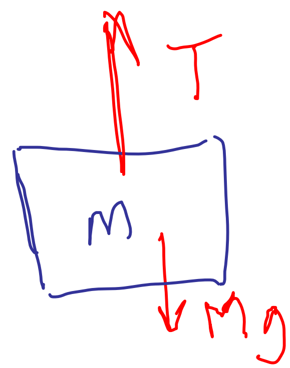
Example, Block & Cylinder

Like 10.9.

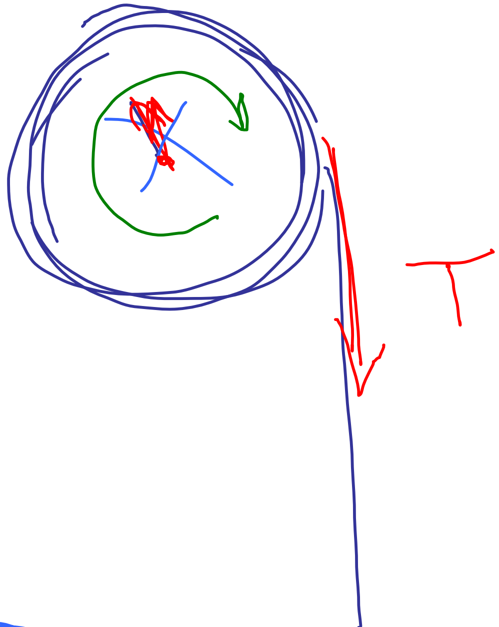
String wrapped around cylinder, I. Mass M hangs from string let it go.

Find accel of the mass.

Two force diagrams



$$\boxed{Mg - T = Ma}$$



Consider torques on pulley

$$\tau_{net} = TR = I\alpha$$

$$a_t = R\alpha = a$$

$$\alpha = a/R$$

$$= I\,a/R$$

$$TR = I\,a/R$$

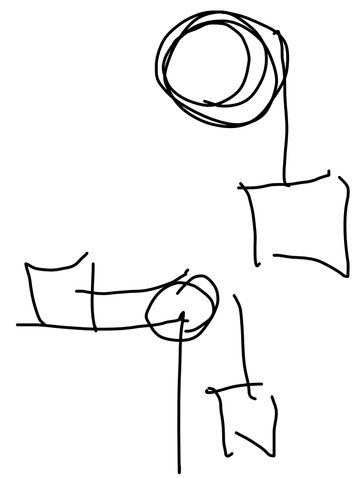
$$\boxed{T = I\,a/R^2}$$

$$Mg - T = Ma$$

$$Mg - I\,{}^{a}/_{R^2} = Ma$$

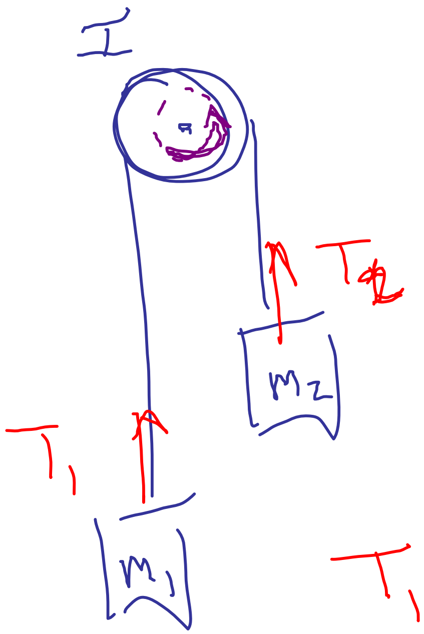
$$Mg = Ma + I\,{}^{a}/_{R^2} = a\left(M + \frac{I}{R^2}\right)$$

$$a = \frac{Mg}{\left(M + {}^{I}/_{R^2}\right)}$$



$I \to 0 \quad a \to g$

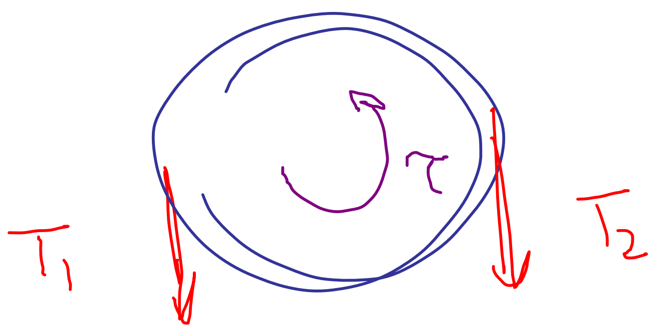
$M \to 0 \quad a \to 0$

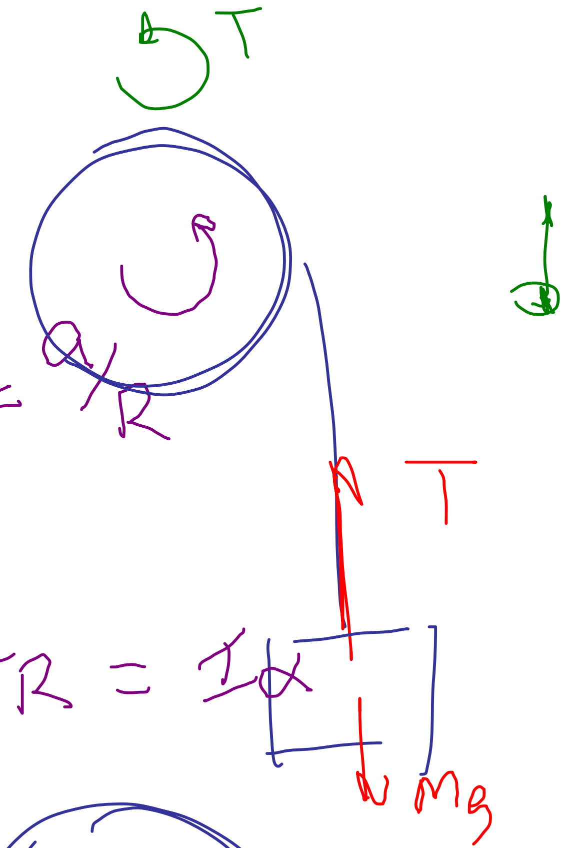


$T_1 = m_1 g$

$T_2 = m_2 g$

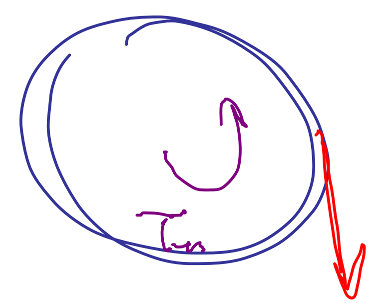
$T_1 \neq T_2$





$\alpha = a/R$

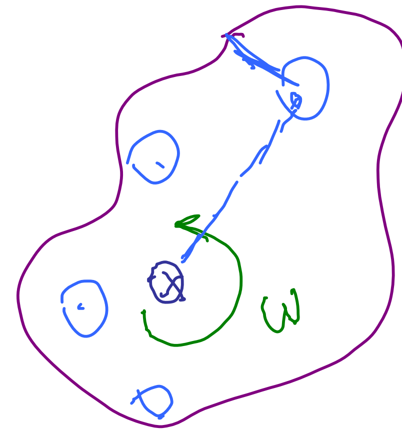
$\tau_{ext} - TR = I\alpha$



Energy

Does it have energy?

Divide up wheel into little bits of mass



I

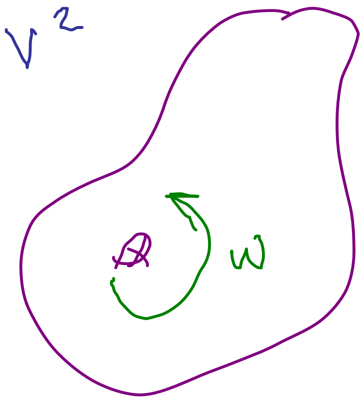
$\tau_{net} = I\alpha$

$$K = \sum_i \frac{1}{2} m_i v_i^2$$

$$= \frac{1}{2} \sum_i m_i (\omega r_i)^2 \quad = \omega^2 r_i^2$$

$$= \frac{1}{2} \omega^2 \sum_i m_i r_i^2 \rightarrow I$$

$K_{rot} = \frac{1}{2} I \omega^2$ $\quad K_{trans} = \frac{1}{2} m v^2$



10.34 A 25-cm diameter saw blade, mass 0.85 kg
uniform cyl.

a) Rot. KE at 3500 rpm

$I = \frac{1}{2} m R^2 \qquad \omega = 3500 \cdot \frac{2\pi}{60} \frac{\text{rad}}{\text{s}} = 367 \frac{\text{rad}}{\text{s}}$

$K = \frac{1}{2} I \omega^2 = 446\ \text{J}$

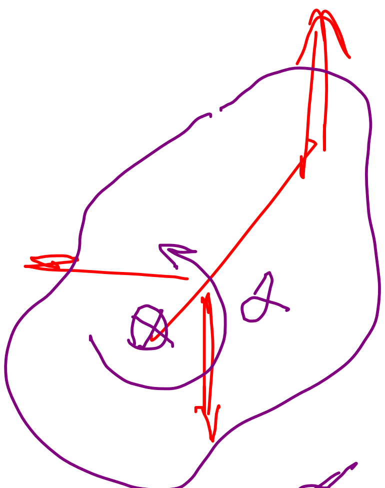
Units:

$\text{kg}\, m^2 \left(\frac{\cancel{\text{rad}}}{s} \right)^2 = \text{J}$

$= \text{kg}\, m^2/s^2$

b) Avg. power applied to bring blade from rest to 3800 rpm in 3.2 s

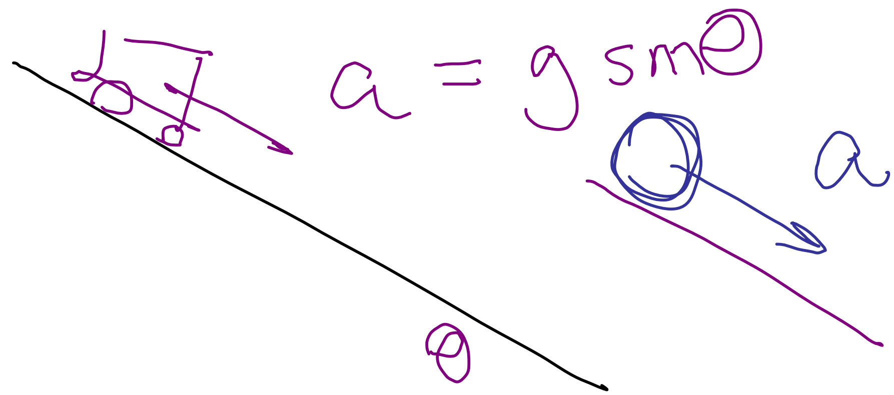
$$\bar{P} = \frac{W}{t} = \frac{\Delta K}{t} = 139\ W$$

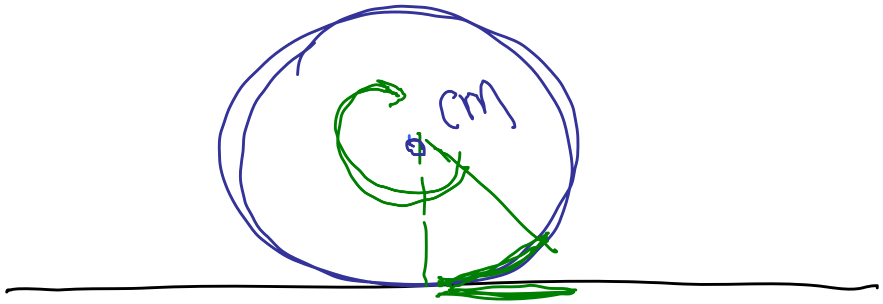


$\tau = I\alpha$

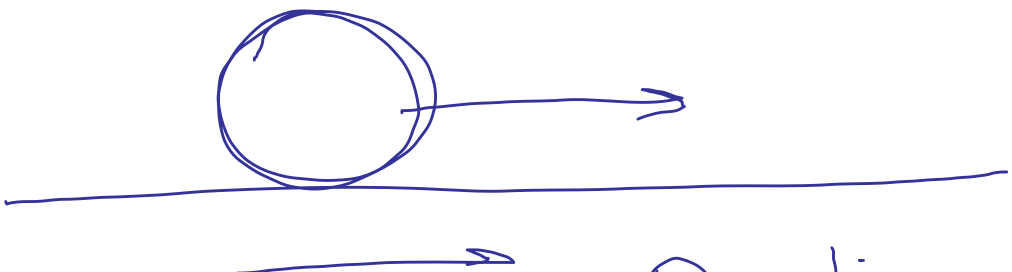
Rolling objects

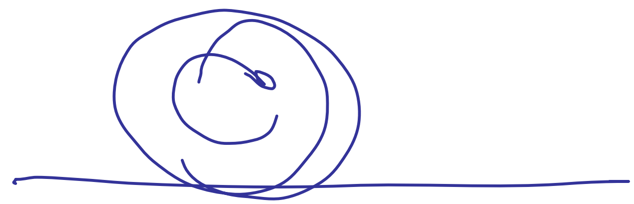
$a = g\sin\theta$





"Rolling" = rolling without slipping





Bowling ball does both.

$$v_{cm} = R\omega$$