Name ____________________

Electric Field Mapping

I. Preliminary Questions

1. Give the definition of Electric Field.

2. Give the definition of Electric Potential.

3. How are equipotential lines and electric field lines related?

NOTE: The next three questions refer to the topographic map you are asked to look up on the web.

4. What is the approximate altitude of Rockwell Cemetery?

5. If you are at West Cemetery, in what direction should you travel if you want to climb most rapidly?

6. What is the highest altitude in the Buck Mountain area? Your answer should be a range of altitudes within which lies the correct answer.

Electric Field Mapping

Grading

Preliminary Questions	9 points
Data Graphs and Tables	12 points
Analysis Questions	19 points

II. Objects of the Laboratory

To understand how to read and use a contour map.
To understand the relationship between voltage and electric field.
To determine electric fields by measurements of voltage.

III. Physical Principles and Description of Equipment

The concept of an electric field is related to the observation that electric charges exert forces on other electric charges. It is perhaps simpler to introduce the concept of electric field by first considering a different field, the gravitational field. You know that if you have an object with mass on earth, a force that we call gravity is being exerted on that object. We can define a quantity called the gravitational field by considering the force that gravity would exert on a point mass. At any point, the gravitational field is the ratio of the gravitational force exerted on the mass to the mass itself:

$$\vec{g} \equiv \frac{\vec{F}_{gravity}}{m}. \tag{1}$$

It is convenient to talk about the field even at points where there is no mass to feel the gravitational force. This is simple enough to do; we just apply equation (1) and understand it to mean the force that would be felt by a mass if it were at that point. Near the earth $\vec{g}$ has a magnitude of approximately 9.8 m/s^2 and direction downward.

An electric field $\vec{E}$ is defined in a similar way; the difference is that it involves electric force rather than gravitational force and electric charge rather than mass. Thus, for a point charge q which feels an electric force $\vec{F}_{electric}$ at a point, the electric field at that point is defined to be

$$\vec{E} \equiv \frac{\vec{F}_{electric}}{q}. \tag{2}$$

An important part of the definition is that the presence of the point charge q (often called a test charge) must not itself affect other electric charges that are the source of the electric force (and thus of the electric field). Most textbooks deal with this issue by adding to the definition of $\vec{E}$ that it is the limit of this ratio as $q \to 0$. Because force is a vector quantity, this definition means that electric field is also a vector.

While the electric field is a crucial aspect of electromagnetism, measuring it directly is a difficult task. It is not practical to place an appropriately small charge at many points and measure the forces there. It is easier to determine the electric field by utilizing measurements of the electric potential. You have already measured differences in potential numerous times in this semester's labs, but you have not made any connection between those measurements and electric fields. The connection is made via the work done by the electric force as the test charge moves. In the same way that electric field is defined as electric force per charge, the electric potential is defined in terms of the work done by that force per charge. If the electric force does work W as a test charge moves from point A to point B, then the difference in electric potential ΔV between points A and B is defined as

$$\Delta V \equiv -\frac{W}{q}. \tag{3}$$

Several aspects of the behavior of electric potential can be determined simply from this definition and from recalling the relationship between work and force:

(1) If the electric force does positive work, as happens when the motion is in the direction of the force, then the change in potential is seen to be negative when the test charge is positive. Combining this observation with the result from equation (2) that the force and the electric field have the same direction for a positive test charge, you should conclude that the electric potential decreases if you move in the direction of the electric field.

(2) In areas of stronger electric field, the electric potential will change more rapidly as one moves along the electric field (since a stronger electric field means a stronger force, which in turn means less distance traveled to do the same amount of work).

(3) For two points to have the same electric potential ($\Delta V = 0$) must mean that the work done by the electric force on a test charge moving between those two points must be zero. One can ensure that the electric force does zero work by having the test charge always move perpendicular to the electric field (recall that no work is done by a force on an object if the force is always perpendicular to the motion of the object).

One way to understand the electric field is to measure the electric potential at many points and map out its behavior. Connecting points which have the same electric potential will produce an equipotential line. Different equipotential lines are plotted for different values of the electric potential. Such a plot, often referred to as a "contour plot," serves the purpose of providing a two-dimensional representation of a three-dimensional graph – the contour lines show the values of electric potential as a function of position coordinates x and y. From the argument above, the electric field must always be perpendicular to an equipotential line, and it must be in the direction which points toward lower potentials.

If you do hiking or other outdoor activities, you may be familiar with topographic maps. "Topo maps" are used to serve a similar purpose – to indicate in two dimensions the value of altitude (the third quantity) as position changes. For an example of a topo map, go to the web site **http://www.topozone.com/map.asp?z=16&n=4003243&e=641116&s=25**. This shows a topo map of the Buck Mountain area east of Cookeville (you should see "Buck Mountain" printed on the map, running nearly vertically). The brown lines represent lines of equal altitude; you should notice that some of them are bolder and have a number somewhere on them. That number represents the height above sea level in feet; for instance in the left center of your map, you should see such a line labeled 1100 near a marker for Buck Cemetery. Lighter brown lines are also contour lines, but they will not have numbers on them, and you must figure out their altitudes by counting from a bold line. On this map, you can see that there are 5 divisions between corresponding sets of bold lines, so each light brown line must represent a 20-foot change in altitude. Since the marker for Buck Cemetery is between the 1100 foot dark brown line and the next light brown line on the downhill side, you should conclude from this map that Buck Cemetery lies between 1080 and 1100 feet in altitude.

Now locate the "o" in the "Mountain" of the Buck Mountain label. Notice that in this area, the contour lines are relatively widely spaced. However as you move away from this part of the map to either the east (right on the map), south (down), or west (left), you quickly run into a region where the contour lines are much closer together. Regions where the contour lines are closer together are steeper (since you do not have to travel as far to change altitude by 20 feet) – these areas must be the sides of the mountain. Notice that the quickest way to change altitude is to move perpendicularly to a contour line.

Now let's return to a discussion of contour maps of electric potential. In this lab, you will measure values of electric potential at specific values of (x,y). You will then use Excel to create contour plots corresponding to your data. Using the fact that electric field lines must cross these contour lines perpendicularly, you will then be able to sketch the behavior of the electric field itself. If two contour lines are a distance Δs apart (measured perpendicularly) and have a difference in potential of ΔV, the magnitude of the electric field is approximately

$$E \approx \left| \frac{\Delta V}{\Delta s} \right| . \tag{4}$$

This approximation becomes more accurate as Δs becomes smaller and becomes exact in the limit $\Delta s \to 0$. In the limit, this is of course some type of derivative, the complete relation between the (vector) electric field $\vec{E}$ and the (scalar) electric potential can be written in terms of a gradient (which should be a topic in Calculus III):

$$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} . \tag{5}$$

To measure voltages in this experiment, you will use a digital multimeter. A picture of the multimeter is shown in Figure 1.

Figure 1. A digital multimeter.

You can use this meter to measure either voltage, electric current, or resistance by changing the way you connect it and the settings you choose. The rotating switch in the center lets you choose not only what quantity you want to measure but also how large a value can be measured on a given setting. For this lab you wish to measure DC voltages up to about 25 V; the DC voltage settings are in the upper right (you will see a symbol V with two horizontal lines next to it), and you will see settings labeled 200m, 2, 20, 200, 1000 alongside the switch. Since you need to measure voltages higher than 20 V but less than 200 V, you should set this switch to the 200 V setting. Connections to this meter are made with a connector called a banana plug that fits snugly into the various connectors along the bottom of the meter. To measure voltages, you must plug one wire (often referred to as a “lead” in this context) into the port labeled V (the far right one) and the other into the port labeled COM (the port which is second from the right edge). A positive reading then means that the voltage lead is at higher potential than the common lead, while a negative reading would indicate that the voltage lead is at lower potential than the common lead (remember the voltmeter is measuring a difference in voltage between its two leads). The COM (common) port is what is often called electric ground, while the lead from the V port may be called the positive lead. Most of the time one chooses ground to be the level of zero electric potential and then just refers to the potential rather than potential difference.

In this experiment, you will use several different metal objects as “electrodes;” when a potential difference is applied to two of these electrodes, an electric field will be created. You will use two flat electrodes, a ring electrode, and a point electrode. A picture of the materials you will use as electrodes is shown in Figure 2.

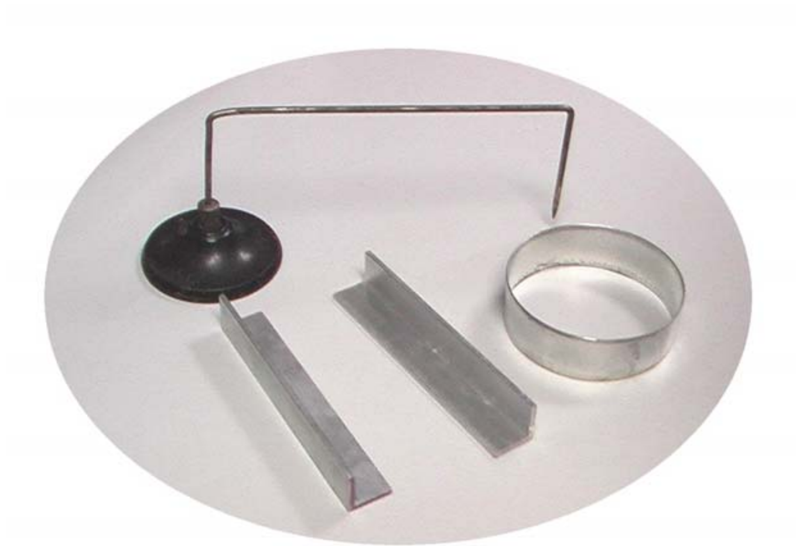

Figure 2. A photograph of the materials used as electrodes in this lab.

IV. References

Fundamentals of Physics, Halliday, Resnick, and Walker, John Wiley and Sons, Inc Sixth Edition, Chapters 23, 25.

V. Equipment

Personal computer
25 V Power supply
Digital multimeter with two leads
Plastic tray
Switch
Metal ring
Metal point electrode
2 pieces aluminum angle iron
Transparent ruler
3 wires

VI. Warnings

Anytime you are not making measurements, open the switch on your circuit to remove the voltage from your electrodes.

Have your TA or instructor check your circuit before closing the switch or turning on the power supply.

During the course of the experiment, do not touch metal objects such as pipes or plumbing fixtures.

Do not touch any bare metal parts of the circuit while the switch is closed.

When you are done collecting data, turn your power supply off.

VII. Procedure

Begin by opening the file **Electric Field Mapping.xlt**. Create a cover page and print a copy for each group member. Print a single copy of the worksheets Grid 1, Grid 2, and Grid 3 to use in your measurements. Fill your plastic tray with approximately 1/8 inch of water – just enough so that the bottom is completely covered when it's sitting on your lab table. Place the copy of Grid 1 under your tray (Grid 1 is the grid with the largest squares) so that the cross in the center of the grid is centered on your tray. Take your two pieces of angle iron and place them at the ends of the tray so that they are as far toward the ends of the tray as possible, so that they are each parallel to the ends of the tray, and so that the vertical part of the angle iron is toward the center of the tray (you can see an approximate setup in Figure 3).

Figure 3. Photograph showing the positioning of the grid and the two angle iron electrodes.

What you are going to do is use the power supply to create a potential difference between the two electrodes and then measure the potential at each grid point using the digital multimeter. Start by wiring the circuit shown in Figure 4.

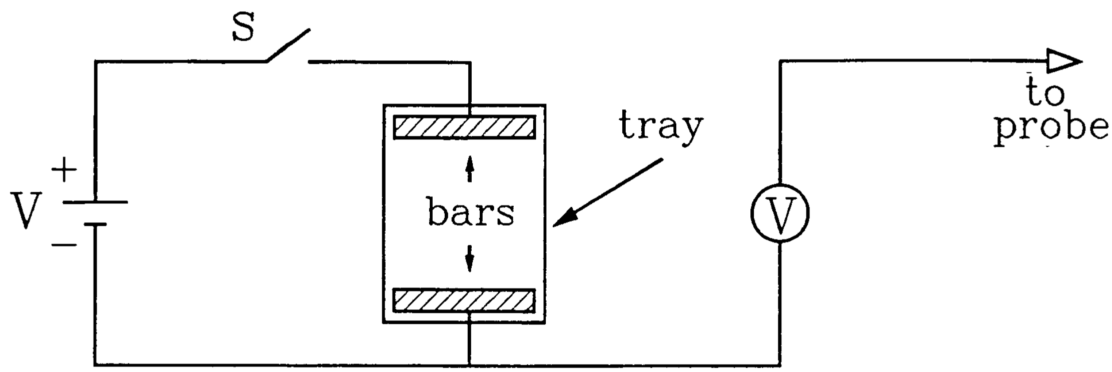

Figure 4

The power supply is represented here by a battery symbol; you should use the connections labeled “0 – 20 V DC” (toward the right of the supply). The connections to the two angle irons (labeled “bars” in Figure 4) should be made just by clipping a wire to each one. The connections for the voltmeter must be made carefully. For the positive lead you should use the wire that has a banana plug on one end and a voltage probe (a sharp metal tip) on the other. For the ground lead, you should use the wire that has a banana plug on one end and an alligator clip on the other. Set up the multimeter to measure voltages up to 200 V. DO NOT turn on either your power supply or your multimeter until you have had your circuit checked by your TA or instructor.

Once your circuit has been approved, check that your tray is still centered on the grid and that your angle iron electrodes are at the ends of the tray. Turn on the power supply and set the voltage to 25 V DC by adjusting the knob and reading the meter on the supply. Turn on your multimeter with the sliding switch on its left side. Close the switch in your circuit. Touch your voltage probe to one electrode and then the other. One should be near 25 V and the other should be near 0 V.

Analysis Question 1. There are two connections on your power supply for DC operation. One is red and one is black. Which one corresponds to the higher voltage? This is the usual convention for power supplies.

In order to have Excel make a contour map of the voltages for you, you must measure the voltage at every point on a rectangular grid. Go to worksheet Sheet1. Locate the positive electrode and find a grid point on your paper that is closest to both the electrode and the side of the tray (do not worry about any grid points that do not lie under the tray). Place your voltage probe into the water so that the point is directly above the point on the grid and the probe is oriented as vertically as possible. Read your voltmeter and record the measurement in cell A1 of your worksheet. Move parallel to the electrode to the next grid point, measure the voltage, and record that reading in cell B1. Continue until you have measured the voltage at each grid point along that line and recorded those values in row 1 of the spreadsheet. Then move to the next line in the grid parallel to the electrode, start at the same side of the tray as you did earlier, and measure at each grid point; record these measurements in row 2. Move to the next line and record those measurements in row 3. Continue in a similar fashion until your worksheet contains values measured at every grid point which is inside the edges of the tray. Open the switch to remove electric bias from the electrodes.

Now you are ready to create the contour plot. Follow these instructions:

1. Select the rectangular area containing your data. Do not include any empty cells.
2. Start the Chart Wizard by choosing the icon that looks like a bar graph in red, yellow, and blue.
3. Under “Chart Type,” choose “Surface.”
4. Under “Chart sub-type,” choose “3-D Surface” in the lower left of the four options.
5. Choose “Next” to advance to Step 2 of the Chart Wizard.
6. Choose “Next” to advance to Step 3 of the Chart Wizard.
7. Enter a title for you graph. Choose “Next” to advance to Step 4 of the Chart Wizard.
8. Choose “As new sheet” for the placement of the graph and choose “Finish.”

You should now have a contour plot of your voltage measurements on the screen. It will look somewhat different from the topo map because Excel colors in each region between contour lines with different colors; at this point you should have five different regions on your graph. The equipotential lines are the lines separating one color from an adjacent color. At the right edge of your graph is the legend which tells you the voltage range to which each color corresponds. The horizontal axis of the graph will be labeled with integers which identify the row in the spreadsheet; the vertical axis is labeled (on the right) with S1, S2, … corresponding to various columns in the spreadsheet. Thus for example, the position corresponding to a horizontal axis label of 4 and vertical axis label of S2 has the value in cell B4 (4^{th} row, 2^{nd} column) of your spreadsheet.

By default Excel creates these plots with five different regions; because your voltages in this case are in the range 0 – 25 V, each of these regions has a range of 5 V. These are shown on the graph’s legend. However, often you can see more details if you have more equipotential lines. You can change the spacing of equipotential lines by the following procedure:

1. Right-click on the legend and choose “Format Legend.”
2. Choose the “Scale” tab.
3. Change the value in “Major Unit” (it will be 5 when you get there). Start by entering 3 in this box.
4. Choose “OK.”

Analysis Question 2. How did your graph change when you changed the spacing between equipotential lines? Which of these two graphs do you prefer and why?

Try some other values for the spacing between equipotential lines. Try values larger than 5 and smaller than 1 to be sure you have a good feel for how this change affects your graph. While there is no right or wrong choice here, one issue that we have to contend with is that Excel depends on color to help distinguish regions. However, your printer is not a color printer, and if there are too many colors it can be difficult to distinguish different regions on a printed copy. We recommend that for this graph, you choose an interval between contour lines that is somewhere around 2 V. Once you settle on a final graph, print a copy for each group member.

Analysis Question 3. Describe the behavior of the equipotential lines that you observe. What does this tell you about the direction of the electric field at different points between the electrodes?

Analysis Question 4. Remove the grid from beneath your tray and use your ruler to determine the size of the grids. Combine this with information from your graph to estimate the magnitude of the electric field near the center of the tray.

Analysis Question 5. Using your graph of equipotential lines and the relationship between voltage and electric field, state how the electric field varies as you move among different points between the electrodes.

Next, you will measure voltage in a different situation. Take your copy of Grid 2 (the middle-sized grid) and center it under your tray. Check to make sure that the electrodes are all the way against the ends. Take the metal ring and place it in the tray so that it is centered on the cross on your grid. Once everything is in place, close the switch. Once again make measurements at each grid point inside the edges of the tray. If a grid point lies underneath the ring, get your probe as close to it as you can WITHOUT MOVING THE RING. Record your values on worksheet Sheet2. Once you have all the points measured, open the switch and turn off the power supply. Generate a surface plot for these voltage measurements. Adjust the interval between equipotential lines to 1.5 V and print a copy of the graph for each group member. Sketch the approximate position of the ring on each graph.

Analysis Question 6. Starting at the positive electrode and ending at the negative (ground) electrode, sketch at least three electric field lines. Remember that electric field lines must always be perpendicular to equipotential lines (including equipotential lines that may not be shown on your plot).

Analysis Question 7. Describe how the presence of the ring changes the electric field (not the potential).

Analysis Question 8. How does the magnitude of the electric field inside the ring compare to the magnitude outside the ring? Include an explanation of how you reached this conclusion.

Analysis Question 9. What do your data say about the direction of the electric field just outside the ring?

Next take Grid 3 and center it under your tray; this grid will not extend under the entire tray. Remove the electrodes from the tray, place them on the paper towels provided, and center your ring on the grid. Wire the circuit shown in Figure 5.

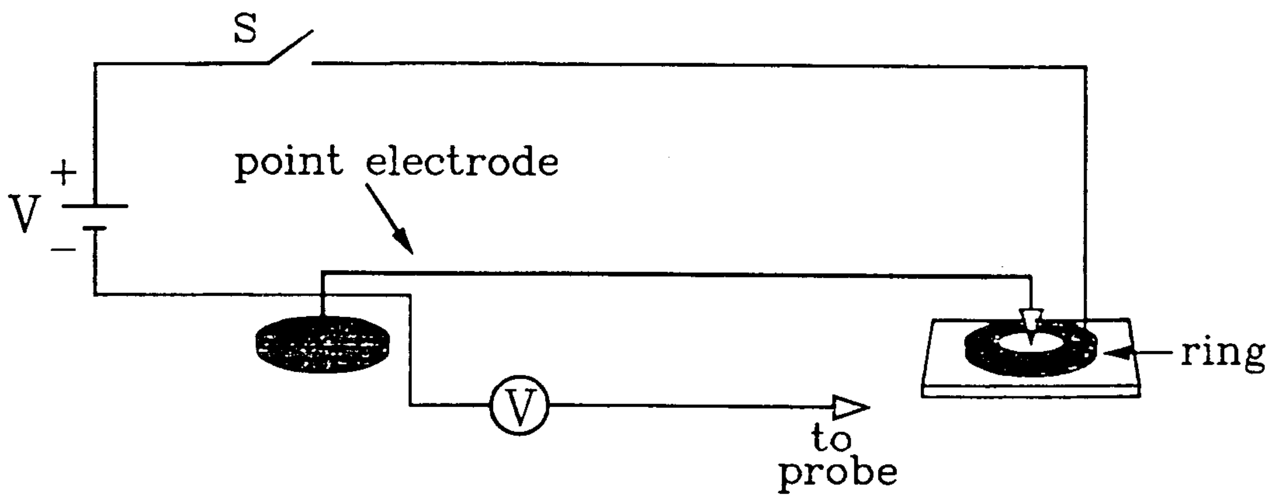

Figure 5. Both the ground side of the power supply and the ground side of the voltmeter should be connected to the point electrode.

Adjust your point electrode so that it is in contact with the water directly above the cross marking the center of the grid. You will probably find it helpful to have the support base of this electrode positioned so that you will not easily bump and move it. Once your circuit is ready, have your TA or instructor check it before proceeding.

Once your circuit has been approved, turn on the power supply and raise its output to approximately 25 V. Once again be careful not to move either the ring or the point electrode while you are making measurements. Measure the voltage at each grid point and record the data on Sheet3. Create a contour plot and determine an appropriate spacing for equipotential lines. Print a copy of the plot for each group member. Sketch the location of the ring on your plot.

Analysis Question 10. What are the shapes of the equipotential lines in this case?

Analysis Question 11. What is the direction of the electric field inside the ring?

Analysis Question 12. What does your data tell you about the electric field outside the ring?

Analysis Question 13. Inside the ring, is the magnitude of the electric field greater near the center or near the edge?

Analysis Question 14. Estimate the magnitude of the electric field at the point electrode and the magnitude at the inner edge of the ring. Be sure to explain clearly how you obtain your answer.