

**Phys 2920, Spring 2012**  
**Problem Set #9**

1. Use a computer program which does “3D graphics”, make a plot of the function

$$f(x, y) = 5 \sin(2x) \sin(5y)$$

for the range  $0 \leq x \leq 2\pi$ ,  $0 \leq y \leq 2\pi$ .

*Hint: In Maple, you can learn how the procedure `plot3d` is used. You may also want to change the option `numpoints` or `grid` to get a better picture.*

2. Using a computer program produce (any way you can) a plot of the function

$$f(\rho, \phi) = 5 \sin(\rho^2/3) \cos(\phi) e^{-\rho^2/10}$$

for the range  $-5 \leq x \leq 5$ ,  $-5 \leq y \leq 5$

3. Find  $\cos(-2 + 5i)$ , using both your calculator and demonstrating the result using only a real-variable (\$10) calculator.

4. Evaluate  $\cos^{-1}(10)$ ; get the decimal value from your calculator and show how this value comes about using the definition of  $\cos^{-1}(z)$  and real-variable math.

5. (CV 3.44) Prove that  $\frac{d}{dz}(z^2 z^*)$  does not exist anywhere.  
(If it were analytic, the C-R equations would have to be satisfied...)

6. (CV 3.47) Verify that the real and imaginary parts of the following functions satisfy the Cauchy-Riemann equations and thus deduce the analyticity of each function:

$$(a) \quad f(z) = z^2 + 5iz + 3 - i, \quad (b) \quad f(z) = ze^{-z}, \quad (c) \quad f(z) = \sin 2z$$

*In other words, use  $z = x + iy$ , find  $u(x, y)$  and  $v(x, y)$  and test the C-R equations.*

7. (CV 3.78) Evaluate

$$(a) \quad \lim_{z \rightarrow 2i} \frac{z^2 + 4}{2z^2 + (3 - 4i)z - 6i} \quad (b) \quad \lim_{z \rightarrow e^{\pi i/3}} (z - e^{\pi i/3}) \left( \frac{z}{z^3 + 1} \right) \quad (c) \quad \lim_{z \rightarrow i} \frac{z^2 - 2iz - 1}{z^4 + 2z^2 + 1}$$

*(A little practice with l'Hopital's rule and derivatives.)*