Phys 3610, Fall 2008
Problem Set #5, Hint-o-licious Hints

1. *Taylor*, **7.16** You should have (show!)

$$\mathcal{L} = \tfrac{1}{2}(m + I/R^2)\dot{x}^2 + mgx\sin\alpha$$

2. *Taylor*, **7.17** You should have

$$\mathcal{L} = \tfrac{1}{2}\left(m_1 + m_2 + \frac{I}{2R^2}\right)\dot{x}^2 + (m_1 - m_2)gx$$

Get $\ddot{x}$ from the Lagrange equation.

3. *Taylor*, **7.31** For $\mathcal{L}$ I got

$$\mathcal{L} = \tfrac{1}{2}(m + M)\dot{x}^2 + \tfrac{1}{2}ML^2\dot{\phi}^2 + ML\dot{x}\dot{\phi}\cos\phi - \tfrac{1}{2}kx^2 + mgL\cos\phi$$

Getting the Lagrange equations for x and ϕ should be fairly easy. To make the approximation of small oscillations in x and ϕ, you can replace $\sin\phi$ by ϕ, $\cos\phi$ by 1 and eliminate any term in the two equations which contains anything other than a *single* power of x or ϕ or their time derivatives. (For example, if a term contains $\dot{\phi}^2\phi$ or $\dot{x}\dot{\phi}\phi$, kill it!)

What left is something much simpler, a SHO-type pair of equations which unfortunately are still coupled. Solving these kinds of systems is, alas, the subject of the coupled oscillators chapter, which I don't think we'll get to.

4. *Taylor*, **7.34** Showing that the kinetic energy of the spring is

$$T_{\rm spr} = \tfrac{1}{6}M\dot{x}^2$$

was done in class; fill in the argument or come up with a clearer one! You should get

$$\mathcal{L} = \frac{1}{2}\left(m + \frac{M}{3}\right)\dot{x}^2 - \tfrac{1}{2}kx^2$$

Find the angular frequency of oscillations.

5. *Taylor*, **7.36** You should get

$$\mathcal{L} = \frac{m}{2}(\dot{r}^2 + r^2\dot{\phi}^2) + mgr\cos\phi - \tfrac{1}{2}(r - \ell_0)^2$$

Get the (messy) r and ϕ equations of motion.

The equilibrium length under gravity is

$$\ell = \ell_0 + \frac{mg}{k}$$

and you will substitute $r = \ell + epsilon$ where ϵ is the new time-dependent variable. Neglecting all the small terms in ϵ and ϕ decouples the equations and makes them very simple.

6. *Taylor*, **7.37** Show that the Lagrangian is

$$\mathcal{L} = m\dot{r}^2 + \tfrac{1}{2}mr^2\dot{\phi}^2 + mg(L - r)$$

which gives equations

$$2\ddot{r} = r\dot{\phi}^2 - g \qquad mr^2\dot{\phi} = \text{constant} \equiv \ell$$

7.