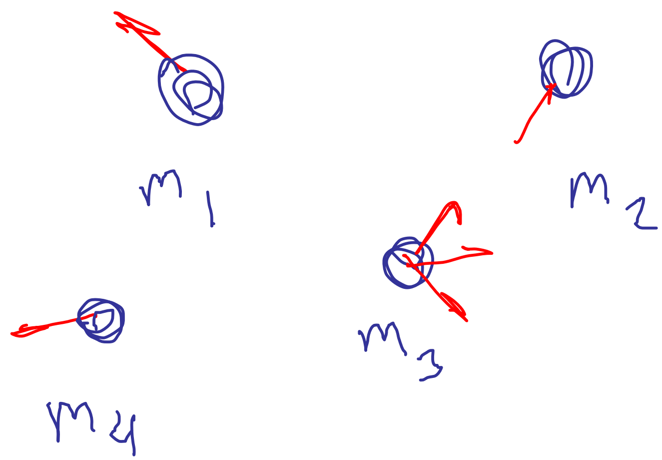
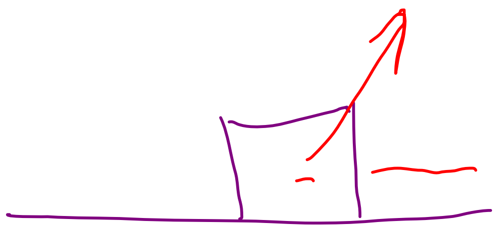
Phys 2110-5 10/17/12

Note Title 10/17/2012

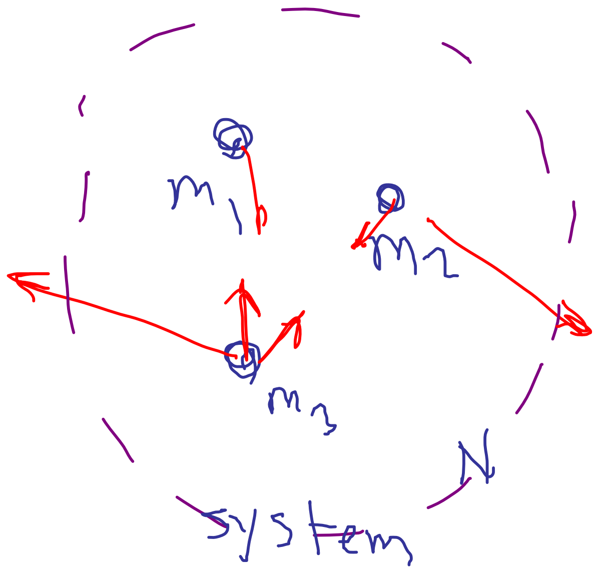
Systems of Particles (Chap 9)

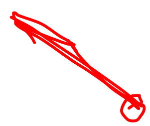




one particle

Momentum $m\vec{v}$





$$\vec{F}_{net,i} = m_i \vec{a}_i = m_i \frac{d^2 \vec{r}_i}{dt}$$

$$= m_i \frac{d\vec{v}_i}{dt} = \frac{d}{dt}(m_i \vec{v}_i)$$

$$= \frac{d}{dt} \vec{p}_i$$

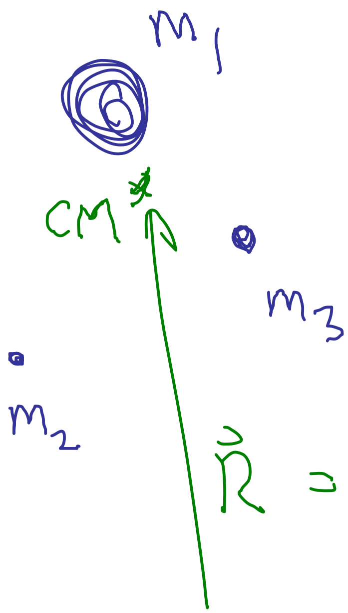
Add up all forces acting on all particles

$$\vec{F}_{Total} = \sum_{i=1}^{N} F_{i,net} = \sum_i \frac{d^2 (m_i \vec{r}_i)}{dt^2} = M \left[\frac{d^2}{dt^2} \sum \frac{(m_i \vec{r}_i)}{M} \right]$$

Total mass, $M = m_1 + m_2 + \cdots m_N$

$$\vec{F}_{Tota} = M\frac{d^2}{dt^2}\left[\sum_{i=1}^{N}\frac{(m_i \vec{r}_i)}{M}\right]$$

Weighted avg of locations of all masses



$$\vec{R} = \sum \frac{(m_i \vec{r}_i)}{M}$$

Define a point in space Center of mass, $\vec{R}$

CM moves a particles do

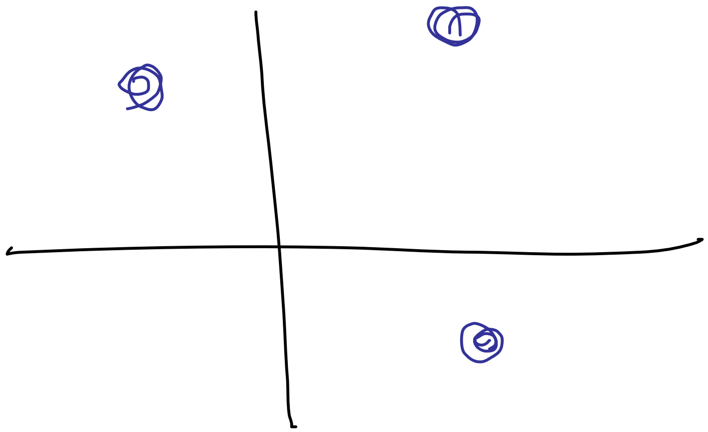
$$\vec{V}_{cm} = \frac{d}{dt}\vec{R} = \sum_{i=1}^{N}\frac{m_i \vec{V}_i}{M}$$

$$\vec{a}_{cm} = \frac{d^2}{dt^2}\vec{R} = \sum_{i=1}^{N}\frac{m_i \vec{a}_i}{M}$$

$$\vec{F}_{Total} = M \vec{a}_{cm}$$

Looks like $\vec{F} = m\vec{a}$, ... M is total mass

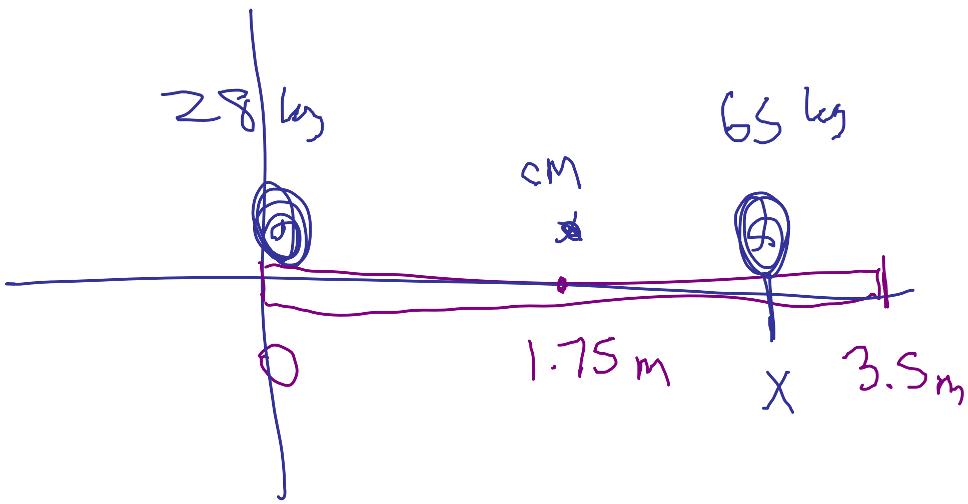
$\vec{a}_{cm}$



$$x_{cm} = \frac{1}{M} \sum m_i x_i$$

$$y_{cm} = \frac{1}{M} \sum m_i y_i$$

9.12 28 kg child sits at one end of 3.5-m long seesaw. Where should 65-kg father sit so that center of mass is at center of seesaw.



want $x_{cm} = 1.75\,m$

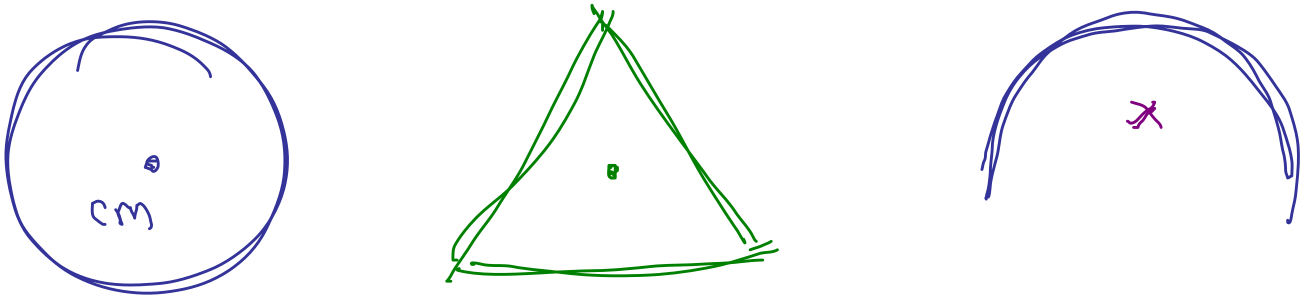
$$\boxed{1.75\,m = \sum \frac{m_i x_i}{M}}$$

$$= \frac{(28\,kg)(0) + (65\,kg)X}{93\,kg}$$

Solve for X

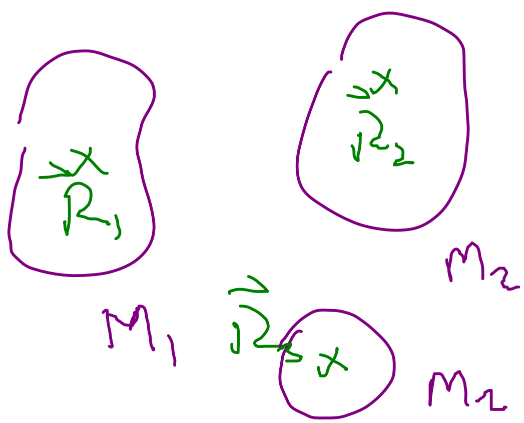
$\longrightarrow$ should sit 0.754 m to right of center of seesaw

In general, can calculate cM of a continuous object



$$x_{cm} = \frac{\int x \, \rho(\vec{r}) \, dV}{M}$$

See Calc III



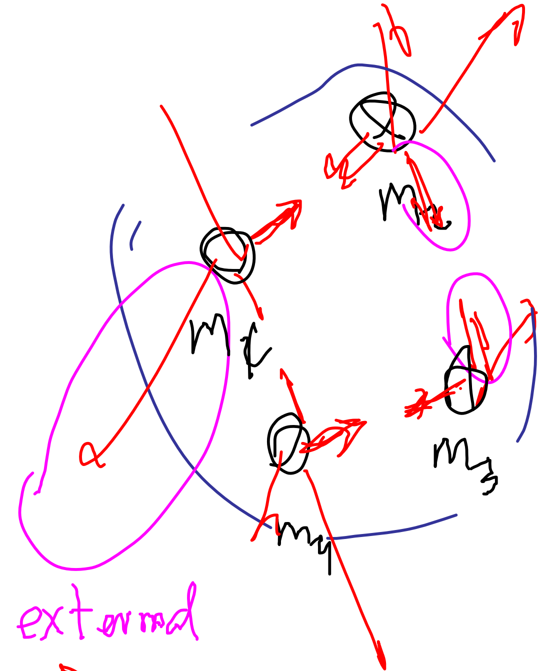
$$R_{syst} = \frac{m_1 \vec{R}_1 + m_2 \vec{R}_2 + m_3 \vec{R}_3}{M}$$

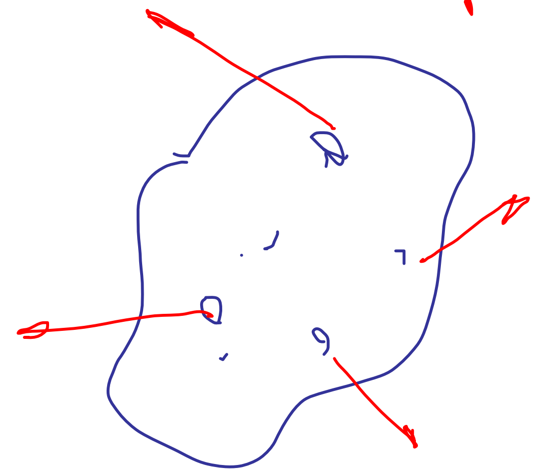
$$\boxed{\vec{F}_{Tot} = M\vec{a}_{cm}}$$

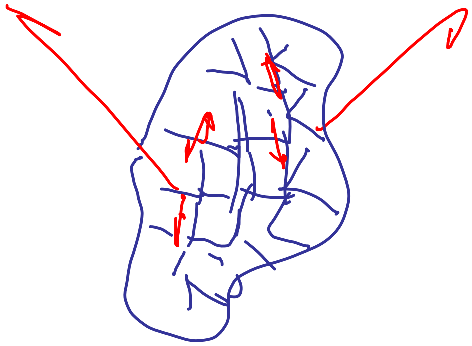
Internal forces cancel, N's 3rd Law.

Only external forces are left

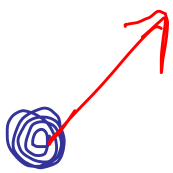
$$\boxed{\vec{F}_{\text{total ext}} = M\vec{a}_{cm}}$$



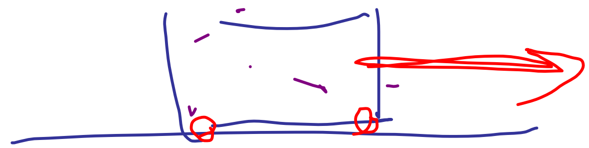




$$F_{\substack{Tot \\ ext}} = M\vec{a}_{cm}$$

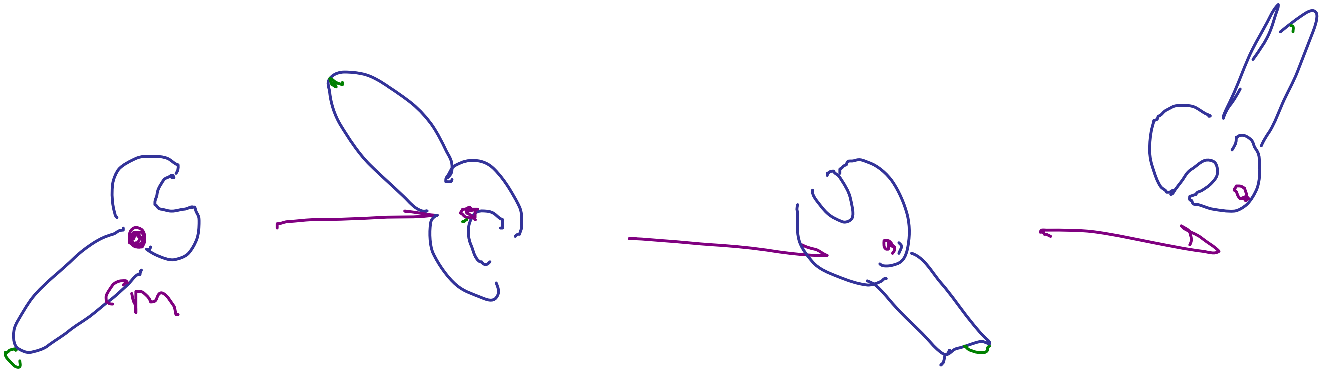


$\vec{F} = m\vec{a}$



We've been doing right thing all along.

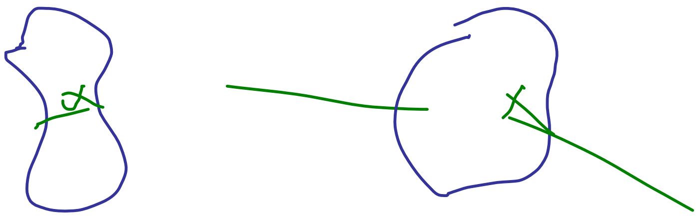
$$\vec{a} = \vec{a}_{cm} \qquad \vec{F} = m\vec{a}$$



$$\vec{F}_{ext} = \vec{0}$$

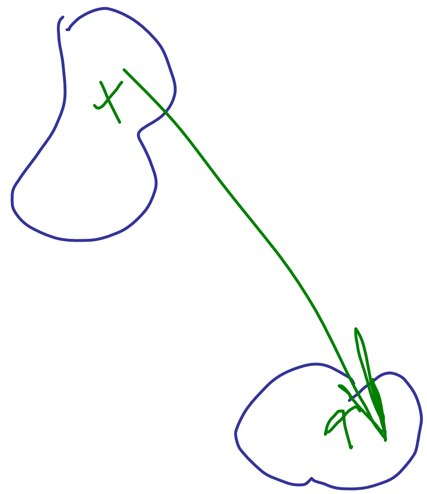
$$\vec{a}_{cm} = 0$$

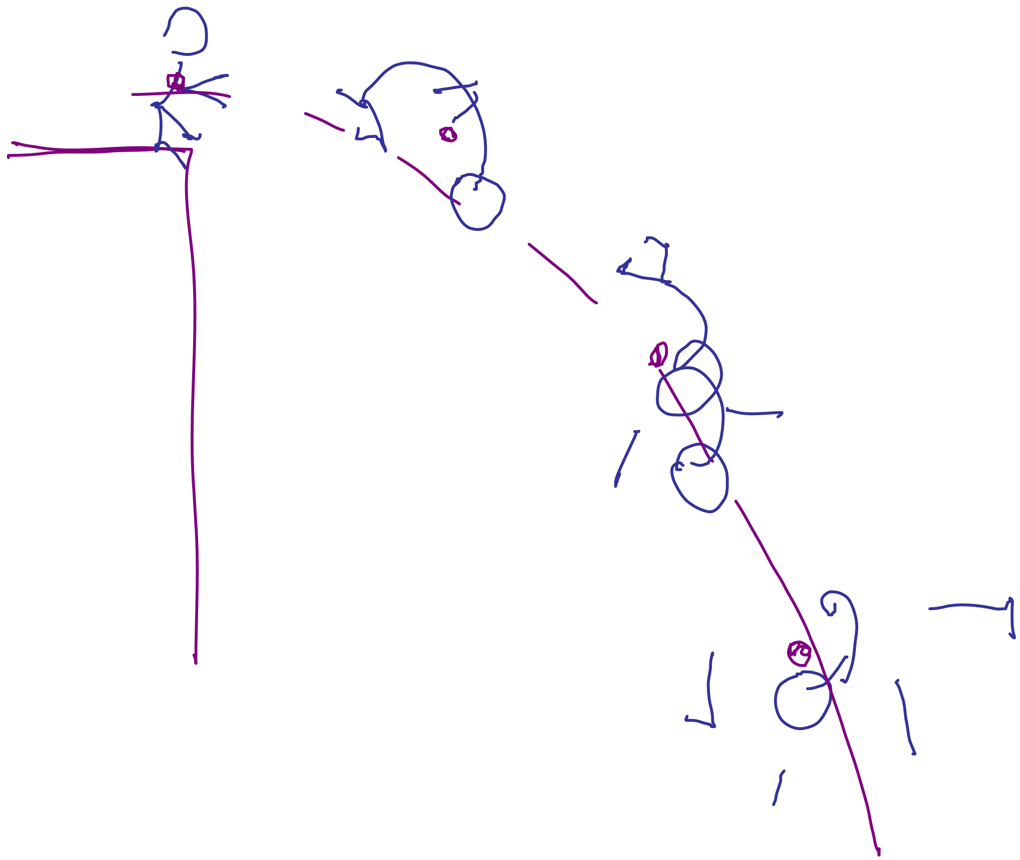
$$\vec{v}_{cm} = \text{constant}$$



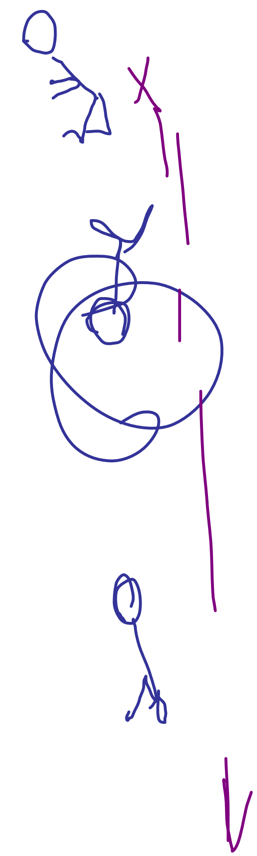
$$F_{ext,\,net} = -Mg\,\hat{j}$$

$$\vec{a}_{cm} = -g\,\hat{j}$$





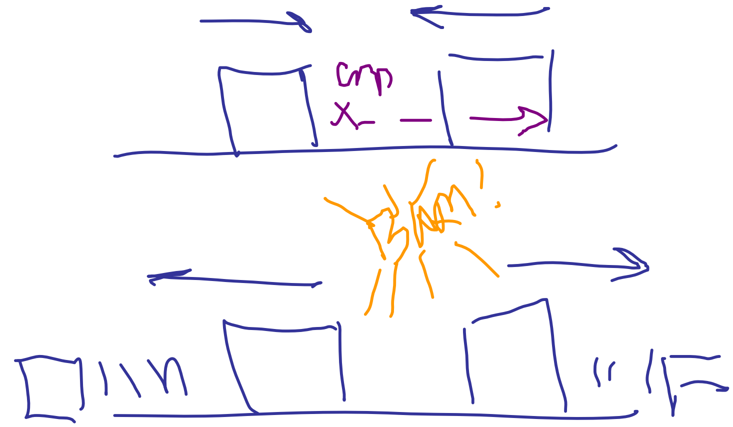
$$F_{ext} = -Mg\,\hat{j}$$



$$\vec{F}_{ext} = M \frac{d^2}{dt^2}\vec{R}$$

$$= M \frac{d}{dt}\vec{V}_{cm}$$

$$\vec{V}_{cm} = \frac{1}{M}\sum_i m_i \vec{V}_i$$



Vector

$P_x = mV_x$

$P_y = mV_y$

Momentum: $\vec{p} = m\vec{v}$

Units? $[m][\vec{v}]$

Units? $kg\,\frac{m}{s} = \frac{kg\,m}{s}$

$$\vec{V}_{cm} = \frac{1}{M}\sum_i \vec{P}_i$$

Total momentum: $\sum_i \vec{P}_i = \vec{P}$

$$\vec{v}_{cm} = \frac{1}{M}\vec{P}$$

$$\vec{P} = M\vec{v}_{cm}$$

$$\vec{p} = m\vec{v}$$

$$\vec{F}_{\substack{ext\\total}} = M\frac{d}{dt}\vec{v}_{cm}$$

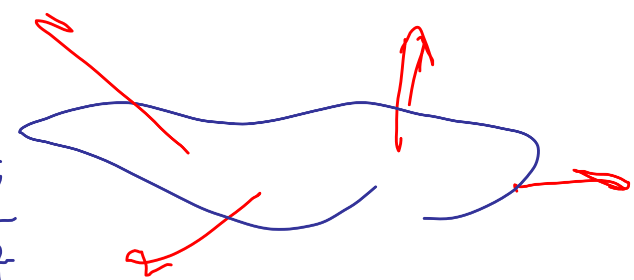
$$= \cancel{M}\frac{d}{dt}\left[\frac{1}{\cancel{M}}\vec{P}\right] = \frac{d}{dt}\vec{P}$$

$$= M\vec{a}_{cm}$$

$$\boxed{F_{\substack{ext\\total}} = \frac{d}{dt}\vec{P}}$$

Eq 9.6 p.139

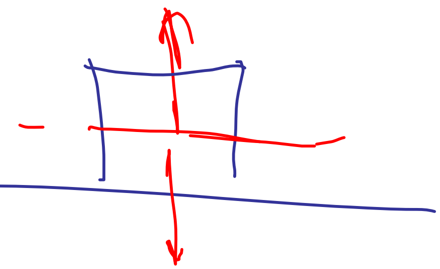
$$\vec{F} = \frac{d\vec{p}}{dt}$$



$$\vec{F}_{net,ext} = \frac{d}{dt}\vec{P}$$

Special case no net ext force

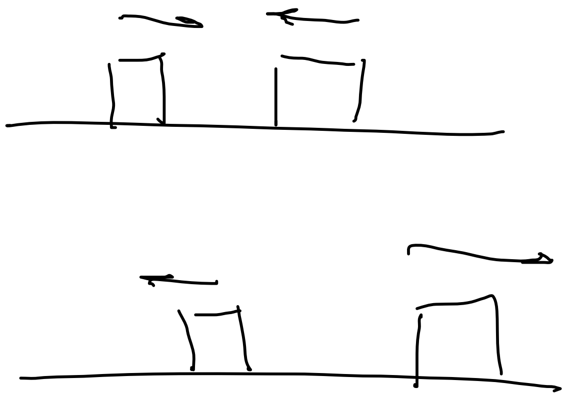
$$\vec{F}_{net,ext} = \vec{0}$$



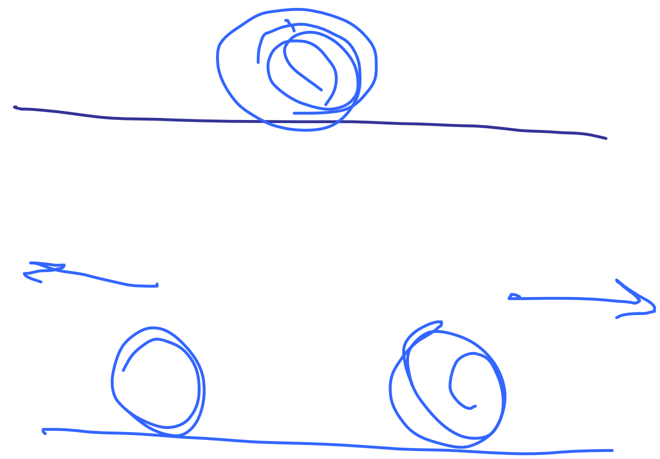
$$\frac{d}{dt}\vec{P} = 0$$

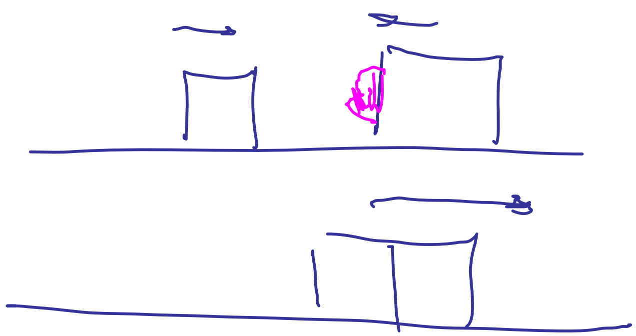
$$\boxed{\begin{aligned} \vec{P} &= \text{const} \\ \vec{P}_1 &= \vec{P}_2 \end{aligned}}$$

Apply this to collisions



$\vec{p}$ is same





$\vec{p}$ is same

p stays same