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Mar. 1, 2011

Phys 2112, Spring 2011
Quiz #1

1. A particle moves counter-clockwise in a circle of radius 2.40 m centered at the origin, with constant speed 1.50 $\frac{\mathrm{m}}{\mathrm{s}}$.

Write down some suitable equations of motion. (That is, $x(t)$ and $y(t)$.)

Here,

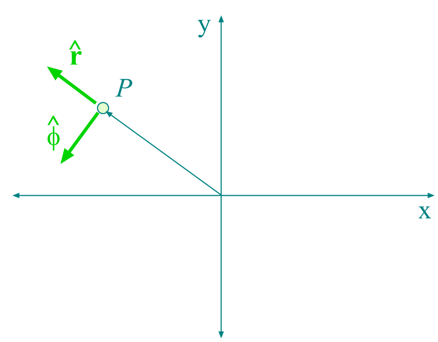
$$R = 2.40\ \mathrm{m} \qquad T = \frac{2\pi R}{v} = \frac{2\pi(2.4\ \mathrm{m})}{(1.50\,\frac{\mathrm{m}}{\mathrm{s}})} = 10.1\ \mathrm{s} \qquad \omega = \frac{2\pi}{T} = 0.625\ \mathrm{s}^{-1}$$

Then

$$x(t) = R\cos(\omega t) = (2.4\ \mathrm{m})\cos[(0.625\ \mathrm{s}^{-1})t] \qquad y(t) = R\sin(\omega t) = (2.4\ \mathrm{m})\sin[(0.625\ \mathrm{s}^{-1})t]$$

2. For the point P shown here, show the directions of the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\phi}}$.

Unit vectors drawn on the figure.



3. Express the kinetic energy $K = \frac{1}{2}mv^2$ of a particle in terms of polar coordinates.

From the components of the $\mathbf{v}$ vector when expressed in polar coordinates, we have

$$v^2 = \dot{r}^2 + (r\dot{\phi})^2 = \dot{r}^2 + r^2\dot{\phi}^2$$

Then

$$K = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2)$$

4. A particle moves in one dimension with a velocity given by

$$v(t) = -3.0\,\tfrac{\mathrm{m}}{\mathrm{s}} - (50\,\tfrac{\mathrm{m}}{\mathrm{s}})e^{-t/(4.0\,\mathrm{s})}$$

where $x(0) = 0$

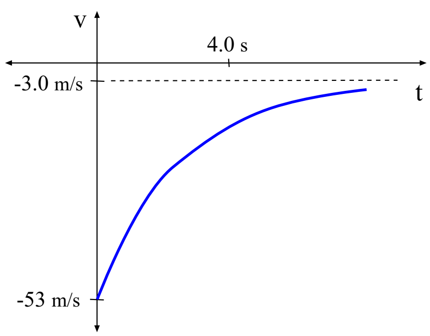
a) Find $x(t)$ and $a(t)$

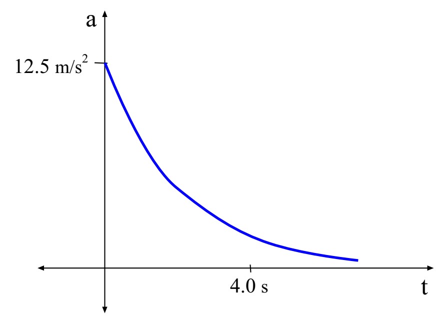
$$a(t) = v'(t) = -(50\,\tfrac{\mathrm{m}}{\mathrm{s}})\left(\frac{-1}{4.0\ \mathrm{s}}\right)e^{-t/(4.0\,\mathrm{s})} = 12.5\,\tfrac{\mathrm{m}}{\mathrm{s}^2}e^{-t/(4.0\,\mathrm{s})}$$

$$\begin{aligned} x(t) &= x(0) + \int_0^t v(t')\,dt' = 0 + \int_0^t \left[-3.0\,\tfrac{\mathrm{m}}{\mathrm{s}} - (50\,\tfrac{\mathrm{m}}{\mathrm{s}})e^{-t'/(4.0\,\mathrm{s})}\right]\,dt \\ &= (-3\,\tfrac{\mathrm{m}}{\mathrm{s}})t'\Big|_0^t - (50\,\tfrac{\mathrm{m}}{\mathrm{s}})(-4.0\ \mathrm{s})e^{-t'/(4.0\,\mathrm{s})}\Big|_0^t \\ &= (-3\,\tfrac{\mathrm{m}}{\mathrm{s}})t + (200\,\tfrac{\mathrm{m}}{\mathrm{s}})\left(e^{-t/(4.0\,\mathrm{s})} - 1\right) \end{aligned}$$

b) Sketch crude graphs of $v(t)$ and $a(t)$ for $t \geq 0$.

They look something like this, with both v and a having a fall-off time of 4.0 s:





Show work for all problems and include the right units!

$$v = \dot{x} \qquad a = \dot{v} \qquad x = x_0 + \int_0^t v(t')\,dt' \qquad v = v_0 + \int_0^t a(t')\,dt'$$

$$x = r\cos\phi \qquad y = r\sin\phi \qquad r = \sqrt{x^2+y^2} \qquad \tan\phi = \frac{y}{x} \qquad v = \frac{2\pi R}{T} \qquad \omega = \frac{2\pi}{T} = 2\pi f$$

$$\mathbf{v} = \dot{r}\,\hat{\mathbf{r}} + r\dot{\phi}\,\hat{\boldsymbol{\phi}} \qquad \mathbf{a} = (\ddot{r} - r\dot{\phi}^2)\,\hat{\mathbf{r}} + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\,\hat{\boldsymbol{\phi}}$$