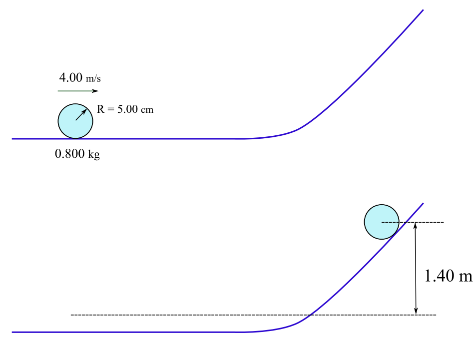
Name____________________________

Apr. 19, 2013

Quiz #4 — Spring 2013

Phys 2110 – Sec 4

1. A round object of mass 0.800 kg and radius 5.00 cm rolls without slipping on a flat surface so that the speed of its center is $4.00\frac{\mathrm{m}}{\mathrm{s}}$. It continues to roll (smoothly) up a ramp so that it reaches a maximum height of 1.40 m.



a) What is the *translational* kinetic energy of the object as it is rolling on the flat part?

$$K_{\mathrm{trans}} = \tfrac{1}{2}mv_{\mathrm{c}}^2 = \tfrac{1}{2}(0.800\ \mathrm{kg})(4.00\tfrac{\mathrm{m}}{\mathrm{s}})^2 = \boxed{6.40\ \mathrm{J}}$$

b) What is the angular velocity (speed) of the object is it rolls on the flat part?

$$\omega = \frac{v_{\mathrm{c}}}{R} = \frac{(4.00\frac{\mathrm{m}}{\mathrm{s}})}{(0.0500\ \mathrm{m})} = \boxed{80.0\tfrac{\mathrm{rad}}{\mathrm{s}}}$$

c) What is the total energy of the object as it rolls on the flat part?

By conservation of energy (no friction forces doing any work) it must be equal to the potential energy of the object in the final position, thus

$$K_{\mathrm{roll}} = mgh = (0.800\ \mathrm{kg})(9.80\tfrac{\mathrm{m}}{\mathrm{s}^2})(1.40\ \mathrm{m}) = \boxed{11.0\ \mathrm{J}}$$

d) What is the *rotational* kinetic energy of the object as it rolls on the flat part?

The sum of translational and rotational KE's gives the total KE, so

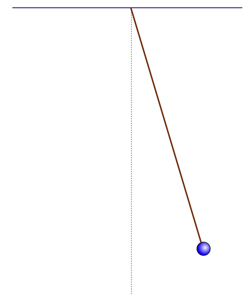
$$K_{\mathrm{rot}} = K_{\mathrm{roll}} - K_{\mathrm{trans}} = \boxed{4.60\ \mathrm{J}}$$

e) What is the moment of inertia of the object (for an axis thru its center)?

Since $K_{\mathrm{rot}} = \frac{1}{2}I\omega^2$ we get

$$I = \frac{2K_{\mathrm{rot}}}{\omega^2} = \frac{2(4.60\ \mathrm{J})}{(80.0\frac{\mathrm{rad}}{\mathrm{s}})^2} = \boxed{1.44\times10^{-3}\,\mathrm{kg}\cdot\mathrm{m}^2}$$

2. Find the length of a simple pendulum which has a period of 5.00 s.



Use

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \Longrightarrow \qquad T^2 = 4\pi^2\frac{L}{g} \qquad \Longrightarrow \qquad L = \frac{gT^2}{4\pi^2}$$

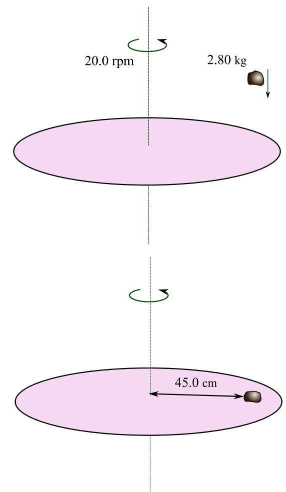
Plug in numbers

$$L = \frac{(9.80\frac{\mathrm{m}}{\mathrm{s}^2})(5.00\ \mathrm{s})^2}{4\pi^2} = \boxed{6.20\ \mathrm{m}}$$

3. A potter's wheel with rotational inertia $6.20\,\mathrm{kg}\cdot\mathrm{m}^2$ is spinning freely at 20.0 rpm. The potter drops a 2.80–kg lump of clay onto the wheel, where it sticks 45.0 cm from the rotation axis.

What's the wheel's subsequent angular speed?

[Hint: Before the collision, the lump has no angular momentum about the rotation axis.]



The initial angular velocity of the wheel was

$$\omega_i = (20.0\ \mathrm{rpm}) = 2.09\frac{\mathrm{rad}}{\mathrm{s}}$$

With the moment of inertia of the wheel being I_{wh}, the initial angular momentum of the *system* was

$$L_i = I_{\mathrm{wh}}\omega_i$$

After the collision the system rotates as one unit with moment of inertia

$$I = I_{\mathrm{wh}} + mr^2 = 6.20\,\mathrm{kg}\cdot\mathrm{m}^2 + (2.80\ \mathrm{kg})(0.450\ \mathrm{m})^2 = 6.77\,\mathrm{kg}\cdot\mathrm{m}^2$$

No net external torque; L is conserved; this gives

$$L_f = I\omega_f = L_i = I_{\mathrm{wh}}\,\omega_i \qquad \Longrightarrow \qquad \omega_f = \frac{I_{\mathrm{wh}}}{I}\,\omega_i = \boxed{1.91\frac{\mathrm{rad}}{\mathrm{s}} = 18.3\ \mathrm{rpm}}$$

You must show all your work and include the right units with your answers!

$$\tau = rF\sin\theta \qquad \tau_{\mathrm{net}} = I\alpha \qquad I = \sum m_i r_i^2 \qquad I_{\mathrm{cyl}} = \tfrac{1}{2}MR^2 \qquad I_{\mathrm{sph,\,sol}} = \tfrac{2}{5}MR^2 \qquad I_{\mathrm{sph,\,hol}} = \tfrac{2}{3}MR^2$$

$$K_{\mathrm{rot}} = \tfrac{1}{2}I\omega^2 \qquad W_{\mathrm{rot}} = \int \tau\, d\theta \qquad v_c = \omega r \qquad a_c = \alpha r \qquad K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = K_{\mathrm{trans}} + K_{\mathrm{rot}}$$

$$I_{\mathrm{rod,\,end}} = \tfrac{1}{3}ML^2 \qquad I_{\mathrm{rod,\,ctr}} = \tfrac{1}{12}ML^2 \qquad L = I\omega \qquad L_{\mathrm{str\ line}} = mvb \qquad \text{No net ext torque} \Rightarrow L\ \text{ cons'd}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x \qquad \omega = \sqrt{\frac{k}{m}} \qquad T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T} \qquad v_{\max} = A\omega \qquad a_{\max} = A\omega^2$$

$$\omega = \sqrt{\frac{g}{L}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \qquad \omega = \sqrt{\frac{mgL}{I}} \qquad T = 2\pi\sqrt{\frac{I}{mgL}} \qquad \omega = \sqrt{\frac{\kappa}{I}}$$