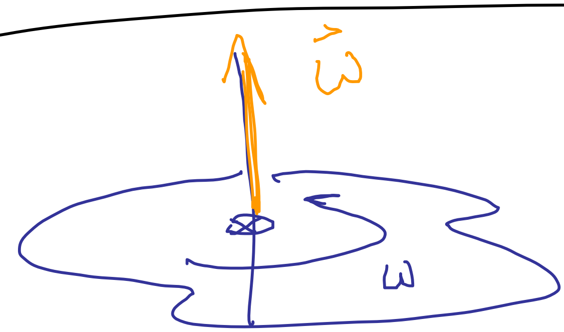
Phys 2110-4 11/9/11

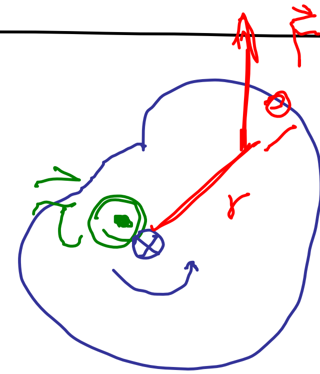
Note Title 11/9/2011

Chap 11 Rotations α, ω, θ

$$\tau = I\alpha \quad K = \frac{1}{2} I \omega^2$$

Rolling $v_{cm} = R\omega$ etc. $K = K_{tran} + K_{rolling}$



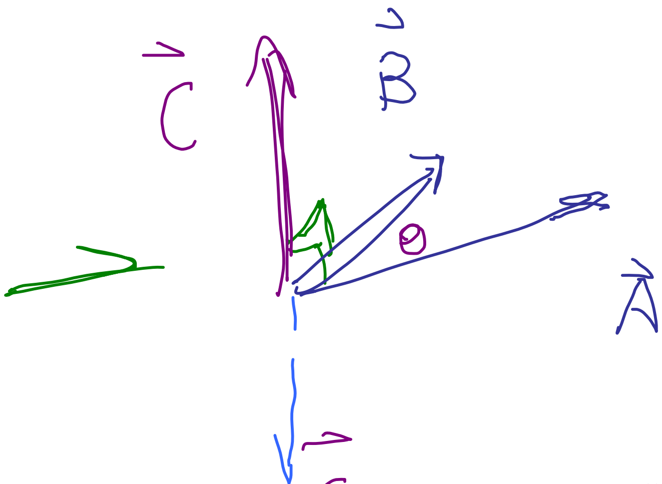


$\tau = rF \sin\theta$

Really

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Cross product $\qquad \vec{C} = \vec{A} \times \vec{B}$

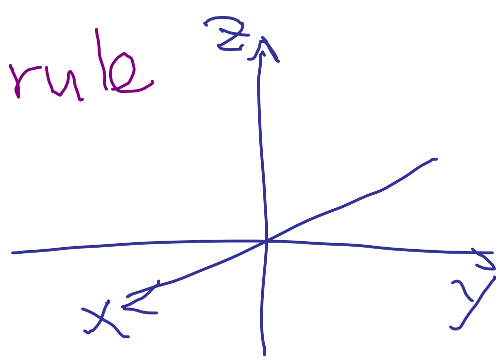


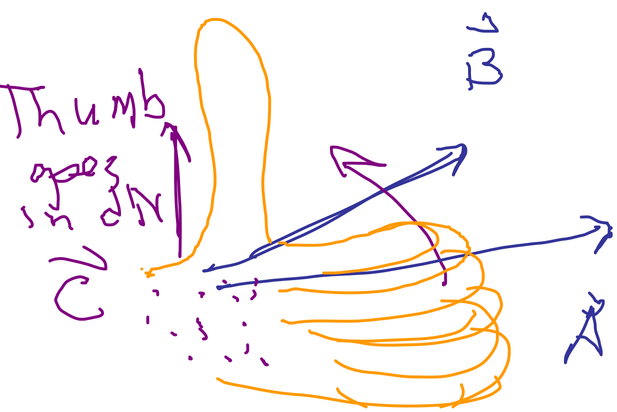
$$|\vec{C}| = AB|\sin\theta|$$

If $\vec{A} \parallel \vec{B}$ then $|\vec{C}| = 0$

$\vec{C}$ is perp to both $\vec{A}$ & $\vec{B}$.

Right hand rule





$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

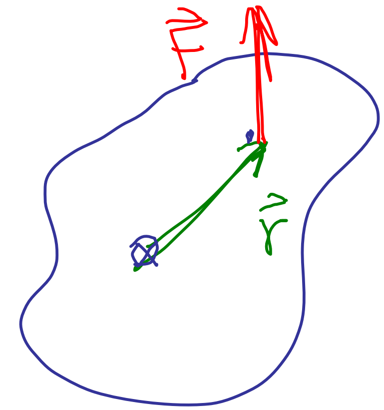
$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\,\hat{i} + (- \text{———})$$

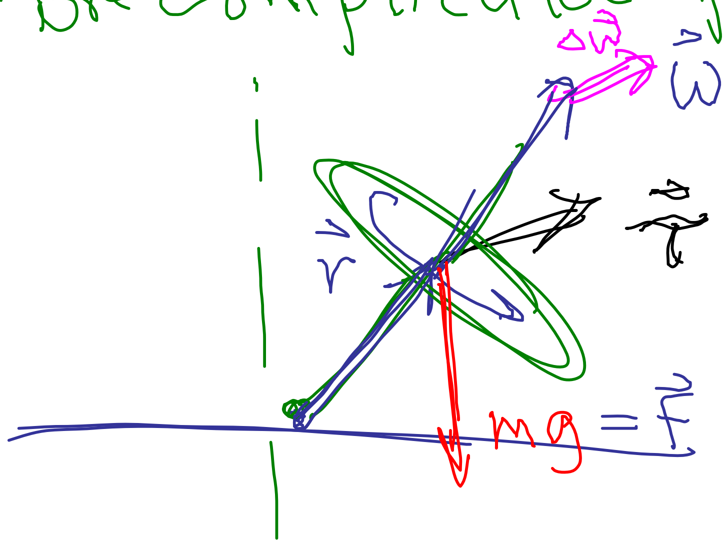
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}\,(A_y B_z - A_z B_y) + \hat{j}\,(\text{———})$$

$\vec{\tau} = \vec{r} \times \vec{F}$

Torque vector comes out of board



More complicated problems



$$\vec{\tau} = I\vec{\alpha}$$
$$= I\frac{\Delta\vec{\omega}}{\Delta t}$$

See local mechanics honchos.

$x \to \theta$

$\dot{x}$

$a \to \alpha$

$F \to \tau$

$m \to I$

$K \to K$

Analog for momentum

$$p_x = m v_x$$

$$L = I\omega$$

Angular momentum

Scalar $\Rightarrow$ Vector

Units: $(kg\, m^2)\left(\frac{1}{sec}\right)$

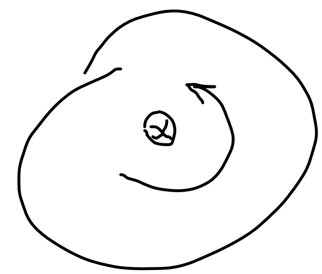
$$[L] = \frac{kg\, m^2}{s} = J \cdot s$$

Actually $\vec{L} = I\vec{\omega}$

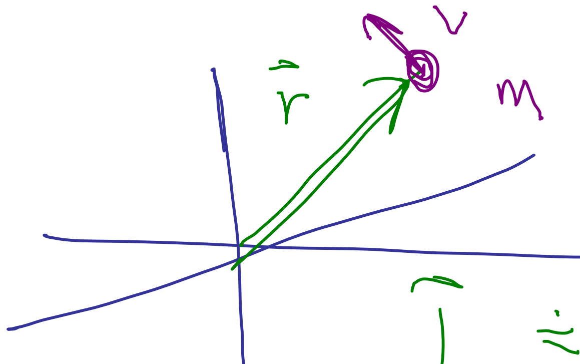
It's more complicated

$$\vec{L} = \begin{pmatrix} I \end{pmatrix} \vec{\omega}$$

matrix !



Really



$\vec{p} = m\vec{v}$

$\vec{L} = \vec{r} \times \vec{p}$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L_z = rp$$
$$= rmv$$
$$= rmr\omega$$
$$= (mr^2)\omega$$
$$= I\omega$$

For us,

$$\vec{L} = I\omega\hat{k}$$

Wuh! What is it good for? Ahh

$\vec{p}$ Isolated system, $\vec{P}$ conserved

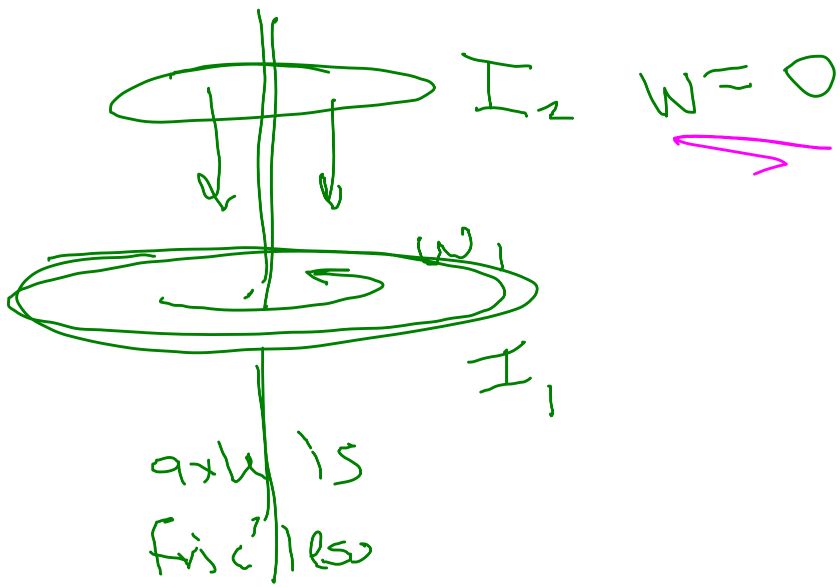
No net forces from outside.

$\vec{L}$ If isolated (rotationally)

[no external net torque]

the $\vec{L}_{Total}$ is conserved.

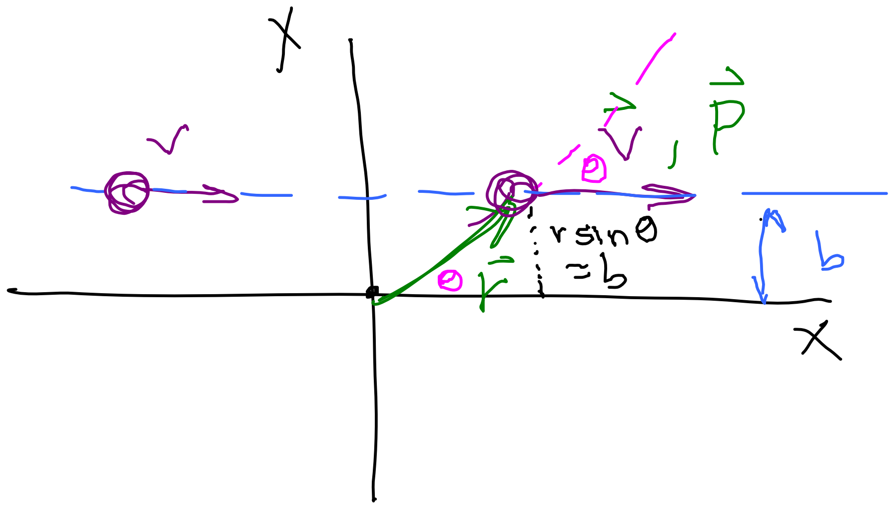
$I\omega$



No external torque $\quad L_{tot}$ conserved

$$I_1\omega_1 = (I_1 + I_2)\,\omega_2$$

$$\omega_2 = \left(\frac{I_1}{I_1 + I_2}\right)\omega_1$$



What is ang mom of this mass

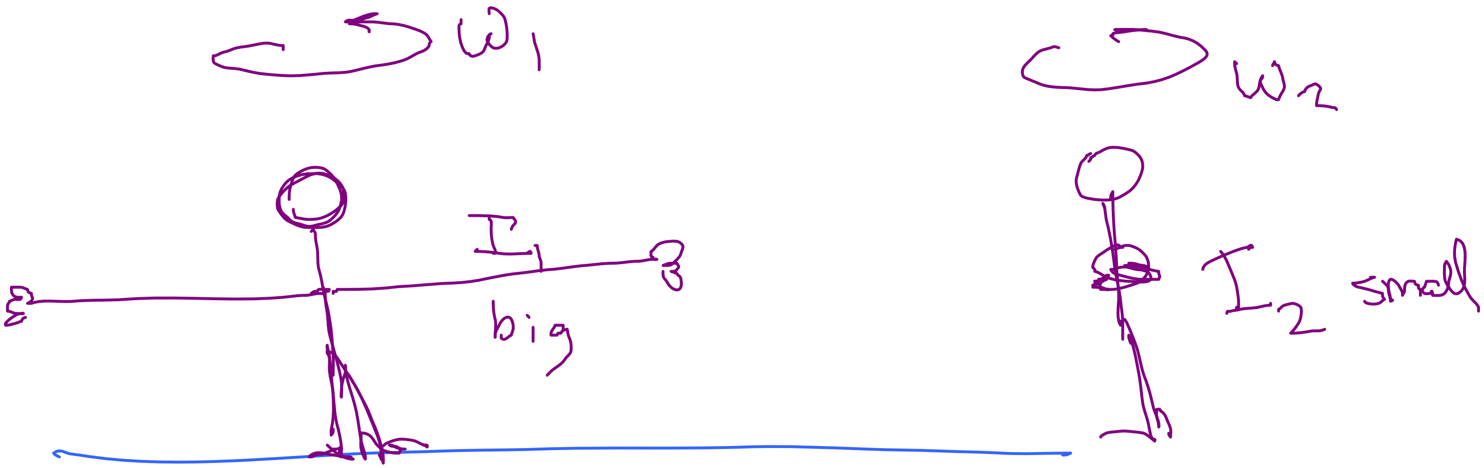
$$\vec{L} = \vec{r} \times \vec{p}$$ with $$|\vec{L}| = rp\sin\theta$$

(into page)

$$= rmv\sin\theta$$

$$= mvb$$

More common example:



No external torq.   Ang mom is conserved.

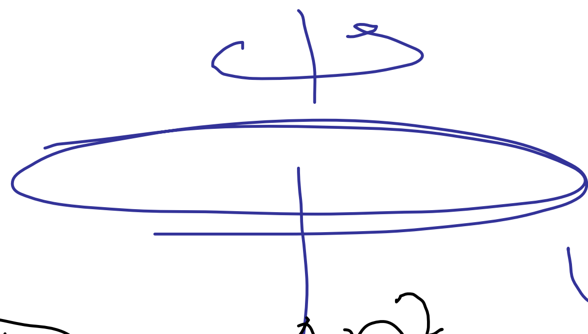
$$L_1 = I_1 \omega_1 = L_2 = I_2 \omega_2$$

E not conserved
gained.

$$\omega_2 = \frac{I_1 \omega_1}{I_2} = \; > \omega_1$$

11.23 A 640 g hoop 90 cm in diameter is rotating at 170 rpm about its central axis. What's its angular momentum

$I = 2.3\ \text{J}\cdot\text{s}$



$$L = I\omega \qquad p = mv$$

$$\omega = \left(170\ \underset{\text{rev}/\cancel{\text{min}}}{\text{rpm}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)$$

$$I = MR^2$$
$$= (0.640\ \text{kg})(0.45\ \text{m})^2 \quad \text{etc}$$

11.38 A turntable of radius 25 cm
rot'l inertia 0.0154 kg m^2
spins freely 22.0 rpm
Mouse walks from edge to
center. Find new rot'n speed.
Work done by mouse