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Oct. 12, 2010

Phys 2112, Fall 2010
Quiz #1

1. The kinetic energy of the electrons in the inner shell of the Ca atom is on the order of 5000 eV. Do we need relativity (in addition to quantum mechanics) to describe their motion?

The KE is

$$K = 5000 \text{ eV} \left(\frac{1.602 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) = 8.01 \times 10^{-16} \text{ J}$$

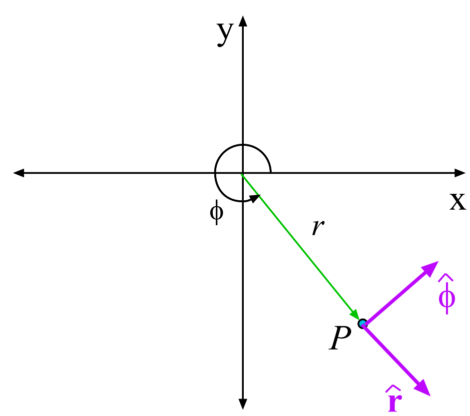
Get the speed from this

$$K = \tfrac{1}{2}mv^2 \qquad \Longrightarrow \qquad v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(8.01 \times 10{-}16 \text{ J})}{(9.11 \times 10^{-31} \text{ kg})}} = 4.2 \times 10^7 \tfrac{\text{m}}{\text{s}}$$

This speed is greater than 10% of the speed of light so that we *do* need to use the relativistic laws of motion to describe these electrons.

2. For the point P shown here, show the directions of the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\phi}}$.

Unit vectors added to the original figure, shown here:



3. A particle moves in a circle of radius 350.0 cm centered at the origin, with constant speed 2.00 $\frac{\text{m}}{\text{s}}$.

Write down some suitable equations of motion. (That is, $x(t)$ and $y(t)$.)

The period of the motion is

$$T = \frac{2\pi R}{v} = \frac{2\pi(3.50 \text{ m})}{(2.0 \, \frac{\text{m}}{\text{s}})} = 11.0 \text{ s}$$

so that the angular frequency is

$$\omega = \frac{2\pi}{T} = 0.571 \text{ s}^{-1} \quad .$$

With this and the mathematical form for circular motion we have

$$x(t) = R\cos(\omega t) = (3.50 \text{ m}) \cos\left((0.571 \text{ s}^{-1})t\right)$$

$$y(t) = R\sin(\omega t) = (3.50 \text{ m}) \sin\left((0.571 \text{ s}^{-1})t\right)$$

4. A particle moves in one dimension with a velocity given by

$$x(t) = (3.0\ \mathrm{m})\left(1 - e^{-t^2/T^2}\right)$$

where $T = 5.0$ s.

a) Find $v(t)$ and $a(t)$

The first time derivative gives

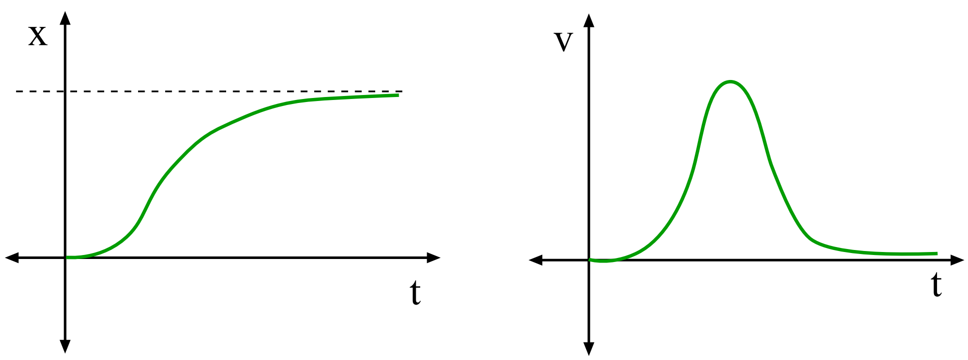
$$v(t) = x'(t) = (3.0\ \mathrm{m})\left(-\frac{2t}{T^2}(-e^{-t^2/T^2})\right) = \frac{(6.0\ \mathrm{m})t}{T^2}e^{-t^2/T^2}$$

The next derivative gives

$$a(t) = v(t) = \frac{(6.0\ \mathrm{m})}{T^2}\left(e^{-t^2/T^2} + \frac{-2t^2}{T^2}e^{-t^2/T^2}\right) = \frac{(6.0\ \mathrm{m})}{T^2}\left(1 - \frac{2t^2}{T^2}\right)e^{-t^2/T^2}$$

b) Sketch crude graphs of $x(t)$ and $v(t)$ for $t \geq 0$.

The graph of x vs. t starts off at $x = 0$ with zero slope; x increases and at large times approaches $x = 3.0$ m. It looks like the first graph shown below. The slope of this graph (v) increases from zero to some maximum value and then approaches zero again:



Show work for all problems and include the right units!

$c = 2.998\times10^8\ \frac{\mathrm{m}}{\mathrm{s}}$ $\quad h = 6.626\times10^{-34}\ \mathrm{J\cdot s}$ $\quad 1\ \mathrm{eV} = 1.602\times10^{-19}\ \mathrm{J}$ $\quad \mathrm{M} \equiv 10^6$ $\quad \mathrm{k} \equiv 10^3$

$m_\mathrm{p} = 1.67\times10^{-27}\ \mathrm{kg}$ $\quad m_\mathrm{e} = 9.11\times10^{-31}\ \mathrm{kg}$ $\quad K = \frac{1}{2}mv^2$ $\quad p = mv$ $\quad \lambda = \frac{h}{p}$

$x = r\cos\phi$ $\quad y = r\sin\phi$ $\quad r = \sqrt{x^2+y^2}$ $\quad \tan\phi = \frac{y}{x}$ $\quad v = \frac{2\pi R}{T}$ $\quad \omega = \frac{2\pi}{T} = 2\pi f$