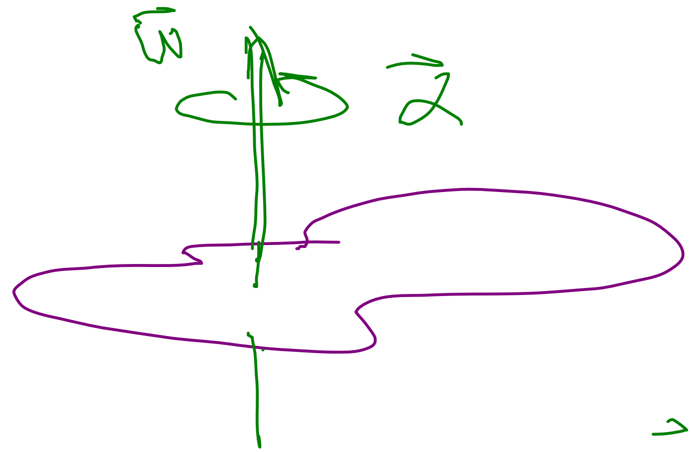
# Rotations (Ch 11)

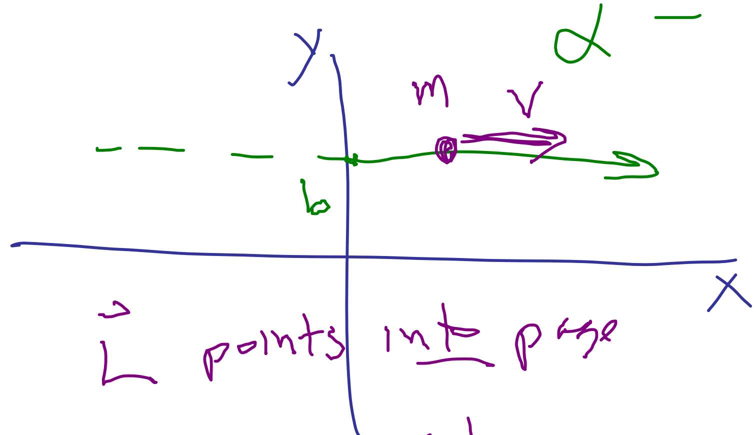



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\alpha} = \frac{\Delta \vec{\omega}}{\Delta t}$$

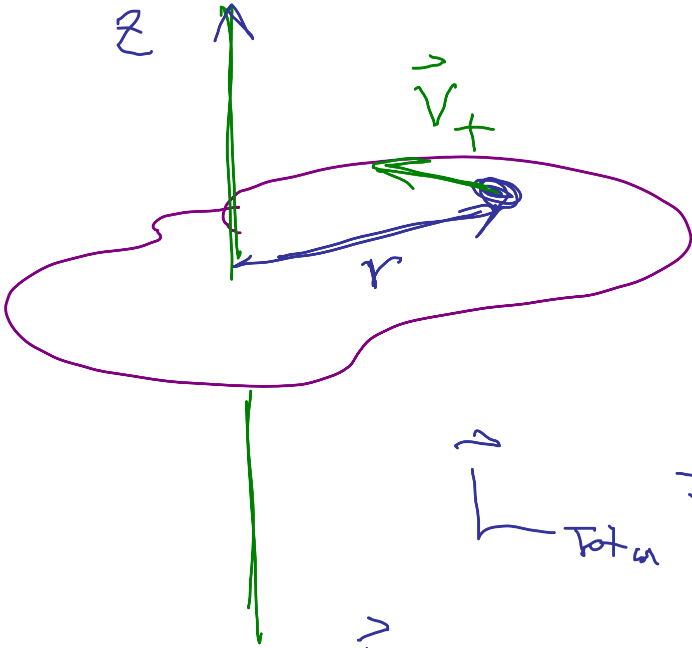
Angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$




$\vec{L}$ points into page

$$L = mvb$$




$$\vec{L} = r(mv)\,\hat{k}$$

$$= r(m\omega r)\,\hat{k}$$

$$\vec{L}_{Tot} = \sum \vec{L}_i = \omega\underbrace{\left(\sum m_i r_i^2\right)}_{I}\hat{k}$$

$$\vec{L}_{Tot} = \omega I\,\hat{k} = I\omega\,\hat{k}$$

$$P_x = mV_x$$

$$\downarrow$$

$$L = I\omega$$

$\vec{L}$ (huh!) what is it good for?

$\vec{F} = d\vec{p}/dt$ Isolated system

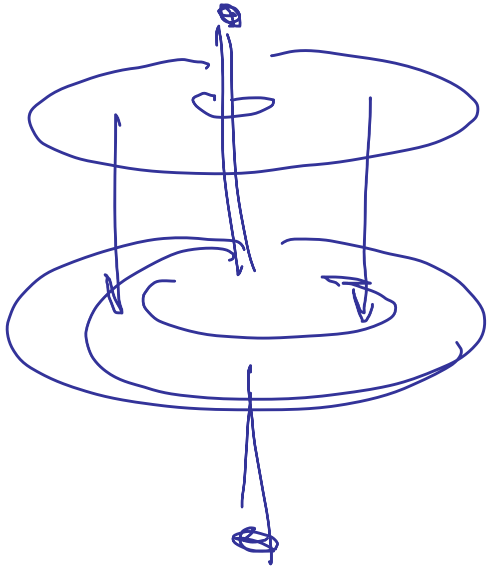
→ total P stays.

$\vec{\tau} = d\vec{L}/dt$ Rotationally isolated system

→ Total L stays same.

No (net) external torques.

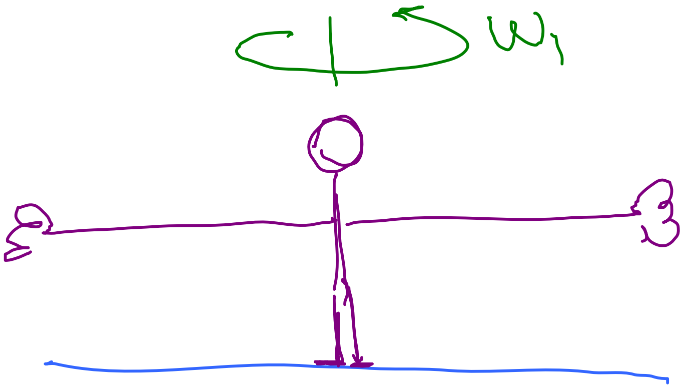
$\sum L$ stays same. $L = I\omega$



Internal torques

No external torque

$$\sum I\omega = \text{constant}$$





$\omega_2 > \omega_1$

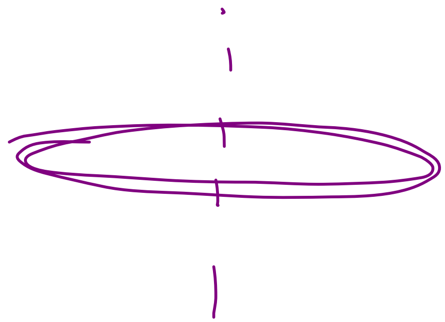
$I_2 < I_1$

$\omega_2 > \omega_1$

$$I_1\omega_1 = I_2\omega_2$$

11.23 640-g hoop 90 cm in diameter rotates at 170 rpm about central

What is ang mom?



$$\underline{I = MR^2}$$

$$\underline{\omega = 170 \text{ rpm} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ m}}{60 \text{ s}}\right)}$$

$$L = \underline{I\omega} = 2.3 \ (\text{J}\cdot\text{s}) \quad \left(\frac{\text{kg m}^2}{\text{s}}\right)$$

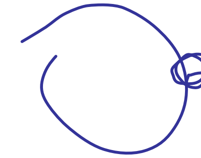
11.38 A turntable of radius 25 cm & rot. inertia 0.0154 kg m$^2$ spins freely at 22.0 rpm around axis, 19.5 g mouse sits at edge. Walks edge to center. Find

a) New rotation speed

b) Work done by mouse.

E not nec. conserved!!!


Rot'ly isolated. Total L is conserved.



$mr^2$

$L_{init}$ : $I_{Total,1}\,\omega_{init}$

$$= \left(0.0154\,kg\,m^2 + (0.0195\,kg)(0.25\,m)^2\right)\omega_{init}$$

$L_{final}$ : $I_{total,2}\,\omega_{final}$

$$= (0.0154\,kg\,m^2)\,\omega_{final}$$

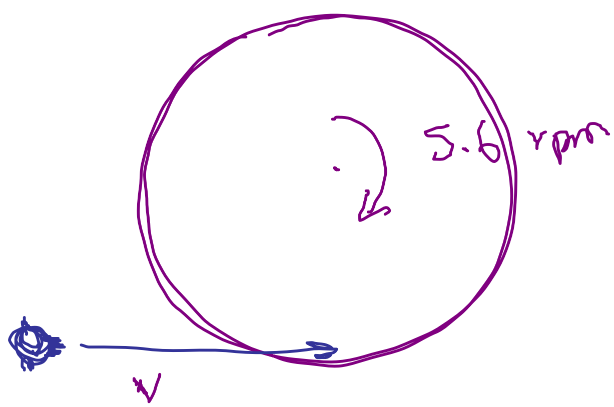
$$\omega_{fin} = \frac{I_1}{I_2}\,\omega_1 = 2.48\,\frac{rad}{s} = 23.7\ rpm$$

b) $KE_{init}$ : $\frac{1}{2} I_1 \omega_1^2 = 0.04405 \text{ J}$

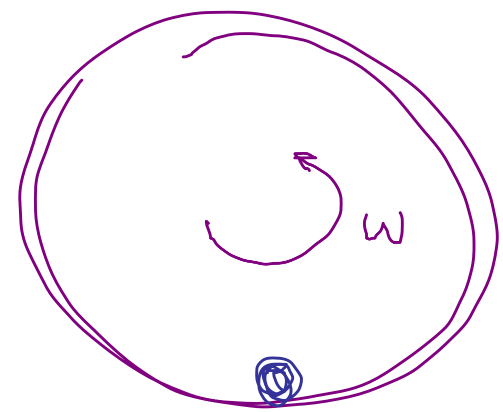
$KE_{final} = \frac{1}{2} I_2 \omega_2^2 = 0.0473 \text{ J}$

$\Delta K = 3.3 \times 10^{-3} \text{ J}$  Gained energy.

11.43 Circular bird feeder 19 cm in radius, rot'l inertia 0.12 $kg\,m^2$, spins slowly at 5.6 rpm. 140 g bird lands on rim, comes in tangent at 1.1 $\frac{m}{s}$ in dir. opposite to feeder's orig. motion. Final rot'n rate.




$L_{Total}$ is conserved.



$$L_{init} = I_{free}\left(0.586\,\tfrac{rad}{s}\right) + (0.140\,kg)(1.1\,\tfrac{m}{s})(0.19\,m)$$

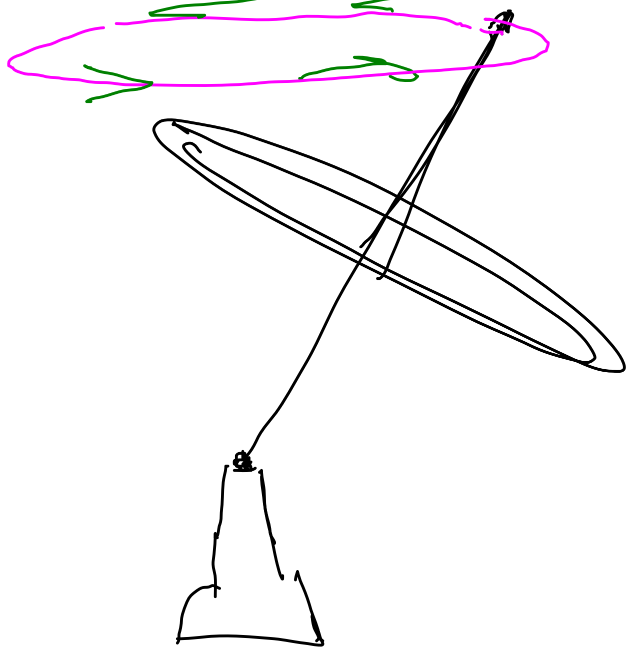
$$\omega = 0.586\,\tfrac{rad}{s}$$

$mvb$

$$L_{fin} = \left(I_{free} + mR^2\right)\omega_f$$

I got (?) $\omega_f = 0.329\,\tfrac{rad}{s}$

# Gyroscopes



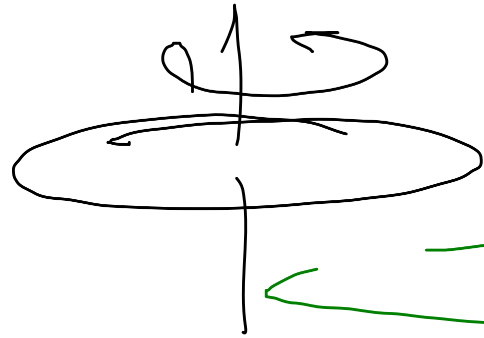
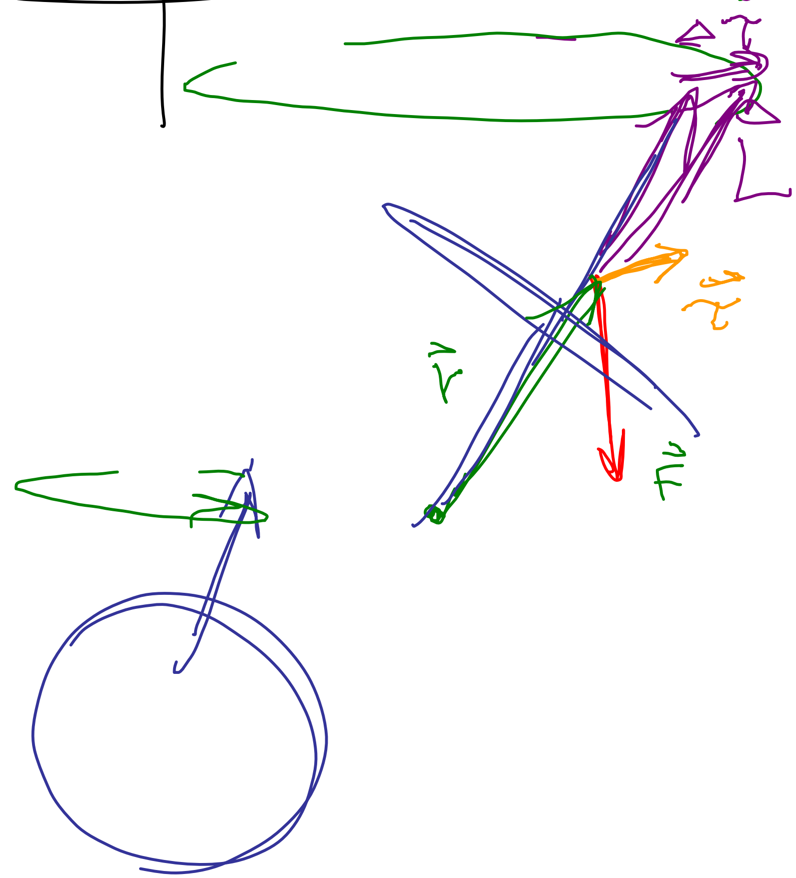


Fig 11.9




$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

Same dir. as $\Delta\vec{L}$