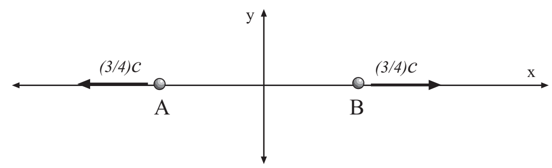
Phys 4620, Spring 2005
Exam #3

1. If we move toward a source of light we still measure the speed of the light as c but we measure a different frequency and wavelength from that measured in the source frame.

Now, we have a formula from Phys 2110 for the change in frequency when we move relative to a source[1]. Why can't we use that formula for light?

2. As seen in frame $\mathcal{S}$ particle A moves at speed $\frac{3}{4}c$ in the $-x$ direction and particle B moves at speed $\frac{3}{4}c$ in the $+x$ direction.

What is the speed of particle B as measured in the frame of particle A?



3. A pion has a lifetime of 2.6×10^{-8} s (in its rest frame). If a pion has a speed of $\frac{4}{5}c$, how far (on average) do we expect it to travel after being created?

4.a) What does it mean when we say that a certain quantity transforms as a "Lorentz vector" ?

b) What does it mean when we say that a certain quantity is "Lorentz invariant"?

5. A proton ($M_{\rm p}c^2 = 938$ MeV) has 800 MeV of kinetic energy.

What is its speed? (You can answer as a fraction of c.)

6. A proton collides with a proton at rest to produce 3 protons and an antiproton (which has the same mass, but opposite charge from the proton):

$$\mathrm{p} + \mathrm{p} \longrightarrow \mathrm{p} + \mathrm{p} + \mathrm{p} + \bar{\mathrm{p}}$$

Find the smallest possible (threshold) kinetic energy for the incident proton for this reaction to take place. Use $M_{\bar{\rm p}}c^2 = M_{\rm p}c^2 = 938$ MeV and express the answer in MeV.

(Hint: Use *invariance* and *conservation* and use the fact that at threshold in the CM frame the final particles are at rest.)

7. In the lab frame we have E and B fields which are both nonzero and point in the same direction ($\hat{\mathbf{z}}$, say). Is there any reference frame where the electric field is zero? (Give this reference frame or explain why there can't be one.)

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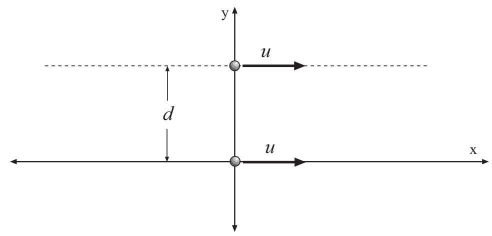
[1] It's

$$f' = \left(\frac{v_0 \pm v_{\rm obs}}{v_0 \mp v_{\rm src}} \right) f \ .$$

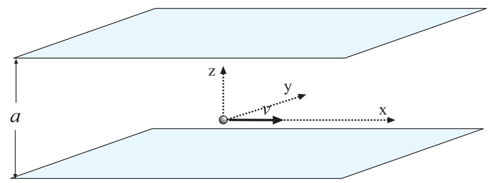
8. In system $\mathcal{S}$ charges q_A and q_B are both flying by at constant speed u on trajectories parallel to the x axis (in the xy plane), q_A on $y = 0$ and q_B on $y = d$.



a) Find the fields at q_B due to q_A and the force that q_A exerts on q_B (as, say, both cross the y axis).

b) The field(s) in the frame of the moving charges is simple! Show that $(E^2 - c^2B^2)$ is the same in both frames.

9. a) There are two charged plates at $z = 0$ and $z = a$ each carrying charge densities $+\sigma$ and $-\sigma$ respectively. What is the electric field between the plates?



b) If a charge q moves in the $+\hat{\mathbf{x}}$ direction with speed v, what are the electric and magnetic fields in the frame of this charge?

Do either 10 or 11:

10. The Maxwell equations can be written in the form

$$\frac{\partial F^{\mu\nu}}{\partial x^{\nu}} = \mu_0 J^{\mu} \qquad\qquad \frac{\partial G^{\mu\nu}}{\partial x^{\nu}} = 0$$

Show how the first of these, with the choice $\mu = 1$ (x) gives a certain component of one of the Maxwell equations.

11. Evaluate $F^{\mu\nu}F_{\mu\nu}$.

It will make things easier to use the antisymmetry of $F^{\mu\nu}$ and to break up the values of $F^{\mu\nu}$ into F^{0i} and F^{ij}. Recall that when you lower a "0" index you get a minus sign.

Useful Equations

$$\int_{\mathbf{a}}^{\mathbf{b}} (\nabla T)\cdot d\mathbf{l} = T(\mathbf{b}) - T(\mathbf{a}) \qquad \int_{\mathcal{V}} (\nabla\cdot\mathbf{v})d\tau = \oint_{\mathcal{S}} \mathbf{v}\cdot d\mathbf{a} \qquad \int_{\mathcal{S}} (\nabla\times\mathbf{v})\cdot d\mathbf{a} = \oint_{\mathcal{P}} \mathbf{v}\cdot d\mathbf{l}$$

Spherical:

$$d\mathbf{l} = dl_r\,\hat{\mathbf{r}} + dl_\theta\,\hat{\boldsymbol{\theta}} + dl_\phi\,\hat{\boldsymbol{\phi}} \qquad d\tau = r^2\ \sin\theta\,dr\,d\theta\,d\phi$$

Gradient:

$$\nabla T = \frac{\partial T}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}} \tag{1}$$

Divergence:

$$\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} \tag{2}$$

Curl:

$$\nabla\times\mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\,v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}} \tag{3}$$

Laplacian:

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2} \tag{4}$$

Cylindrical:

$$d\mathbf{l} = dl_s\,\hat{\mathbf{s}} + dl_\phi\,\hat{\boldsymbol{\phi}} + dl_z\,\hat{\mathbf{z}} \qquad d\tau = s\,ds\,d\phi\,dz$$

Gradient:

$$\nabla T = \frac{\partial T}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial T}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}} \tag{5}$$

Divergence:

$$\nabla\cdot\mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z} \tag{6}$$

Curl:

$$\nabla\times\mathbf{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}} \tag{7}$$

Laplacian:

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial\phi^2} + \frac{\partial^2 T}{\partial z^2} \tag{8}$$

More Math

Gradients:

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$$

Divergences:

$$\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$

Product Rules:

(1) $\nabla\cdot(\nabla T)$ (Divergence of curl)

$$\nabla\cdot(\nabla T) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \nabla^2 T$$

(2) $\nabla\times(\nabla T)$ (Curl of gradient)

$$\nabla\times(\nabla T) = 0$$

(3) $\nabla(\nabla\cdot\mathbf{v})$ (Gradient of divergence)

Nothing interesting about this; does not occur often.

(4) $\nabla\cdot(\nabla\times\mathbf{v})$

$$\nabla\cdot(\nabla\times\mathbf{v}) = 0$$

(5) $\nabla\times(\nabla\times\mathbf{v})$ (Curl of a curl)

$$\nabla\times(\nabla\times\mathbf{v}) = \nabla(\nabla\cdot\mathbf{v}) - \nabla^2\mathbf{v}$$

Physics:

$$\begin{aligned} \nabla\cdot\mathbf{E} &= \frac{1}{\epsilon_0}\rho \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \\ \nabla\cdot\mathbf{B} &= 0 \qquad \nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t} \end{aligned}$$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad \nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t} \qquad \mathcal{E} = -\frac{d\Phi}{dt}$$

$$\mathbf{E} = -\nabla V - \frac{\partial\mathbf{A}}{\partial t} \qquad \mathbf{A}' = \mathbf{A} + \nabla\lambda \qquad V' = v - \frac{\partial\lambda}{\partial t}$$

$$\text{Coulomb}: \quad \nabla\cdot\mathbf{A} = 0 \qquad \text{Lorentz}: \quad \nabla\cdot\mathbf{A} = -\mu_0\epsilon_0\frac{\partial V}{\partial t}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}', t_r)}{\mathscr{r}}\,d\tau' \qquad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}', t_r)}{\mathscr{r}}\,d\tau'$$

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0}\frac{qc}{\mathscr{r}c - \boldsymbol{\mathscr{r}} \cdot \mathbf{v}} \qquad \mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi}\frac{qc\mathbf{v}}{(\mathscr{r}c - \boldsymbol{\mathscr{r}} \cdot \mathbf{v})} = \frac{\mathbf{v}}{c^2}V(\mathbf{r}, t)$$

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0}\frac{\mathscr{r}}{(\boldsymbol{\mathscr{r}} \cdot \mathbf{u})^3}[(c^2 - v^2)\mathbf{u} + \boldsymbol{\mathscr{r}} \times (\mathbf{u} \times \mathbf{a})] \qquad \mathbf{B}(\mathbf{r}, t) = \frac{1}{c}\boldsymbol{\mathscr{r}} \times \mathbf{E}(\mathbf{r}, t)$$

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0}\frac{1 - v^2/c^2}{\left(1 - v^2\sin^2\theta/c^2\right)^{3/2}}\frac{\hat{\mathbf{R}}}{R^2} \qquad \mathbf{B} = \frac{1}{c}(\hat{\boldsymbol{\mathscr{r}}} \times \mathbf{E}) = \frac{1}{c^2}(\mathbf{v} \times \mathbf{E})$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \Delta\bar{t} = \sqrt{1 - v^2/c^2}\Delta t \qquad \Delta\bar{x} = \frac{1}{\sqrt{1 - v^2/c^2}}\Delta x$$

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + (v_{AB}v_{BC}/c^2)} \qquad \bar{t} = \gamma\left(t - \frac{v}{c^2}x\right) \qquad \bar{x} = \gamma(x - vt) \qquad \bar{y} = y \qquad \bar{z} = z$$

$$\bar{x}^\mu = \sum_{\nu=0}^{3}(\Lambda^\mu_\nu)x^\nu \qquad \Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\eta^\mu = \gamma(c, v_x, v_y, v_z) \qquad \mathbf{p} = \frac{m\mathbf{u}}{\sqrt{1 - u^2/c^2}} \qquad p^\mu = (E/c, p_x, p_y, p_z) \qquad E = \gamma mc^2$$

$$p^\mu p_\mu = -m^2c^2 \qquad E^2 = p^2c^2 + m^2c^4$$

$$K^\mu = \frac{dp^\mu}{d\tau} \qquad J^\mu = (c\rho, J_x, J_y, J_z) \qquad A^\mu = (V/c, A^x, A^y, A^z)$$

$$\begin{aligned} \bar{E}_x &= E_x \qquad \bar{E}_y = \gamma(E_y - vB_z) \qquad \bar{E}_z = \gamma(E_z + vB_y) \\ \bar{B}_x &= B_x \qquad \bar{B}_y = \gamma(B_y + \frac{v}{c^2}E_z) \qquad \bar{B}_z = \gamma(B_z - \frac{v}{c^2}E_y) \end{aligned}$$

$$F^{\mu\nu} = \left\{ \begin{array}{cccc} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{array} \right\} \qquad F^{\mu\nu} = \frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A^\mu}{\partial x_\nu}$$

Invariants: $\mathbf{E} \cdot \mathbf{B}$, $(E^2 - c^2B^2)$

$$\frac{\partial J^\mu}{\partial x^\mu} = 0 \qquad \frac{\partial F^{\mu\nu}}{\partial x^v} = \mu_0 J^\mu \qquad \frac{\partial G^{\mu\nu}}{\partial x^\nu} = 0 \qquad K^\mu = q\eta_\nu F^{\mu\nu}$$