

**Phys 3810, Spring 2011**  
**Problem Set #6, Hints**

- 1. Griffiths, 4.23** Apply the (analytic) raising operator to  $Y_2^1(\theta, \phi)$  but also show from (4.120) and (4.121)

$$L_+ Y_2^1 = 2\hbar Y_2^2$$

- 2. Griffiths, 4.26** Multiply a lot of little matrices.

Show that if  $j \neq k$  then

$$\sigma_j \sigma_k = i \sum_l \epsilon_{jkl} \sigma_l$$

and if  $j = k$  then  $\sigma_j \sigma_k = \sigma_j^2 = \mathbf{1}$ . But results are contained in

$$\sigma_j \sigma_k = \delta_{jk} + i \sum_l \epsilon_{jkl} \sigma_l$$

- 3. Griffiths, 4.27** Normalization should be easy. Remember that the condition is  $\chi^\dagger \chi = 1$  and for  $\chi^\dagger$  you have to do a complex conjugation. In part (b) you should get

$$\langle S_x \rangle = 0 \quad \langle S_y \rangle = -\frac{12}{25}\hbar \quad \langle S_z \rangle = -\frac{7}{50}\hbar$$

On (c) you should get

$$\sigma_{S_z} = \frac{12}{25}\hbar$$

- 4. Griffiths, 4.29** Show that the eigenvectors of  $S_y$  (well, one choice for them) are

$$\chi_+^{(y)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \quad \chi_-^{(y)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

- 5. Griffiths, 4.35** This one is a short easy (?) answer. Recall that spins  $s_1$  and  $s_2$  can “add” to give all spins from

$$s_1 + s_2 \quad \text{down to} \quad |s_1 - s_2|$$