Phys 3820, Fall 2009
Problem Set #2, Hint-o-licious Hints

1. *Griffiths*, **6.11** Fairly easy algebra.

2. *Griffiths*, **6.14** The relativistic perturbation to the Hamiltonian is

$$H' = -\frac{p^4}{8m^3c^2}$$

so we want to calculate

$$E^1_{n,\mathrm{rel}} = \left\langle n \left| -\frac{p^4}{8m^3c^2} \right| \right\rangle$$

Use

$$p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-) \qquad \Longrightarrow \qquad p^2 = -\frac{\hbar m\omega}{2}(a_+^2 + a_-^2 - a_+a_- - a_-a_+)$$

in

$$E^1_{n,\mathrm{rel}} = -\frac{1}{8m^3c^2}\langle p^2\psi^0_n | p^2\psi^0_n\rangle$$

and use orthogonality of the HO wave functions. Get

$$E^1_{n,\mathrm{rel}} = -\frac{3\hbar^2\omega^2}{32mc^2}(2n^2 + 2n + 1)$$

3. *Griffiths*, **6.17** Take the individual terms from Eqs. (6.57) and (6.65) and consider individually the cases $j = l \pm \frac{1}{2}$. Do the algebra (which is a little messy) and show that in both cases you get simple result in terms of j.

4. *Griffiths*, **6.20** Use $|\mathbf{L}| \approx \hbar$ in (6.59) With this, the critical size of the B field is around 12 T.

5. *Griffiths*, **6.21** The fine structure is larger than the Zeeman contribution to the energy. The zero-field value of the energies is given by (6.67). Since it included the spin-orbit splitting, the energies depend on j (and n).

Now, for $n = 2$ we have the states $l = 0$ and $l = 1$. We must have states of "good" j, so we note that the $l = 0$ state is a $j = \frac{1}{2}$ states while the $l = 1$ state give $j = \frac{1}{2}$ and $j = \frac{3}{2}$.

Calculate the Landé g factor g_J for each state note, it depends on j and l, and then the weak–field Zeeman energy is

$$E^1_Z = \mu_B g_J B_{\mathrm{ext}} m_j$$

If you plot E vs. $\mu_B B_{\mathrm{ext}}$, the slope of the line is $g_J m_j$.

For the $j = \frac{1}{2}$ states we then have a pair for the state that came from $l = 0$ (with $m_j = \pm\frac{1}{2}$) and a pair for the state that came from $l = 1$. The $j = \frac{3}{2}$ state came from $l = 1$ and with $m_j = -\frac{3}{2} \ldots \frac{3}{2}$, there are four lines with their own slopes.

6. *Griffiths*, **6.28** This one is trickier than it seems because you need to do some tracing to tell how to replace the various factors in the formula for the hyper-fine splitting ΔE for the various (interesting!) two-body bound systems given. It is best to ignore the formula (6.92) and go back to (6.89), which basically says

$$\Delta E \propto \frac{g_p}{m_p m_e a^3}$$

because in this expression each symbol appears for a particular reason. (In (6.92) the two factors of m_e appear for two different reasons so they have different replacements!) Actually, this expression should also have a factor g_e in the numerator, but as long as the "orbiting" particle is electron-like (as the muon is), it's always 2 so it won't change.

Make replacements, noting: g_p is the "g–factor" of the positive particle. m_p is the mass of the positive particle. m_e is the *true mass* of the orbiting negative particle, since it comes from an expression with the gyromagnetic ratio.

Most complicated is a. This factor arises from the wave function of the two–particle system and is inversely proportional to the *reduced mass* of the system:

$$a \propto \frac{1}{m(\text{red})}$$

For example the reduced mass of positronium case is $\frac{m_e}{2}$. The reduced mass for the H atom is *very close* to m_e.

7. *Griffiths*, **6.29** The perturbation is

$$H' = \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{b}\right)$$

and use this in

$$E^1_{\text{gs}} = \int \psi^*(\mathbf{r}) H' \psi(\mathbf{r}) d^3\mathbf{r}$$

As explained in class, you can approximate the exponential as 1 to get a lowest–order answer, and that all we need to check the order of magnitude of the result. With this, show

$$\frac{E^1_{\text{gs}}}{|E^1_{\text{gs}}|} = \frac{4}{3}\left(\frac{b}{a}\right)^2$$

Compare with value with those from fine structure and the hyperfine splitting! (Use Table 6.1).