Phys 4610, Fall 2004
Exam #3

1. In the calculation of the energy stored in an electrostatic system, we first derived the expression $W = \frac{\epsilon_0}{2}\int E^2\, d\tau$. Later when considering linear dielectric media we found the expression $W = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}\, d\tau$.

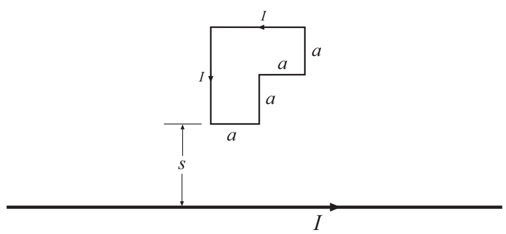
What is the difference between these two expressions? What parts(s) of the energy is accounted for in each one?

A polarized medium conists of positive and negative charges which have been stretched away from their "normal" positions so as to give a net buildup of charge on the surfaces of the dielectrics and/or in their interiors. There may also be free charges in the system.

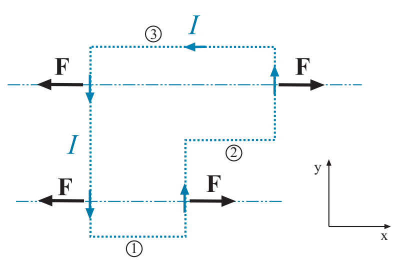
The expression $W = \frac{\epsilon_0}{2}\int E^2\, d\tau$ (to be used with continuous charge distributions!) gives the energy required to assemble the free and bound charge densities and *nothing else*. The expression $W = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}\, d\tau$ gives the energy to assemble the free and bound charge densities and *also* to pull the charges of the medium apart, even if the stretching results in no volume charge density. The second one *does* give the complete system energy when linear dielectrics are present.

2. A planar loop with six 90° bends in it is coplanar with a very long wire which carries a current I. The loop also carries a current I, as shown. (Three sides of the loop are parallel to the long wire.)

Find the magnitude and direction of the net force on the loop.



First we note that there is no net force on the parts of the loop that run in the y direction. This is because for corresponding current elements at the same distance from the long wire the forces have the same magnitude but are in opposite directions, as shown here. Summing over the elements for the "y" legs gives zero net force.



The force on side 1 of the loop is

$$I\mathbf{l}_1 \times \mathbf{B}_1 = I(a\hat{\mathbf{x}}) \times \frac{\mu_0 I}{2\pi s}\hat{\mathbf{z}} = -\frac{\mu_0 a I^2}{2\pi s}\hat{\mathbf{y}}$$

The force on side 2 of the loop is

$$I\mathbf{l}_2 \times \mathbf{B}_2 = I(a\hat{\mathbf{x}}) \times \frac{\mu_0 I}{2\pi(s+a)}\hat{\mathbf{z}} = -\frac{\mu_0 a I^2}{2\pi(s+a)}\hat{\mathbf{y}}$$

The force on side 3 is

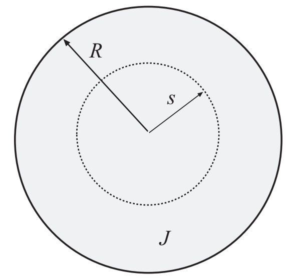
$$I\mathbf{l}_3 \times \mathbf{B}_3 = I(-2a\hat{\mathbf{x}}) \times \frac{\mu_0 I}{2\pi(s+2a)}\hat{\mathbf{z}} = +\frac{\mu_0 a I^2}{2\pi(s+2a)}\hat{\mathbf{y}}$$

Adding these, the total force is

$$\mathbf{F}_{\text{net}} = -\frac{\mu_0 a I^2}{2\pi}\left(\frac{1}{s} + \frac{1}{(s+a)} - \frac{2}{(s+2a)}\right)\hat{\mathbf{y}}$$

3. A long straight wire of radius R carries a total current I, but the current density J is nonuniform; it falls off linearly to zero at the surface, i.e.

$$J = k(R - s)$$



a) Find k.

The integral of $J(s)$ over the x-sec area must give I, hence

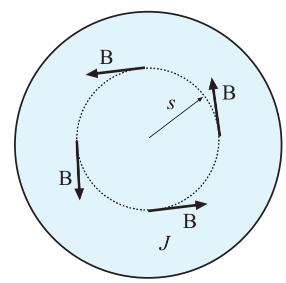
$$\begin{aligned}\int J(s)\,da &= 2\pi\int_0^R k(R-s)s\,ds = 2\pi k\left[R\frac{s^2}{2} - \frac{s^3}{3}\right]\Big|_0^R \\ &= 2\pi k R^3\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{\pi k R^3}{3} = I\end{aligned}$$

So that

$$k = \frac{3I}{\pi R^3}$$

b) Find the magnitude of magnetic field inside and outside the wire. Give its direction, though you don't need to repeat the reasons for this.

For $s < R$ take an Wmperian loop of radius s. With the current coming out of the page, the B field is tangential as shown here, with



$$\begin{aligned}\oint \mathbf{B}\cdot d\mathbf{l} &= (2\pi s)B = \mu_0 I_{\text{enc}} \\ &= \mu_0\int_0^{2\pi}\int_0^s J(s')s'\,ds'\,d\phi' = \mu_0 2\pi k\int_0^s (R-s')s'\,ds' \\ &= \mu_0 2\pi k\left[R\frac{s'^2}{2} - \frac{s'^3}{3}\right]\Big|_0^s = \mu_0 2\pi\frac{3I}{\pi R^3}s^2\left[\frac{R}{2} - \frac{s}{3}\right] \\ &= \frac{6\mu_0 I s^2}{R^3}\frac{(3R-2s)}{6} = \frac{\mu_0 I s^2(3R-2s)}{R^3}\end{aligned}$$

Divide by $2\pi s$ and include the direction of $\mathbf{B}$, then

$$\mathbf{B} = \frac{\mu_0 I s(3R-2s)}{2\pi R^3}\hat{\boldsymbol{\phi}}$$

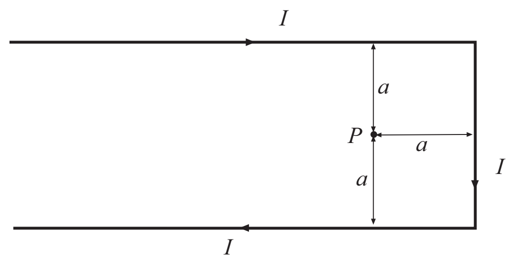
for $s \le R$.

For $s > R$, an Amperiain loop encloses *all* the curent so the result is the same as for a long thin wire,

$$\mathbf{B}\frac{\mu_0 I}{2\pi s}\hat{\boldsymbol{\phi}}$$

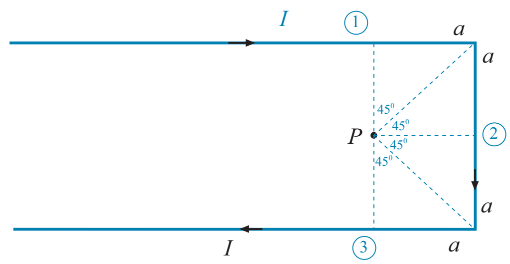
Note, the $s \le R$ and $s > R$ results agree at $s = R$.

4. A very long wire has two right–angle bends in it and carries current I, as shown. The point P lies in the same plane as the wire; it lies at a distance a from the sides of the wire, as shown



a) Find the magnitude and direction of the magnetic field at P. (Hint: Explicit integration is not needed.)

Divide up the wire into 3 parts as shown; for each part of the wire we can use the rule for the B field due to a wire segment (given at back of exam),



$$B == \frac{\mu_0 I}{4\pi s} |\sin\theta_2 - \sin\theta_1|$$

For part 1, with $\theta_2 = 90°$ and $\theta_1 = -45°$ (it doesn't matter which is which here) we get

$$B_1 = \frac{\mu_0 I}{4\pi a}\left(1 + \frac{1}{\sqrt{2}}\right)$$

and the direction is *into the page*. This is also the contribution of segment 3.

For segment 2 use $\theta_2 = +45°$ and $\theta_1 = -45°$, then

$$B_2 = \frac{\mu_0 I}{4\pi a}(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}) = \frac{\mu_0 I}{4\pi a}\left(\frac{2}{\sqrt{2}}\right)$$

and the direction is likewise into the page.

The total field is

$$B_1 + B_2 + B_3 = \frac{\mu_0 I}{4\pi a}\left(2 + \frac{4}{\sqrt{2}}\right) \qquad \text{into the page}$$

b) If $I = 5.0$ A and $a = 2.0$ cm, what is the strength of the magnetic field at P?

For $I = 5.0$ A and $a = 2.0$ cm, we get

$$B = \frac{(4\pi \times 10^{-7}\,\frac{\mathrm{N}}{\mathrm{A}^2})(5.0\ \mathrm{A})}{4\pi(2.0 \times 10^{-2}\ \mathrm{m})}\left(2 + \frac{4}{\sqrt{2}}\right) = 1.2 \times 10^{-4}\ \mathrm{T}$$

5. A magnetic field in a certain region of space is uniform and points in the y direction: $\mathbf{B} = B\hat{\mathbf{y}}$.

Find an expression for the magnetic vector potential. Comment on the uniqueness of this answer.

Since $\mathbf{B} = \nabla \times \mathbf{A}$, that is,

$$\mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix}$$

We see that we can get the given $\mathbf{B}$ field with

$$A_y = A_z = 0, \quad \frac{\partial A_x}{\partial z} = B \quad \Rightarrow \quad A_x = zB, \quad \text{so} \quad \mathbf{A} = zB\hat{\mathbf{x}}$$

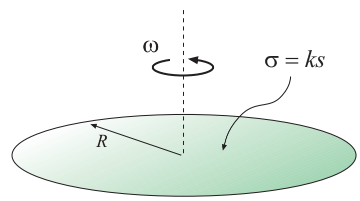
But we can also get it with

$$A_x = A_y = 0, \quad -\frac{\partial A_z}{\partial x} = B \quad \Rightarrow \quad A_z = -xB, \quad \text{so} \quad \mathbf{A} = -xB\hat{\mathbf{z}}$$

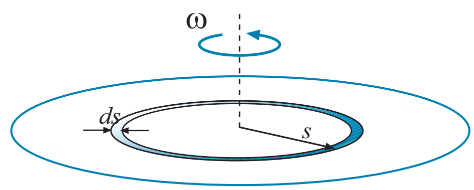
Both of these solutions give $\nabla \times \mathbf{A} = \mathbf{B}$ for the given $\mathbf{B}$, and also $\nabla \cdot \mathbf{A} = 0$. So the vector potnetial has a remaining (non-trivial!) ambiguity even after imposing the condition $\nabla \cdot \mathbf{A} = 0$.

6. A rotating disk of radius R has a charge density given by $\sigma = ks$. The disk rotates with angular speed ω.

Find the magnetic moment of the rotating disk.



Consider a thin ring of the disk at radius s of width ds. We can reason out the current dI for this ring as follows:



Charge contained: $\quad dq = (2\pi s)ds\sigma = 2\pi ks^2 ds$

The linear charge density of the rotating ring is:

$$d\lambda = \frac{dq}{\text{Circ}} = \frac{2\pi ks^2\, ds}{2\pi s} = ks\, ds$$

Speed of the moving charge is $v = \omega s$ so that

$$dI = v d\lambda = k\omega s^2 ds$$

Area of the loop is πs^2 so

$$dm = (\pi s^2) dI = k\pi\omega s^4 ds$$

Now integrate from $s = 0$ to $s = R$:

$$m = \int a\, dI = k\pi\omega \int_0^R s^4\, ds = k\pi\omega \frac{R^5}{5}$$

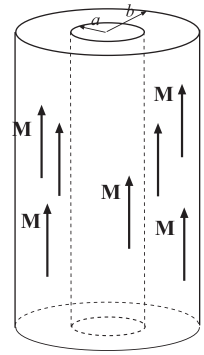
7. What do we mean when we say a medium acquires a magnetization $\mathbf{M}$?

The quantity $\mathbf{M}$ gives the *magnetic dipole moment per unit volume of the material*. The currents in the material coordinate themselves so as to give a *net* magnetic dipole for a volume $d\tau$ of the material. The magnetic moment of this volume is then

$$d\mathbf{m} = \mathbf{M} d\tau$$

The vector function $\mathbf{M}$ can depend on location.

8. A very long cylindrical shell has outer radius b and inner radius a. The material carries a uniform magnetic polarization $\mathbf{M}$ which points along the direction of the cylinder's axis.



a) Find the volume and surface bound currents $\mathbf{J}_b$ and $\mathbf{K}_b$. (Be careful with the directions for $\mathbf{K}_b$.)

Within the volume of the material, $\mathbf{M}$ is uniform, so

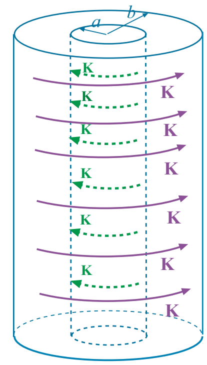
$$\mathbf{J}_b = \nabla \times \mathbf{M} = 0$$

On the inner surface, $\hat{\mathbf{n}} = -\hat{\mathbf{s}}$, so

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} = M(\hat{\mathbf{z}} \times (-\hat{\mathbf{s}})) = -M\hat{\boldsymbol{\phi}}$$

On the outer surface, $\hat{\mathbf{n}} = \hat{\mathbf{s}}$, so

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} = M(\hat{\mathbf{z}} \times \hat{\mathbf{s}}) = +M\hat{\boldsymbol{\phi}}$$



b) Find the direction and magnitude of the magnetic field $\mathbf{B}$ for $0 < s < a$, $a < s < b$ and $b < s$.

The surface currents create a $\mathbf{B}$ field equivalent to solenoids with currents going in opposite directions! Use the equivalence $K = nI$ for a solenoid.

For $s < a$ the fields from the solenoids cancel, so $B = 0$.

For $a < s < b$ we get only the field from the outer solenoid, so

$$B_z = \mu_0 n I \hat{\mathbf{z}} = \mu K \hat{\mathbf{z}} = \mu_0 M \hat{\mathbf{z}}$$

For $b < s$ we get no field from either solenoid, so $B = 0$.

Useful Equations

$$\int_{\mathbf{a}}^{\mathbf{b}} (\nabla T)\cdot d\mathbf{l} = T(\mathbf{b}) - T(\mathbf{a}) \qquad \int_{\mathcal{V}} (\nabla\cdot\mathbf{v}) d\tau = \oint_{\mathcal{S}} \mathbf{v}\cdot d\mathbf{a} \qquad \int_{\mathcal{S}} (\nabla\times\mathbf{v})\cdot d\mathbf{a} = \oint_{\mathcal{P}} \mathbf{v}\cdot d\mathbf{l}$$

Spherical:

$$d\mathbf{l} = dl_r\,\hat{\mathbf{r}} + dl_\theta\,\hat{\boldsymbol{\theta}} + dl_\phi\,\hat{\boldsymbol{\phi}} \qquad d\tau = r^2\ \sin\theta\, dr\, d\theta\, d\phi$$

Gradient:

$$\nabla T = \frac{\partial T}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial T}{\partial\theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial\phi}\hat{\boldsymbol{\phi}} \tag{1}$$

Divergence:

$$\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} \tag{2}$$

Curl:

$$\nabla\times\mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}} \tag{3}$$

Laplacian:

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2} \tag{4}$$

Cylindrical:

$$d\mathbf{l} = dl_s\,\hat{\mathbf{s}} + dl_\phi\,\hat{\boldsymbol{\phi}} + dl_z\,\hat{\mathbf{z}} \qquad d\tau = s\, ds\, d\phi\, dz$$

Gradient:

$$\nabla T = \frac{\partial T}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial T}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}} \tag{5}$$

Divergence:

$$\nabla\cdot\mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z} \tag{6}$$

Curl:

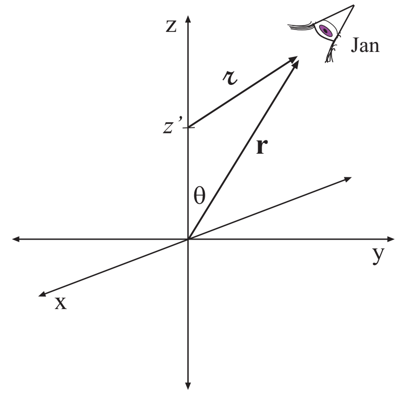
$$\nabla\times\mathbf{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}} \tag{7}$$

Laplacian:

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial\phi^2} + \frac{\partial^2 T}{\partial z^2} \tag{8}$$

More Math:

In the figure at the right,



$$\mathscr{r} = \sqrt{r^2 + z'^2 - 2rz'\cos\theta}$$

If $x < 1$ then

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \cdots$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = 2\cos^2\theta - 1$$

$$\int \sin^2 x\, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x$$

$$\int \cos^2 x\, dx = \frac{1}{2}x + \frac{1}{4}\sin 2x$$

$$\int_0^a \sin(n\pi y/a)\sin(n'\pi y/a)\, dy = \begin{cases} 0, & \text{if } n' \neq n \\ \frac{a}{2} & \text{if } n' = n \end{cases}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r'}{r}\right)^n P_n(\cos\theta') \qquad V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

$$P_0(x) = 1 \qquad P_1(x) = x \qquad P_2(x) = (3x^2-1)/2 \qquad P_3(x) = (5x^3 - 3x)/2$$

$$\int_{-1}^{1} P_l(x)P_{l'}(x)dx = \int_0^\pi P_l(\cos\theta)P_{l'}(\cos\theta)\sin\theta\, d\theta = \begin{cases} 0 & \text{if } l' \neq l \\ \frac{2}{2l+1} & \text{if } l' = l \end{cases}$$

Physics:

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}^2}\,\hat{\mathscr{r}} \qquad \mathbf{F} = Q\mathbf{E} \qquad \mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q\,\hat{\mathscr{r}}}{\mathscr{r}^2} \qquad V(\mathbf{r}) = -\int_{\mathcal{O}}^{\mathbf{r}} E\cdot d\mathbf{l}$$

$$\nabla\times\mathbf{E} = 0 \qquad \mathbf{E} = -\nabla V \qquad -\nabla^2 V = \nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0} \qquad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r}')}{\mathscr{r}}\,d\tau'$$

$$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{\sigma}{\epsilon_0} \qquad \mathbf{E}^{\parallel}_{\text{above}} = \mathbf{E}^{\parallel}_{\text{below}} \qquad W = \frac{1}{8\pi\epsilon_0}\sum_{\substack{i,j\\ i\neq j}}^{n}\frac{q_i q_j}{\mathscr{r}_{ij}}$$

$$W = \frac{1}{2}\int\rho V\,d\tau = \frac{\epsilon_0}{2}\int E^2\,d\tau \qquad \mathbf{f} = \frac{1}{2\epsilon_0}\sigma^2\hat{\mathbf{n}} \qquad P = \frac{\epsilon_0}{2}E^2 \qquad C \equiv \frac{Q}{V}$$

$$\mathbf{p} \equiv \int \mathbf{r}'\rho(\mathbf{r}')\,d\tau' \qquad V_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2} \qquad \mathbf{E}_{\text{dip}}(r,\theta) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}})$$

$$\mathbf{p} = \alpha\mathbf{E} \qquad \mathbf{N} = \mathbf{p}\times\mathbf{E} \qquad \mathbf{F} = (\mathbf{p}\cdot\nabla)\mathbf{E} \qquad U = -\mathbf{p}\cdot\mathbf{E}$$

$$\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}} \qquad \rho_b = -\nabla\cdot\mathbf{P} \qquad \mathbf{P} = \epsilon_0\chi_e\mathbf{E}$$

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon\mathbf{E} \qquad \nabla\cdot\mathbf{D} = \rho_f \qquad \oint\mathbf{D}\cdot d\mathbf{a} = Q_{f,\text{enc}}$$

$$\mathbf{F}_{\text{mag}} = Q(\mathbf{v}\times\mathbf{B}) \qquad \mathbf{F}_{\text{mag}} = \int I(d\mathbf{l}\times\mathbf{B}) \qquad \mathbf{K} \equiv \frac{d\mathbf{I}}{dl_\perp} \qquad \mathbf{J} \equiv \frac{d\mathbf{I}}{da_\perp} \qquad \nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{I}\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}dl' = \frac{\mu_0}{4\pi}I\int\frac{d\mathbf{l}'\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} \qquad \mu_0 = 4\pi\times10^{-7}\,\frac{\text{N}}{\text{A}^2} \qquad 1\ \text{T} = 1\,\frac{\text{N}}{\text{A}\cdot\text{m}}$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{B} = \mu_0\mathbf{J} \qquad \oint\mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \qquad \mathbf{B} = \nabla\times\mathbf{A}$$

$$\nabla\cdot\mathbf{A} = 0 \qquad \nabla^2\mathbf{A} = -\mu_0\mathbf{J} \qquad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}')}{\mathscr{r}}\,d\tau'$$

$$B^\perp_{\text{above}} = B^\perp_{\text{below}} \qquad \mathbf{B}_{\text{above}}-\mathbf{B}_{\text{below}} = \mu_0(\mathbf{K}\times\hat{\mathbf{n}}) \qquad \mathbf{A}_{\text{above}} = \mathbf{A}_{\text{below}} \qquad \frac{\partial\mathbf{A}_{\text{above}}}{\partial n}-\frac{\partial\mathbf{A}_{\text{below}}}{\partial n} = -\mu_0\mathbf{K}$$

$$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2} \qquad \text{where} \qquad \mathbf{m} \equiv I\int d\mathbf{a} = I\mathbf{a}$$

$$\mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\boldsymbol{\phi}} \qquad \mathbf{B}_{\text{dip}}(\mathbf{r}) = \nabla\times\mathbf{A} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{\mathbf{r}}+\sin\theta\,\hat{\boldsymbol{\theta}})$$

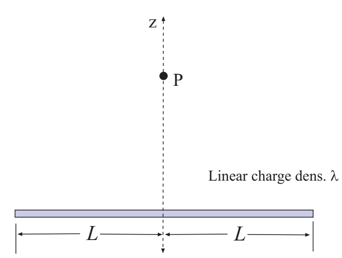
$$\mathbf{N} = \mathbf{m}\times\mathbf{B} \qquad \mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_{\mathcal{V}}\frac{\mathbf{J}_b(\mathbf{r}')}{\mathscr{r}}\,d\tau' + \frac{\mu_0}{4\pi}\oint_{\mathcal{S}}\frac{\mathbf{K}_b(\mathbf{r}')}{\mathscr{r}}\,da' \qquad \text{where} \qquad \mathbf{J}_b = \nabla\times\mathbf{M} \qquad \text{and} \qquad \mathbf{K}_b = \mathbf{M}\times\hat{\mathbf{n}}$$

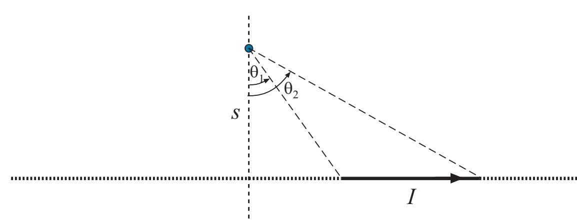
$$\mathbf{J} = \mathbf{J}_b + \mathbf{J}_f \qquad \mathbf{H} \equiv \frac{1}{\mu_0}\mathbf{B}-\mathbf{M}$$

Specific Results:

$$E_z = \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}}$$



$$B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$$



$$B = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}$$