Phys 4900mp, Spring 2013
Problem Set #1

1. Using whatever software you want, make a plot of $J_3(x)$ and $J_4(x)$ (together on the same graph) from $x = 0$ out to $x = 10$.

2. Using whatever software you want, make a plot of $Y_1(x)$ from $x = 0.5$ out to $x = 10$.

3. Using whatever software you want, make a surface plot of the function

$$z(s, \phi) = J_1(\alpha_{11} s) \cos\phi$$

(which satisfies the condition $z(1, \phi) = 0$) for s from 0 to 1 and all ϕ. Use $\alpha_{11} = 3.8317$.

4. (a) *(Schaum FA) 6.51 (a))* Show that

$$\int_0^\infty e^{-ax} J_0(bx)\, dx = \frac{1}{\sqrt{a^2 + b^2}}$$

(b) (Easier.) *(Schaum FA 6.52)* Use (a) or any other method to show that

$$\int_0^\infty J_0(x)\, dx = 1$$

5. Find the lowest three (electromagnetic) resonant frequencies for a conducting cylindrical resonating cavity which has a very small height and radius 5.00 cm.

6. Find the lowest three energy eigenvalues for an electron moving in a two-dimensional box of radius 1.50×10^{-8} m.

7. Suppose the lowest resonant frequency of a drumhead of radius 0.420 m is 100.0 Hz. What is the speed of waves on the membrane? Find the next two lowest resonant frequencies of the drumhead.

8. *(Schaum FA 6.72)* Solve

$$4xy'' + 4y' + y = 0$$

9. Show that the solution of

$$\frac{d^2y}{dx^2} + \lambda x y = 0$$

which is regular at $x = 0$ is a multiple of $x^{1/2} J_{1/3}\left(\frac{2}{3}\sqrt{\lambda x^3}\right)$.

This is the problem that (I assume) all of you wanted to do! The DE is of the form of the Schrödinger equation for a linear potential, and in the QM class we *called* the solutions Airy functions but also claimed they were related to Bessel functions.

This is problem 18.9 in RHB, who want you to grind out the answer, but you can use the formula given in the notes for the general DE which is reducible to Bessel's DE.

RHB = Riley, Hobson, Bence, Mathematical Methods for Physics and Engineering, 3rd ed
Schaum FA = Spiegel, Fourier Analysis