Phys 3610, Fall 2009
Problem Set #5, Hint-o-licious Hints

1. *Taylor*, **8.6** Use the relations on p. 299 Substitute for $\mathbf{r}_1$ and $\mathbf{p}_1 = m_1\dot{\mathbf{r}}_1$ for the values they have in the CM frame, i.e.

$$\mathbf{r}_1 = \mathbf{R} + \frac{m_2}{M}\mathbf{r} \ .$$

Do some algebra and use

$$\mathbf{L} = \mathbf{r} \times \mu\dot{\mathbf{r}}$$

2. *Taylor*, **8.10** I get, with $m_1 = m_2 = m$,

$$\mathcal{L} = m\dot{\mathbf{R}}^2 + \frac{m}{4}\dot{\mathbf{r}}^2 - kR^2 - \frac{k}{2}(\alpha + \tfrac{1}{4})r^2$$

4. *Taylor*, **8.16** This is similar to the case of the hyperbola that I gave out in class except that here $1 - \epsilon^2$ is positive.

5. *Taylor*, **8.23** With the force law as given in the problem, the DE for u is

$$u'' = -u - \frac{m}{\ell^2 u^2}(-ku^2 + \lambda u^3)$$

which you can get into the form

$$u'' = -\left(1 + \frac{mk}{\ell^2}\right)\left(u - \frac{mk}{\ell^2 + m\lambda}\right)$$

You can make the solution clearer by using $w = u + C$ where C is some appropriate constant and eventually get

$$r = u^{-1} = \frac{(\ell^2 + m\lambda)/mk}{1 + \frac{A(\ell^2+m)\lambda}{mk}\cos(\beta\phi)}$$

where

$$\beta = \sqrt{1 + \frac{m\lambda}{\ell^2}}$$

which is indeed of the form

$$r = \frac{c}{1 + \epsilon\cos(\beta\phi)}$$

and if $\epsilon < 1$ the orbit is bounded but in general since β could be anything the orbit may not be closed. You can show that it is only for the cases that

$$\beta = \frac{m}{n} \qquad \text{i.e. a rational number}$$

that one gets a closed orbit, that is one where r has *some* period.

6. *Taylor*, **8.28** Fairly easy. Use the formula

$$r(\phi) = \frac{c}{1 + \epsilon\cos\phi}$$