Phys 2112, Spring 2011
Problem Set #3

1. For the motion given by

$$x = (2\text{ m})\cos(3\text{ s}^{-1}t) \qquad y = (2\text{ m})\sin(3\text{ s}^{-1}t)$$

find the speed of the particle and the magnitude of its acceleration. How would you describe the direction of particle's velocity and acceleration?

2. Write down the equations for $x(t)$ and $y(t)$ for a particle which goes around the origin (counterclockwise) in a circle of radius 0.800 m (centered on the origin) with a constant speed of $2.0\,\frac{\text{m}}{\text{s}}$.

3. How would the answer for 2 be different if I wanted the particle to go around the origin in a *clockwise* direction?

4. Show that the trajectory specified by the equations of motion

$$x = a\cos(\omega t) \qquad y = b\sin(\omega t)$$

where a and b are positive constants is an ellipse centered at the origin. (Hint: can you find a fairly simple relation between x and y using a trig identity?)

5. Show (by substituting for r and ϕ and doing some math) that the trajectory given in polar coordinates by

$$r = \frac{c}{1+\cos\phi}$$

(where c is some positive constant) is a parabola.

6. Using the expressions for the polar unit vectors

$$\hat{\mathbf{r}} = \cos\phi\,\hat{\mathbf{i}} + \sin\phi\,\hat{\mathbf{j}} \qquad \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{i}} + \cos\phi\,\hat{\mathbf{j}}$$

Find expressions for the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\phi}}$ in terms of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ (and ϕ).

7. In polar coordinates, demonstrate that the expression for the velocity is

$$\mathbf{v} = \dot{r}\,\hat{\mathbf{r}} + r\dot{\phi}\,\hat{\boldsymbol{\phi}}$$

8. In polar coordinates, demonstrate that the expression for the acceleration is

$$\mathbf{a} = (\ddot{r} - r\dot{\phi}^2)\,\hat{\mathbf{r}} + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\,\hat{\boldsymbol{\phi}}$$