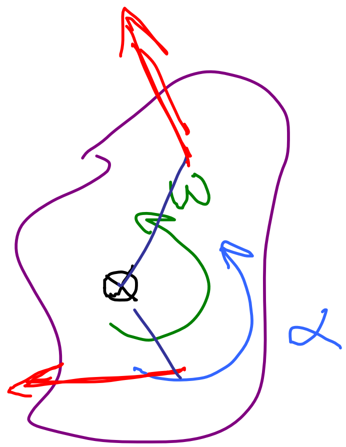
Phys 2110-4 3/26/12

Note Title 3/26/2012

Rotational Mechanics

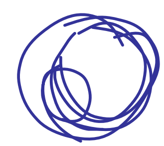


$$\boxed{\tau_{net} = I\alpha}$$

$$I = \sum m r^2$$

$$I = \frac{2}{5} m R^2$$

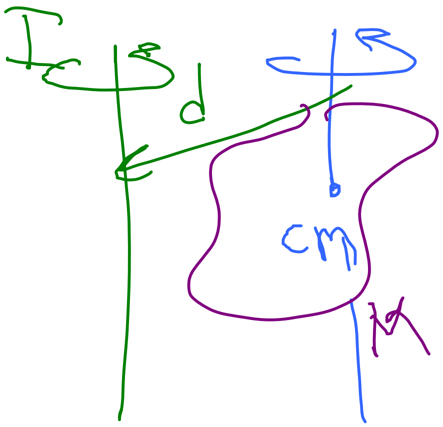
Disk $I = \frac{1}{2} m R^2$



$$\boxed{K_{rot} = \frac{1}{2} I \omega^2}$$

$$K = \frac{1}{2} m v^2$$

Parallel Axis Thm



I_{cm}

(10.17)

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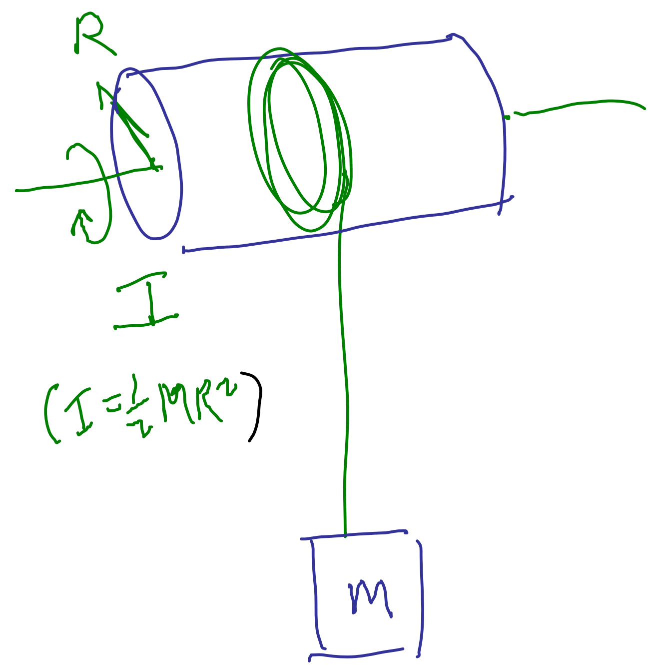
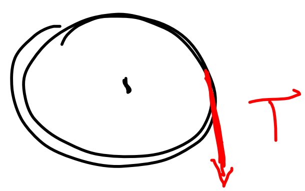
$$I = I_{cm} + M d^2$$

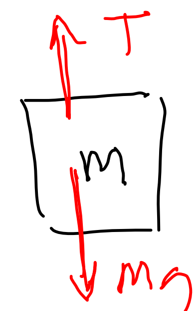
Example: p.164 ex 10.9

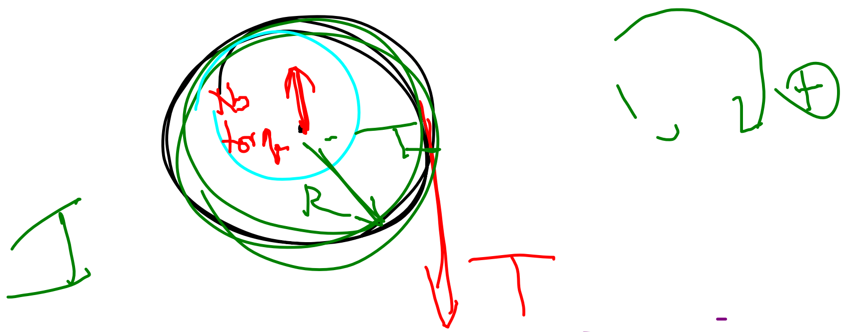
Solid cylinder mom of in. I, string wrapped around it, hang mass m. Release it. Find accel of the mass.

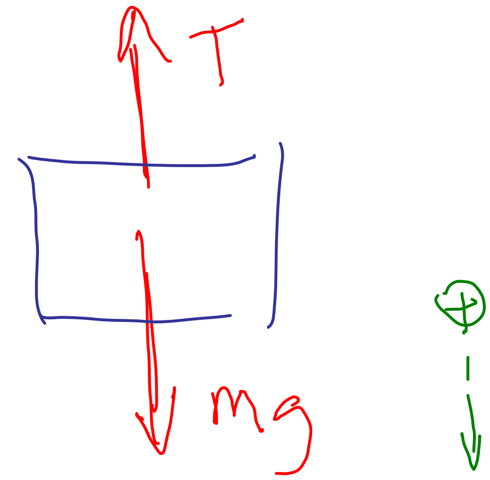


Free-body diagram, draw damn picture.









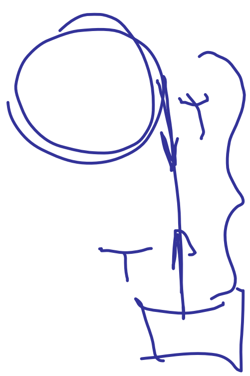
$$\boxed{\tau = TR = I\alpha}$$

$$\boxed{mg - T = ma}$$

$$\boxed{a = a_T = R\alpha}$$

a_T

a



τ = torque

T = tension

$$TR = I\alpha \qquad mg - T = ma$$

$$a = R\alpha$$

$$TR = I\frac{a}{R} \quad \Rightarrow \quad \boxed{T = I\frac{a}{R^2}}$$

$$mg - I\frac{a}{R^2} = ma$$

$$mg = ma + I\frac{a}{R^2} = a\left(m + \frac{I}{R^2}\right)$$

$$a = \frac{mg}{\left(m + I/R^2\right)}$$

If

$$I = \frac{1}{2} M R^2$$

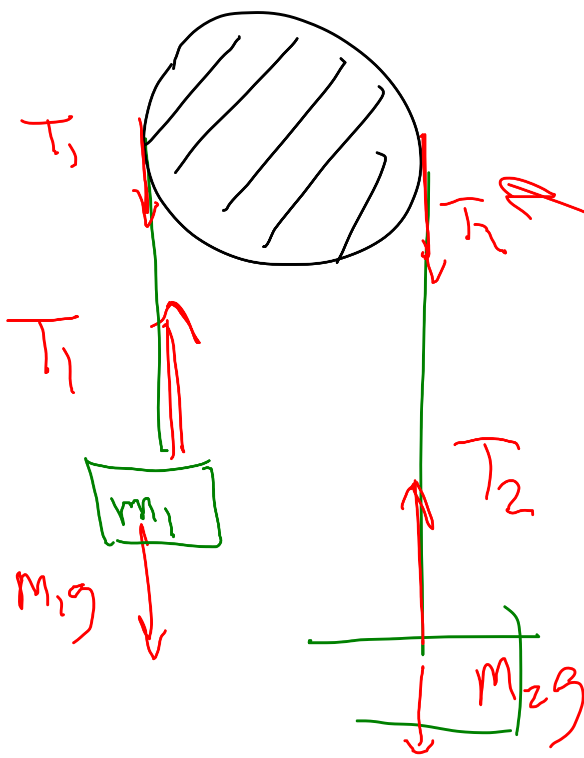
$$a = \frac{mg}{\left(m + M/2\right)}$$

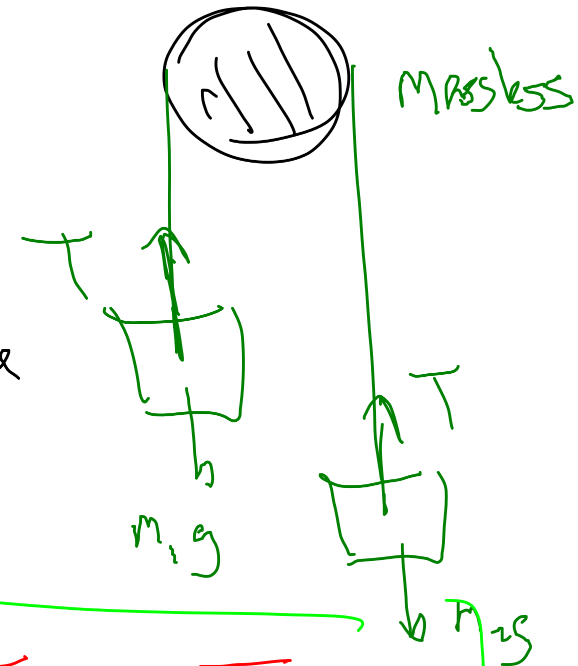
$M = 0$

Other pulley problems:

If pulley not massless

tensions not same



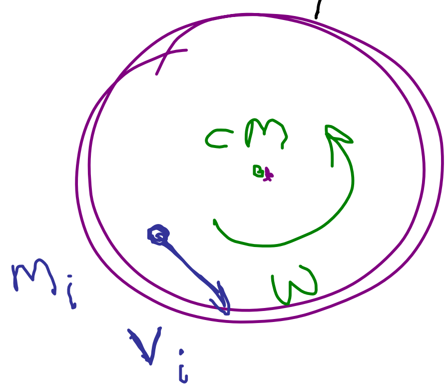


$$\tau_{\text{net, clockwise}} = T_2 R - T_1 R$$

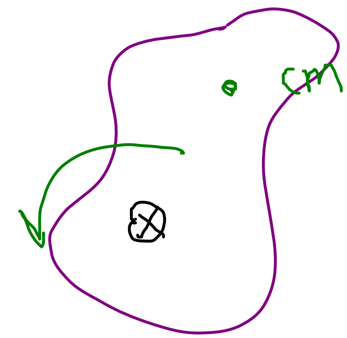
$$= (T_2 - T_1) R$$

$$= I\alpha$$

Energy in a rotating system.



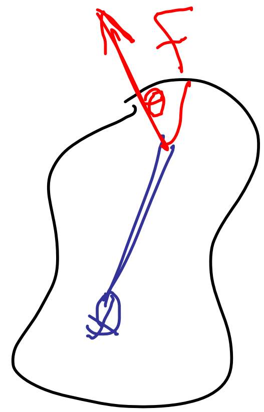
$$v_i = r_i \omega$$



$$KE_{\substack{\text{entire}\\ \text{wheel}}} = \sum_i \tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2} \sum m_i (r_i \omega)^2$$

$$= \tfrac{1}{2} \left(\sum_i m_i r_i^2 \right) \omega^2 = \tfrac{1}{2} I \omega^2$$

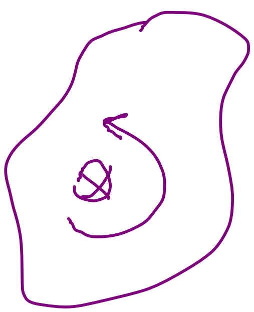
$$K = \tfrac{1}{2} m v^2$$

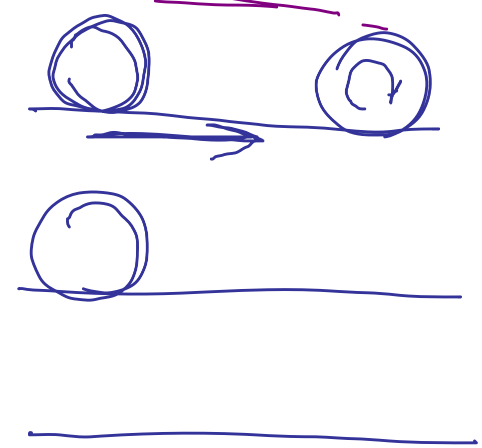


$$\tau = rF\sin\theta$$

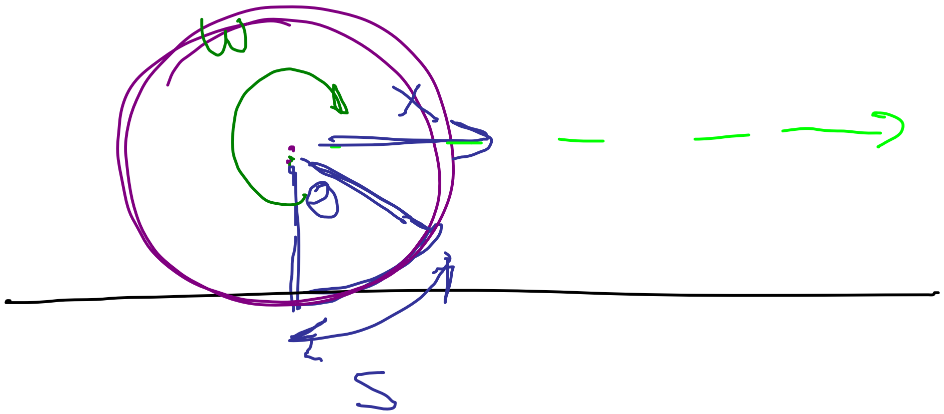
$$W_{rot} = \int_{\theta_1}^{\theta_2} \tau(\theta)\, d\theta = \Delta K_{rot}$$

$\Delta U = 0$





Rolling w/o Slipping.



$$s = x = R\theta$$

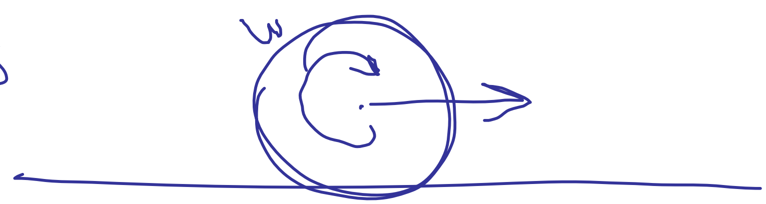
$$\Delta x = R\,\Delta\theta$$

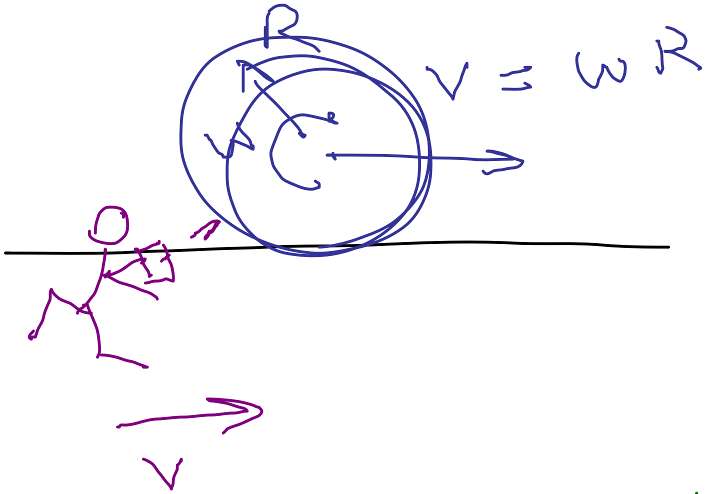
$$\frac{\Delta x}{\Delta t} = R\frac{\Delta\theta}{\Delta t}$$

$$V_{cm} = R\omega$$

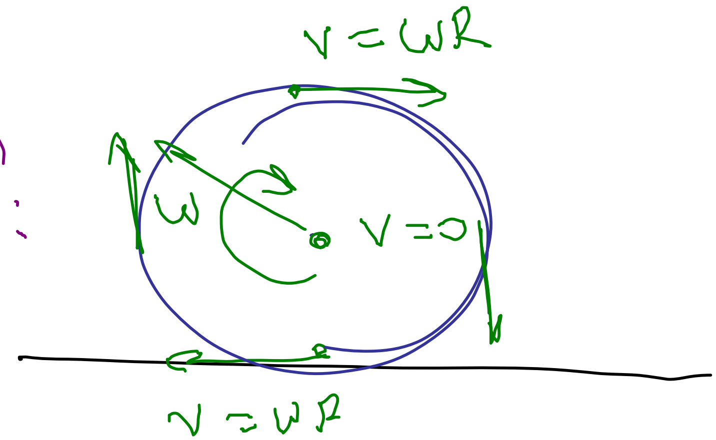
$$a_{cm} = R\alpha$$

Diff parts of ball
have diff speeds

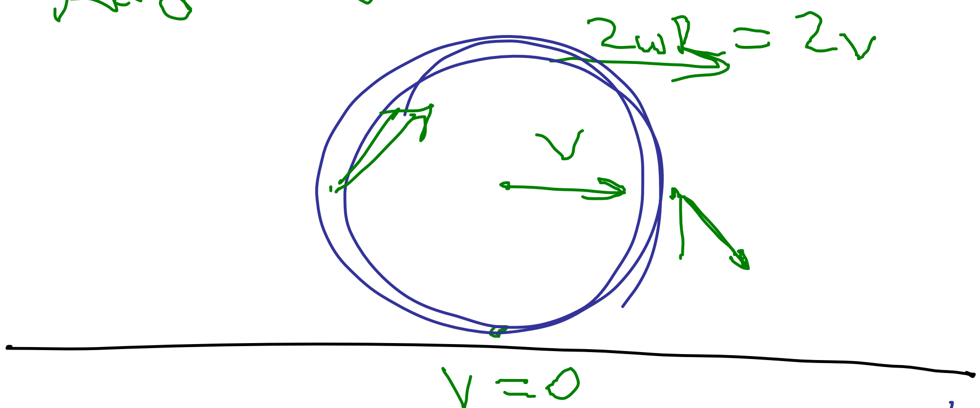




Little man sees:



Add +v onto these velocities



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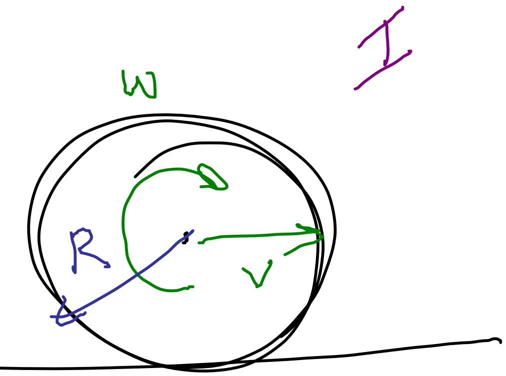
Energy of rolling object

$$K_{roll} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= K_{trans} + K_{rot}$$



$$v = R\omega$$

$$\omega = v/R$$

10.39 What fraction of a solid uniform disk's kinetic energy is rotational if it's rolling w/o slipping.

$$K = K_{tran} + K_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}\underbrace{\left(\frac{1}{2}M\cancel{R^2}\right)}_{I}\left(\frac{v}{\cancel{R}}\right)^2$$

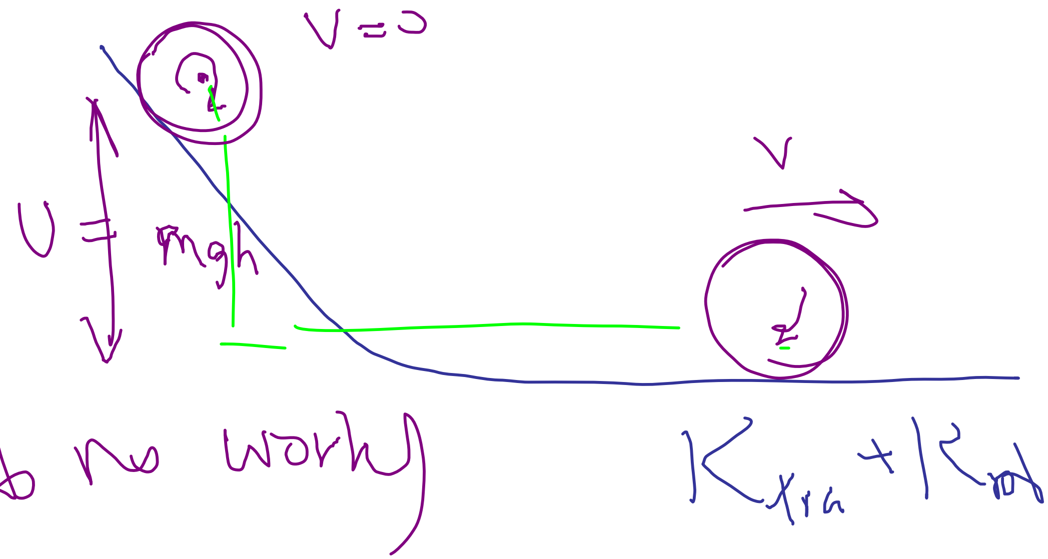
$\omega = v/R$

$$= \frac{1}{2}mv^2 + \underbrace{\frac{1}{4}Mv^2}_{K_{rot}} = \frac{3}{4}Mv^2$$

$$frac = K_{rot}/K = \frac{\frac{1}{4}Mv^2}{\frac{3}{4}Mv^2} = \frac{1}{3}$$

Rolling Motion

Cons of energy.
(fric forces do no work)



$$E_1 = E_2$$