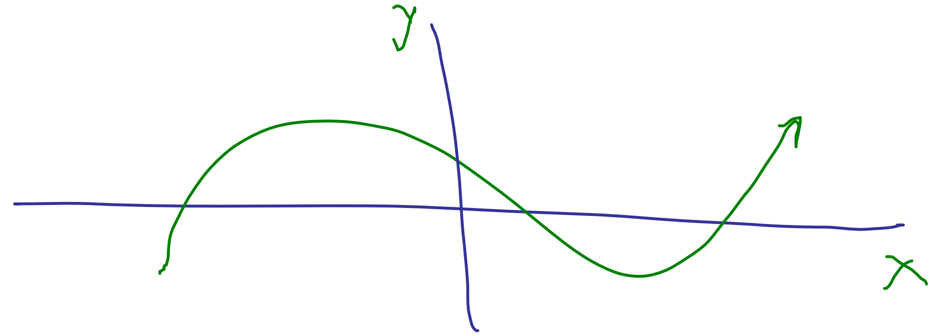
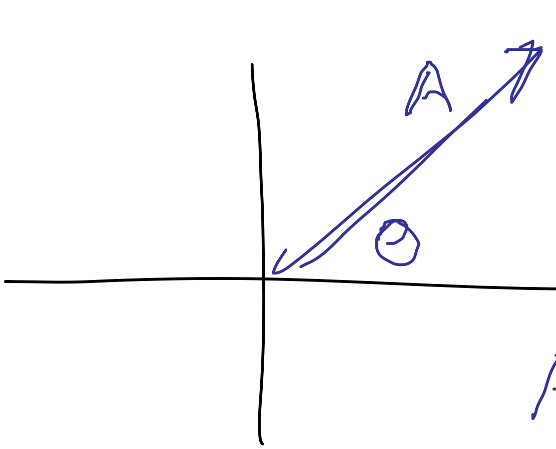
Phys 2110-5 9/10/12

Note Title 9/10/2012

2D Motion

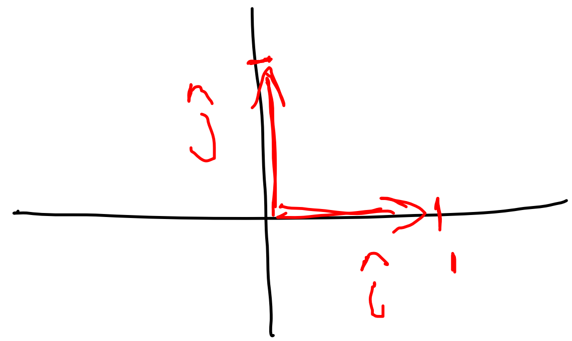
$x(t)$ $y(t)$

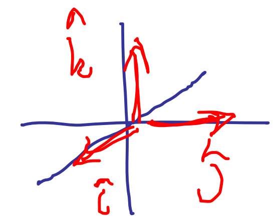




$\vec{A}$

A_x, A_y





$\hat{x}, \hat{y}$

2D motion

$\vec{r} = x\hat{i} + y\hat{j}$

$\Delta\vec{r}$ = displacement

$= \Delta x\,\hat{i} + \Delta y\,\hat{j}$

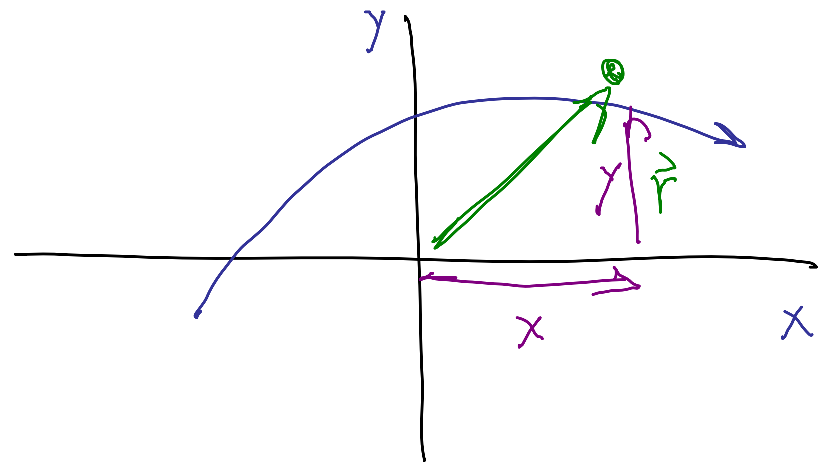
Average velocity

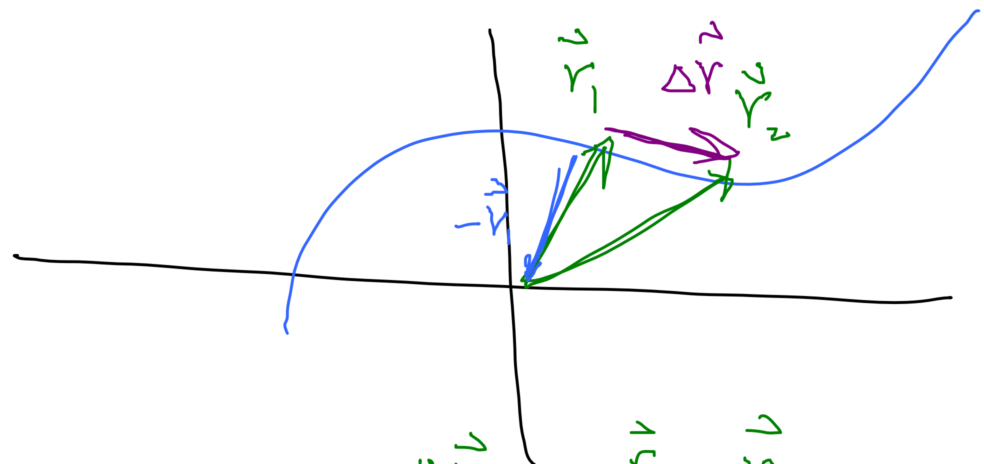
$\vec{v}_{avg} = \dfrac{\Delta\vec{r}}{\Delta t}$

$= \dfrac{\Delta x}{\Delta t}\hat{i} + \dfrac{\Delta y}{\Delta t}\hat{j}$

$v_{x\,avg}$

$v_{y\,avg}$





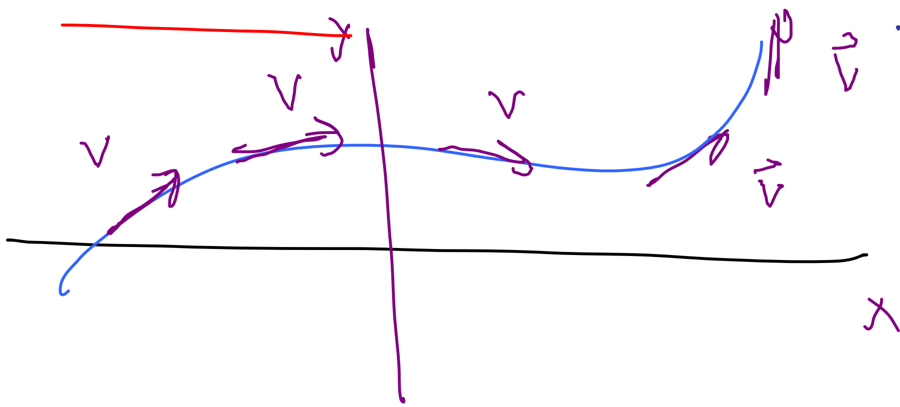
$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

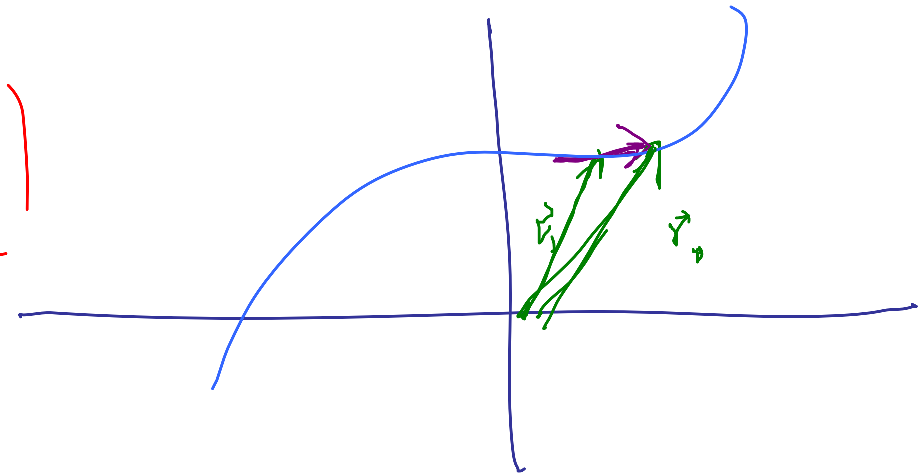
Instantaneous velocity

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = d\vec{r}/dt = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$= v_x \hat{i} + v_y \hat{j}$$

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt}$$





Vel vector is tangent to path

If velocity is constant (both components are constant

$$v_x = \frac{dx}{dt} = \text{const}$$

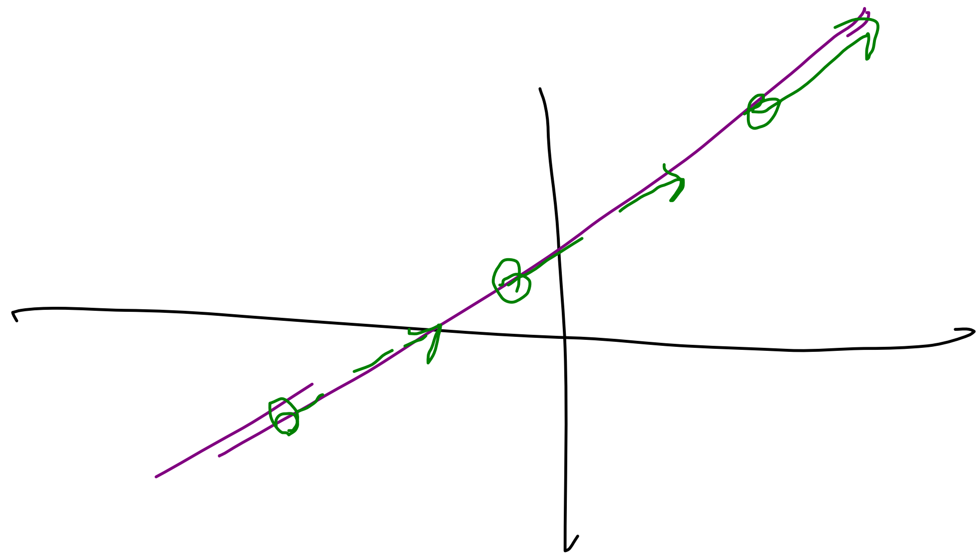
$$v_y = \frac{dy}{dt} = \text{const}$$

$$x = x_0 + v_x t$$

$$y = y_0 + v_y t$$

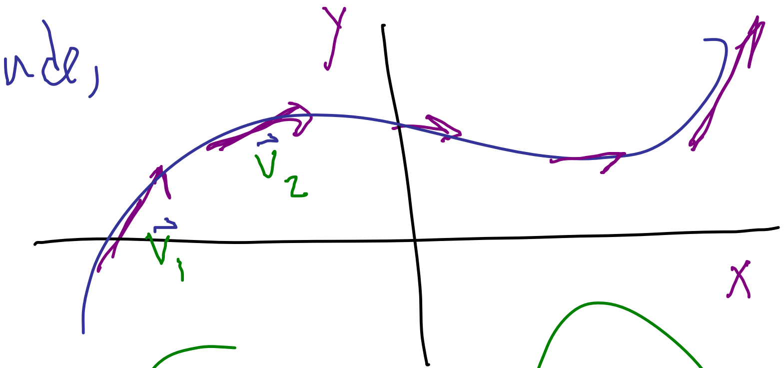
2 components of init. positn

comp's of velocity.



Velocity (vector) changes with time

Velocity can keep same magnitude, but direction changes!



Acceleration:

Avg. accel. $\vec{a}_{avg} = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\Delta v_x}{\Delta t}\hat{i} + \dfrac{\Delta v_y}{\Delta t}\hat{j}$

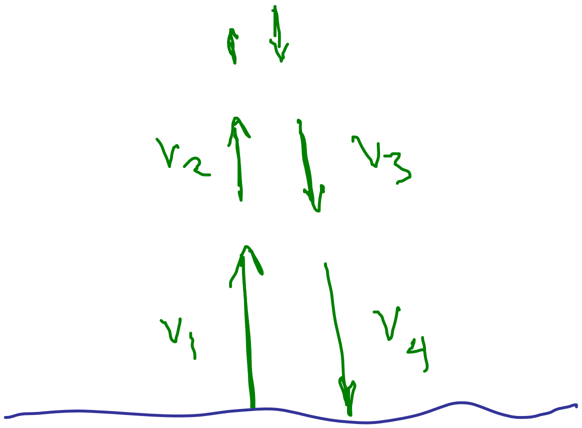
$a_{x,\,avg}$ $\quad$ $a_{y,\,avg.}$

Instantaneous accel.

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

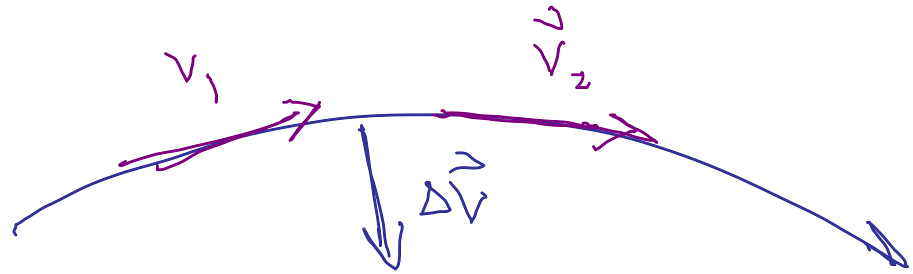
$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

$$a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$$

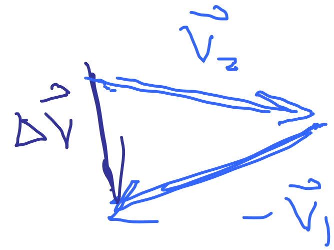


$\vec{v}_2 - \vec{v}_1 = \downarrow$

$\vec{v}_4 - \vec{v}_3 = \downarrow$



$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$



$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$

Here, $\Delta \vec{v}$ (also $\vec{a}$) points <u>inward</u>!

Circular Motion

Constant speed

Speed = s

$s = |\vec{v}|$

$= \sqrt{v_x^2 + v_y^2}$

3.30 The position of an object as a fn of time is $\vec{r} = \underbrace{(3.2t + 1.8t^2)}_{x}\,\hat{i} + \underbrace{(1.7t - 2.4t^2)}_{y}\,\hat{j}$

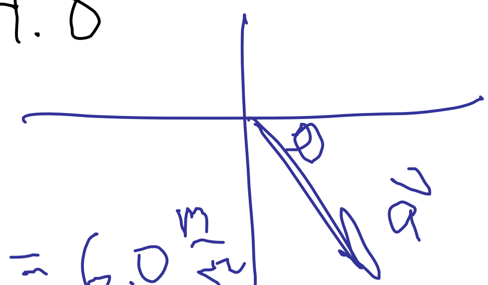
(in m, t in seconds)

Find object's accel. vector.

$$V_x = \frac{dx}{dt} = 3.2 + 3.6t \qquad V_y = 1.7 - 4.8t$$

$$a_x = \frac{dv_x}{dt} = 3.6 \qquad a_y = -4.8$$

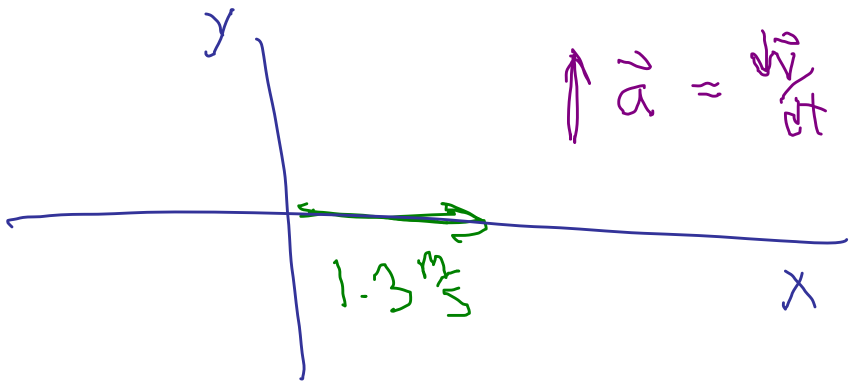
$$\vec{a} = 3.6\,\frac{m}{s^2}\,\hat{i} - 4.8\,\frac{m}{s^2}\,\hat{j}$$



$$|\vec{a}| = 6.0\,\frac{m}{s^2}$$

3.25 An object is moving in the x-direction at 1.3 $\frac{m}{s}$. Undergoes an acceleration of $\vec{a} = 0.52\,\frac{m}{s^2}\,\hat{j}$. Find its velocity after 4.4 s.

constant



$$\vec{a} = \frac{d\vec{v}}{dt} = 0.52\,\frac{m}{s^2}\,\hat{j}$$

$$\frac{dv_x}{dt} = 0 = a_x$$

v_x constant! $1.3\,\frac{m}{s}$

$$\frac{dv_y}{dt} = 0.52\,\frac{m}{s^2}$$

constant

$$v_y = \cancel{v_{0y}} + a_y t = \left(0.52\,\frac{m}{s^2}\right)t$$

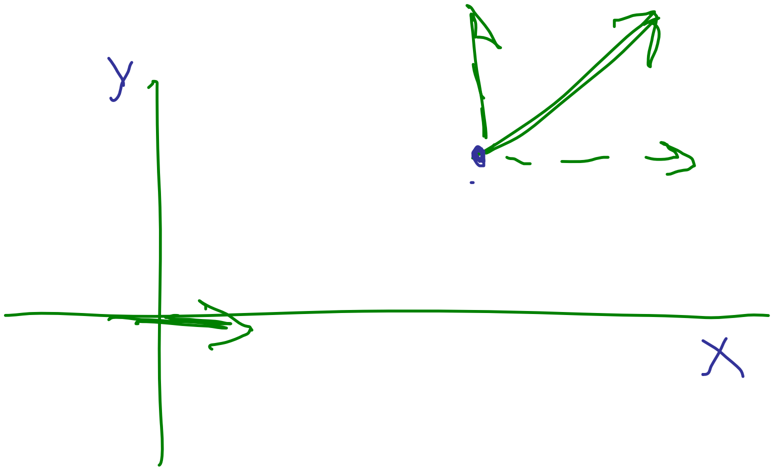
None!

at 4.4 s

$$= \left(0.52\,\frac{m}{s^2}\right)(4.4\,s)$$

$$= 2.28\,\frac{m}{s}$$

At 4.4 s, $v_x = 1.3\frac{m}{s}$ $v_y = 2.28\frac{m}{s}$



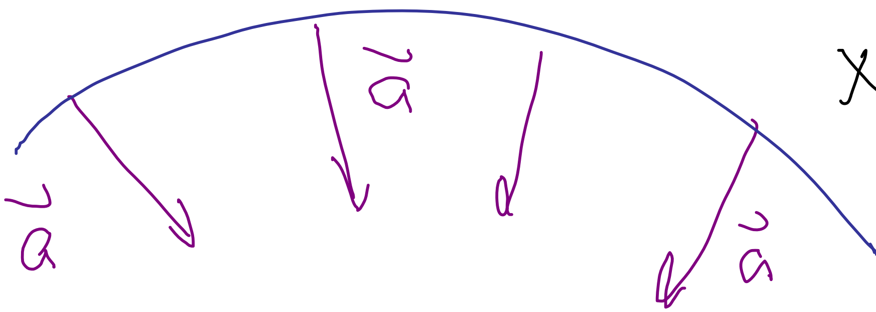
$\vec{v} = 1.3\frac{m}{s}\hat{i} + 2.28\frac{m}{s}\hat{j}$

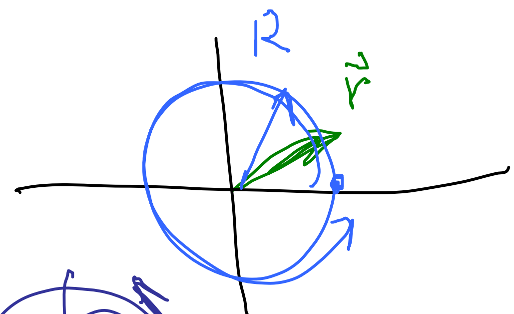
Mag & direction...

Circular motion:

$x = R\cos(\omega t)$

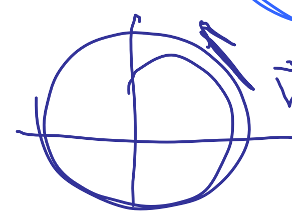
$y = R\sin(\omega t)$





Velocity: $v_x = -\omega R \sin \omega t$

$v_y = \omega R \cos \omega t$

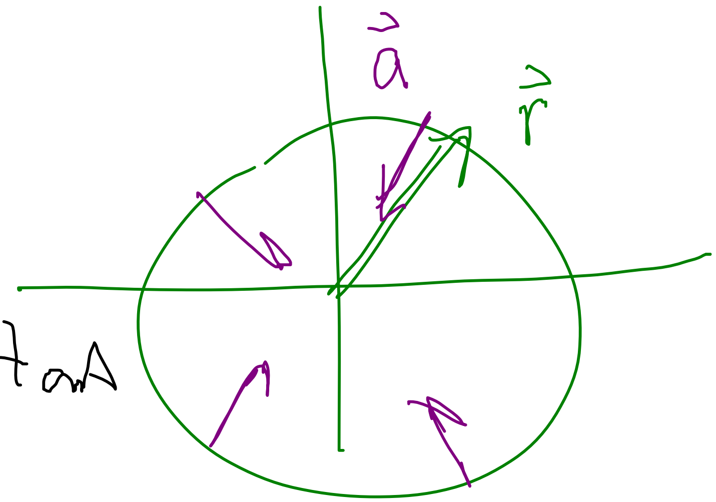


$$a_x = \frac{dv_x}{dt} = -\omega^2 R \cos(\omega t)$$

$$a_y = \frac{dv_y}{dt} = -\omega^2 R \sin(\omega t)$$

$$x = R\cos(\omega t)$$

$$y = R\sin(\omega t)$$



Special Case: $\vec{a}$ is constant

a_x is constant

a_y is constant

$$a_x = \frac{dv_x}{dt} \qquad v_x = a_x t + C$$

$$\boxed{V_x = V_{x0} + a_x t}$$

$$a_y = \frac{dv_y}{dt} \qquad \boxed{V_y = V_{y0} + a_y t}$$

Initial vel. components

$a_x \neq a_y$

$V_{x0} \neq V_{y0}$

or

2 ~~Eq~~ equation.

Integrate again:

$$\boxed{\begin{aligned} X &= X_0 + V_{x0} t + \tfrac{1}{2} a_x t^2 \\ Y &= Y_0 + V_{y0} t + \tfrac{1}{2} a_y t^2 \end{aligned}}$$

Again, diff. numbers!

Also get:

Constant accel!

$$\boxed{\begin{aligned} v_x^2 &= v_{x0}^2 + 2a_x(x - x_0) \\ v_y^2 &= v_{y0}^2 + 2a_y(y - y_0) \end{aligned}}$$

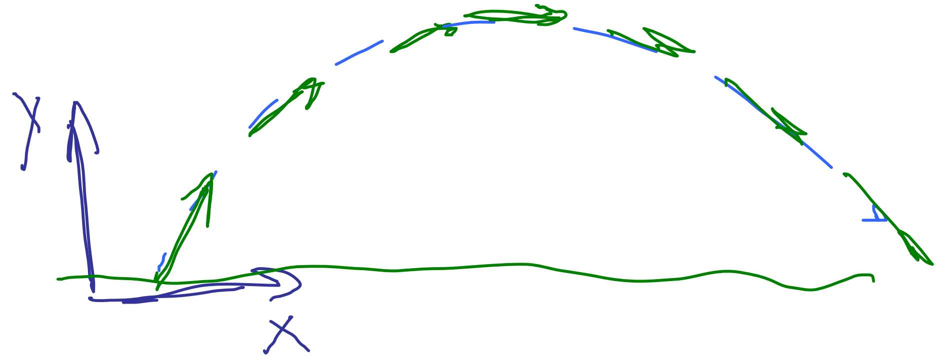
$$\boxed{\begin{aligned} x &= x_0 + \tfrac{1}{2}(v_{x0} + v_x)t \\ y &= y_0 + \tfrac{1}{2}(v_{y0} + v_y)t \end{aligned}}$$

$\vec{a}$ constant!

Free-fall

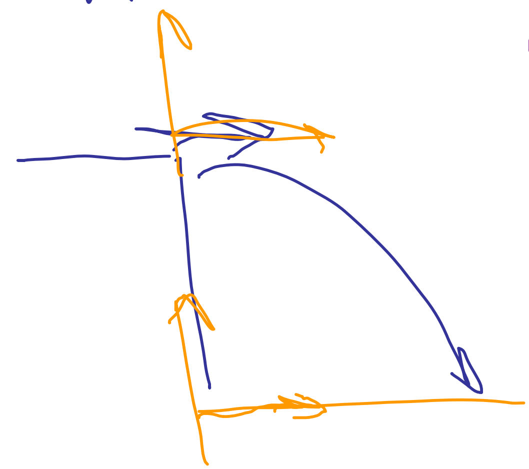
x-comp of $\vec{v}$ doesn't change.

y-comp does change.



Things fall down not sideways

ignore the air!!!



$$a_x = 0$$

$$a_y = -9.8\,\frac{m}{s^2} = -g$$

$$\vec{a} = -9.8\,\frac{m}{s^2}\,\hat{j} = -g\,\hat{j}$$

$$x = x_0 + v_{x0} t$$

$$y = y_0 + v_{y0} t - \frac{1}{2} g t^2 \qquad a_y = -g$$

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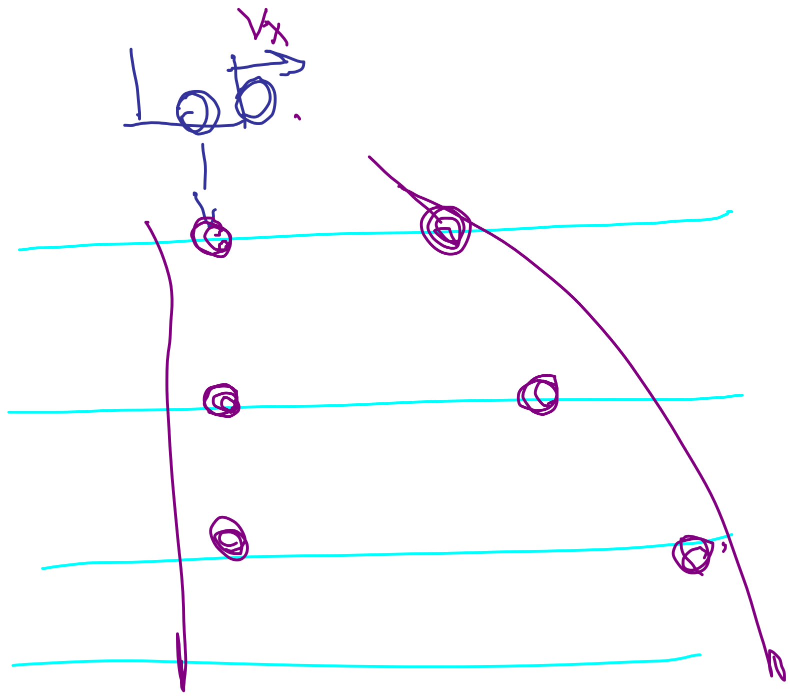

Both $v_{y0} = 0$

y coord's of both same

3.33 A carpenter tosses shingle horizontally off 8.8-m high roof at 11 m/s.
How long it take to reach ground?
How far it go horizontally?