Derivation of (3.37) in Griffiths, 3rd ed.
Or: "Shee— How Did He Do Dat???"

Griffiths finds a closed-form expression for the solution to the "slot" problem where the potential on the strip at $x = 0$ is constant and equal to V_0:

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5\ldots} \frac{1}{n} e^{-n\pi x/a} \sin(n\pi y/a) \tag{1}$$

which is Eq. (3.36) in the text.

Use

$$\sin(n\pi y/a) = \frac{1}{2i}\left(e^{in\pi y/a} - e^{-in\pi y/a}\right)$$

and multiply through by the factor $e^{-n\pi x/a}$. Then 1 becomes:

$$V(x,y) = \frac{4V_0}{2i\pi} \sum_{n=1,3,5\ldots} \frac{1}{n} \left[e^{\left(-\frac{n\pi x}{a}+i\frac{n\pi y}{a}\right)} - e^{\left(-\frac{n\pi x}{a}+i\frac{n\pi y}{a}\right)}\right] \tag{2}$$

and in the exponentials bring the n outside the argument as a power and split up the sum:

$$V(x,y) = \frac{2V_0}{i\pi}\left\{\sum_{n=1,3,5\ldots} \frac{1}{n}\left(e^{-\pi x/a+i\pi y/a}\right)^n - \sum_{n=1,3,5\ldots} \frac{1}{n}\left(e^{-\pi x/a-i\pi y/a}\right)^n\right\} \tag{3}$$

Now define

$$x_+ \equiv e^{-\pi x/a+i\pi y/a} \qquad \text{and} \qquad x_- \equiv e^{-\pi x/a-i\pi y/a} \tag{4}$$

and note from the real part of the arguments (which are negative) we have

$$|x_+| < 1 \quad \text{and} \quad |x_-| < 1\,.$$

Then we can use the Taylor series for $\tanh^{-1} x$ (see CRC Math),

$$\tanh^{-1} x = \frac{x}{1} + \frac{x^3}{3} + \frac{x^5}{5} + \cdots$$

on both sums in 3. We get:

$$V(x,y) = \frac{2V_0}{i\pi}\left\{\tanh^{-1} x_+ - \tanh^{-1} x_-\right\} \tag{5}$$

There is also a relation for the difference of two $\tanh^{-1}$'s. Again from CRC Math,

$$\tanh^{-1} x - \tanh^{-1} y = \tanh^{-1}\left(\frac{x-y}{1-xy}\right)$$

Using this in 5,

$$V(x,y) = \frac{2V_0}{i\pi} \tanh^{-1}\left(\frac{x_+ - x_-}{1 - x_+x_-}\right) \tag{6}$$

Now pulling out a common factor we have

$$x_+ - x_- = e^{-\pi x/a}\left(e^{i\pi y/a} - e^{-i\pi y/a}\right) = 2ie^{-\pi x/a}\sin\left(\frac{\pi y}{a}\right)$$

and

$$1 - x_+x_- = 1 - e^{-2\pi x/a}$$

Putting these into 6,

$$V(x,y) = \frac{2V_0}{i\pi}\tanh^{-1}\left(\frac{2ie^{-\pi x/a}\sin\left(\frac{\pi y}{a}\right)}{1 - e^{-2\pi x/a}}\right) \tag{7}$$

Multiply top and bottom of the argument here by $e^{+\pi x/a}$ and use $\sinh z = \frac{1}{2}(e^z - e^{-z})$, then

$$V(x,y) = \frac{2V_0}{i\pi}\tanh^{-1}\left(\frac{2i\sin\left(\frac{\pi y}{a}\right)}{e^{\pi x/a} - e^{-\pi x/a}}\right) = \frac{2V_0}{i\pi}\tanh^{-1}\left(\frac{i\sin\left(\frac{\pi y}{a}\right)}{\sinh\left(\frac{\pi x}{a}\right)}\right) \tag{8}$$

Finally, use $\tanh^{-1}(ix) = i\tan^{-1}x$. Then the factor of i cancels, and 8 becomes

$$V(x,y) = \frac{2V_0}{\pi}\tan^{-1}\left(\frac{\sin\left(\frac{\pi y}{a}\right)}{\sinh\left(\frac{\pi x}{a}\right)}\right) \tag{9}$$

which is the closed form in Eq. (3.37)