Phys 3810, Spring 2009
Problem Set #5, Hint-o-licious Hints

1. *Griffiths*, **4.19** Part (a) is a simple application of the basic commutation relations

$$[x, p_x] = xp_x - p_x x = i\hbar, \qquad [x, p_y] = 0, \qquad \text{etc.}$$

For (b), pull the commuting factors out in front to simplify things, and use the results in (a) get the answers. For (c), write out the string of operators and use, e.g.

$$L_z x = xL_z + i\hbar y \qquad L_z y = yL_z - i\hbar x$$

to switch the order of the operators and get cancellation.

2. *Griffiths*, **4.20** You need to use

$$\frac{d}{dt}\langle \mathbf{L} \rangle = \frac{i}{\hbar}\langle [H, \mathbf{L}] \rangle$$

where

$$H = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + V(\mathbf{r})$$

to get the result. Just choose any component of this equation, say z. Show that

$$\frac{1}{2m}[p_x^2, xp_y - yp_x] = \frac{-i\hbar}{m} p_x p_y \qquad \frac{1}{2m}[p_y^2, xp_y - yp_x] = \frac{+i\hbar}{m} p_x p_y$$

so these terms will cancel. Then show

$$[V(\mathbf{r}), yp_y - yp_x] = i\hbar \left(x\frac{\partial V}{\partial y} - y\frac{\partial V}{\partial x} \right) = i\hbar (\mathbf{r} \times \nabla V)_z$$

This will give

$$\frac{d}{dt}\langle L_z \rangle = \langle N_z \rangle$$

3. *Griffiths*, **4.23** Apply the (analytic) raising operator to $Y_2^1(\theta, \phi)$ but also show from (4.120) and (4.121)

$$L_+ Y_2^1 = 2\hbar Y_2^2$$

4. *Griffiths*, **4.26** Multiply a lot of little matrices.
Show that if $j \neq k$ then

$$\sigma_j \sigma_k = i \sum_l \epsilon_{jkl} \sigma_l$$

and if $j = k$ then $\sigma_j \sigma_k = \sigma_j^2 = \mathbf{1}$. But results are contained in

$$\sigma_j \sigma_k = \delta_{jk} + i \sum_l \epsilon_{jkl} \sigma_l$$

5. *Griffiths*, **4.27** Normalizaton should be easy. Remember that the condition is $\chi^\dagger \chi = 1$ and for $\chi^\dagger$ you have to do a complex conjugation. In part (b) you should get

$$\langle S_x \rangle = 0 \qquad \langle S_y \rangle = -\frac{12}{25}\hbar \qquad \langle S_z \rangle = -\frac{7}{50}\hbar$$

On (c) you should get

$$\sigma_{S_z} = \frac{12}{25}\hbar$$

6. *Griffiths*, **4.29** Show that the eigenvectors of S_y (well, one choice for them) are

$$\chi_+^{(y)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \qquad \chi_-^{(y)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$