

**Phys 2920, Spring 2009**  
**Problem Set #5**

1. For a frame  $\mathcal{S}$  which measures events as  $(x, y, z, t)$  and a frame  $\mathcal{S}'$  which moves at speed  $v$  along the  $+x$  axis with respect to  $\mathcal{S}$  and measures events as  $(x', y', z', t')$ , the relation between the coordinates is

$$\begin{aligned}x' &= \gamma(x - vt) \\y' &= y \\z' &= z \\t' &= \gamma\left(t - \frac{v}{c^2}x\right)\end{aligned}$$

where

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

This set of relations are the **Lorentz transformations**.

Write this set of equation as a matrix/vector relation, of the form

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

where  $\Lambda$  is a  $4 \times 4$  matrix. It is really a function of  $v$ , so it is  $\Lambda(v)$ . The matrix  $\Lambda$  will just contain  $\gamma$ ,  $v$  and  $c$  but not  $x$  or  $t$ .

b) Show that  $\Lambda(v)\Lambda(-v) = \mathbf{1}$ . (The second matrix is just gotten by substituting  $-v$  for  $v$  everywhere in your matrix  $\Lambda(v)$ ; this does not change  $\gamma$ .)

2. (VA 4.42) If  $\phi = 2xz^4 - x^2y$ , find  $\nabla\phi$  and  $|\nabla\phi|$  at the point  $(2, -2, -1)$ .

3. Show that

$$\nabla\left(\frac{1}{r}\right) = -\frac{\hat{\mathbf{r}}}{r^2}$$

using Cartesian coordinates, wherein we have

$$r = \sqrt{x^2 + y^2 + z^2}$$

Though this is an important gradient to take, this is not the easiest way to do it but all we've got so far is Cartesian coordinates so use *them*. Of course,  $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$

4. (VA 4.64) In what direction from the point  $(1, 3, 2)$  is the directional derivative of  $\phi = 2xz - y^2$  a maximum? What is the magnitude of this maximum?

5. (VA 4.63) Find the directional derivative of  $P = 4e^{2x-y+z}$  at the point  $(1, 1, -1)$  in a direction toward the point  $(-3, 5, 6)$ .

6. (VA 4.71) Evaluate  $\nabla \cdot (2x^2z\mathbf{i} - xy^2z\mathbf{j} + 3yz^2\mathbf{k})$ .
7. (VA 4.72) If  $\phi = 3x^2z - y^2z^3 + 4x^3y + 2x - 3y - 5$ , find  $\nabla^2\phi$ .
8. (VA 4.77) Prove that  $\nabla^2(\phi\psi) = \phi\nabla^2\psi + 2\nabla\phi \cdot \nabla\psi + \psi\nabla^2\phi$
9. (VA 4.96) If  $\mathbf{a} = yz^2\mathbf{i} - 3xz^2\mathbf{j} + 2xyz\mathbf{k}$  and  $\mathbf{b} = 3x\mathbf{i} + 4z\mathbf{j} - xy\mathbf{k}$ , and  $\phi = xyz$ , find:
- a)  $\mathbf{a} \times (\nabla \times \mathbf{b})$
- c)  $(\nabla \times \mathbf{a}) \times \mathbf{b}$