

2D Motion

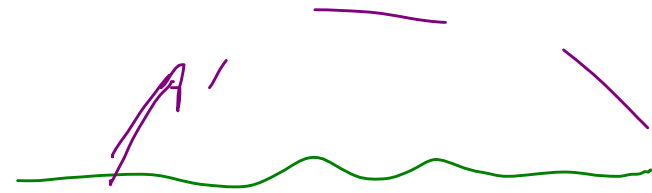
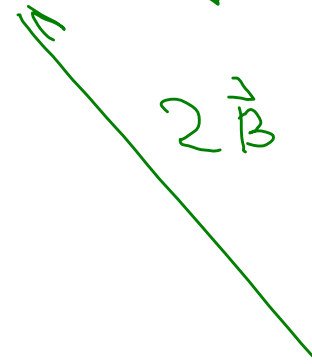
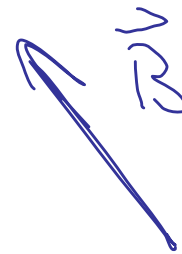
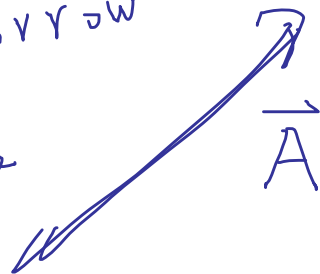
But first...

Vectors

Direction, magnitude

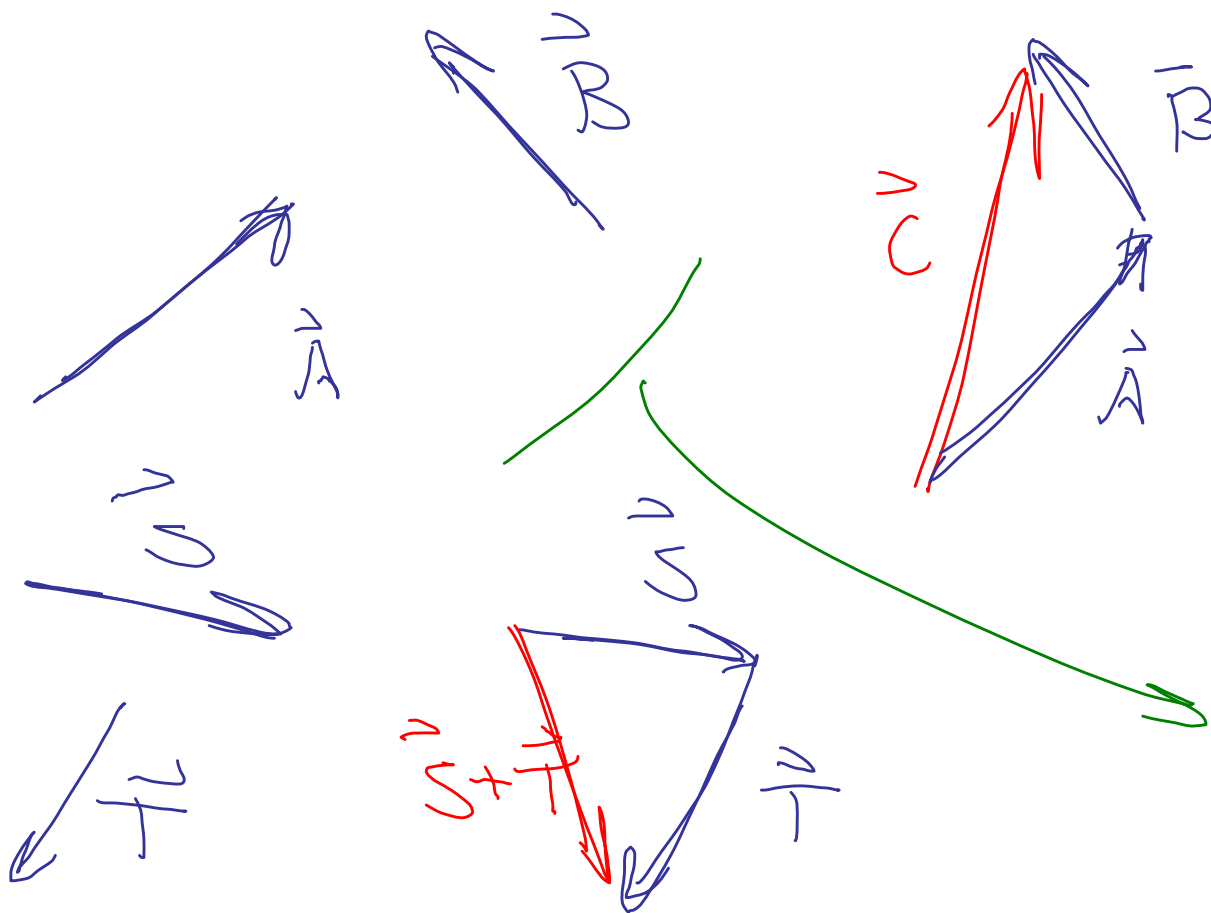
Repr'd by arrow

Dir & magnitude

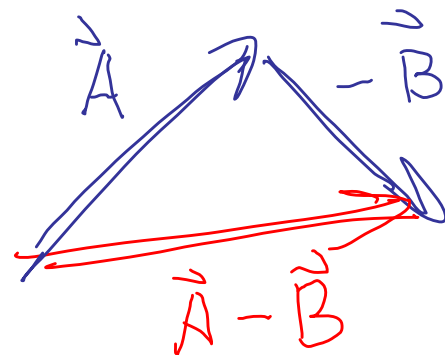


Adding vectors

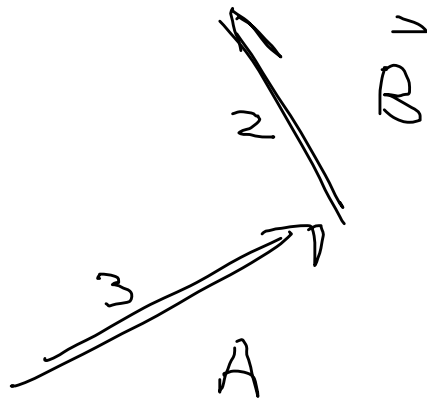
adding arrows



$$\vec{C} = \vec{A} + \vec{B}$$



"Adding vectors graphically"



Tell me the magnitude
of $\vec{A} + \vec{B}$

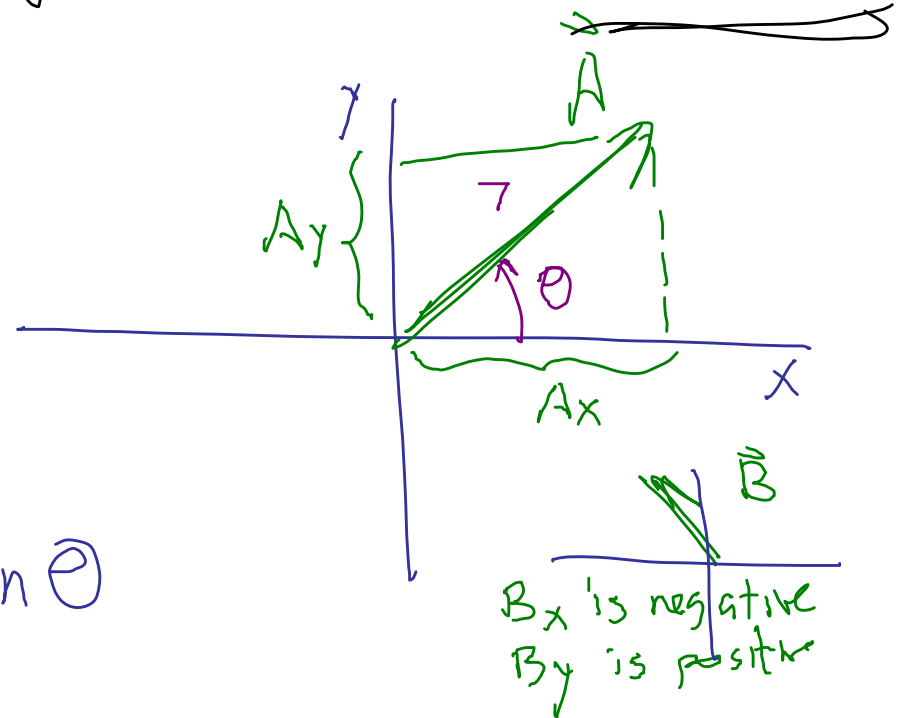
Not 5

Use components

Direction, magnitude

θ , A

$$A_x = A \cos \theta \quad A_y = A \sin \theta$$



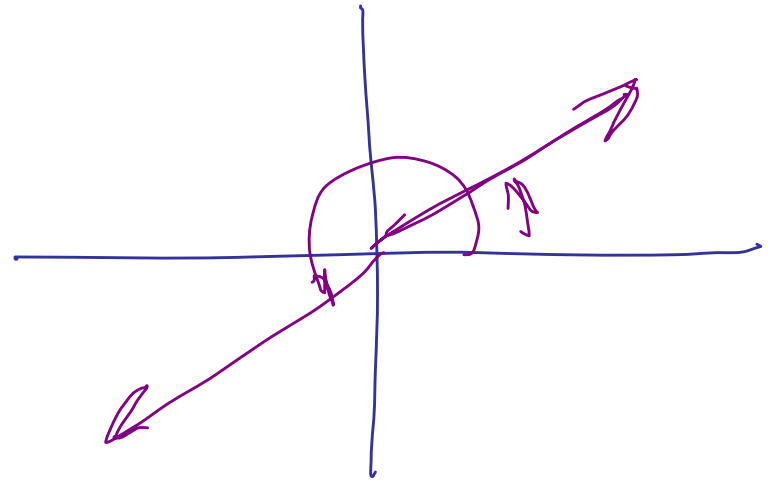
$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan \theta = \frac{A_y}{A_x}$$

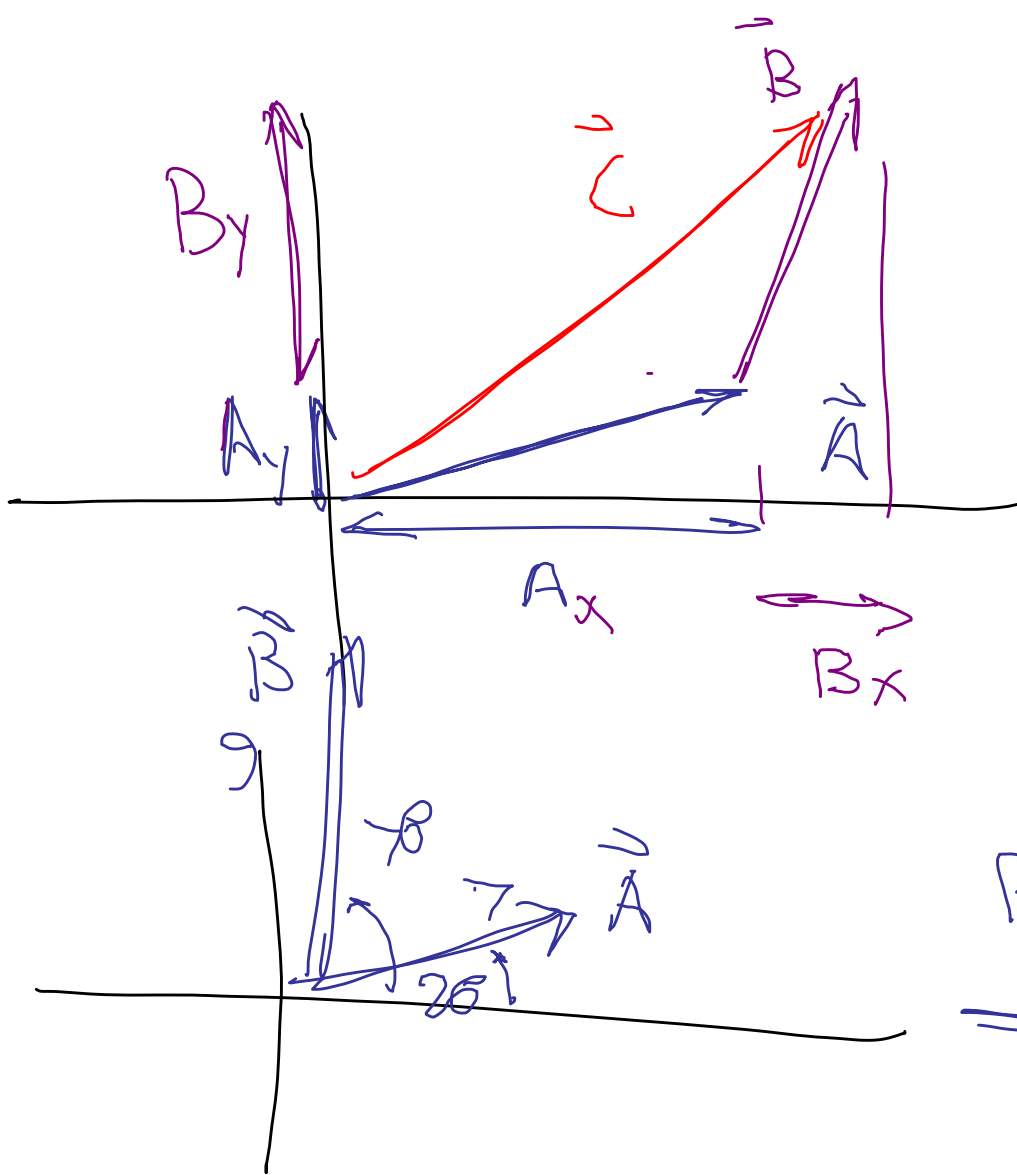
$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

can give wrong
answer

$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right) + \text{Brain}$$



Add vectors, find components of vectors
Add components separately.



$$C_x = A_x + B_x$$

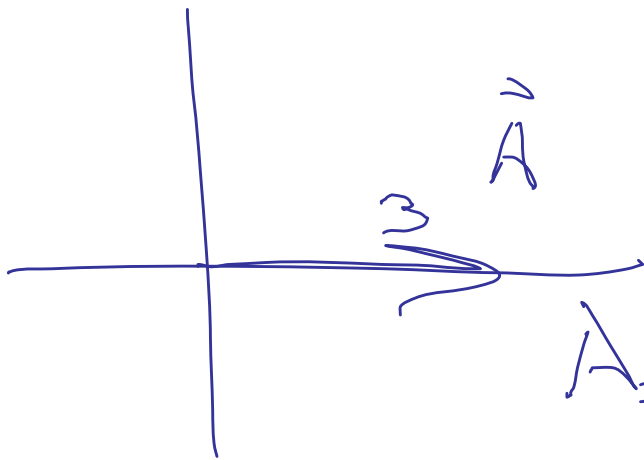
$$C_y = A_y + B_y$$

& use this to find
dir & mag of \vec{C}

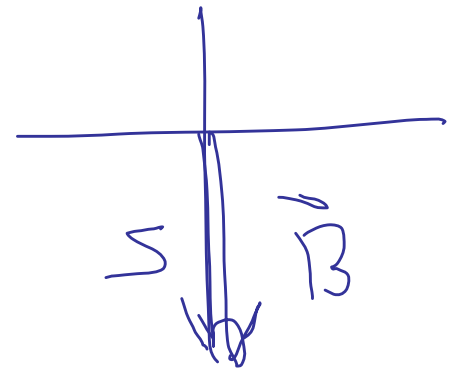
Find $\vec{A} + \vec{B}$. $A_x = 7 \cos 20^\circ$
 $\Rightarrow A_x, B_x$ $A_y = 7 \sin 20^\circ$
 A_y, B_y

$$C_x = A_x + B_x \quad C_y = A_y + B_y$$

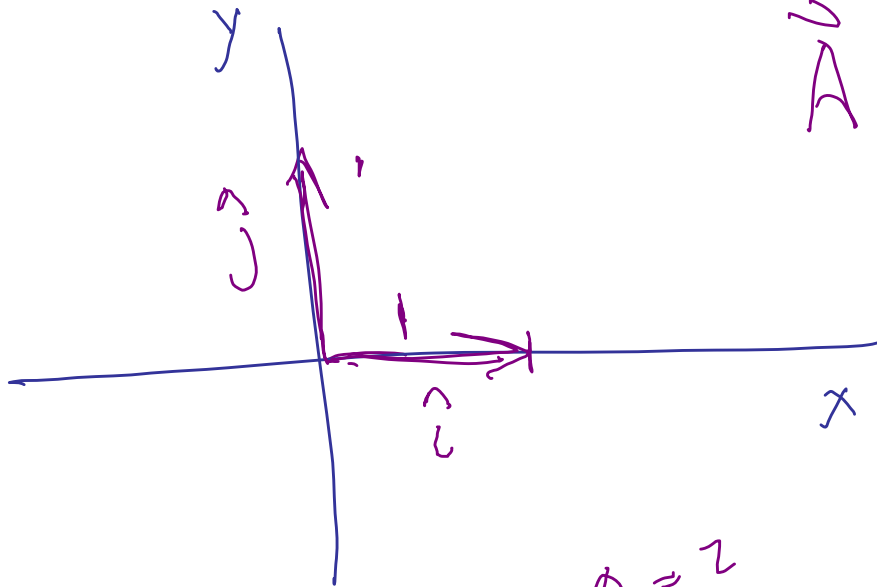
C, θ



$$A_x = 3 \quad A_y = 0$$



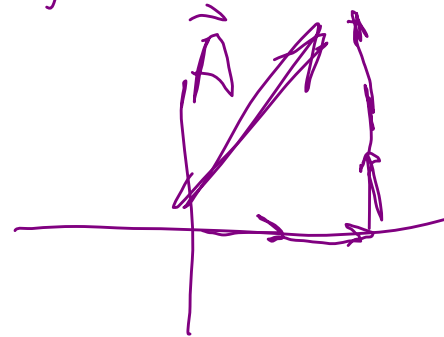
$$B_x = 0 \quad B_y = -5$$



$$A_x = 2 \\ A_y = 3$$

$$\vec{A} \quad A_x, A_y$$

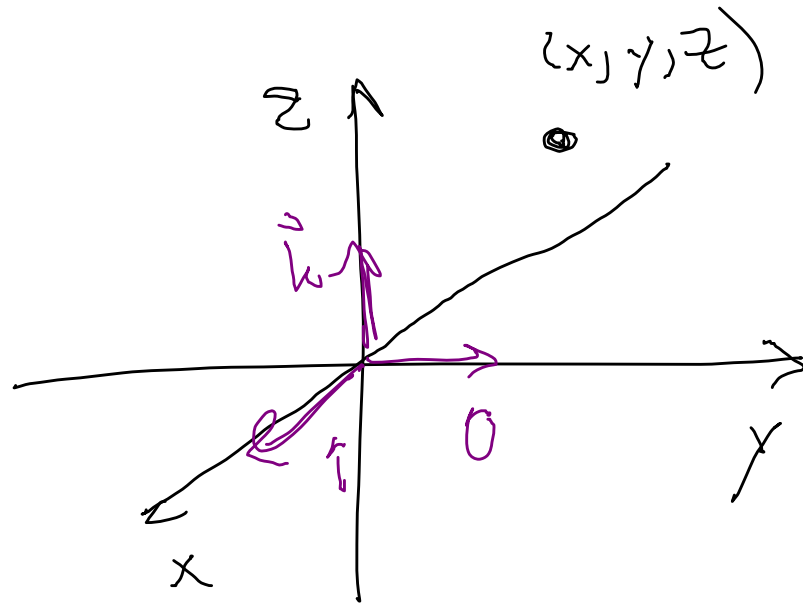
$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$



$$= (2, 3)$$

$$\vec{A} = 2\hat{i} + 3\hat{j}$$

3-Dimensions



Motion in 2D

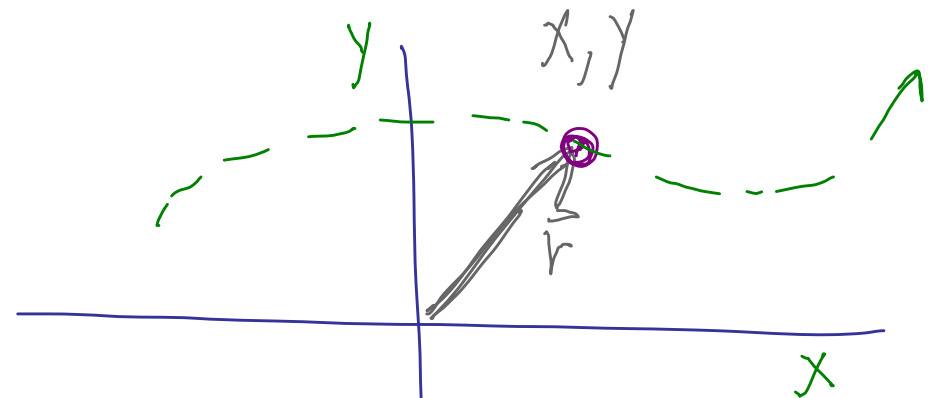
$$\vec{r} = x\hat{i} + y\hat{j}$$

Gravity Attracting

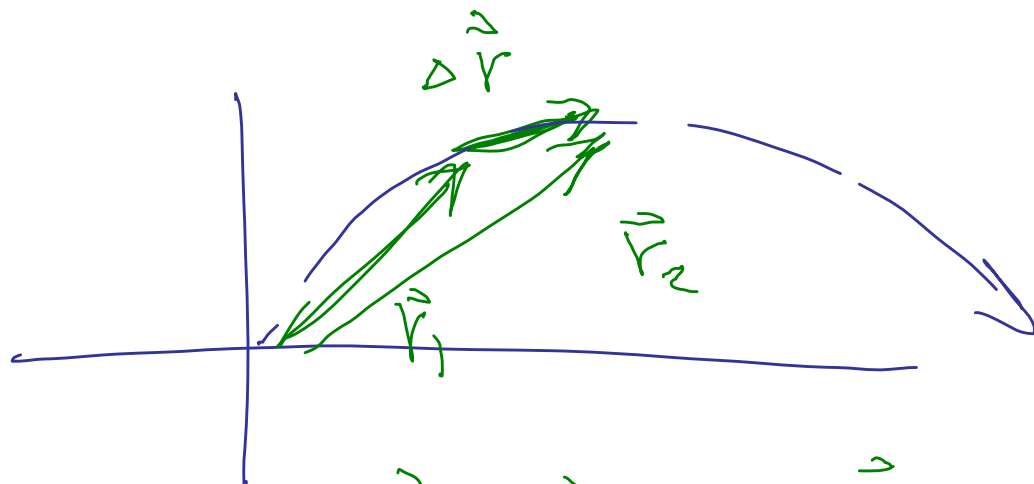
Dot Product

Cross Product

"small" particle.



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad \text{p. 33}$$



How rapidly does
position change?
velocity!

$$\vec{r}_2 - \vec{r}_1 = \Delta \vec{r}$$

$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x \hat{i} + \Delta y \hat{j}}{\Delta t}$$

$$= \left(\frac{\Delta x}{\Delta t} \right) \hat{i} + \left(\frac{\Delta y}{\Delta t} \right) \hat{j} = \bar{v}_x \hat{i} + \bar{v}_y \hat{j}$$

Instantaneous velocity.

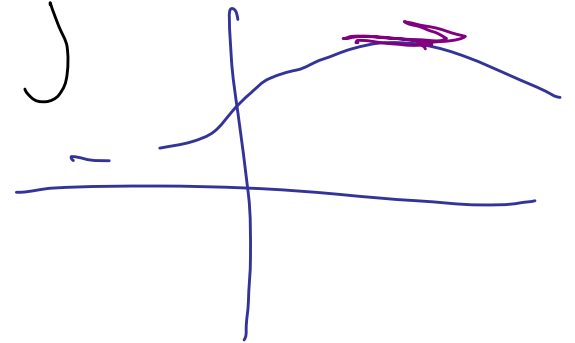
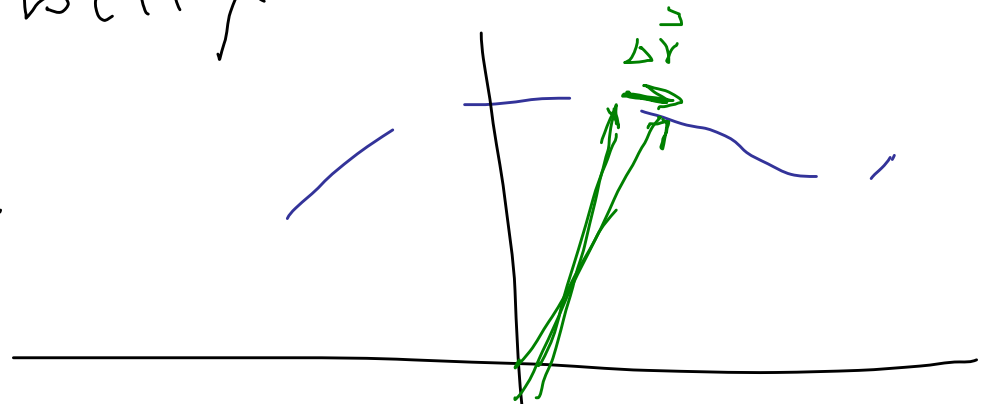
$\Delta t \rightarrow$ verrrrry small

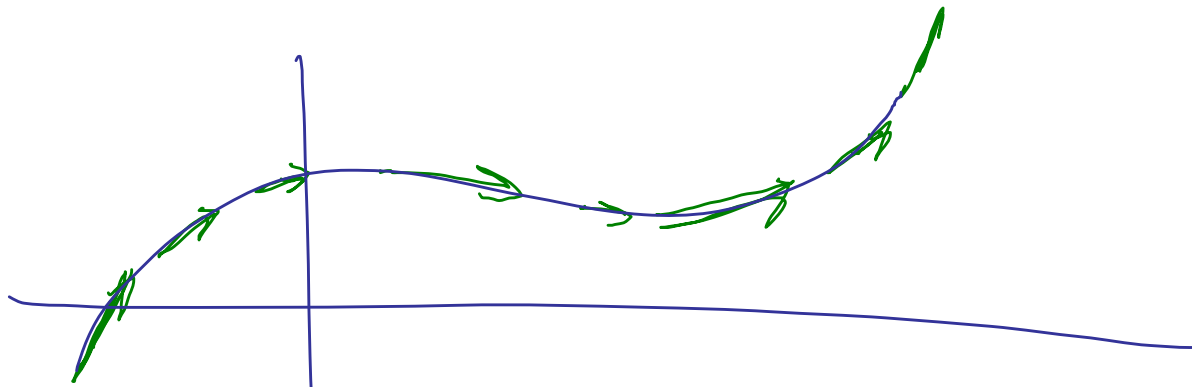
$$\lim_{\Delta t \rightarrow 0}$$

$$\begin{aligned}\vec{v} &= \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \left(\frac{dx}{dt} \right) \hat{i} + \left(\frac{dy}{dt} \right) \hat{j} \\ &= v_x \hat{i} + v_y \hat{j}\end{aligned}$$

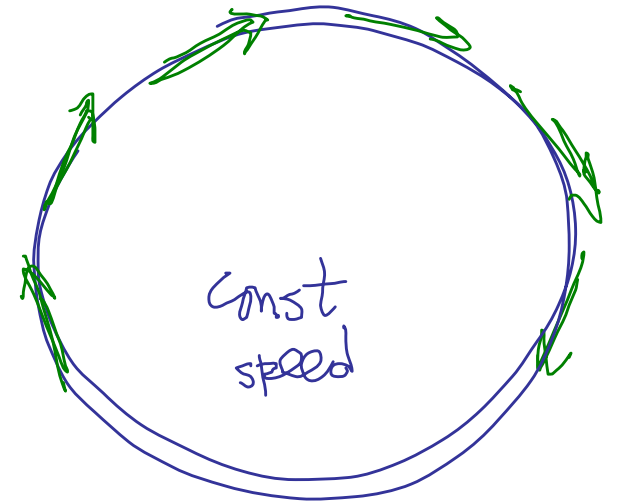
$$v_x = \frac{dx}{dt}$$

$$v_y = \frac{dy}{dt}$$

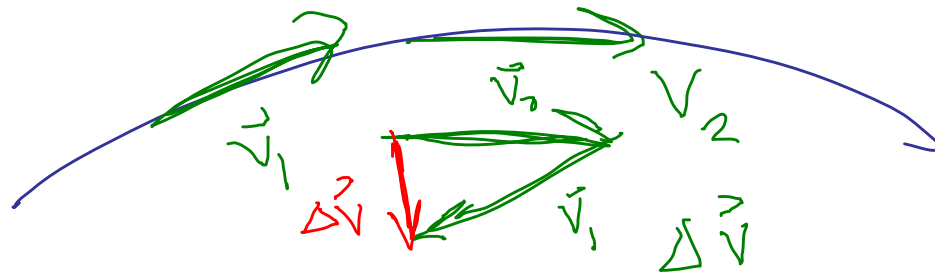




$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$



Velocity is changing



$\Delta \vec{v} \Rightarrow \vec{a}$
points inward

Velocity vector changes. \Rightarrow Acceleration

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

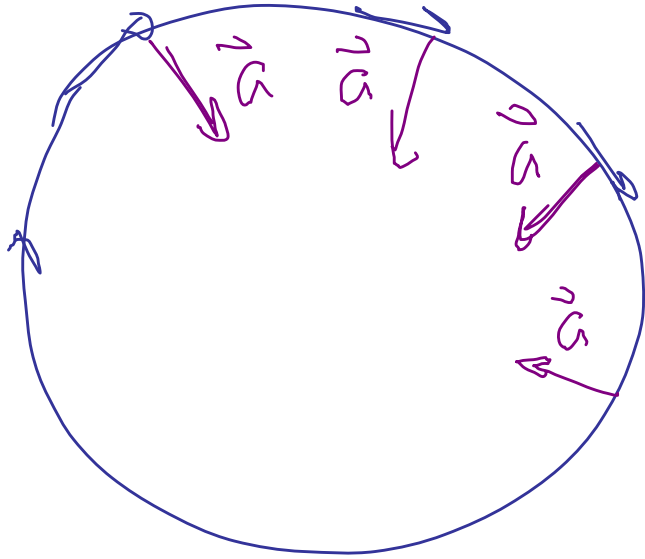
p. 33

$$= \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j}$$

$$= a_x \hat{i} + a_y \hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$a_x = \frac{dv_x}{dt}$	$a_y = \frac{dv_y}{dt}$
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Not constant
acceleration.

$$v_x = \frac{dx}{dt} \quad \frac{m}{s}$$

$$a_x = \frac{dv_x}{dt} \quad \frac{m}{s^2}$$