Phys 3810, Spring 2011
Problem Set #4, Hint-o-licious Hints

1. *Griffiths*, **3.27** The first measurement puts the system in state ψ_1. Now the probabilities for measuring b_1 and b_2 can be read off from the coefficients (squared). The B measurement was made, but *you are not told* the result of the measurement.

Now when A is measured again, you are *not* assured of getting a_1 and that is because the B measurement —whatever its outcome— has changed the state of the system. Write the ϕ's in terms of the ψ's and then consider that if the B measurement had given b_1 what the probability of a_1 would be. Then consider that if the B measurement had given b_2 what the probability of a_1 would be. Combine all the probabilities together.

The answer to the question is $\frac{337}{625}$ but show how you get it! I had to stop and think for a while why the answer is not 1. But the fact that the measurement of B was made does have a physical effect *even if we don't know the result of that measurement*. (The answer does employ the fact that we don't know what it was.)

2. *Griffiths*, **3.31** You'll need to evaluate

$$[H, xp] = \left[\frac{p^2}{2m}, xp\right] + [V(x), xp]$$

The first of these is

$$\left[\frac{p^2}{2m}, xp\right] = -2i\hbar\frac{p^2}{2m}$$

and the second is

$$[V(x), xp] = -\frac{\hbar}{i}x\frac{dV}{dx}$$

3. *Griffiths*, **3.37** Get the eigenvalues and eigenvectors of H. (They are c and $a \pm b$, and some simple vectors with 1's and -1's. Show all of this!) Part (c) has the system start in a state which is not an eigenstate, so you must decompose it in terms of eigenstates (can be done "by inspection") and then attach the usual oscillatory time dependence with the energy eigenvalues.

4. *Griffiths*, **4.2** Show that the energies are given by

$$E_{(n_x.n_y,n_z)} = \frac{\pi^2\hbar^2}{2ma^2}(n_x^2 + n_y^2 + n_z^2)$$

The latter part of the problem is a math–puzzle sort of thing where you figure out how many ways one can get the same E with different sets of integers.

You can find the values and the degeneracies from E_1 up to E_{14}, but if you run out of patience, I'll tell you that

$$E_{14} = \frac{\pi^2\hbar^2}{2ma^2}(27)$$

So what's different about this value from the first few?

5. *Griffiths*, **4.38** The separated solution is

$$\psi(\mathbf{r}) = X(x)Y(y)Z(z)$$

Note that when you write out the Schrödinger equation in cartesian coordinates for this potential with the separated solution (substitute and then divide by $\psi = XYZ$) it becomes a sum of three terms each of which depends only on x y or z. This means that you can write three separate Schrödinger equations, i.e.

$$-\frac{\hbar^2}{2m}\frac{1}{X}\frac{d^2X}{dx^2} + \tfrac{1}{2}m\omega^2 = E_x \qquad \text{, etc.}$$

which we know how to solve. (Be as clear as you can about this part.) The total energy is just sum of energies from each separate Schrödinger equation:

$$E = E_x + E_y + E_z = \hbar\omega(n_x + n_y + n_z + \tfrac{3}{2})$$

The second part involves more thought. The energy of the oscillator state just depends on

$$n \equiv n_x + n_y + n_z \qquad \text{for} \qquad n_x = 0, 1, 2, \ldots \quad \text{etc.}$$

so the question is how many ways can you get n from the three separate indices. That's a sort of puzzle–math problem. First, see if you can spot the pattern for the lowest n's. (To give you one: $n = 3$ has 10 possible states.)

6. *Griffiths*, **4.8** Test that the $u(r)$ radial solution

$$u_1(r) = Arj_1(kr)$$

really does solve Eq. (4.41).

For the infinite spherical well, the boundary condition $u(a) = 0$ (continuity of the wave function) leads to $ka = \tan(ka)$. (Show this, of course.) Use a graph to demonstrate that for the higher solutions, we have

$$ka \approx \frac{(2n+1)}{2}\pi \qquad \text{for} \quad n = 1, 2, 3, \ldots$$