Phys 2920, Spring 2010
Problem Set #1

1. (a) Express the complex number $7 + 3i$ in polar form ($\rho e^{i\phi}$). (b) Express the complex number $(8.0)e^{4.7i}$ in $a + bi$ form.

2. Find the unit vector in the direction of the vector $4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.

3. In each case, determine whether vectors are linearly independent or linearly dependent:

a) $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} - 4\mathbf{k}$, $\mathbf{c} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$.

b) $\mathbf{a} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 4\mathbf{j} - \mathbf{k}$, $\mathbf{c} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

4. (a) Prove that the vectors $\mathbf{a} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\mathbf{b} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{c} = 4\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}$ can form the sides of a triangle. (b) Find the lengths of the medians of the triangle.

5. For a set of N point charges q_i, the dipole moment is defined as

$$\mathbf{p} = \sum_{i=1}^{N} q_i \mathbf{r}_i$$

Suppose we change to a new coordinate system (with an origin shifted with respect to the old one by $\mathbf{R}$) such that the new position vectors are

$$\mathbf{r}'_i = \mathbf{r}_i - \mathbf{R}$$

What is the condition on the charges such that $\mathbf{p}$ has the same value in the new coordinates?

6. Find the angle between $\mathbf{c} = 4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{d} = 3\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$.

7. For what values of a are $\mathbf{A} = a\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 2a\mathbf{i} + a\mathbf{j} - 4\mathbf{k}$ perpendicular?

8. Find the work done in moving an object along a straight line from $(3, 2, -1)$ to $(2, -1, 4)$ in a force field given by $\mathbf{F} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$.

9. If $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$, find a vector of magnitude 5 perpendicular to both $\mathbf{a}$ and $\mathbf{b}$

10. Simplify $(\mathbf{a} + \mathbf{b}) \cdot [(\mathbf{b} + \mathbf{c}) \times (\mathbf{c} + \mathbf{a})]$.

11. Show (any way you can) that for any vectors $\mathbf{a}$ and $\mathbf{b}$,

$$|\mathbf{a} - \mathbf{b}| \geq |\mathbf{a}| - |\mathbf{b}|$$

12. Use the "trick" with δ_{ij} and ϵ_{ijk} to show

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$$