

Phys 2920, Spring 2011
Problem Set #5

1. For a frame \mathcal{S} which measures events as (x, y, z, t) and a frame \mathcal{S}' which moves at speed v along the $+x$ axis with respect to \mathcal{S} and measures events as (x', y', z', t') , the relation between the coordinates is

$$\begin{aligned}x' &= \gamma(x - vt) \\y' &= y \\z' &= z \\t' &= \gamma\left(t - \frac{v}{c^2}x\right)\end{aligned}$$

where

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

This set of relations are the **Lorentz transformations**.

Write this set of equation as a matrix/vector relation, of the form

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

where Λ is a 4×4 matrix. It is really a function of v , so it is $\Lambda(v)$. The matrix Λ will just contain γ , v and c but not x or t .

b) Show that $\Lambda(v)\Lambda(-v) = \mathbf{1}$. (The second matrix is just gotten by substituting $-v$ for v everywhere in your matrix $\Lambda(v)$; this does not change γ .)

2. (VA 4.42) If $\phi = 2xz^4 - x^2y$, find $\nabla\phi$ and $|\nabla\phi|$ at the point $(2, -2, -1)$.

3. Show that

$$\nabla\left(\frac{1}{r}\right) = -\frac{\hat{\mathbf{r}}}{r^2}$$

using Cartesian coordinates, wherein we have

$$r = \sqrt{x^2 + y^2 + z^2}$$

Though this is an important gradient to take, this is not the easiest way to do it but all we've got so far is Cartesian coordinates so use *them*. Of course, $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$

4. (VA 4.64) In what direction from the point $(1, 3, 2)$ is the directional derivative of $\phi = 2xz - y^2$ a maximum? What is the magnitude of this maximum?

5. (VA 4.63) Find the directional derivative of $P = 4e^{2x-y+z}$ at the point $(1, 1, -1)$ in a direction toward the point $(-3, 5, 6)$.

6. (VA 4.71) Evaluate $\nabla \cdot (2x^2z\mathbf{i} - xy^2z\mathbf{j} + 3yz^2\mathbf{k})$.
7. (VA 4.72) If $\phi = 3x^2z - y^2z^3 + 4x^3y + 2x - 3y - 5$, find $\nabla^2\phi$.
8. (VA 4.77) Prove that $\nabla^2(\phi\psi) = \phi\nabla^2\psi + 2\nabla\phi \cdot \nabla\psi + \psi\nabla^2\phi$
9. (VA 4.96) If $\mathbf{a} = yz^2\mathbf{i} - 3xz^2\mathbf{j} + 2xyz\mathbf{k}$ and $\mathbf{b} = 3x\mathbf{i} + 4z\mathbf{j} - xy\mathbf{k}$, and $\phi = xyz$, find:
 - a) $\mathbf{a} \times (\nabla \times \mathbf{b})$
 - c) $(\nabla \times \mathbf{a}) \times \mathbf{b}$