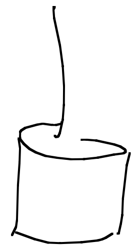
Phys 2110-3 11/22/10

Note Title 11/22/2010

Osc's :



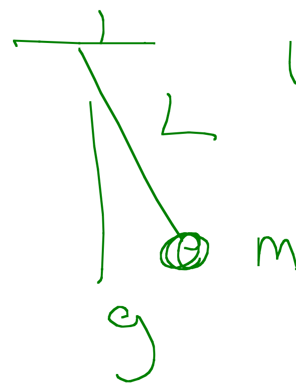
Torsional pendulum

$\omega = \sqrt{K/I}$

$\frac{d^2x}{dt^2} = -\frac{k}{m}x$

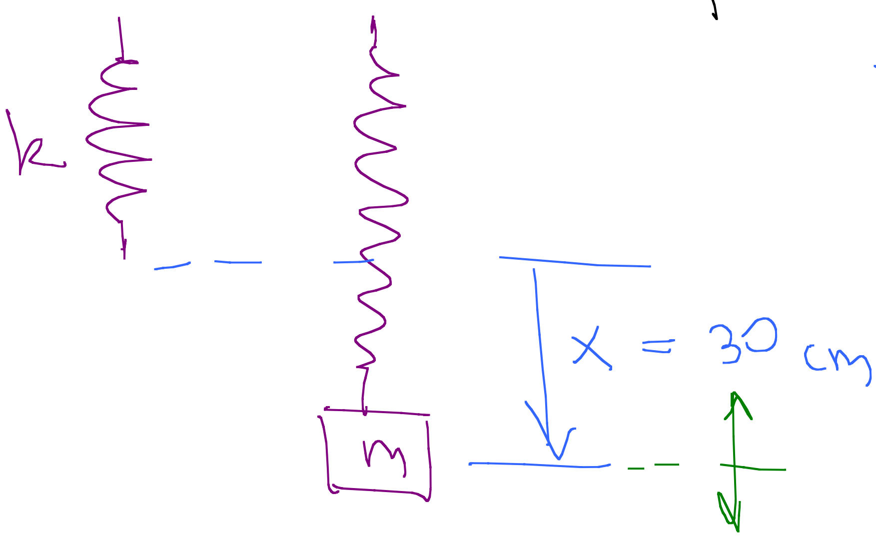
$\frac{k}{m} \rightarrow \omega^2$

$T = 1/f \quad f = \omega/2\pi \ldots$



$\omega = \sqrt{g/L}$

13.46 340-g mass is attached to a vertical spring & lowered slowly to a new equilibr. position 30 cm below orig. equilibr. The system is set into motion, harmonic. What is the period?



$mg = kx$

$k = \frac{mg}{x} = \frac{(0.340\ kg)(9.8\ \frac{m}{s^2})}{(0.30\ m)}$

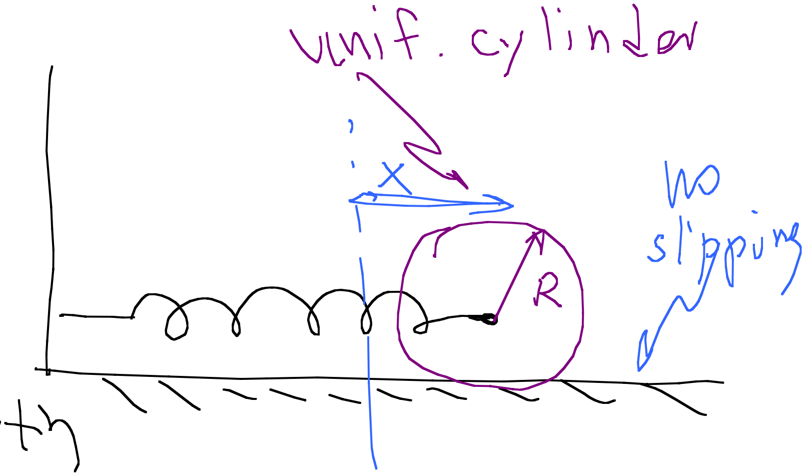
$= 11.1\ N/m$

$T = 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi\sqrt{\frac{0.340\ kg}{11.1\ N/m}}$

$\boxed{1.1\ s}$

13.60 Solid cylinder (M, R) axle is attached to horiz spring, k, cylinder rolls back & forth no slipping. Find ang. freq. of motion.



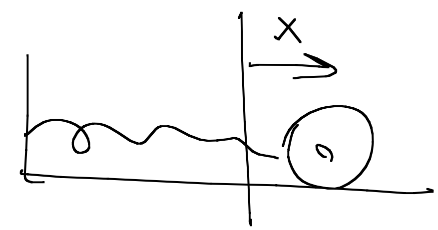
→ Consider forces & torques on cylinder, get force & torques eqns, combine.

→ $a_x = \frac{d^2x}{dt^2} = -\bigcirc\, x$

→ ω^2

Consider energy!

Kinetic, potential energy, conserved!



$$U = \frac{1}{2}kx^2$$

$$K_{roll} = K_{trans} + K_{rot}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$\boxed{v = \frac{dx}{dt}}$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

$$K + U = \boxed{\text{const}} = \boxed{\frac{1}{2}kx^2 + \frac{3}{4}mv^2}$$

Take deriv. with resp. to time

$$0 = \cancel{\frac{1}{2}} k(\cancel{2}x)\frac{dx}{dt} + \frac{3}{4} m (2v)\frac{dv}{dt}$$

$$kx\cancel{v} + \frac{3}{2} m\cancel{v} a = 0$$

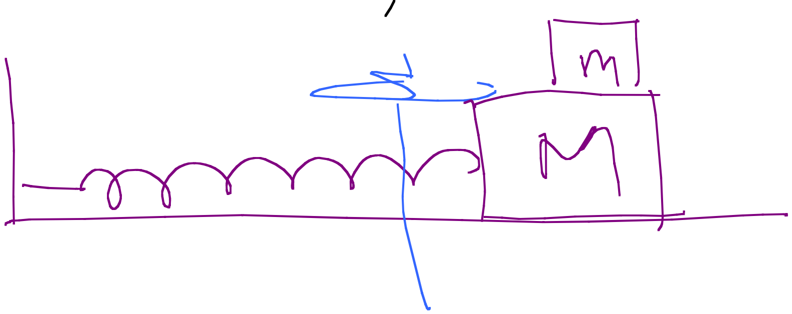
$$kx = -\frac{3}{2} m \frac{d^2x}{dt^2}$$

$$\cancel{a} = \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\underbrace{\frac{2}{3}\frac{k}{m}}_{\omega^2} x$$

$$\omega = \sqrt{\frac{2}{3} k/m}$$

13.70 A 500 g block on a frictionless surface is attached limp spring const. $k = 8.7 \frac{N}{m}$. Second block rests on top of first block and whole system ex's SHM w/ period 1.8 s. When ampl. is incr'd to 35 cm, upper block begins to slip. What is coeff. of static fric (between blocks)



Static fric makes small one oscillate.

At large enough A, a_{max} (f_{max}) is so large, it is bigger than limit of static fric.

First case
move as unit

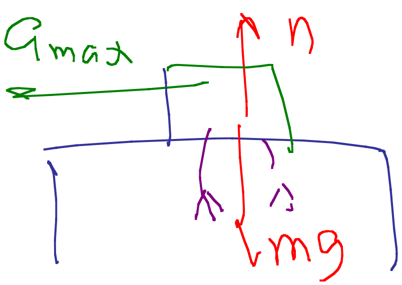
$T = 1.8\ s$

8.7 N/m

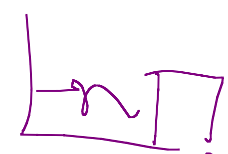
0.500 kg

$$\omega = \sqrt{\frac{k}{M+m}} \Longrightarrow m = 0.2139$$

Second case $A = 0.35\,m$, slips



$$f_{max} = m a_{max} = \boxed{\cancel{m}(A\omega^2)}$$
$$= \mu_s n = \boxed{\mu_s \cancel{m} g}$$



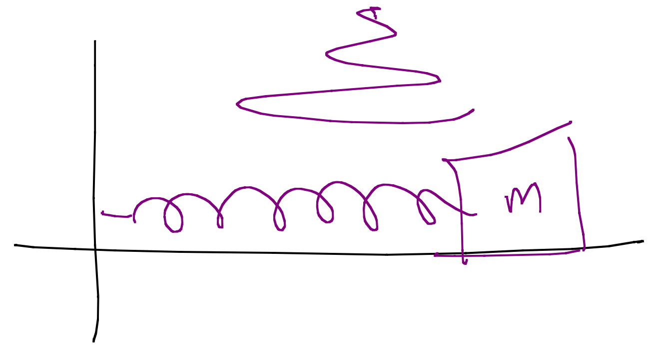
$a_{max} = \omega^2 A$

$$\mu_s g = \omega^2 A$$

$$\mu_s = \frac{\omega^2 A}{g} \qquad \text{Get} \qquad \boxed{\mu_s = 0.435}$$

More topics

more realistic system



Damped oscillations

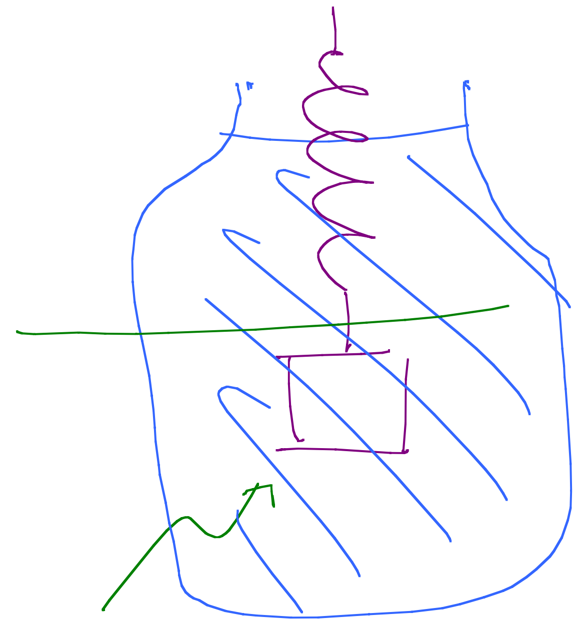
Consider fric force which oppose motion
depends on speed

$$f_{x,\text{resis}} = -b v_x$$

N's 2nd law

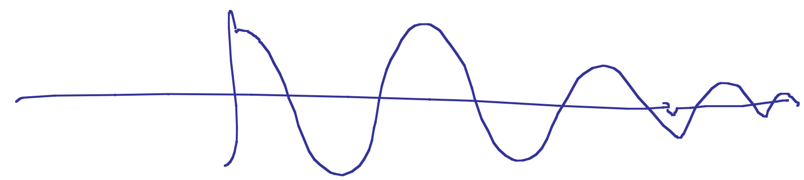
spr   Fric

$$F_{net,x} = ma_x = -kx - bv$$



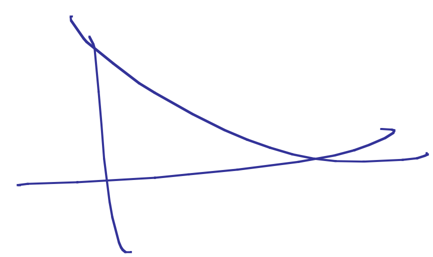
$$\boxed{m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}}$$

Harder diff eqn



Sol'n has form

$$x(t) = Ae^{-\frac{b}{2m}(t)}\cos(\omega t + \phi)$$

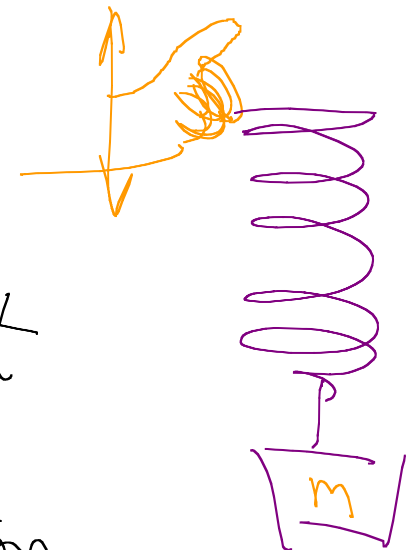


Another possibility: Apply another force.

Extra force also oscillate, freq ω_0

$\omega = \sqrt{\frac{k}{m}}$ $\qquad F_{x,\text{extra}} = F_0 \cos \omega_0 t$

Can be accomplished



$$m \frac{d^2x}{dt^2} = -kx - bv + F_0 \cos \omega_0 t$$

Driven oscillation