Phys 3610, Fall 2009
Problem Set #2, Hint-o-licious Hints

1. *Taylor*, **2.31** Find $v_{\rm ter}$ using $c = \gamma D^2$. Get the time to fall from (2.58) with $y = 30$ m and then v from (2.56).

2. *Taylor*, **2.38** The equation of motion for the trip upward (y axis points upward) is

$$m\frac{dv}{dt} = -mg - cv^2$$

Do the usual separation of variables and do the integration. Here you can integrate the t side from $t' = 0$ to $t' = t$ and the v side from $v' = v_0$ to $v' = v$. This will give an arctangent function of v. Do some algebra to get it into the desired form.

3. *Taylor*, **2.41** Now we want to throw the baseball upward but solve the differential equation in the variables v and y using

$$\frac{dv}{dt} = \frac{dx}{dt}\frac{dv}{dx} = v\frac{dv}{dx}$$

Here the integral gives a log function.

4. *Taylor*, **2.42** Now do the downward journey in the variables v and y. (In the chapter it was done in term of v and t.) When we consider the cases of "very little" and "very much" air resistance, we mean that $v_0 \ll v_{\rm ter}$ (little) or $v_0 \gg v_{\rm ter}$ (much).

5. *Taylor*, **3.14** After using $m = m_0 - kt$, the equation you want to solve has the form

$$(m_0 - kt)\frac{dv}{dt} = kv_{\rm ex} - bv$$

solve by the usual separation of variables. (Get logs then exponentiate these.) When you get an equation with a constant to determine, apply the condition $v = 0$ at $t = 0$ to fix the constant. Do some algebra to get it into the desired form; you can substitute m for $m_0 - kt$.

6. *Taylor*, **3.16** Use

$$x_{\rm cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

The result is surprisingly small (hundreds of kilometer) since the Sun is so damn big.

7. *Taylor*, **3.32** In the usual spherical coordinates, the distance of an element of the sphere from the axis is $r\sin\theta$. The mass element is $\rho r^2 dr \sin\theta d\theta d\phi$ and the density of the sphere is

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

8. *Taylor*, **3.37** Parts (a) and (b) which deal with coordinates as measured from the CM of the system:

$$\mathbf{r}'_\alpha = \mathbf{r}_\alpha - \mathbf{R}$$

are relatively easy. For (c), the rate of change of the angular momentum *measured about the CM* is

$$\begin{aligned}\frac{d}{dt}\sum_{\alpha=1}^{N}\mathbf{r}'_\alpha \times \mathbf{p}'_\alpha &= \frac{d}{dt}\sum_{\alpha=1}^{N}(\mathbf{r}_\alpha - \mathbf{R}) \times m_\alpha \mathbf{v}'_\alpha \\ &= \frac{d}{dt}\sum_{\alpha=1}^{N}(\mathbf{r}_\alpha - \mathbf{R}) \times m_\alpha (\dot{\mathbf{r}}_\alpha - \dot{\mathbf{R}})\end{aligned}$$

and now the work is in manipulating the time derivatives and cross products. (Use the theorems!) There is some cancellation from the property of the cross product and you can use

$$\sum_{\alpha=1}^{N}\mathbf{r}_\alpha \times m_\alpha \ddot{\mathbf{r}}_\alpha = \sum_{\alpha=1}^{N}\mathbf{r}_\alpha \times \mathbf{F}_\alpha = \mathbf{\Gamma}^{\text{ext}}$$

from p. 94, as well as

$$\sum_{\alpha=1}^{N} m_\alpha \mathbf{r}_\alpha = M\mathbf{R} \qquad \text{and} \qquad M\ddot{\mathbf{R}} = \mathbf{F}^{\text{ext}}$$

also

$$\sum_{\alpha=1}^{N}\mathbf{F}_\alpha = \sum_{\alpha=1}^{N}\mathbf{F}_\alpha^{\text{ext}} \equiv \mathbf{F}^{\text{ext}}$$

What you're after is

$$\frac{d}{dt}\mathbf{L}(CM) = \sum_{\alpha=1}^{N}(\mathbf{r}_\alpha - \mathbf{R}) \times \mathbf{F}^{\text{ext}} = \mathbf{\Gamma}^{\text{ext}}(CM)$$

This is the theorem that says we're *allowed* to work with torques and angular accelerations around an accelerating center of mass.