Phys 3810, Spring 2012 Problem Set #6, Hint-o-licious Hints

1. Griffiths, 4.14 The trick here is that the probability to find the electron at a certain radius r (within dr) is not $R(r)^2 dr$. A hint comes from how R(r) is normalized (as used in Prob 6, Griff 4.11); the integral of any probability distribution must give 1.

When you do get the right probability function, find its maximum to get the most probable value.

2. Griffiths, 4.16 Note that only place in the whole derivation where the electric charges showed up was in the potential

$$V(r) = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

Obviously if the central charge is replaced by +Ze, then the e^2 in the potential gets replaced by Ze^2 , but then e^2 should be replaced *everywhere* by this. See how the result for the energy levels changes. (Is it proportional to Z? To Z^2 ?)

3. Griffiths, 4.20 You need to use

$$\frac{d}{dt}\langle \mathbf{L} \rangle = \frac{i}{\hbar} \langle [H, \mathbf{L}] \rangle$$

where

$$H = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + V(\mathbf{r})$$

to get the result. Just choose any component of this equation, say z. Show that

$$\frac{1}{2m}[p_x^2, xp_y - yp_x] = \frac{-i\hbar}{m}p_x p_y \qquad \frac{1}{2m}[p_y^2, xp_y - yp_x] = \frac{+i\hbar}{m}p_x p_y$$

so these terms will cancel. Then show

$$[V(\mathbf{r}), yp_y - yp_x] = i\hbar \left(x \frac{\partial V}{\partial y} - y \frac{\partial V}{\partial x} \right) = i\hbar (\mathbf{r} \times \nabla V)_z$$

This will give

$$\frac{d}{dt}\langle L_z\rangle = \langle N_z\rangle$$

4. Griffiths, **4.23** Apply the (analytic) raising operator to $Y_2^1(\theta, \phi)$ but also show from (4.120) and (4.121)

$$L_{+}Y_{2}^{1} = 2\hbar Y_{2}^{2}$$

5. Griffiths, **4.26** Multiply a lot of little matrices. Show that if $j \neq k$ then

$$\sigma_j \sigma_k = i \sum_l \epsilon_{jkl} \sigma_l$$

and if j=k then $\sigma_j\sigma_k=\sigma_j^2=\mathbf{1}$. But results are contained in

$$\sigma_j \sigma_k = \delta_{jk} + i \sum_l \epsilon_{jkl} \sigma_l$$

6. Griffiths, 4.27 Normalizaton should be easy. Remember that the condition is $\chi^{\dagger}\chi=1$ and for χ^{\dagger} you have to do a complex conjugation. In part (b) you should get

$$\langle S_x \rangle = 0$$
 $\langle S_y \rangle = -\frac{12}{25}\hbar$ $\langle S_z \rangle = -\frac{7}{50}\hbar$

On (c) you should get

$$\sigma_{S_z} = \frac{12}{25}\hbar$$