Phys 2110 - 4 11/14/11

Note Title 11/14/2011

Ch 10

Work - Energy Theorem

$$\Delta K = W_{net} = \int_a^b F_x \, dx$$

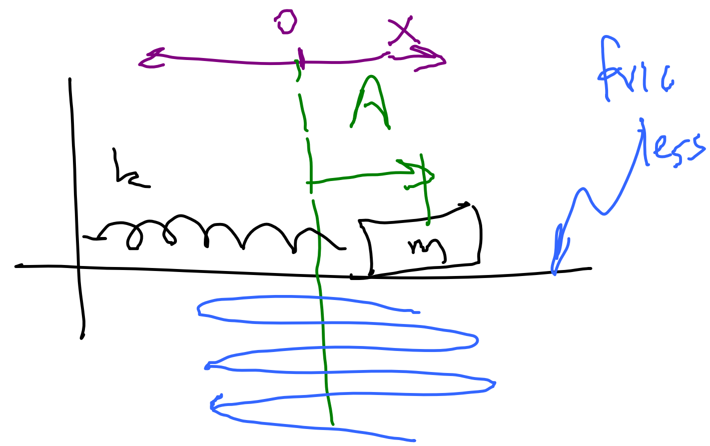
Rotational version

$$\Delta K_{rot} = \underset{\text{by torque}}{W_{net}} = \int_{\theta_1}^{\theta_2} \tau_{ext} \, d\theta \qquad (10.19)$$

$$K_{rot} = \frac{1}{2} I \omega^2$$

Ch 13 Oscillations

$$F_x = -kx = m\underbrace{\frac{d^2x}{dt^2}}_{a}$$



$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

$$\omega^2 = k/m$$

$$x = C_1\sin(\omega t) + C_2\cos(\omega t)$$

$t = 0$ $\quad \underline{\underline{x = A}}$ also $\quad v_x = \frac{dx}{dt} = 0$

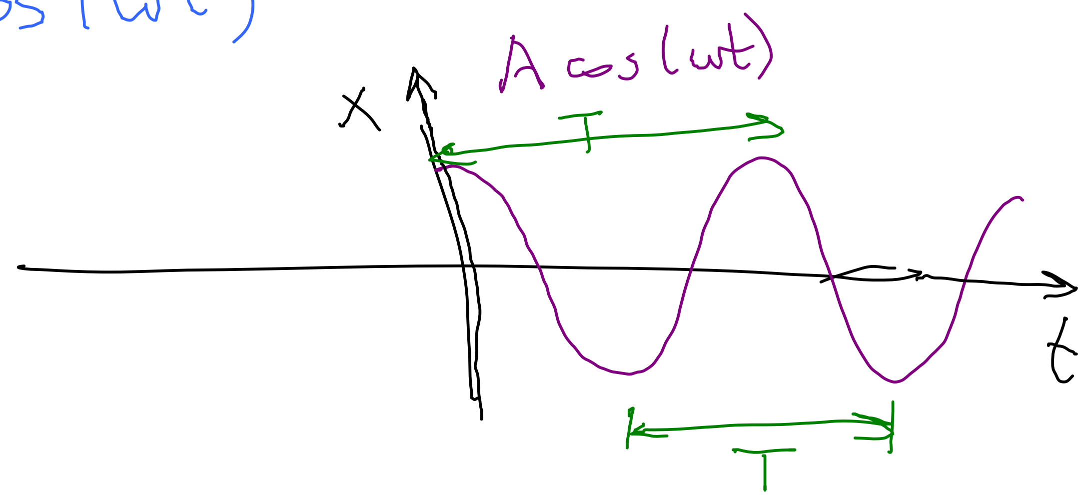
(Initial cond's)

$$x(t) = A\cos(\omega t)$$

Frequency, f

Period T

Angular frequency ω



$$A\cos(\omega t) = A\cos(\omega(t+T))$$

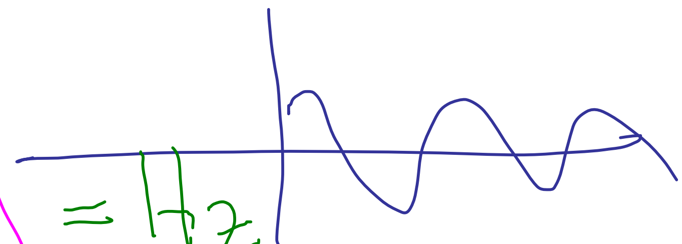
$$\longrightarrow \omega T = 2\pi \qquad \omega = 2\pi / T$$

$\omega = 2\pi / T \qquad f = 1/T$

$\Rightarrow \omega = 2\pi f = \dfrac{\#\text{ radians}}{\text{time}}$ = angular frequency

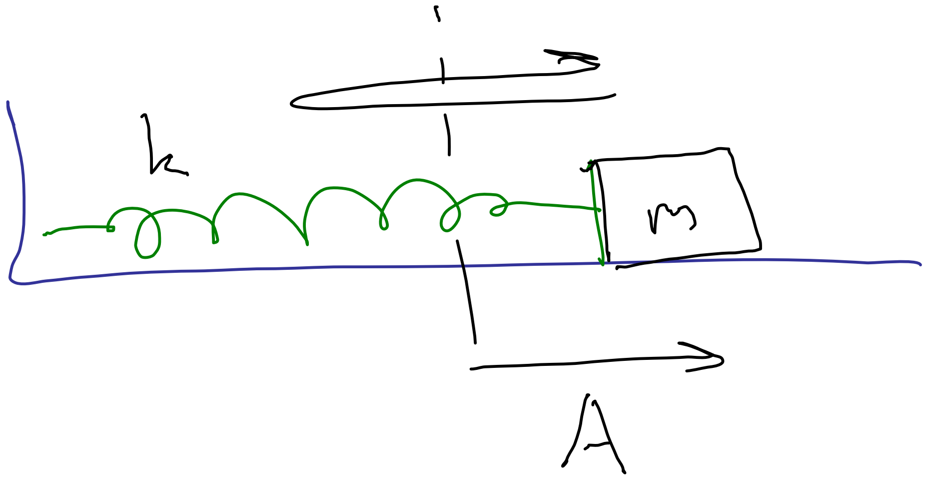
$x(t) = A\cos(\omega t)$

Units: T sec



$f = 1/T = \boxed{\text{osc}/\text{sec}} = \text{Hz}$

$\omega = 2\pi f = \boxed{\dfrac{\text{rad}}{\text{sec}}} = \boxed{1/\text{sec}}$



$$A\cos(\omega t)$$

$$T = 2\pi/\omega$$

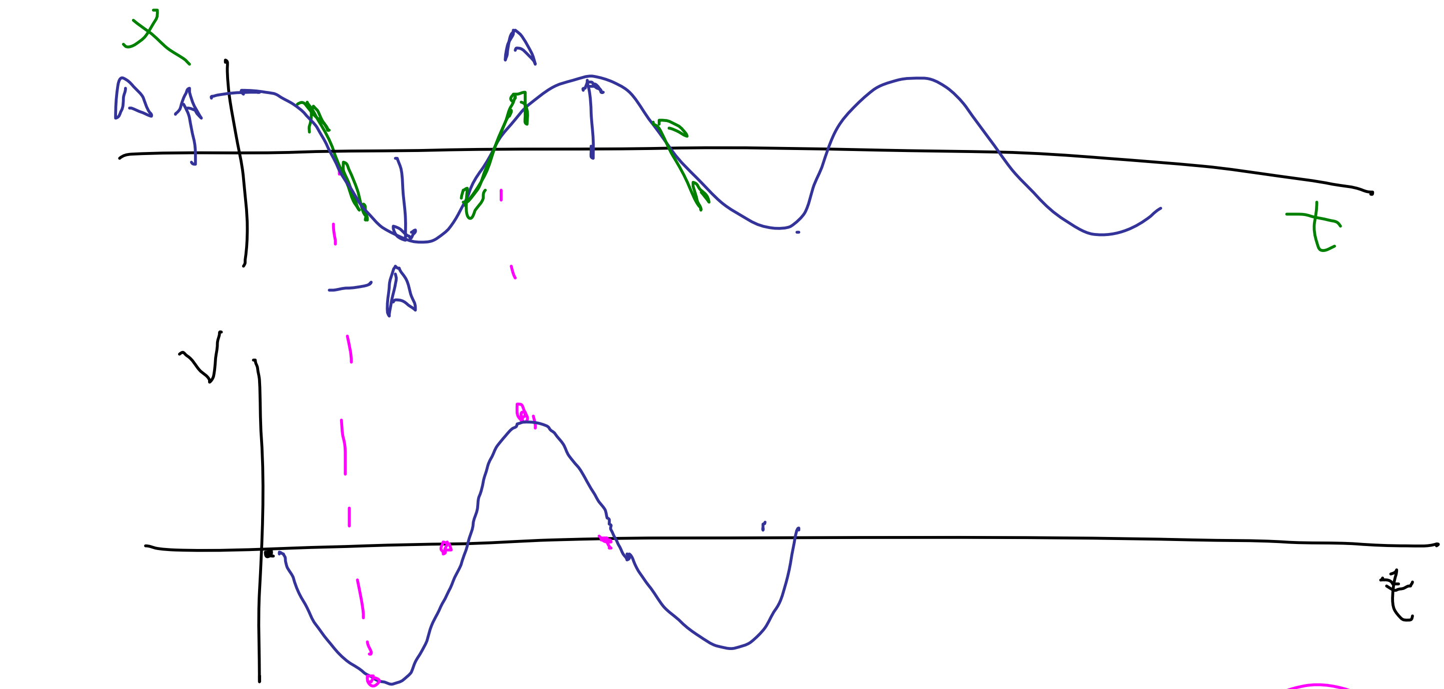
$$= 2\pi\sqrt{m/k}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{k/m}$$

ω, T, f indpt of A



$v = dx/dt$

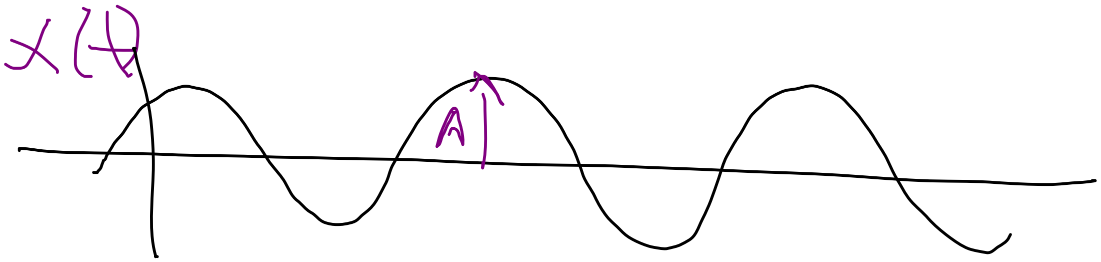
$x = A\cos(\omega t)$

$v = -\omega A \sin(\omega t)$

$x_m = A$ $\quad v_{max} = \omega A$

$a = -\omega^2 A \cos(\omega t)$

$a_{max} = \omega^2 A$



Generally, the sol'n: $x(t) = C_1 \sin(\omega t) + C_2 \cos(\omega t)$

$$x(t) = A\cos(\omega t + \phi) \qquad (13.8)$$

phase constant

If given $x(0)$, $v(0)$

you can find A, ϕ

13.25 A 50-g mass is attached to a spring & undergoes simple harmonic motion. Max accel. is $15\,\frac{m}{s^2}$ and max speed is $3.5\,\frac{m}{s}$

Determine a) angular frequency b) spring const c) Amplitude

$$a_{max} = 15\,\frac{m}{s^2} = \omega^2 A$$

$$V_{max} = 3.5\,\frac{m}{s} = \omega A$$

$$\frac{a_{max}}{V_{max}} = \frac{\omega^2 A}{\omega A} = \omega = \frac{15\,\frac{m}{s^2}}{3.5\,\frac{m}{s}}$$

$$= 4.29\ \frac{1}{s}$$

sec^{-1}

$$f = \frac{\omega}{2\pi} = 0.683\ \frac{osc}{s}$$

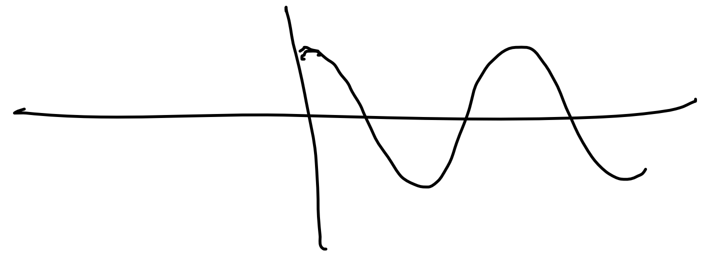
b) $\omega = \sqrt{\frac{k}{m}}$ $\quad \omega^2 = \frac{k}{m}$ $\quad k = \omega^2 m$

~~Dont~~

c) $V_{max} = \omega A$ $\quad A = \frac{V_{max}}{\omega} = \frac{3.5\,\frac{m}{s}}{4.29\,\frac{1}{s}} = 0.920\ \frac{N}{m}$

kg/s^2

Energy:



Fully stretched: Max U, Min K

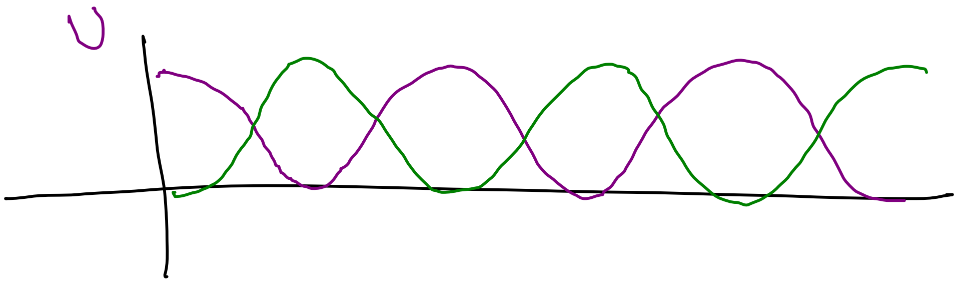
At the equilibr pt: Min U, Max K

$\omega^2 m = k$

$$E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$= \frac{1}{2}\omega^2 m A^2 \cos^2(\omega t) + \frac{1}{2} m (A\omega)^2 \sin^2(\omega t)$$

$$= \frac{1}{2}\omega^2 m A^2 = \boxed{\frac{1}{2}kA^2}$$

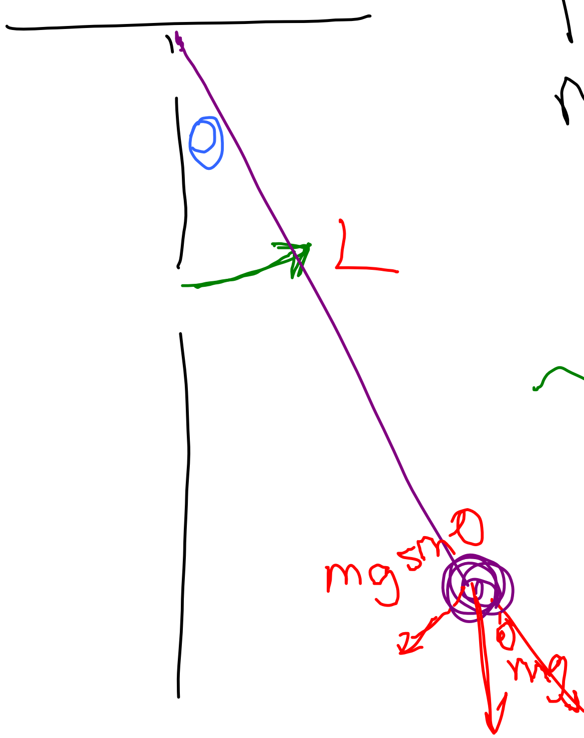


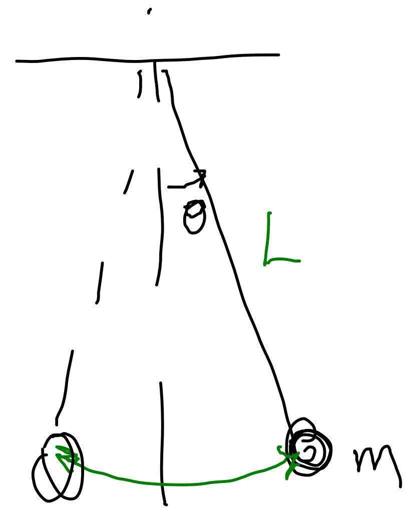
Sum = const

Pendulum

Mass m string

length L





$$\tau = -(mg\sin\theta)L$$

$$= I\alpha = mL^2\frac{d^2\theta}{dt^2}$$

$$\cancel{m} L^{\cancel{2}} \frac{d^2\theta}{dt^2} = -\cancel{m}g \sin\theta \cancel{L}$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

Diff equation

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

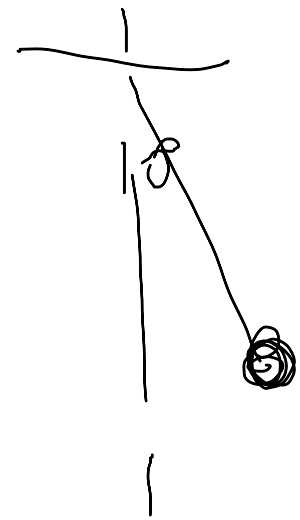
θ is in radians

when θ is small $\sin\theta \approx \theta$

Taylor series $\sin x \approx x - \cancel{\frac{x^3}{3}} \cancel{+} \cancel{\cdots}$

θ, deg	θ, rad	$\sin\theta$
5.73°	0.10	0.09983
11.45°	0.20	0.19866

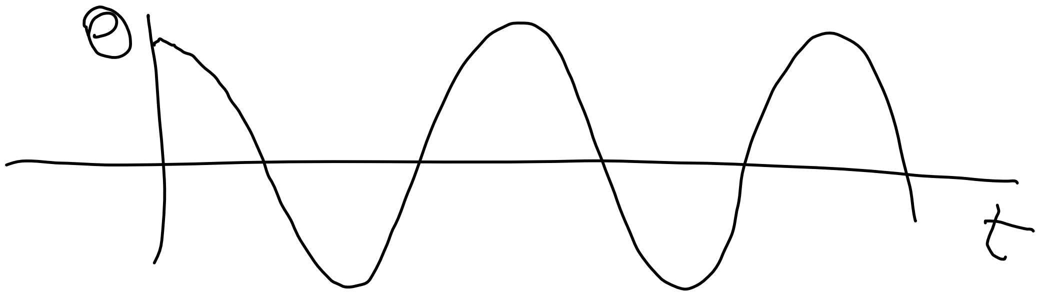
$$\sin\theta \longrightarrow \theta$$



$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta = -\omega^2\theta$$

Pull back to θ_0

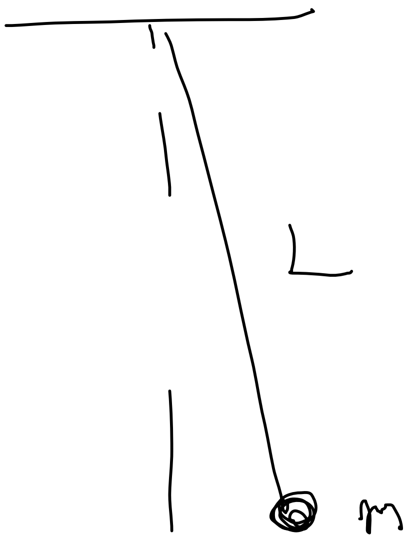
$$\theta(t) = \theta_0 \cos(\omega t) \qquad \omega = \sqrt{\frac{g}{L}}$$



$$\omega = \sqrt{g/L} \qquad f = \frac{1}{2\pi}\sqrt{g/L}$$

$$T = \frac{1}{f} = 2\pi\sqrt{L/g}$$

What is period of a pendulum (simple) of 1m in length



$$T = 2\pi\sqrt{\frac{(1m)}{(9.8\frac{m}{s^2})}} = 2.01s$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Indep't of m

$L \to \times 2$ $T \to \sqrt{2}$