Phys 2110-5 11/19/12

Note Title 11/19/2012

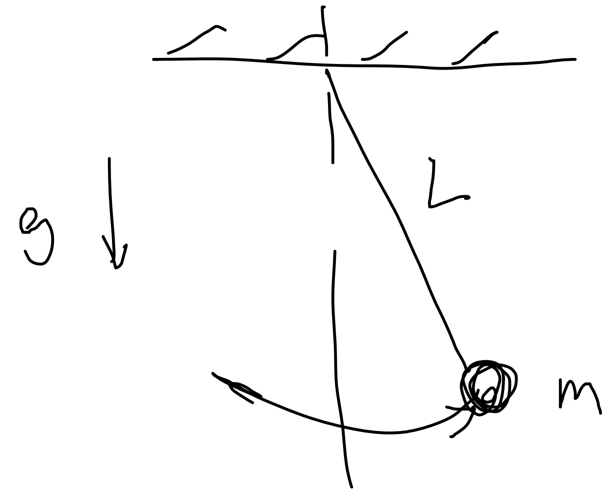
Oscillations

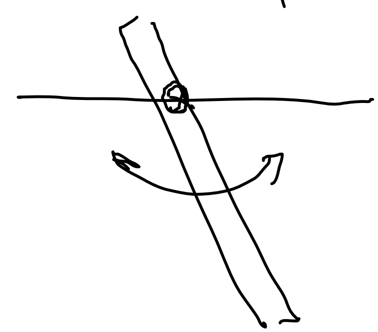
$$\omega = \sqrt{g/L}$$

$$f = \omega/2\pi \qquad T = 1/f$$

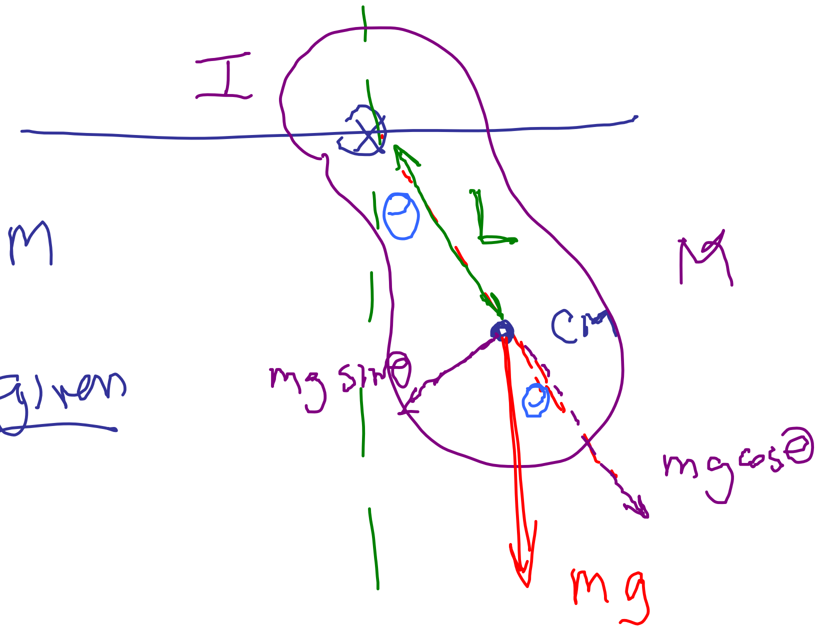
more general pendulum

Simple pendulum





General pendulum



L = dist from pivot to cm

I = Mom inertia around given axis

M = mass of object

$$\tau = \underbrace{-}_{\text{Torque}} Lmg\sin\theta = I\alpha = I\frac{d^2\theta}{dt^2}$$

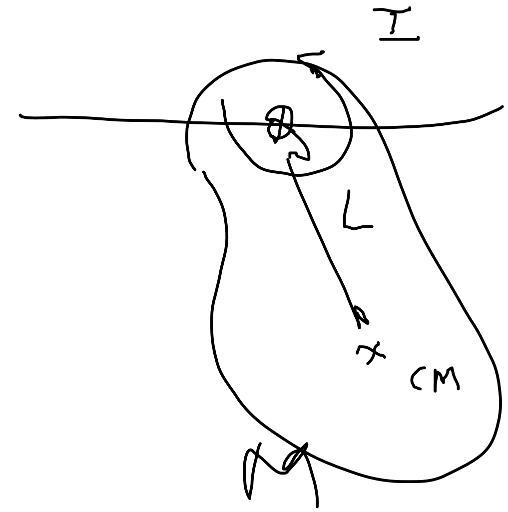
$$\frac{d^2\theta}{dt^2} = -\frac{Lmg}{I}\sin\theta = -\frac{Lmg}{I}\theta$$

$$\theta \approx \sin\theta$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

ω^2

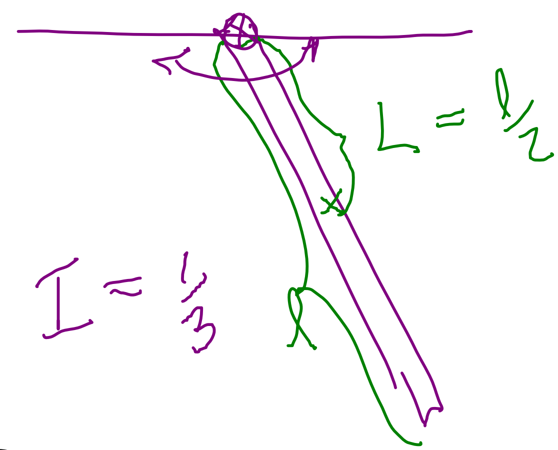
$$\longrightarrow \quad \omega^2 = \frac{Lmg}{I}$$



$$\omega = \sqrt{\frac{mgL}{I}} \qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{mgL}{I}}$$

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{I}{mgL}}$$

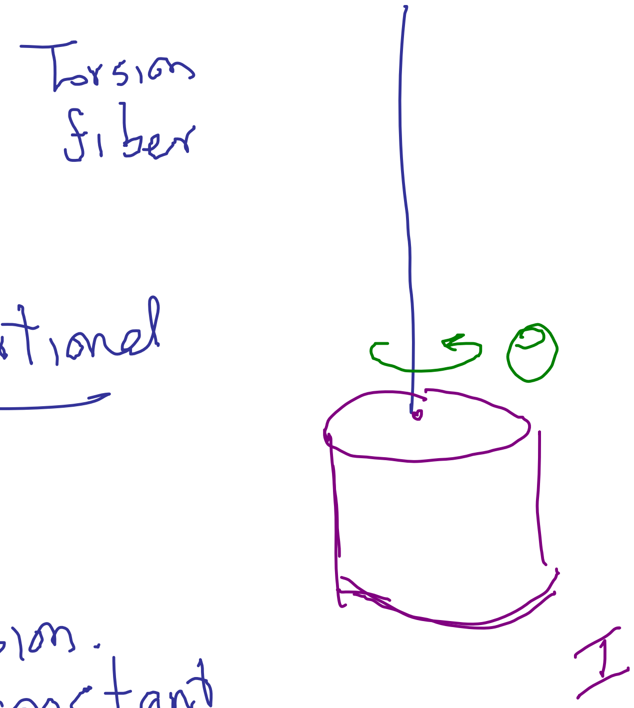
Period of 1 m stick about its end.



$$T = 2\pi\sqrt{\frac{\frac{1}{3}\cancel{m}\,l^2}{\cancel{m}\,g\left(\frac{l}{2}\right)}} = 2\pi\sqrt{\frac{2}{3}\frac{l}{g}}$$

$l = 1\,m$

Another oscillator



Torsional fiber:

Gives torque in opp dir, proportional to angle θ of twist

$$\tau = -K\theta$$

K = torsion constant

units: $N \cdot m / rad$

When in motion

$$\tau = -K\theta = I\frac{d^2\theta}{dt^2}$$

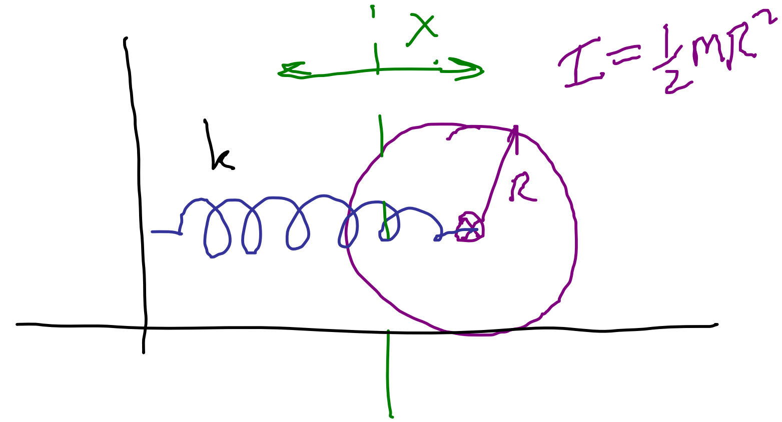
$$\frac{d^2\theta}{dt^2} = -\frac{K}{I}\theta$$

$\frac{K}{I} \rightarrow \omega^2$ etc.

13.63 Solid cylinder mass M, radius R, rolls back & forth ... on a spring of constant k.

Find angular freq. of motion

Do it by energy methods.



Spring is extended by $x \longrightarrow U = \frac{1}{2}kx^2$

$$K = K_{tot} = K_{trans} + K_{rot} = \underbrace{\frac{1}{2}Mv^2}_{\frac{1}{2}Mv^2} + \underbrace{\frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2}_{\frac{1}{2}I\omega^2}$$

$$= \frac{3}{4}Mv^2$$

Energy is cons'd $\quad K + U = \text{const}$

$$\frac{1}{2}kx^2 + \frac{3}{4}Mv^2 = \text{const}$$

Take deriv of this $\left(\frac{d}{dt}\right)$

$$\cancel{\frac{1}{2}}k(\cancel{2}x)\frac{dx}{dt} + \frac{3}{\cancel{4}_2}M(\cancel{2}v)\frac{dv}{dt} = 0$$

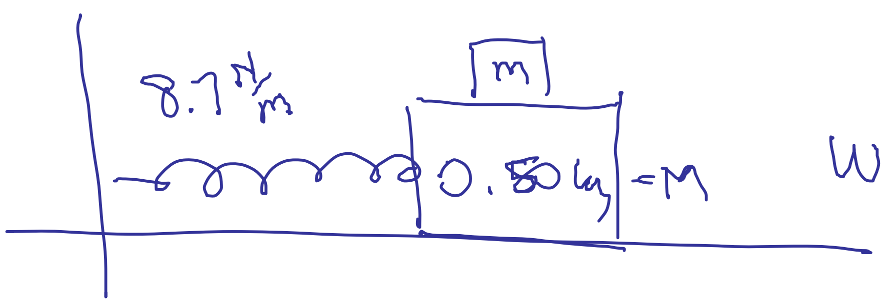
$$kx\cancel{v} + \frac{3}{2}M\cancel{v}a = 0$$

$$a = -\frac{2}{3}\frac{k}{M}x \qquad \frac{d^2x}{dt^2} = -\frac{2}{3}\frac{k}{M}x$$

$$\omega = \sqrt{\frac{2k}{3M}} \quad \text{etc.} \qquad \omega^2$$

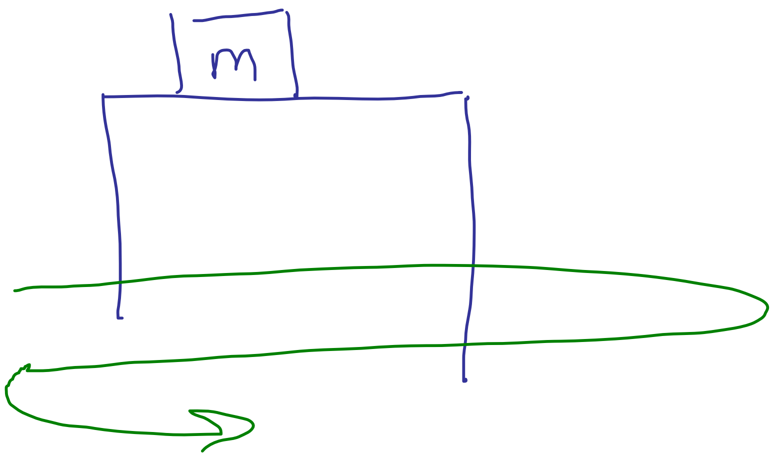
13.74 A 500 g block on fricless horiz surface attached to rather limp spring $k = 8.7\ \frac{N}{m}$ Second rests on the first whole system executes SHM w/ period 1.8 s. When amplitude of motion is incr'd to 35 cm upper block begins to slip. What's coeff. of static friction?

First part blocks move together as one unit



$$\omega = 3.49\ s^{-1} = \sqrt{\frac{k}{M+m}}$$

$$\Longrightarrow m = 0.214\ kg$$



Block slips when max force is bigger than $f_s^{(max)} = \mu_s n$

$= \mu_s m g$

What is max force

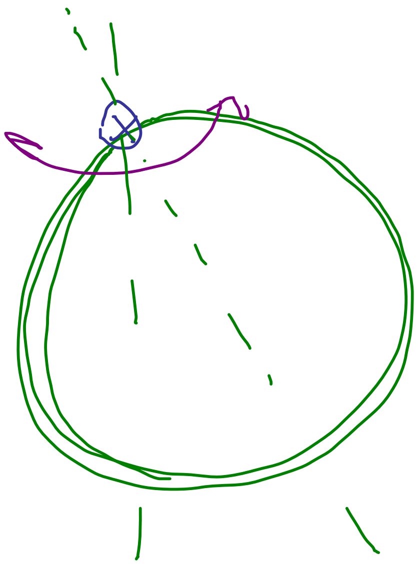
$= m a_{max} = m \underbrace{A \omega^2}$

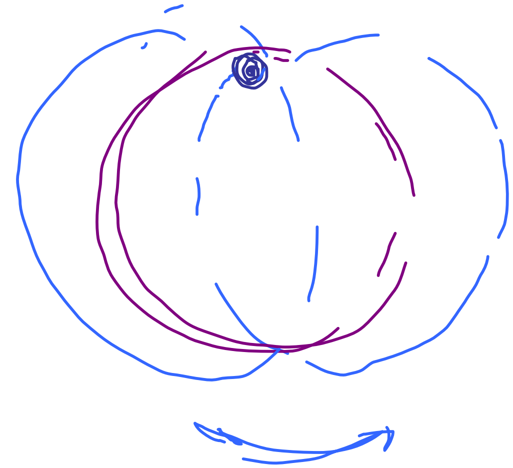
Then:

$\cancel{m} A \omega^2 = \mu_s \cancel{m} g$

$\mu_s = \frac{A\omega^2}{g} = \frac{(0.35 m)(3.49\ s^{-1})^2}{9.8 \frac{m}{s^2}}$ $\boxed{= 0.435}$

13.58 Thin uniform hoop of mass M radius R suspended from horiz rod
set osc w/ small amplitude
Show period is $2\pi\sqrt{\frac{2R}{g}}$





Physical pendulum

$$\omega = \sqrt{\frac{mgL}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

$$I = MR^2 + MR^2 = 2MR^2$$

par axis. thm

$L = R$

$$T = 2\pi\sqrt{\frac{I}{mgL}} = 2\pi\sqrt{\frac{2\cancel{M}R^{\cancel{2}}}{\cancel{M}g\cancel{R}}}$$

$$\boxed{= 2\pi\sqrt{2R/g}}$$

More generally, friction term.

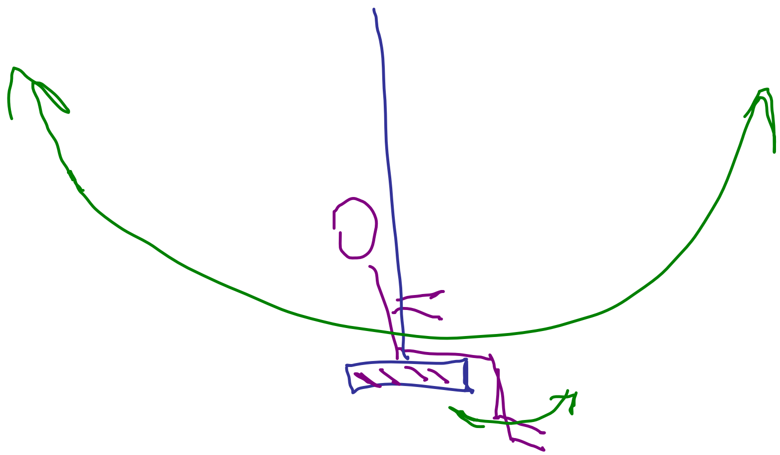
I mean really friction $\propto$ speed

p. 315

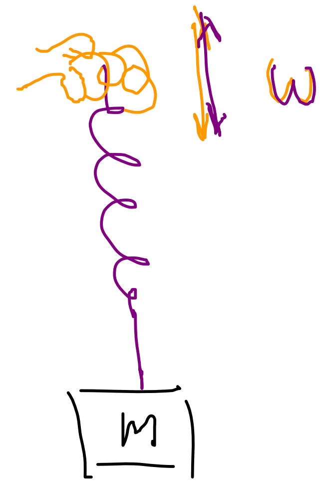
Damped harmonic motion. $F_x = -bv_x$

Harder DE to solve.

Driven oscillations



Resonance



$\omega_0 = \sqrt{\frac{k}{m}}$