Phys 3810, Spring 2010
Problem Set #3, Hint-o-licious Hints

1. *Griffiths*, **2.29** Actually, this one isn't so bad. You just need to go through the finite–well derivation in the book and make the appropriate changes for the asymmetric state.

The main results you should get to are the results of applying the boundary conditions:

$$-\kappa = \ell \cot(\ell a)$$

and, using the same definitions of z and z_0, the new condition for finding the energies (graphically, perhaps)

$$-\cot(z) = \sqrt{(z_0/z)^2 - 1}$$

which is *not* guaranteed to have a root since the right side is positive and the left side starts off being negative for small z.

2. *Griffiths*, **3.7** (a) is pretty easy; (b) is also easy; just try the simplest linear combinations of $f(x)$ and $g(x)$ you can think of. Big hint, they're common functions accessed on calulators with a key marked "hyp".

3. *Griffiths*, **3.12** Some steps: Start with

$$\langle x \rangle = \int_{-\infty}^{\infty} \Psi(x,t)^* x \Psi(x,t)\, dx$$

and substitute (3.55),

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{ipx/\hbar} \Phi(p,t)\, dp$$

Watch for complex conjugates and be sure that the dummy variable of p integration is *different* in the two integrals. When you do this you have a big integral over three variables: x, p and p'.

Do what Griffiths says, use

$$x e^{ipx/\hbar} = \frac{\hbar}{i} \frac{d}{dp} e^{ipx/\hbar}$$

and then do an integration by parts. Be careful with with variable names to know what the derivatives are acting on.

Use (and review) the result of problem 2.26,

$$\int_{-\infty}^{\infty} dx\, e^{iqx} = 2\pi\delta(q)$$

Use this for one of the integrations, but as usual be careful with the variables.

The next integration collapses the delta function and gives the desired result.

4. *Griffiths*, **3.23** As discussed in class, from considering the action of this H on the state $|1\rangle$ and $|2\rangle$, one can show that

$$H = \begin{pmatrix} E & E \\ E & -E \end{pmatrix}$$

With all of the $\sqrt{2}$'s flying around, the normalization is a little funky.

5. *Griffiths*, **3.27** The first measurement puts the system in state ψ_1. Now the probabilities for measuring b_1 and b_2 can be read off from the coefficients (squared). The B measurement was made, but *you are not told* the result of the measurement.

Now when A is measured again, you are *not* assured of getting a_1 and that is because the B measurement —whatever its outcome— has changed the state of the system. Write the ϕ's in terms of the ψ's and then consider that if the B measurement had given b_1 what the probability of a_1 would be. Then consider that if the B measurement had given b_2 what the probability of a_1 would be. Combine all the probabilities together.

The answer to the question is $\frac{337}{625}$ but show how you get it! I had to stop and think for a while why the answer is not 1. But the fact that the measurement of B was made does have a physical effect *even if we don't know the result of that measurement*. (The answer does employ the fact that we don't know what it was.)

6. *Griffiths*, **3.31** You'll need to evaluate

$$[H, xp] = \left[\frac{p^2}{2m}, xp\right] + [V(x), xp]$$

The first of these is

$$\left[\frac{p^2}{2m}, xp\right] = -2i\hbar\frac{p^2}{2m}$$

and the second is

$$[V(x), xp] = -\frac{\hbar}{i}x\frac{dV}{dx}$$

7. *Griffiths*, **3.37** Get the eigenvalues and eigenvectors of H. (They are c and $a \pm b$, and some simple vectors with 1's and -1's. Show all of this!) Part (c) has the system start in a state which is not an eigenstate, so you must decompose it in terms of eigenstates (can be done "by inspection") and then attach the usual oscillatory time dependence with the energy eigenvalues.