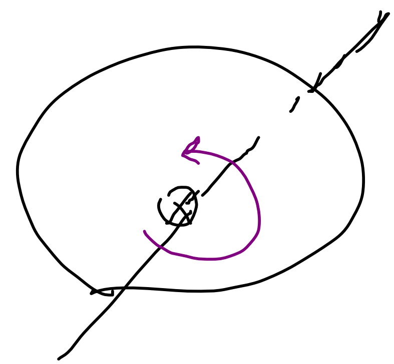
Phys 2110-4 11/4/11

Note Title 11/4/2011

Rotations α, τ, I



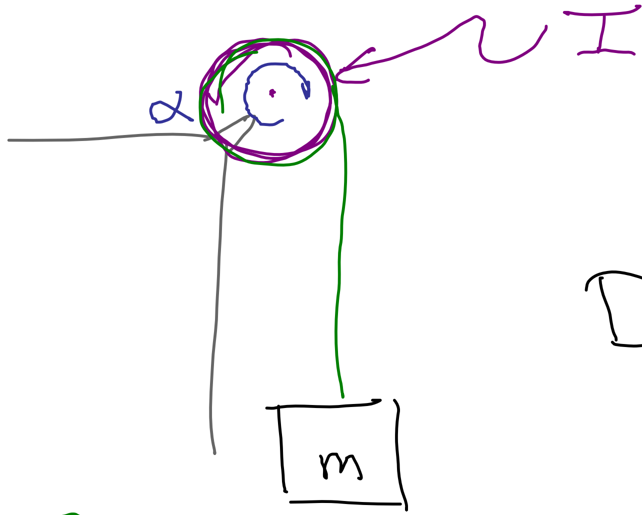
N's 2nd Law for Rotations

$F = ma$

$$\boxed{\tau = I\alpha}$$

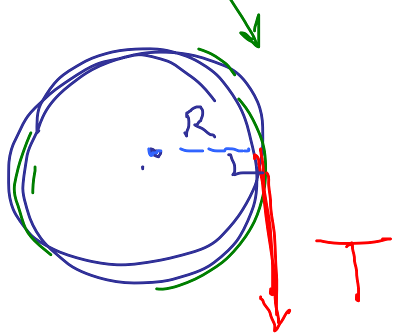
Example 10.9

Buckets, Well.



Draw pictures

$$a = a_t = R\alpha \qquad \alpha = \frac{a}{R}$$



$$\boxed{mg - T = ma}$$

$$\tau = TR \cdot (1) \overset{2^{nd}}{=} I\alpha = I\frac{a}{R}$$

$$mg - T = ma$$

$$\tau = TR = I\frac{a}{R} \qquad \boxed{T = I\frac{a}{R^2}}$$

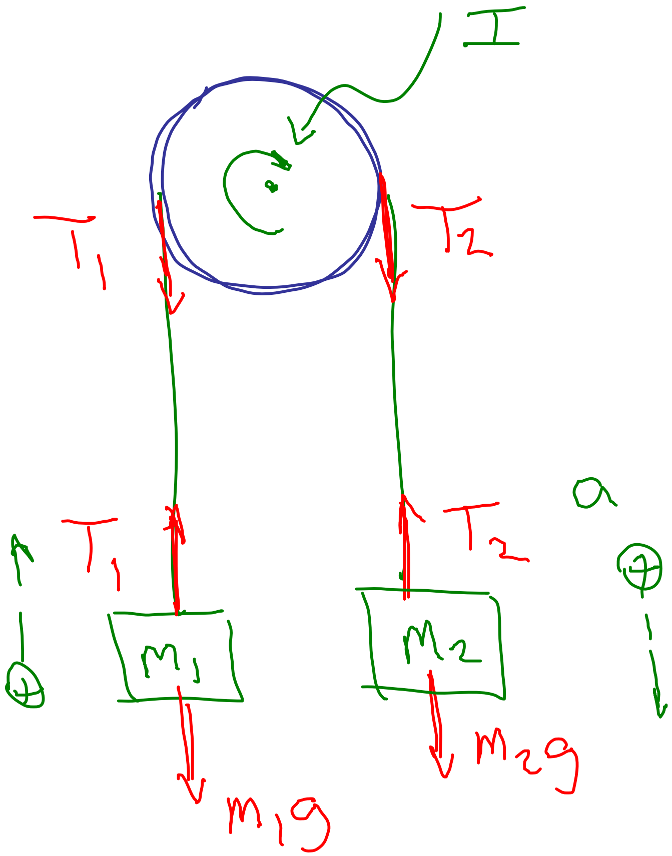
Substitute

$$mg - I\frac{a}{R^2} = ma$$

$$mg = a\left(m + \frac{I}{R^2}\right) \qquad \boxed{a = \frac{mg}{\left(m + \frac{I}{R^2}\right)}}$$

$I = 0$

Other problems involving pulley w/ mass



Now, tension is diff. on both sides

4 eqns
4 unk's

$$\boxed{T_1} - m_1 g = m_1 \boxed{a}$$

$$m_2 g - \boxed{T_2} = m_2 a$$

$$\tau = T_2 R - T_1 R = I \boxed{\alpha}$$

$$a = a_t = \alpha R$$

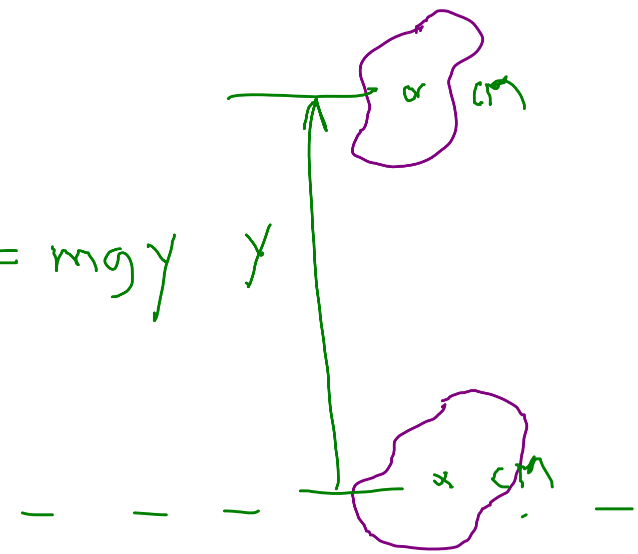
Solve this!

Energy!

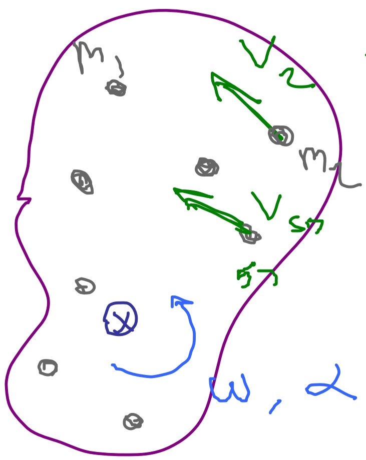
For a large object

Potential E

$$U = mgy$$



Rotating Object



$$v_2 = r_2 \omega$$

$$v_{57} = r_{57}\,\omega$$

$$K = \sum \left(\tfrac{1}{2} m_i v_i^2\right)$$

$$K = \sum_i \frac{1}{2}(m_i v_i^2)$$

$$v_i = r_i \omega$$

$$= \frac{1}{2} \sum_i m_i (r_i \omega)^2$$

$$= \frac{\omega^2}{2} \sum_i m_i r_i^2 = \frac{\omega^2}{2} I$$

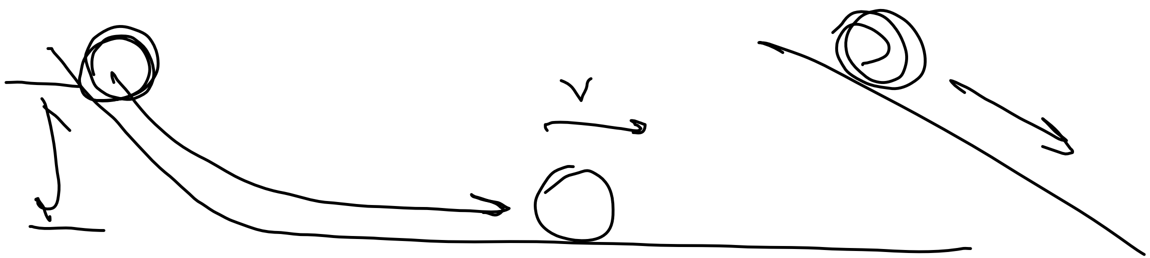
Moment of inertia, I

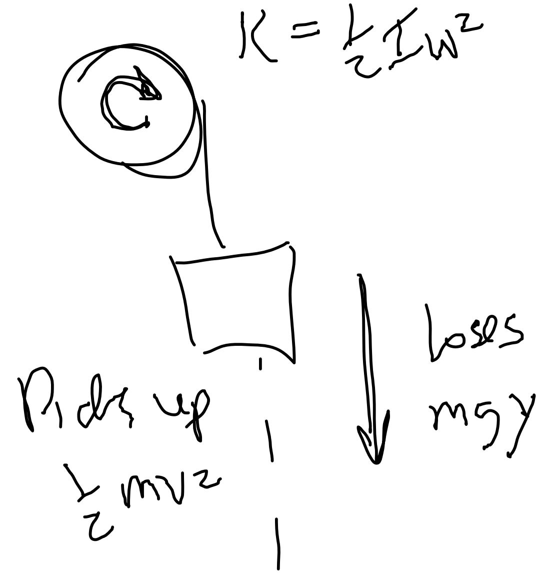
$$K = \frac{1}{2} I \omega^2$$

$$K = \frac{1}{2} m v^2$$

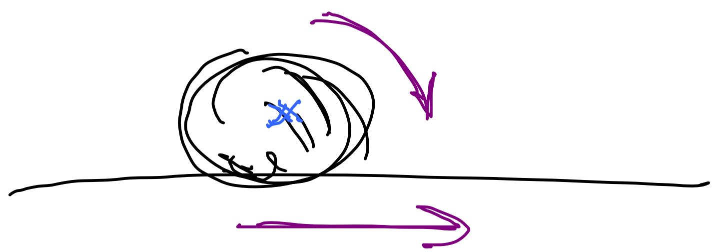
$$J = (kg \cdot m^2)\left(\frac{\cancel{rad}}{s}\right)^2 = \frac{kg \cdot m^2}{s^2} = J$$

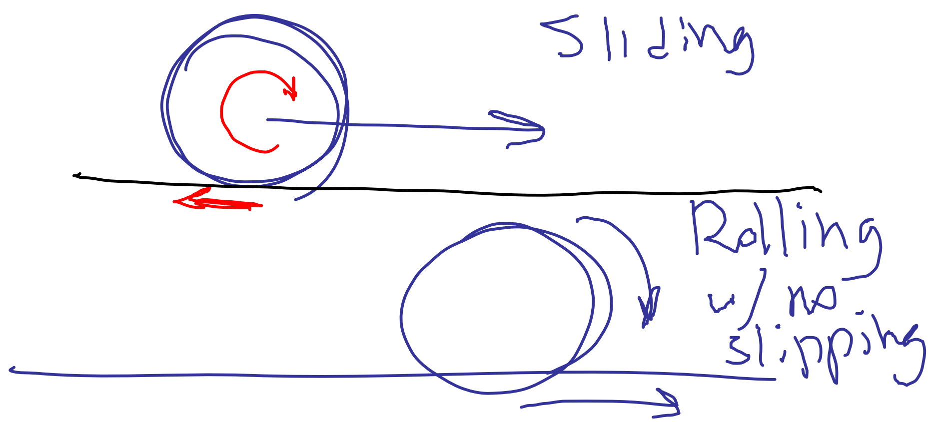
Cons of energy still holds!

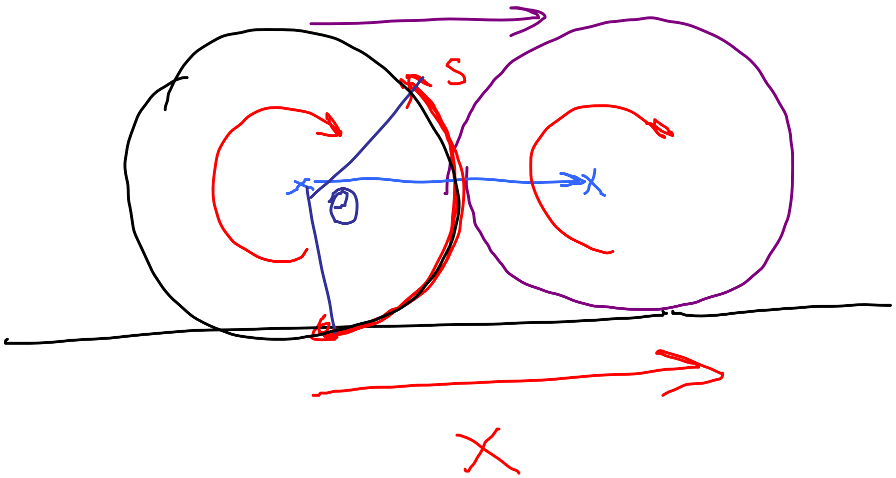




Rolling Motion!







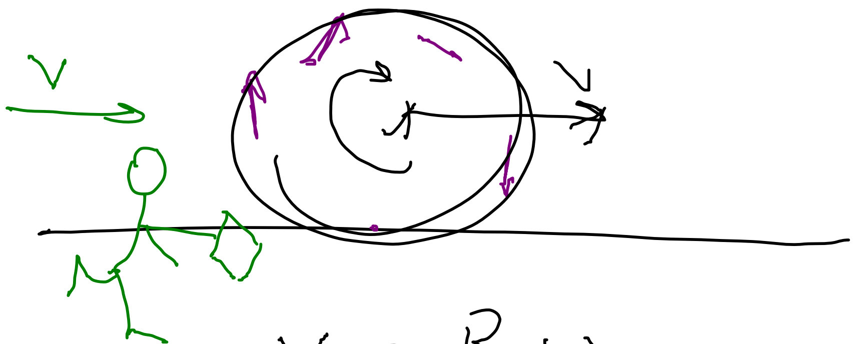
$$S = X$$

$$R\theta = S = X$$

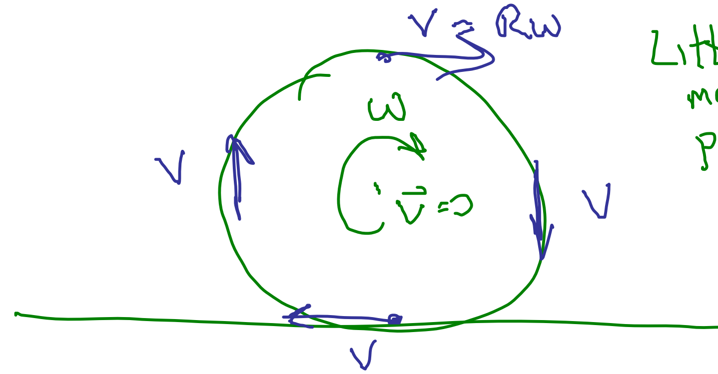
$$\boxed{\rightarrow R\omega = \frac{dx}{dt} = V_{cm}}$$

$$\boxed{\rightarrow R\alpha = a_{cm}}$$

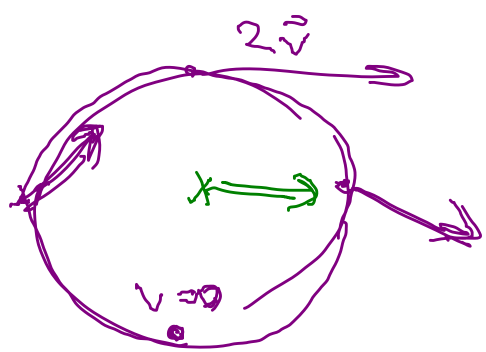
Parts of wheel have different velocities



$v = R\omega$



What are velocities



To get the "ground" $\vec{v}$'s of bits of wheel

add $\xrightarrow{\vec{v}}$

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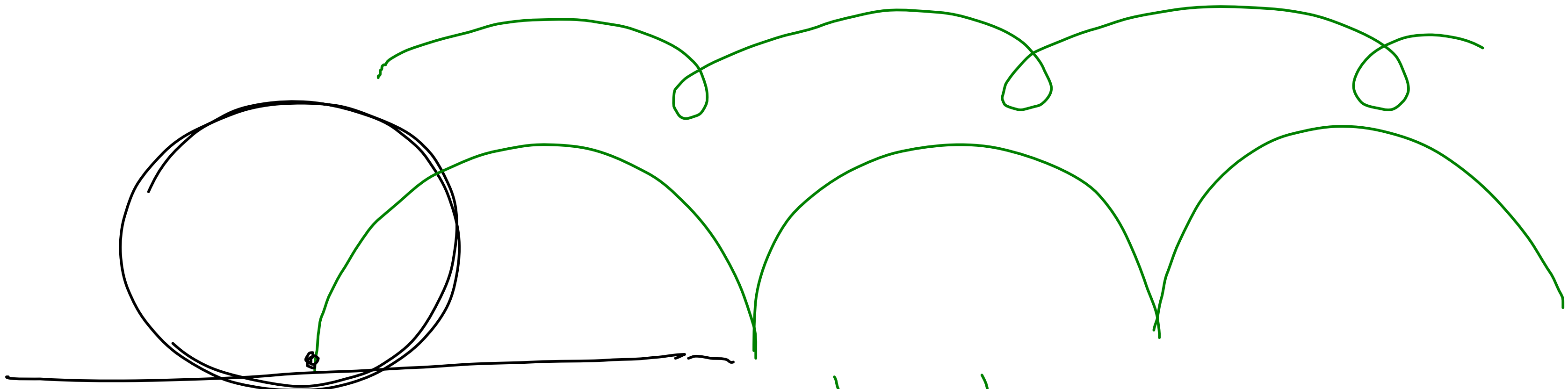
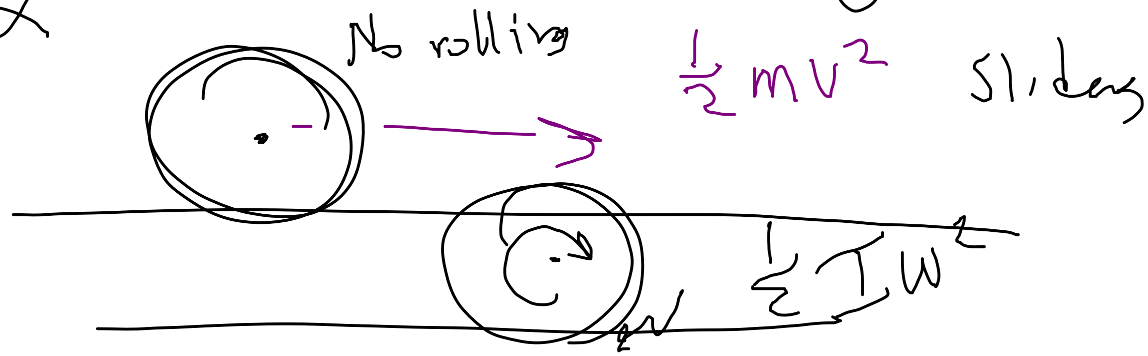


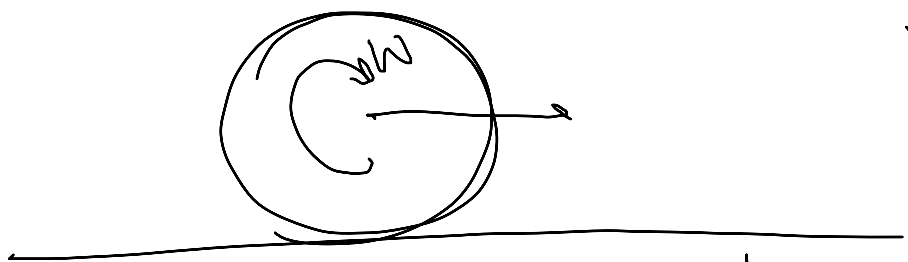
cycloid

$$V_{cm} = R\omega$$

$$a_{cm} = R\alpha$$

KE of rolling object





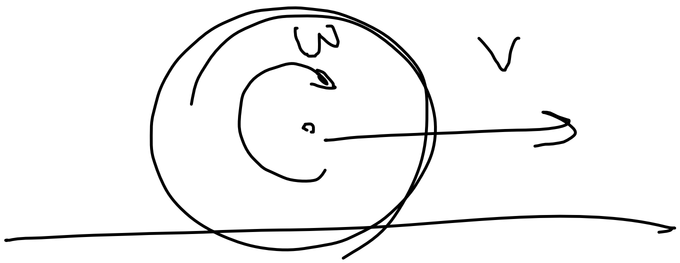
$v_{cm} = R\omega$ (10.20)

$$K_{rolling} = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \omega^2$$

$$= \underset{\text{trans}}{K_{cm}} + \underset{\text{rot}}{K_{internal}}$$

$\omega = v_{cm}/R$

10.39 What fraction of a solid disk's kinetic energy is <u>rotational</u>?
(Rolls w/o slipping)



$$I = \frac{1}{2}MR^2$$

$$v = R\omega$$

$$K_{Tot} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2$$

$$= \frac{3}{4}Mv^2$$

$$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\cdot\frac{1}{2}MR^2\left(\frac{v}{R}\right)^2 = \frac{1}{4}Mv^2$$

$$\frac{K_{rot}}{K_{Tot}} = \frac{\frac{1}{4}Mv^2}{\frac{3}{4}Mv^2} = \boxed{\frac{1}{3}}$$