Phys 3820, Fall 2009
Problem Set #4, Hint-o-licious Hints

1. *Griffiths*, **8.4** For U^{238} I find that the KE of the alpha is 4.3 MeV giving a speed of $1.44 \times 10^7\ \frac{\mathrm{m}}{\mathrm{s}}$. It gives a lifetime of the very rough order of 3×10^{15} yr which is several orders of magnitude to high, but at least it's not "short".

For Po^{212} I find, using the *nuclear* masses from my source, an alpha speed of $1.94 \times 10^7\ \frac{\mathrm{m}}{\mathrm{s}}$ and a lifetime of 0.75 s, which is way off from the experimental value of microseconds, but again, it is "small".

Maybe you do better using *atomic* masses for the nuclei. But be sure you know what masses you are using; for differences in masses the elctrons masses are important.

2. *Griffiths*, **8.7** Here, $V(x) = \frac{1}{2}m\omega^2x^2$. We will use the result of Example 8.4 even though we didn't go through the jive with connection formulae. For a potential well which has smooth sides and where the turning points of the motion are x_1 and x_2, we have

$$\int_{x_1}^{x_2} p(x)\,dx = (n - \tfrac{1}{2})\pi\hbar \quad \text{where} \quad p(x) = \sqrt{2m(E - V(x))}$$

(This formula is also one would have been *postulated* in the pre–Schrödinger quantum mechanics of the years 1915–1925. The early QM gave some correct results but ultimately led nowhere.)

The integral involves an inverse trig function and ultimately gives the correct answer for the HO energy levels.

3. *Griffiths*, **9.1** You jest need to evaluate matrix elements of the form

$$H'_{ij} = H'_{2lm,100} = \langle \psi_{2lm} | H' | \psi_{100} \rangle = eE \int \psi_{2lm}(\mathbf{r}) z \psi(\mathbf{r})\, d^3\mathbf{r}$$

but since $z = r\cos\theta$ and $\cos\theta$ is proportional to Y_1^0, that angular integral here is

$$\int Y_l^{m*}(\theta,\phi) Y_1^0(\theta,\phi)\, d\Omega$$

which from orthogonality is zero unless $l = 1$ and $m = 0$ so there's really only one matrix element to evaluate.

For the diagonal matrix elements, write down the angular integral and you can argue that from the antisymmetry in θ about $\theta = \frac{\pi}{2}$ the result is zero.

4. *Griffiths*, **9.5**

5. *Griffiths*, **9.8** Use (9.47), (9.56) and (9.52) to find the ratio

$$\frac{R_{b\to a}}{A}$$

You'll note that the messy dipole matrix element cancels out as well as nearly everything else, leaving only

$$\frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}$$

From this, at a particular T find the value of ω where the ratio R/A is one, i.e. the rates from stimulated and spontaneous emission are equal.

6. *Griffiths*, **9.10** A common, basic problem. Deduce that

$$t_{\frac{1}{2}} = (\ln 2)\tau$$

7. *Griffiths*, **9.11** You are out to calculate

$$\tau = \frac{1}{A} = \frac{3\pi\epsilon_0\hbar c^3}{\omega_0^3|\mathbf{p}|^2} \qquad \text{where} \qquad \mathbf{p} = q\langle\psi_b|\mathbf{r}|\psi_a\rangle$$

where ψ_1 is the ground state of the H atom and ψ_b is any one of the 4 states with $n = 2$.

First off, you can easily show that

$$\langle\psi_{200}|x|\psi_{100}\rangle = 0 \qquad \text{etc. for } y \text{ and } z$$

so from this, the lifetime of the 200 state is infinite!

Next, consider the 210 state. You can show that the matrix elements

$$\langle\psi_{210}|x|\psi_{100}\rangle \qquad \text{and} \qquad \langle\psi_{210}|y|\psi_{100}\rangle$$

are zero, and that the z matrix element is

$$\langle\psi_{210}|z|\psi_{100}\rangle = -\frac{256}{243\sqrt{2}}ea \approx -(0.74493)ea$$

and with only one component to include in $|\mathbf{p}|^2$, this gives a lifetime of the 210 state of 1.6 ns.

So now consider the 21 ± 1 states. One can show that for these, the z matrix element is zero:

$$\langle\psi_{21\,\pm1}|z|\psi_{100}\rangle = 0$$

so we need only the x and y elements. The sum of the *absolute squares* of these give $|\mathbf{p}|^2$. I found

$$p_x = \langle\psi_{21\,\pm1}|x|\psi_{100}\rangle = \pm\frac{128}{243}ea \quad \text{and} \quad p_y = \langle\psi_{21\,\pm1}|y|\psi_{100}\rangle = -i\frac{128}{243}ea$$

which when put into $|\mathbf{p}|^2$ give the same value as in the 210 case and hence the same lifetime... which we would really expect. So the lifetime of any of the 2p states is 1.6 ns.