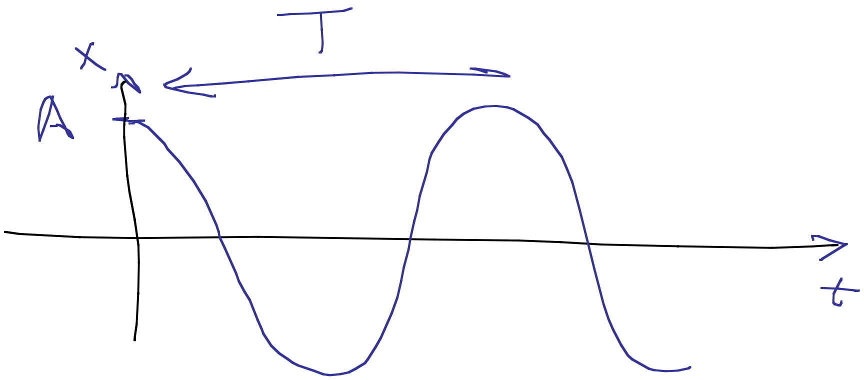
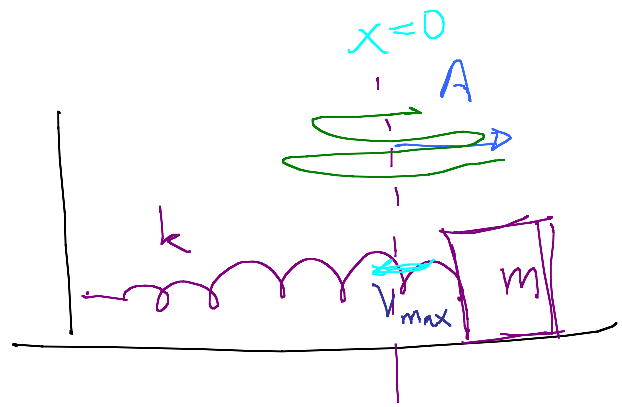
Phys 2110-3 11/17/10

Note Title 11/17/2010

Oscillations Ch 13





$$f = 1/T \qquad \omega = 2\pi f$$

$\frac{osc}{s} = Hz \qquad \frac{rad}{s} \qquad \frac{1}{s}$

$$E = \frac{1}{2} k A^2$$

$$= \frac{1}{2} m V_{max}^2$$

N's 2nd Law

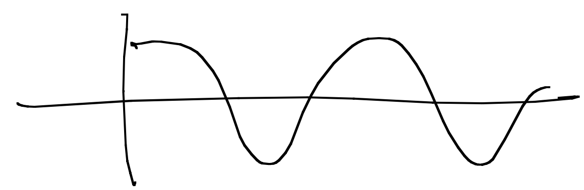
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

Gen. solution

$$x(t) = C_1 \sin(\omega t) + C_2 \cos(\omega t)$$

<u>Released</u> at $x = A$ and $\frac{dx}{dt} = 0$ at $t = 0$

$C_1 = 0 \quad C_2 = A$



$$x(t) = A\cos(\omega t)$$

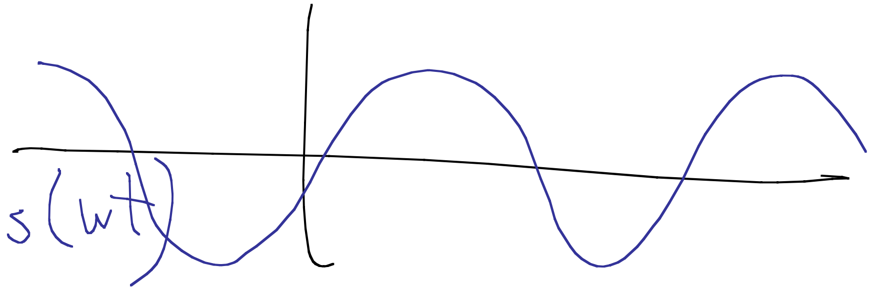
Motion repeats when ωt increases by 2π

so $\Delta(\omega t) = 2\pi$

$\omega \Delta t = 2\pi = \omega T$

$\omega = 2\pi/T = 2\pi f$

More generally



$$x(t) = C_1 \sin(\omega t) + C_2 \cos(\omega t)$$

$$= A \cos(\omega t + \phi)$$

Amplitude

phase constant

To find these,

know 'initial conditions'

$$v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$v_{max} = \omega A$$

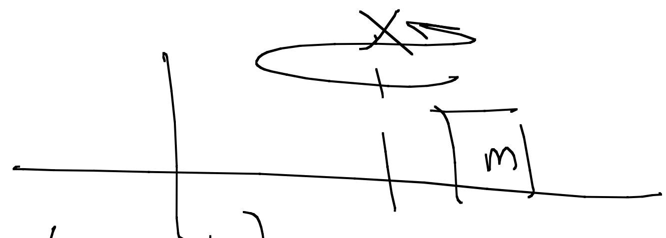
$$-\omega^2 x(t)$$

$$a_{max} = \omega^2 A$$

$$a(t) = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi)$$

$$m a_x = m(-\omega^2 x) = m\left(-\frac{k}{m}\right)x = -kx$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}} \qquad f = \omega/2\pi \qquad T = 1/f$$



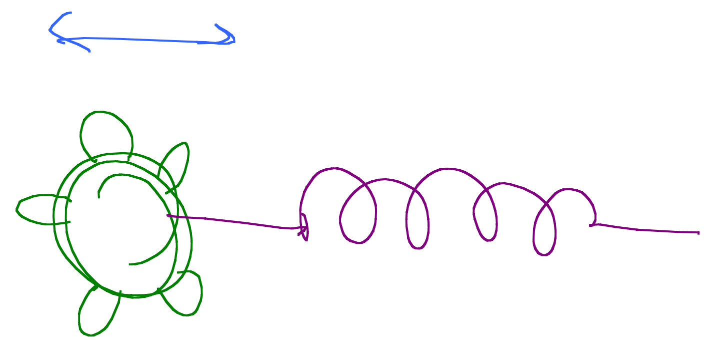
Energy:

$$U = \frac{1}{2}kx^2 = \frac{k}{2}A^2\cos^2(\omega t+\phi)$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\underbrace{\omega^2}_{k/m}\sin^2(\omega t+\phi)$$

$$U+K = \frac{1}{2}kA^2 = E \qquad = \frac{1}{2}kA^2\sin^2(\omega t+\phi)$$

13.22 Astronaut is "weighed" by being attached to spring $k = 400\ \mathrm{N/m}$ set into simple harmonic motion. Osc period is 2.5 s what is ast's mass?



$$T = 2.5\ \mathrm{s}$$

$$\omega = \frac{2\pi}{T} = 2.51\ \mathrm{s}^{-1}$$

$$= \sqrt{\frac{k}{m}} \qquad \omega^2 = \frac{k}{m} \qquad m = \frac{k}{\omega^2} = \frac{400\ \mathrm{N/m}}{(2.51\ \mathrm{s}^{-1})^2}$$

$$= 63.5\ \mathrm{kg}$$

$$\omega = 2\pi f$$

13.25 A 50g mass attached to spring & undergoes SHM. Its max. accel. 15 $\frac{m}{s^2}$ and max speed is 3.5 $\frac{m}{s}$. Determine

a) Ang. freq. b) Spring constant

c) Amplitude

$$a_{max} = \omega^2 A \qquad V_{max} = \omega A$$

$$\frac{a_{max}}{V_{max}} = \frac{\omega^2 A}{\omega A} = \omega = \frac{15 \frac{m}{s^2}}{3.5 \frac{m}{s}} = 4.29\ s^{-1}$$

$$\omega = \sqrt{k/m} \qquad \omega^2 m = k = (4.29\ s^{-1})^2 (0.050\ kg)$$

$$= 0.920\ N/m$$

$$A = \frac{v_{max}}{\omega} = \frac{3.5\ m/s}{(4.29\ s^{-1})} \qquad v_{max} = \omega A$$

$$= 0.817\ m$$

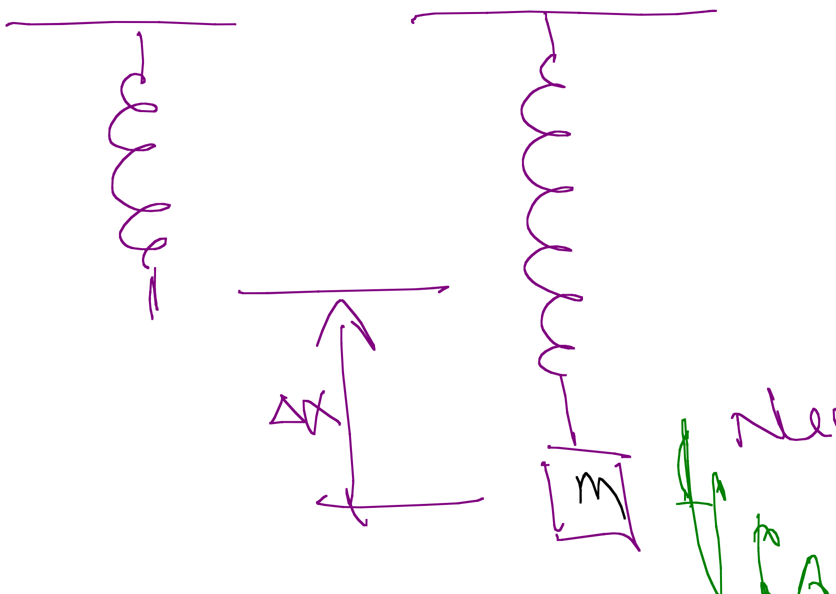
13.35 450 g mass on a spring is osc'ing at 1.2 Hz. The total energy is 0.51 J. Find amplitude

$$E = \frac{1}{2} k A^2 \qquad \omega = 7.54\ s^{-1} \ (= 2\pi f) \qquad \omega = \sqrt{k/m} \qquad k = 25.6\ N/m$$

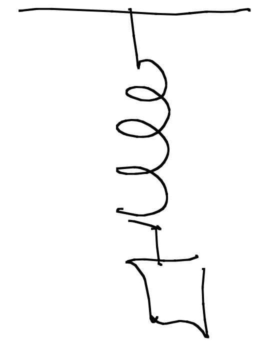
$$\longrightarrow A = 20.\ cm$$

Comments:

Vertical spring



$\Delta x = \frac{mg}{k}$



New equilibr. position

$\omega = \sqrt{\frac{k}{m}}$

To deal w/ mass of spring

$m \to m + m_s/3 \qquad \omega = \sqrt{\frac{k}{m + m_s/3}}$