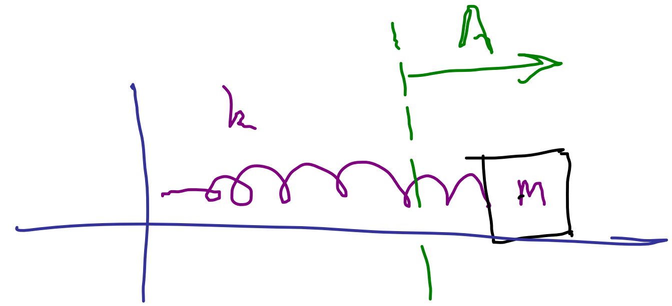
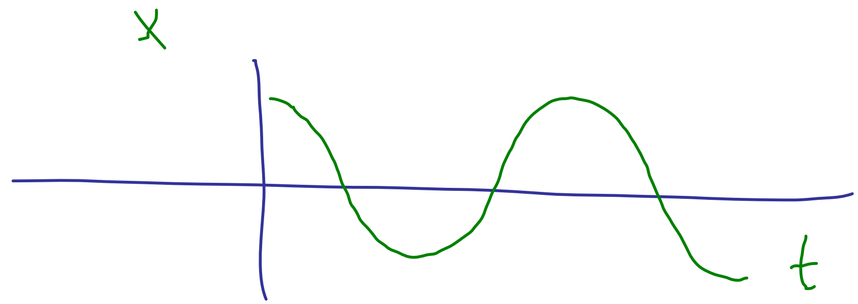
Phys 2110-4   4/15/2013

Note Title   4/15/2013

Oscillations



$$\omega = \sqrt{k/m} \qquad \text{rad/s}$$



$$f = \omega/2\pi \qquad \text{Hz}$$

$$T = 1/f$$

Recall eqn:

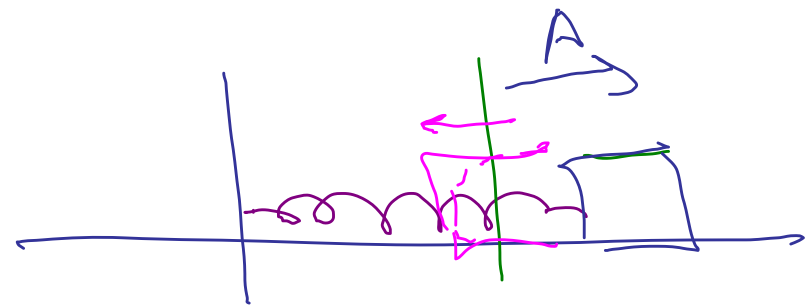
$$\frac{d^2x}{dt^2} = -\omega^2 x$$

General eqn $\longrightarrow A\cos(\omega t)$

$$x(t) = A\cos(\omega t)$$

$$\left(= A\cos(\omega t + \phi)\right)$$

$v(t)$, $a(t)$



$$U(t) = \frac{1}{2}kx^2$$
$$= \frac{1}{2}kA^2\cos^2(\omega t)$$

$$K(t) = \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t)$$

$\omega^2 = k/m$

$$= \frac{1}{2}kA^2\sin^2\omega t$$

$$E(t) = \frac{1}{2}kA^2\left[\cos^2(\omega t) + \sin^2(\omega t)\right]$$
$$= \frac{1}{2}kA^2$$

$$E_i = \frac{1}{2}kA^2$$
$$E_f = \frac{1}{2}mv_{max}^2$$

$$v_{max} = \omega A$$

$$\frac{1}{2}m(\omega A)^2$$
$$= \frac{1}{2}A^2 m\omega^2$$
$$= \frac{1}{2}A^2 m\frac{k}{m} = \frac{1}{2}kA^2$$

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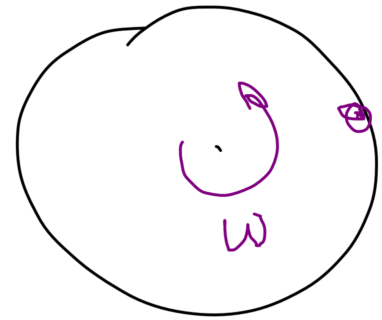
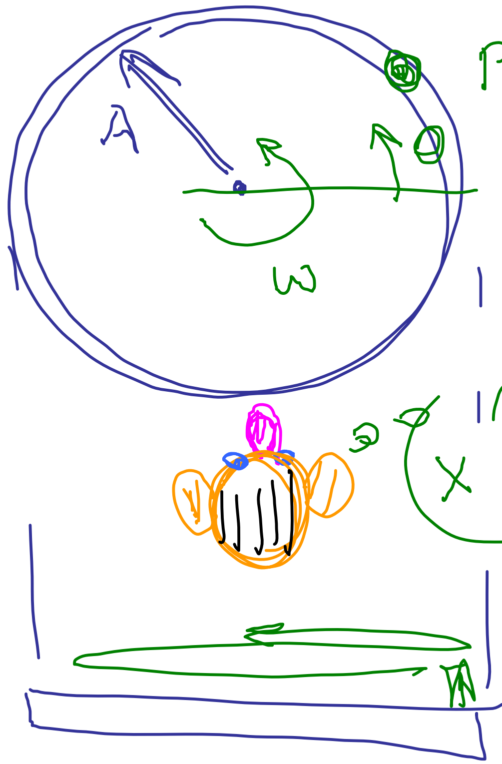
Correspondence between (simple) harmonic motion & circular motion

$\omega = \text{constant}$





$\theta = \omega t$

x component $= x(t) = A\cos\theta$

$= A\cos(\omega t)$

Same symbol, same idea.

rad/s

13.9

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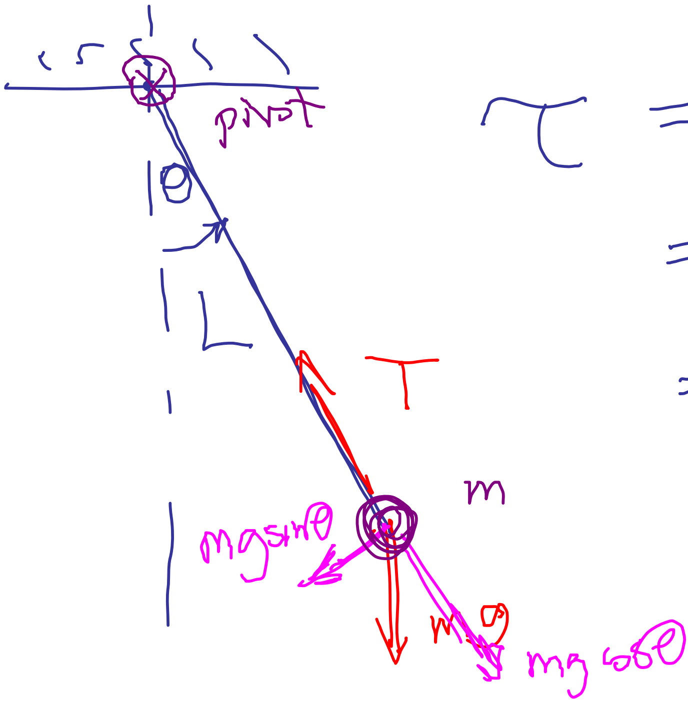
"Reference Circle"

Simplest Oscillating System

Period, freq, ang freq.

Solve using rot'l mechanics



$r F \sin\theta$

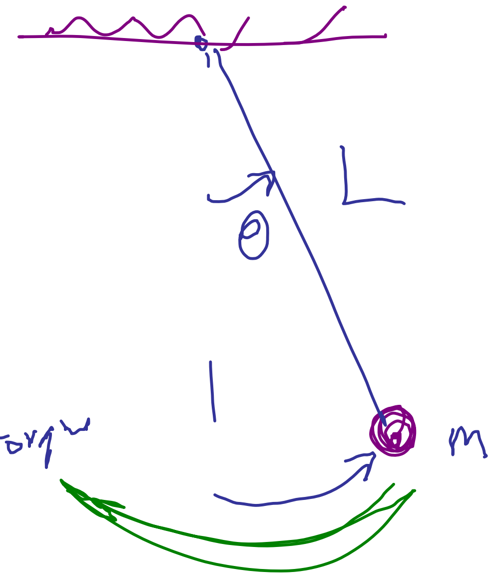
T gives no torque

$$\tau = -Lmg\sin\theta$$

$$= I\alpha$$

$$= (mL^2)\frac{d^2\theta}{dt^2}$$

$$(\cancel{m}L^{\cancel{2}})\frac{d^2\theta}{dt^2} = -\cancel{L}\cancel{m}g\sin\theta$$



Simple Pendulum

$$L\frac{d^2\theta}{dt^2} = -g\sin\theta$$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$\theta(t)$

θ radians. If θ is small $\sin\theta \approx \theta$

θ, deg	θ, rad	$\sin\theta$
5.73°	0.10	0.09983
11.45°	0.20	0.19866

Taylor series.

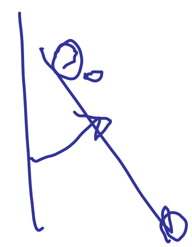
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - $$

Keep θ small.

$$\frac{d^2\theta}{dt^2} = -\underbrace{\frac{g}{L}}_{\omega^2}\theta \qquad \frac{d^2x}{dx^2} = -\omega^2 x$$

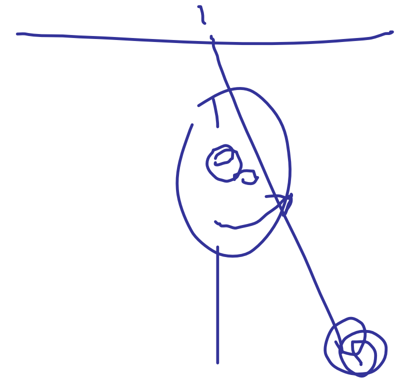
$$\omega^2 = \frac{g}{L} \qquad \omega = \sqrt{\frac{g}{L}} \qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

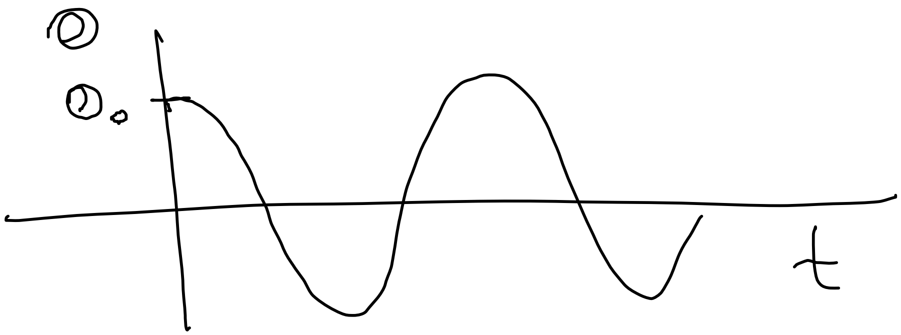
$$T = 1/f.$$



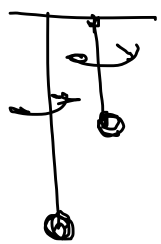
Does not depend on:
$$\begin{cases} m \\ \text{Assuming } \theta \text{ is small} \\ \theta_0 \end{cases}$$

$$\theta(t) = \theta_0 \cos(\omega t)$$





$$\omega = \sqrt{\frac{g}{L}} \qquad T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$



Ratio of periods $= \sqrt{2}$

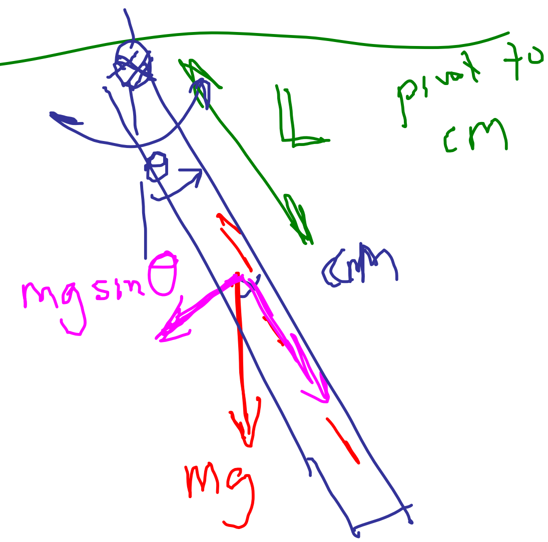
$= 1.414$

What is period of simple pendulum 1 m long?

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.00\ \cancel{m}}{9.80\ \cancel{m}/s^2}}$$

$$= 2.01\ s$$

More general pendulum

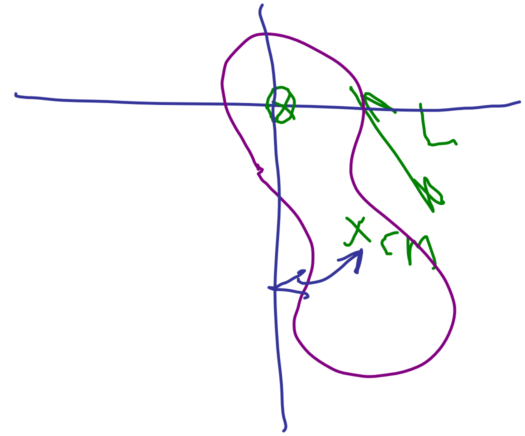


Solve problem

$$\tau = -L\,mg\sin\theta$$

$$= I\alpha = I\frac{d^2\theta}{dt^2}$$

$$I\frac{d^2\theta}{dt^2} = -Lmg\sin\theta$$



θ is small

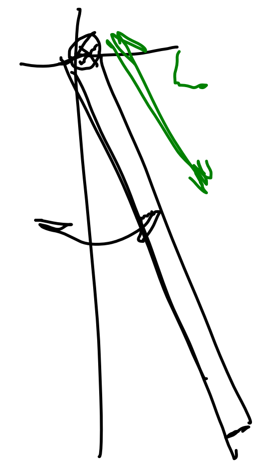
$$\frac{d^2\theta}{dt^2} = -\frac{Lmg}{I}\theta$$

ω^2

$$\omega = \sqrt{\frac{Lmg}{I}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{Lmg}}$$

Example: Uniform stick length of stick $= l$

Osc's about end

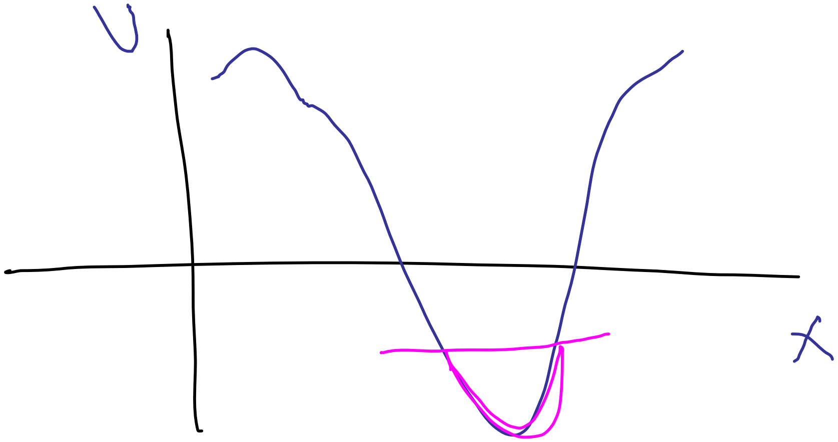


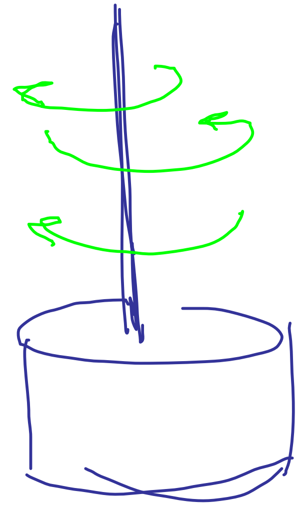
$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

$$= 2\pi\sqrt{\frac{\frac{1}{3}\cancel{m}\,l^2}{\cancel{m}\,g\,l/2}}$$

$$= 2\pi\sqrt{\tfrac{2}{3}\,l/g} \qquad \text{etc. } \omega,\ f$$

physical pendulum.





$$U = \frac{1}{2} k x^2$$

$$F = -kx$$