Phys 3820, Fall 2010
Problem Set #2, Hint-o-licious Hints

1. *Griffiths*, **6.20** Use $|\mathbf{L}| \approx \hbar$ in (6.59) With this, the critical size of the B field is around 12 T.

2. *Griffiths*, **6.21** The fine structure is larger than the Zeeman contribution to the energy. The zero-field value of the energies is given by (6.67). Since it included the spin-orbit splitting, the energies depend on j (and n).

Now, for $n = 2$ we have the states $l = 0$ and $l = 1$. We must have states of "good" j, so we note that the $l = 0$ state is a $j = \frac{1}{2}$ states while the $l = 1$ state give $j = \frac{1}{2}$ and $j = \frac{3}{2}$.

Calculate the Landé g factor g_J for each state note, it depends on j and l, and then the weak–field Zeeman energy is

$$E^1_Z = \mu_B g_J B_{\text{ext}} m_j$$

If you plot E vs. $\mu_B B_{\text{ext}}$, the slope of the line is $g_J m_j$.

For the $j = \frac{1}{2}$ states we then have a pair for the state that came from $l = 0$ (with $m_j = \pm\frac{1}{2}$) and a pair for the state that came from $l = 1$. The $j = \frac{3}{2}$ state came from $l = 1$ and with $m_j = -\frac{3}{2} \dots \frac{3}{2}$, there are four lines with their own slopes.

3. *Griffiths*, **6.29** The perturbation is

$$H' = \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{b}\right)$$

and use this in

$$E^1_{\text{gs}} = \int \psi^*(\mathbf{r}) H' \psi(\mathbf{r}) d^3\mathbf{r}$$

As explained in class, you can approximate the exponential as 1 to get a lowest–order answer, and that all we need to check the order of magnitude of the result. With this, show

$$\frac{E^1_{\text{gs}}}{|E^1_{\text{gs}}|} = \frac{4}{3}\left(\frac{b}{a}\right)^2$$

Compare with value with those from fine structure and the hyperfine splitting! (Use Table 6.1).

4. *Griffiths*, **7.1** (a) With $\psi(x) = Ae^{-bx^2}$ for the linear potential $V(x) = \alpha|x|$ I get

$$\langle H \rangle = \frac{\hbar^2 b}{2m} + \frac{\alpha}{\sqrt{2\pi b}}$$

which has a minimum at

$$b = \left[\frac{m^2\alpha^2}{2\pi\hbar^4}\right]^{1/3}$$

and give a bound of

$$\langle H \rangle_{\text{min}} = \frac{3}{2}\left(\frac{\alpha^2\hbar^2}{2\pi m}\right)^{1/3}$$

(b) For $V(x) = \alpha x^4$ I get

$$\langle H \rangle = \frac{\hbar^2 b}{2m} + \frac{3\alpha}{16b^2}$$

which has a minimum at

$$b = \left(\frac{3m\alpha}{4\hbar^2}\right)^{1/3}$$

and gives a bound

$$\langle H \rangle_{\min} = \frac{3}{2}\left(\frac{3\alpha\hbar^2}{4m^2}\right)^{1/3}$$

5. *Griffiths*, **7.4** Proof of the theorem goes as described in class; with the trial function ψ expanded as

$$\psi = \sum_{n=0}^{\infty} c_n \psi_n \qquad \text{where} \qquad H\psi_n = E_n \psi_n$$

and $n = 0$ stands for the non-degenerate ground state and $n = 1$ stands for any one of the states of the first excited energy. Find the condition on the c_n's implied by normalization.

From $\langle \psi | \psi_0 \rangle = 0$ show that $c_n = 0$ then show

$$\langle H \rangle = \sum_{n=1} |c_n|^2 E_n \geq E_1$$

For the trial function of the form

$$\psi(x) = Ae^{-bx^2}$$

show that normalization gives

$$A^2 = \sqrt{\frac{32b^3}{\pi}}$$

and then after lots of careful algebra

$$\langle T \rangle = \frac{3A^2\hbar^2}{8m}\sqrt{\frac{\pi}{2b}} \qquad\qquad \langle V \rangle = \frac{A^2 m\omega^2}{b^2}\frac{3}{32}\sqrt{\frac{\pi}{2b}}$$

These lead to

$$\langle H \rangle = \frac{3}{2}\left(\frac{\hbar^2 b}{m} + \frac{m\omega^2}{4b}\right)$$

and minimizing this with respect to b gives the minimal value of $\langle H \rangle$ hence an upper bound on the first excited state. Of course, you get the exact answer as you can understand from Example 2.4.