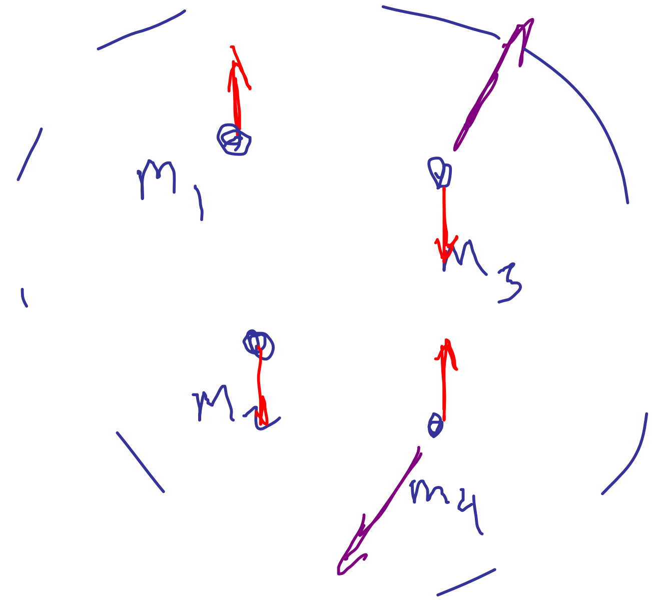
Phys 2110-4 3/8/13

Systems of particles



$$\vec{F}_{\text{tot force}} = \sum_{\text{particle}} \vec{F}_{\text{net}}$$

$$= M \frac{d^2}{dt^2} \vec{R}(t)$$

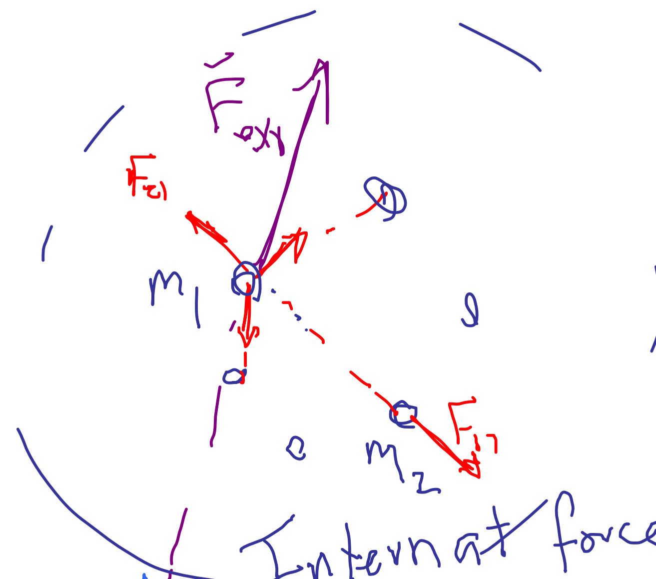
$$\vec{R} = \text{pos of } cm_{\text{center of mass}} = \frac{1}{M} \sum m_i \vec{r}_i = \frac{\iiint \vec{r} \rho \, dV}{M}$$

$$\vec{F}_{Tot} = M\vec{a}_{cm}$$

$$\sum_{part} \vec{F}_{net}$$

$$= \underbrace{\sum_{part_i} F_i^{int}}_{\substack{\text{sum to give zero} \\ \text{N's Third law}}} + \sum_{part_i} F^{ext}$$

$$= \sum_i F_i^{ext}$$

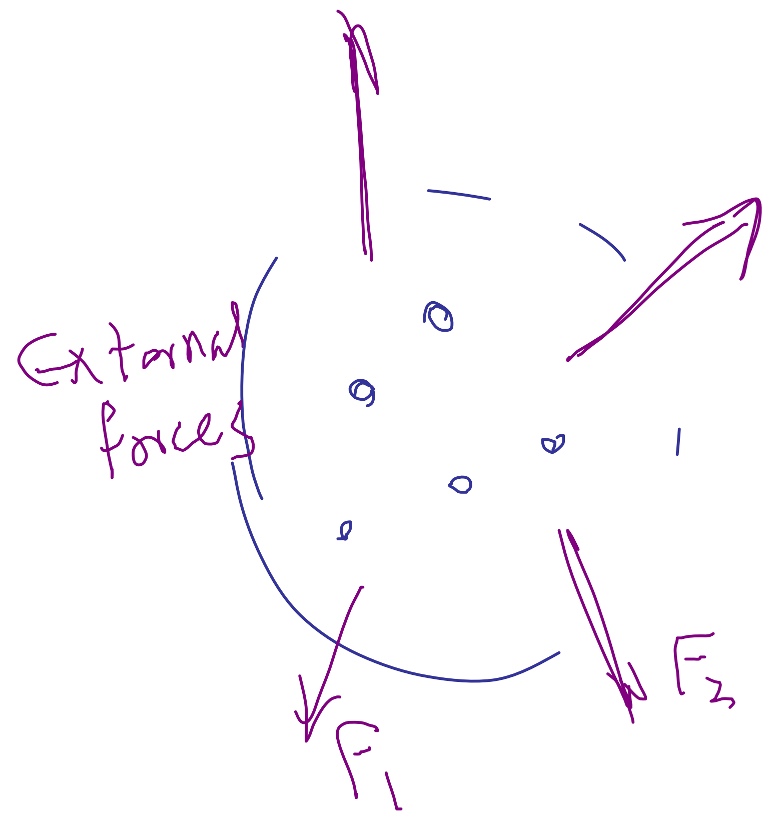


Internal forces are those between masses in the system

External forces come from outside

$$\sum_i \vec{F}_i^{ext} = M\vec{a}_{cm}$$

$$= \vec{F}_{Tot}$$



Isolated system:

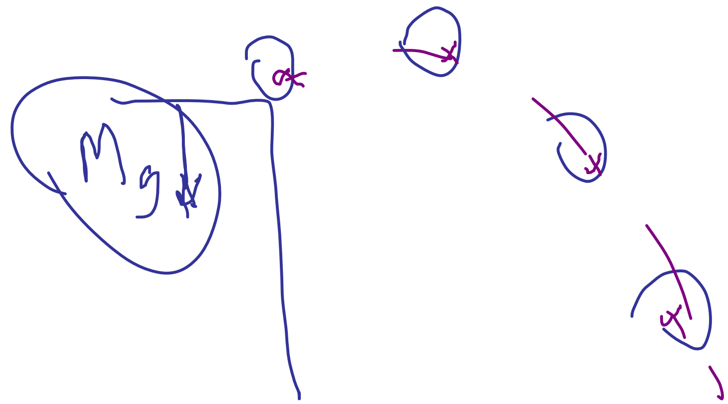
A system in which no net external force.

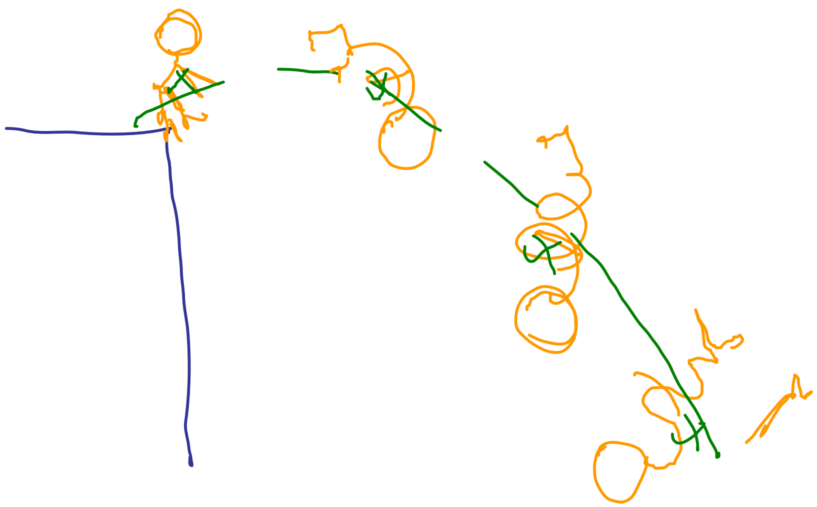
If this is true $M\vec{a}_{cm} = 0$

$$\vec{v}_{cm} = \text{const}$$

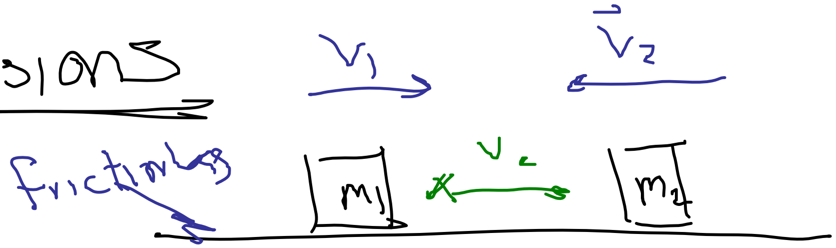
Wrench, isolated system.

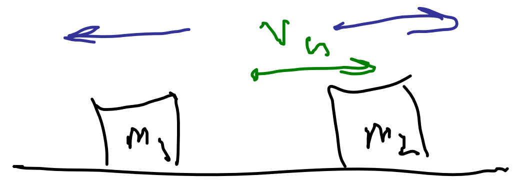
$$F_{net}^{ext} = M\vec{a}_{cm}$$





Collisions





Isolated system. $\vec{V}_{cm}$ stay same.

Definition $\vec{p} = m\vec{v}$ Vector $p_x = mv_x$

$p_y = mv_y$

$p_z = mv_z$

Units: $kg\frac{m}{s} = \frac{kg\, m}{s}$

$$\vec{v}_{cm} = \frac{1}{M}\sum m_i \vec{v}_i \qquad \vec{a}_{cm} = \frac{1}{M}\sum m_i \vec{a}_i$$

$$\vec{v}_{cm} = \frac{1}{M} \sum_{i} m_i \vec{v}_i = \frac{1}{M} \sum_i \vec{p}_i$$

particles

Total momentum $= \vec{P} = \sum_i \vec{p}_i$

Vector

$$P_x = \sum_i p_{ix}$$

$\vdots$

$$\vec{v}_{cm} = \frac{1}{M} \vec{P}$$

$$\vec{P} = M \vec{v}_{cm}$$ for a system

What is $d\vec{P}/dt$?

$$\frac{d\vec{P}}{dt} = M\frac{d\vec{v}_{cm}}{dt} = M\vec{a}_{cm} = \cancel{M}\cdot\frac{1}{\cancel{M}}\vec{F}^{ext}_{net}$$

$$= \vec{F}^{ext}_{net}$$

$$\vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d}{dt}\vec{p}$$

$$\boxed{\vec{F}^{ext}_{net} = \frac{d\vec{P}}{dt}}$$

Gen Thm

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Isolated system

$F^{ext}_{net} = 0$ $\longrightarrow$

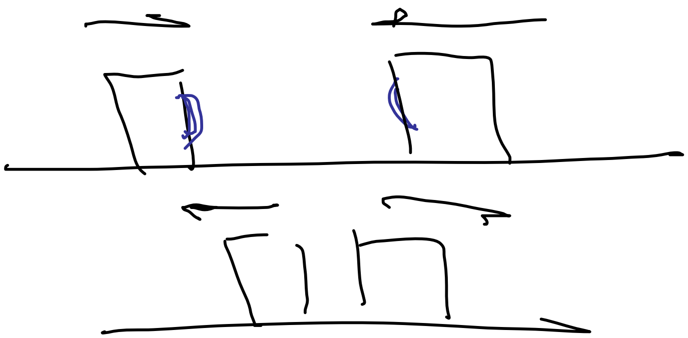
$$\boxed{\frac{d\vec{P}}{dt} = 0 \quad \vec{P} \text{ is constant}}$$

Isolated system

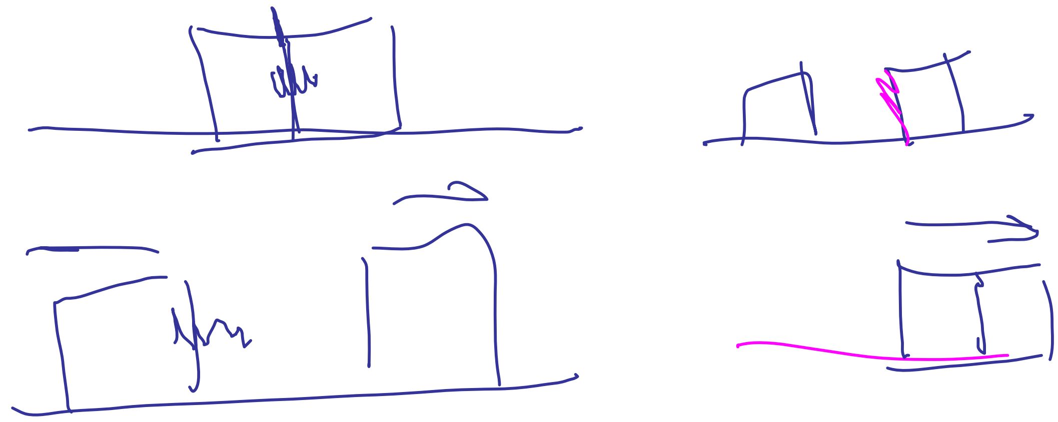
Special case

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 \cdots$$

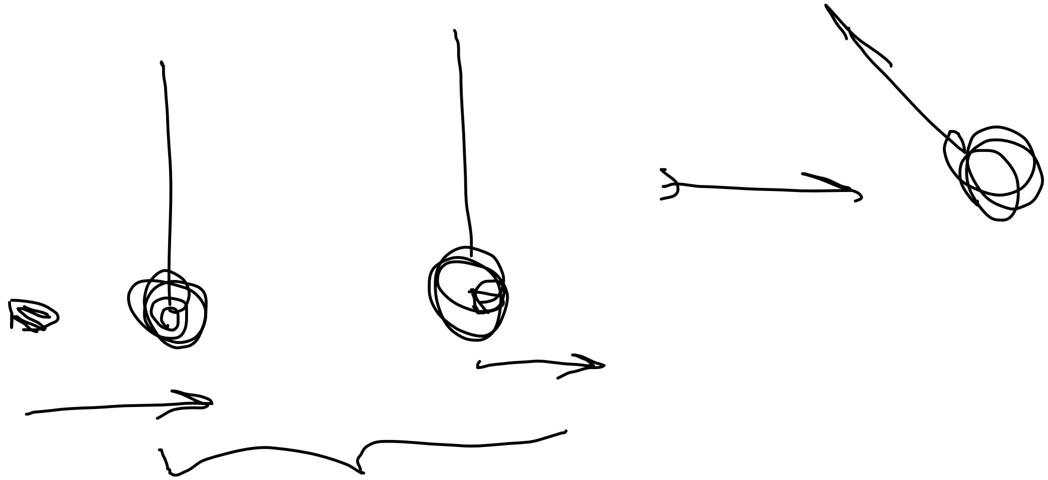
Collisions, Explosions



Measure over a time for which ext forces negligible enough



explosion.

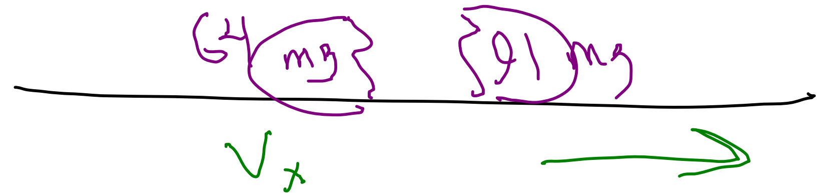


Isolated
Isolated.

9.17 A popcorn kernel at rest in a hot pan bursts into two pieces with masses 91 mg, 64 mg. More massive piece moves horiz at 47 $\frac{cm}{s}$. Describe motion of second piece.

$v = 0$

$m = 155\,mg$



$V_x = 47\,\frac{cm}{s}$

Principle:

Isolated system, total mom is conserved

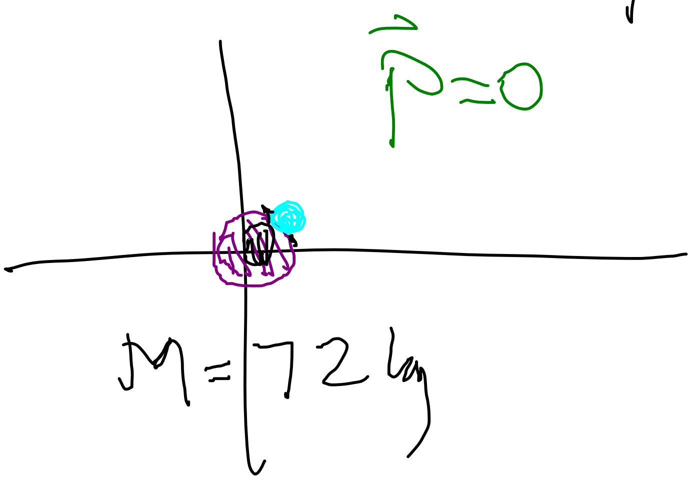
$$\vec{P}_{before} = 0 \qquad \vec{P}_{aft} = (64\,mg)V_x + (91\,mg)\left(47\,\frac{cm}{s}\right)$$

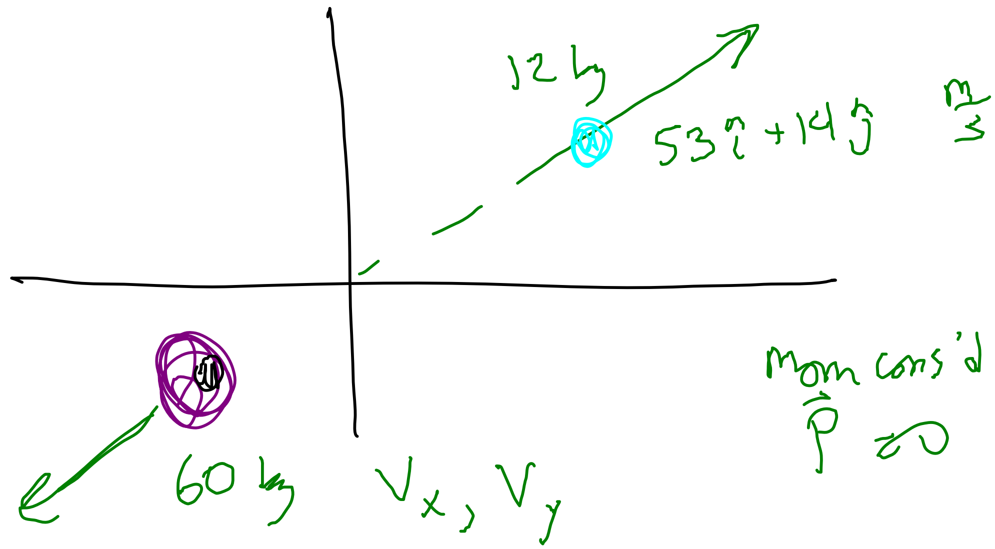
same

$$= 0$$

$$V_x = -\frac{(91\,mg)\left(47\,\frac{cm}{s}\right)}{(64\,mg)} = -67\,\frac{cm}{s}$$

9.18 60 kg skater at rest on a fricless ice surf. tosses 12 kg snowball with velocity $\vec{V} = 53.0\hat{i} + 14.0\hat{j}$ $\left(\frac{m}{s}\right)$. (in horiz. plane) Find skater's subs. velocity

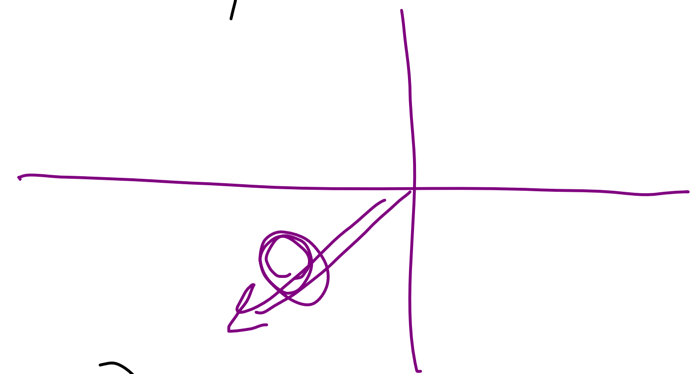




P_x is cons'd

$$0 = (60\,\text{kg})\,v_x + (12\,\text{kg})\left(53\,\frac{\text{m}}{\text{s}}\right)$$

$$v_x = -10.6\,\frac{\text{m}}{\text{s}}$$



P_y is cons'd

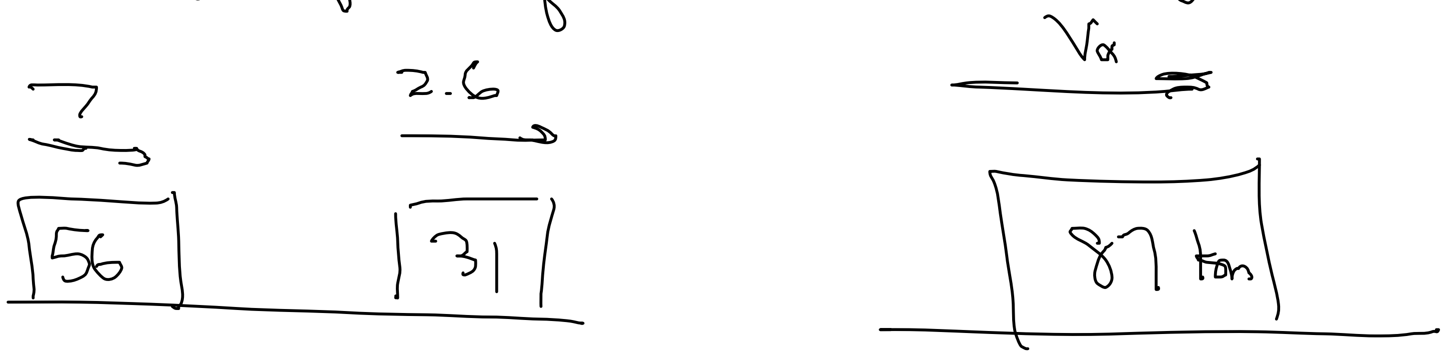
$$0 = (60\,\text{kg})\,V_y + (12\,\text{kg})\left(14\,\frac{\text{m}}{\text{s}}\right)$$

$$V_y = -2.8\,\frac{\text{m}}{\text{s}}$$

Find mag & dir. of $\vec{v}$

9.26 In railroad yard, 56-ton freight car sent at $7.0 \frac{mi}{hr}$ toward 31 ton fr. car moving at $2.6 \ mi/hr$.

a) Speed of car after they couple?



$$(56 \cancel{ton})(7 \tfrac{mi}{hr}) + (31 \cancel{ton})(2.6 \tfrac{mi}{hr}) = (87 \cancel{ton}) V_x$$

Get: $\boxed{V_x = 5.4 \frac{mi}{h}}$