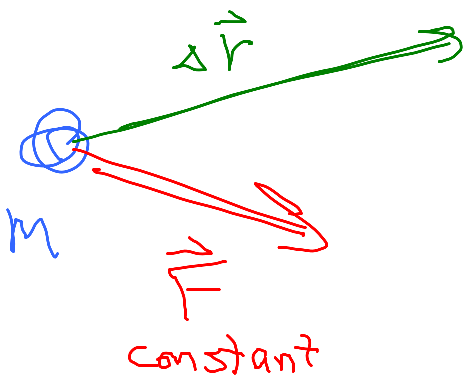
Phys 2110-4 10/7/11

Note Title 10/7/2011

Chap 6 Work & Energy & Power

Definitions, Theorems

Work

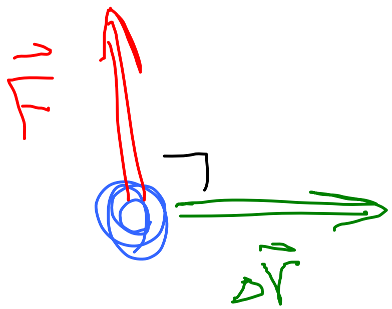


$$W = \vec{F} \Delta \vec{r}$$

$$= F \, |\Delta \vec{r}| \cos\theta$$

Scalar

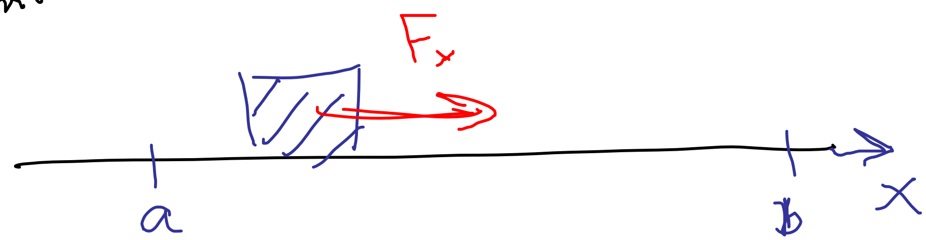
Units $= J = \frac{kg\, m^2}{s^2}$



If $\theta = 90$, no work is done.

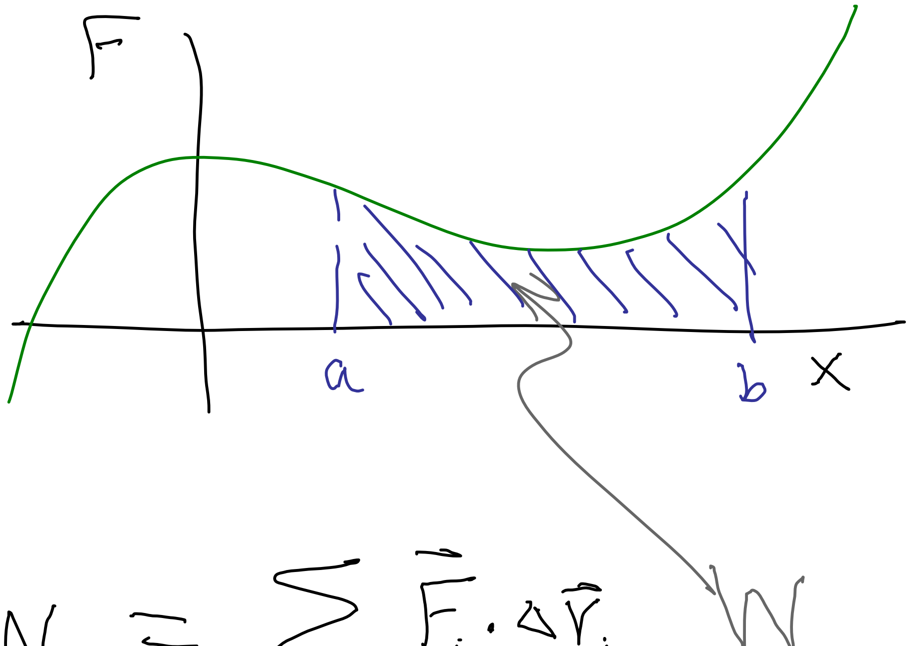
If force is not constant.

Force function of x / s



$F_x \Delta x$

$$W = \sum_{\text{little bits of math}} \vec{F}_i \Delta x_i = \int_a^b F_x(x)\, dx$$

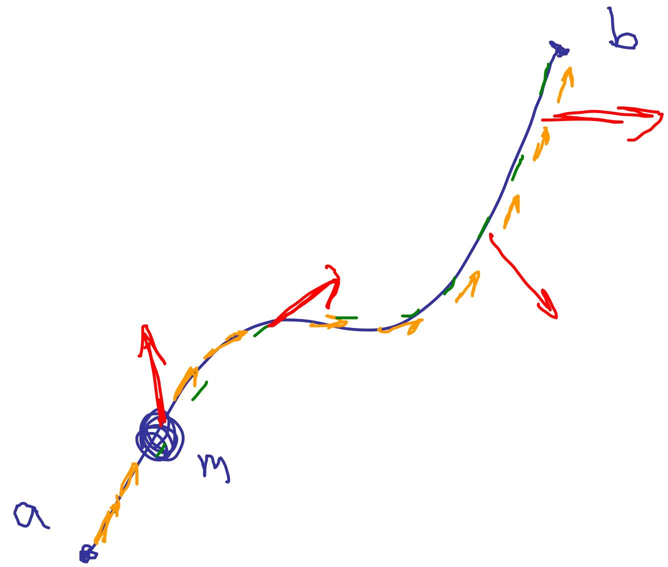


$$W = \int_a^b F(x)\,dx$$

$$W = \sum \vec{F}_i \cdot \Delta\vec{r}_i$$

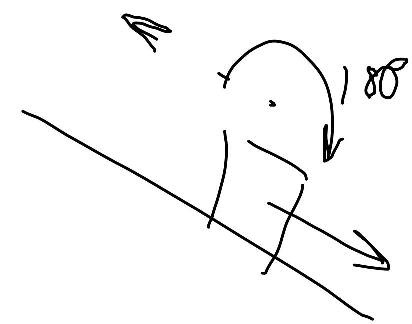
$$= \int_a^b \vec{F} \cdot d\vec{r}$$

"Line integral"



Example

As mass moves as shown, what is work done by gravity



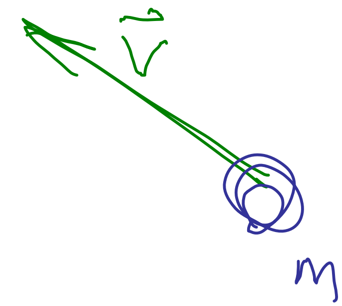
$$W = (mg)(x)\cos(\phi + 90^\circ)$$

$$= mgx(-\sin\phi)$$

$$= -mgx\sin\phi$$

Definition:

Kinetic Energy



$$K = \frac{1}{2} m v^2$$

v = speed of mass $\frac{m}{s}$

Scalar

$$\text{Units} = kg\left(\frac{m}{s}\right)^2 = \frac{kg\, m^2}{s^2} = J$$

same units as W

Always positive number

6.25 What is Kin. Energy of 2.4×10^5 kg airplane cruising at 900 $\frac{km}{h}$?

$v = 250 \frac{m}{s}$

$m = 2.4 \times 10^5$ kg

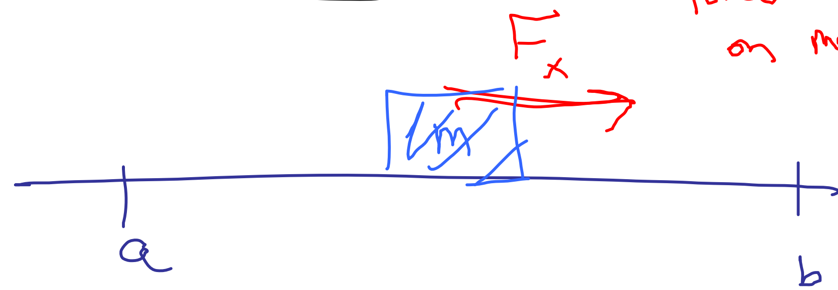
$$K = \frac{1}{2}(2.4 \times 10^5 \text{ kg})(250 \tfrac{m}{s})^2$$

$$= 7.5 \times 10^9 \text{ J} = 7.5 \text{ GJ}$$

What is this good for?

Derivation of Work-Energy Thm

(F is total force on mass)



$$W = \int_a^b F_x \, dx \qquad F_x \leftarrow ma_x$$

$$= m \int_a^b a_x \, dx$$

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx} \cdot \frac{dx}{dt} = \frac{dv_x}{dx} v_x = v_x \frac{dv_x}{dx}$$

$$= m \int_a^b \left[v_x \frac{dv_x}{dx} \right] dx = m \int_a^b \frac{1}{2} \frac{d}{dx}\left[v_x^2 \right] dx$$

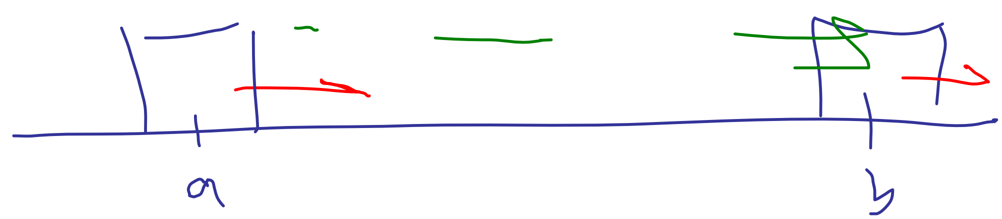
chain rule

$$= \frac{m}{2} \int_a^b \frac{d}{dx}\left[v_x^2\right] dx$$

$$= \frac{m}{2} v_x^2 \Big|_a^b = \frac{1}{2} m\left[v_x(b)\right]^2 - \frac{1}{2} m\left[v_x(a)\right]^2$$

$$= K_b - K_a = \Delta K$$

$$W_{net} = \Delta K$$

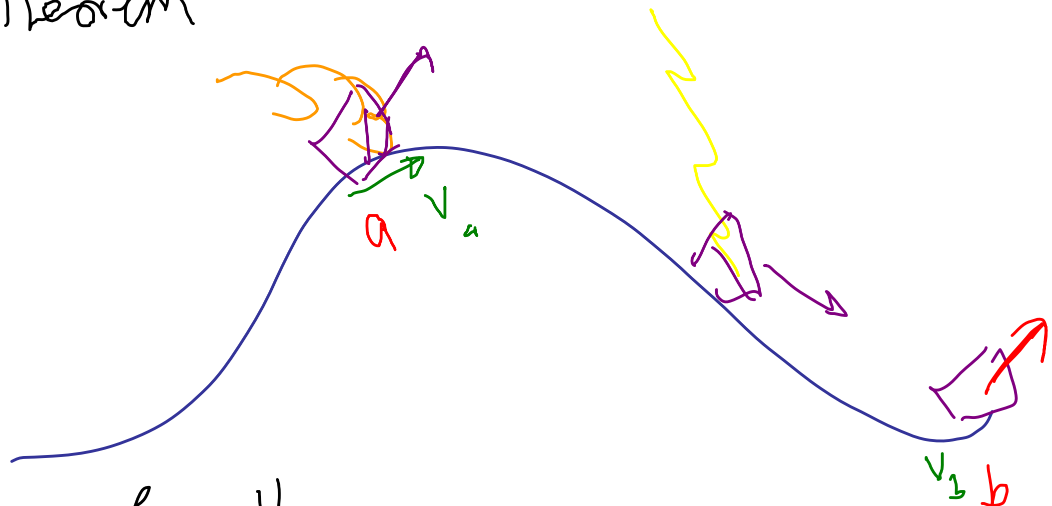


Work-Energy Theorem

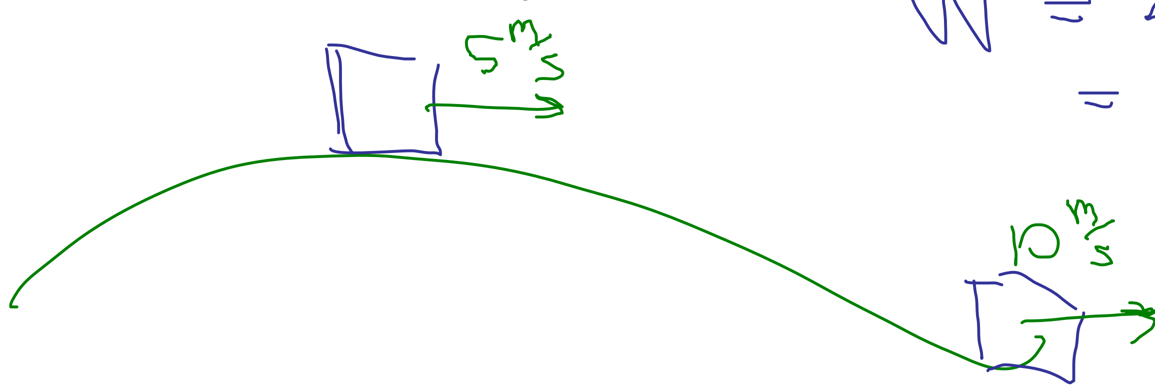
True in general

$$W = \Delta KE$$

Irregardless of shape of path ..



6.28 A 60-kg skateboarder comes over top of hill at $5.0 \frac{m}{s}$ and reaches $10 \frac{m}{s}$ at bottom. Find the total work done on skateboarder between top & bottom of hill

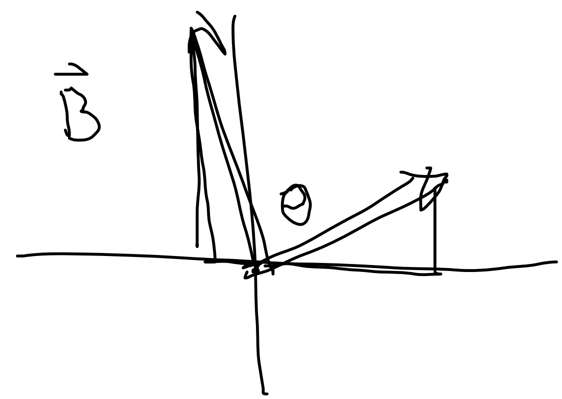


$$W = \Delta K$$
$$= \frac{1}{2}(60\,kg)\left(\left(10 \frac{m}{s}\right)^2 - \left(5 \frac{m}{s}\right)^2\right)$$
$$= 2250\,J$$

Exercise w/ dot product:

Find angle between $\vec{A} = 3\hat{i} + 2\hat{j}$

$\vec{B} = -\hat{i} + 6\hat{j}$



$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$

$= 9$

Also

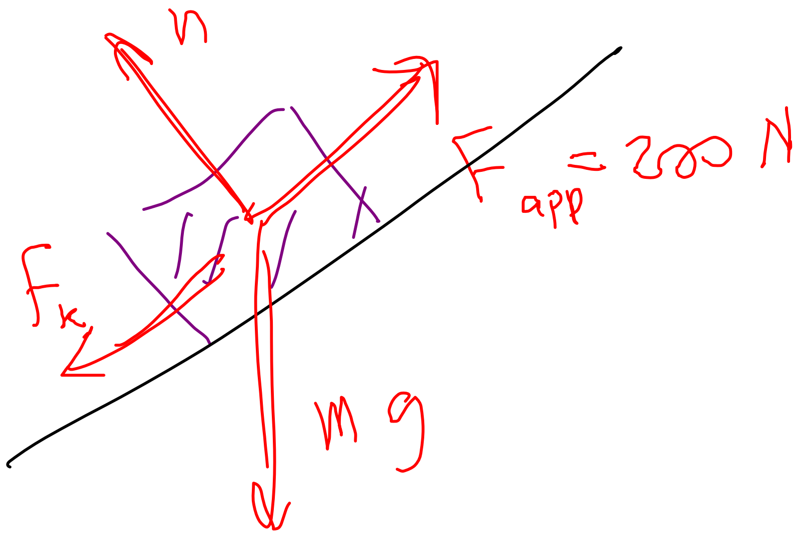
$= AB\cos\theta = \sqrt{13}\sqrt{37}\cos\theta$

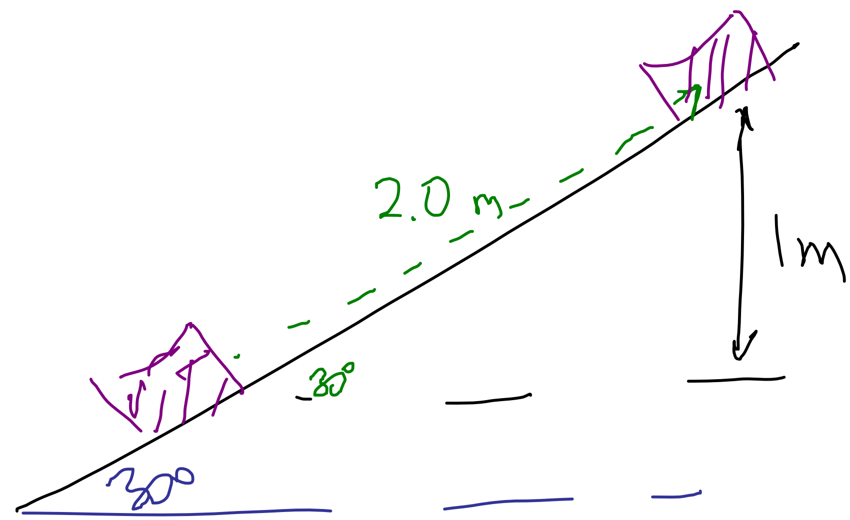
Do it. $\Rightarrow \theta = 65.8°$

6.41 You slide a box of books at constant speed up a 30° ramp, applying a force of 200 N dir'd up slope.
Coeff of fric is 0.18

a) How much work have you done when the box has risen 1m vertically?

b) What's the mass of the box?





a) You do $\quad W = (200\text{ N})(2.0\text{ m})\cos(0°)$

$$= 400\text{ J}$$

b) Use W-E thm

$$W_{net} = \Delta K = 0$$

$$= (200\,N)(2\,m) + f_k(2.0)(-1)$$

$$+ mg(\cos 120^\circ) + 0$$

$$f_k = \mu_k mg \cos\theta$$

Do it.

$$\boxed{m = 31.1\,kg}$$