Phys 3820, Fall 2009
Problem Set #3, Hint-o-licious Hints

1. *Griffiths*, **7.1**

2. *Griffiths*, **7.4** Proof of the theorem goes as described in class; with the trial function ψ expanded as

$$\psi = \sum_{n=0}^{\infty} c_n \psi_n \qquad \text{where} \qquad H\psi_n = E_n \psi_n$$

and $n = 0$ stands for the non-degenerate ground state and $n = 1$ stands for any one of the states of the first excited energy. Find the condition on the c_n's implied by normalization.

From $\langle \psi | \psi_0 \rangle = 0$ show that $c_n = 0$ then show

$$\langle H \rangle = \sum_{n=1} |c_n|^2 E_n \geq E_1$$

For the trial function of the form

$$\psi(x) = Ae^{-bx^2}$$

show that normalization gives

$$A^2 = \sqrt{\frac{32b^3}{\pi}}$$

and then after lots of careful algebra

$$\langle T \rangle = \frac{3A^2\hbar^2}{8m}\sqrt{\frac{\pi}{2b}} \qquad\qquad \langle V \rangle = \frac{A^2 m\omega^2}{b^2}\frac{3}{32}\sqrt{\frac{\pi}{2b}}$$

These lead to

$$\langle H \rangle = \frac{3}{2}\left(\frac{\hbar^2 b}{m} + \frac{m\omega^2}{4b}\right)$$

and minimizing this with respect to b gives the minimal value of $\langle H \rangle$ hence an upper bound on the first excited state. Of course, you get the exact answer as you can understand from Example 2.4.

3. *Griffiths*, **7.6**

4. *Griffiths*, **7.13**

5. *Griffiths*, **7.19**