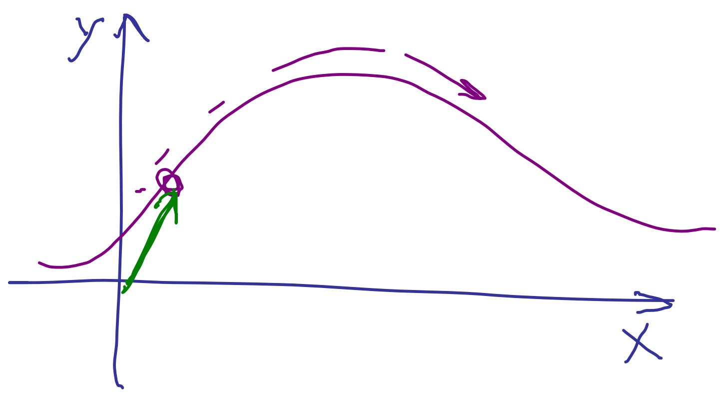
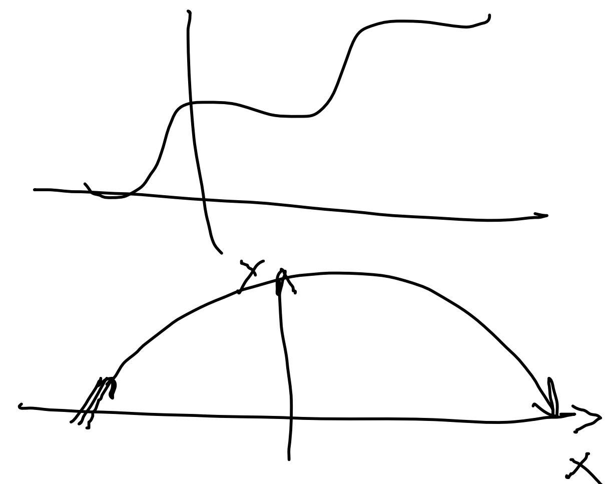
Phys 2110-4 9/12/11

Note Title 9/12/2011

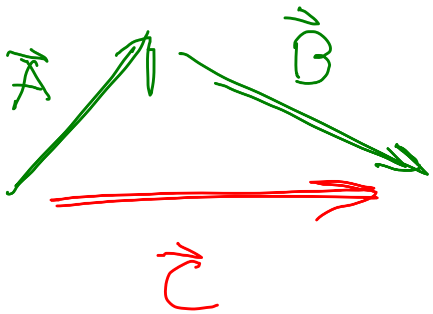
Chap 3 2D Motion

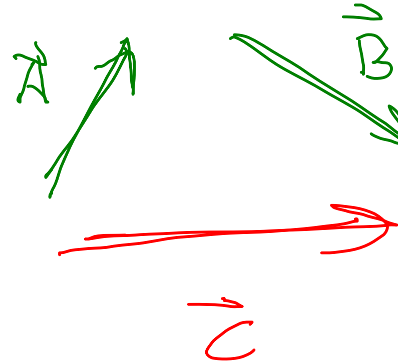
Coord. system:

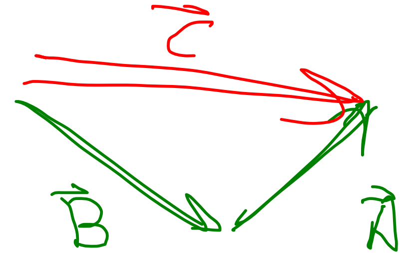




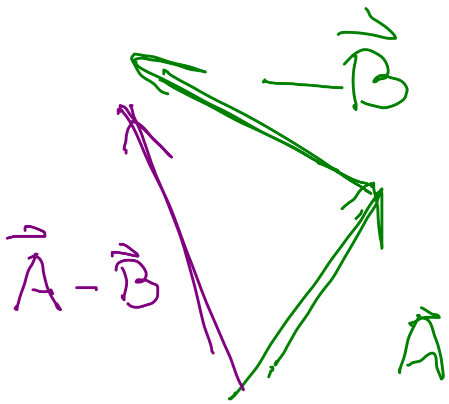
$\Rightarrow$ Vectors

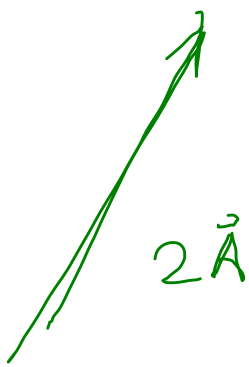




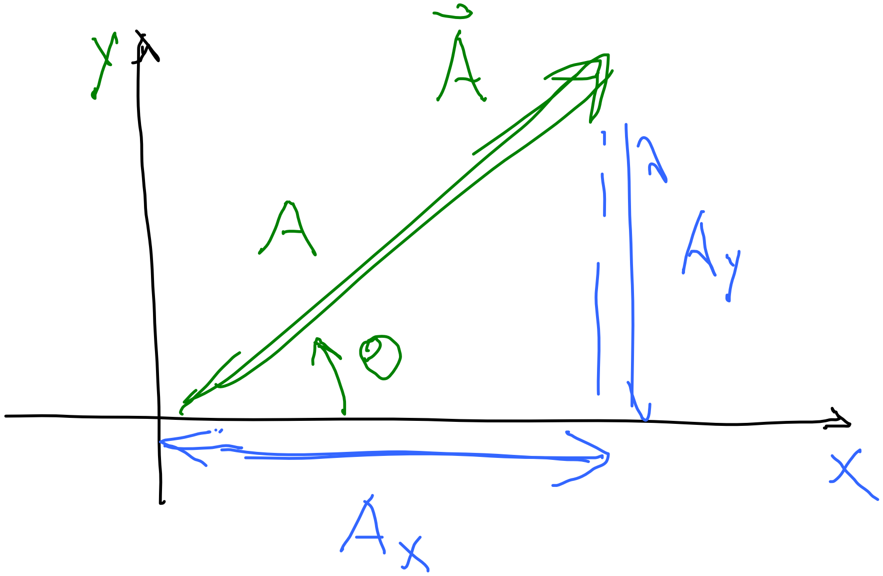


Geometric addition





Components



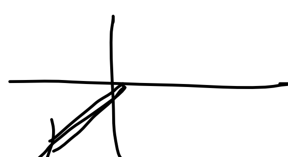
A_x, A_y are x, y components

A, θ are magnitude & direction.

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

Might be negative!

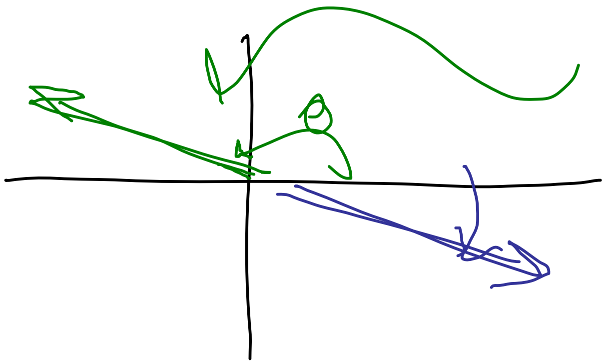


$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = A_y / A_x$$

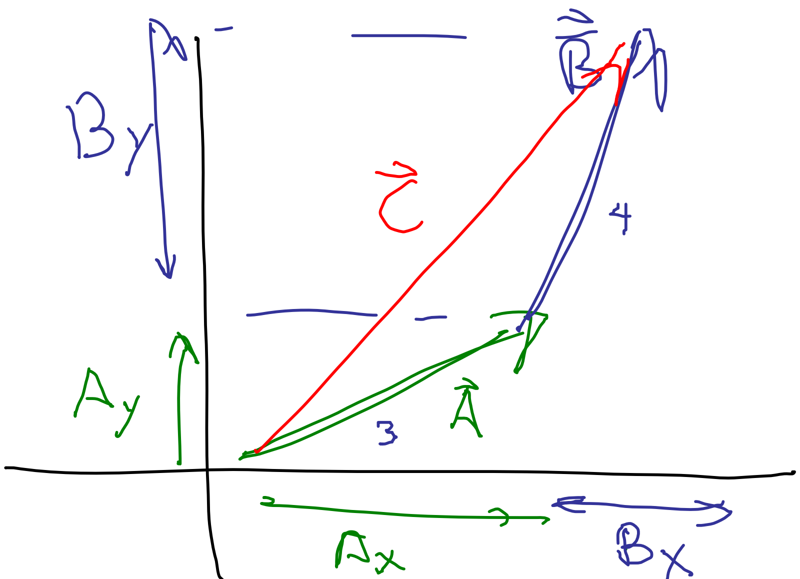
$$\overset{?}{\Longrightarrow} \theta = \tan^{-1}\left(A_x / A_x\right)$$

Watch out!



$A_y / A_x = \tan\theta$ = negative

Calc can give wrong answer!
Maybe add 180



$C_x = A_x + B_x$
$C_y = A_y + B_y$

Can now find the mag. & dir of C

These just add!

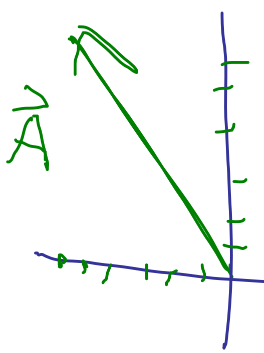
To add vectors: Decompose into components

Add the components separately.

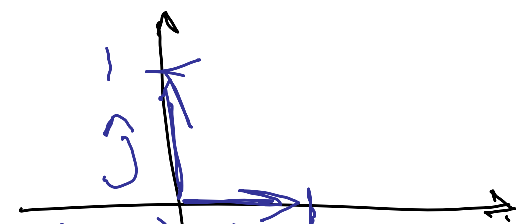
Then can find mag & dir of the result.

Example: Find the mag & direction of

$-5\hat{i} + 6\hat{j} = \vec{A}$

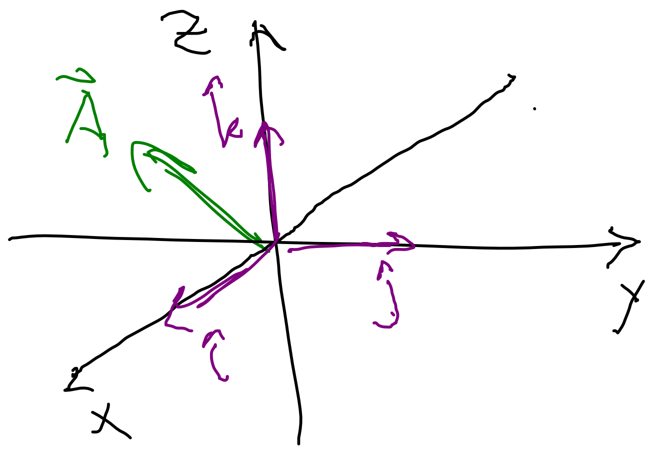


$A = \sqrt{(-5)^2 + 6^2} = 7.8$



Dir: $\theta = \tan^{-1}(6/-5) = \tan(-1.2) \longrightarrow -50^\circ + 180 = 130$

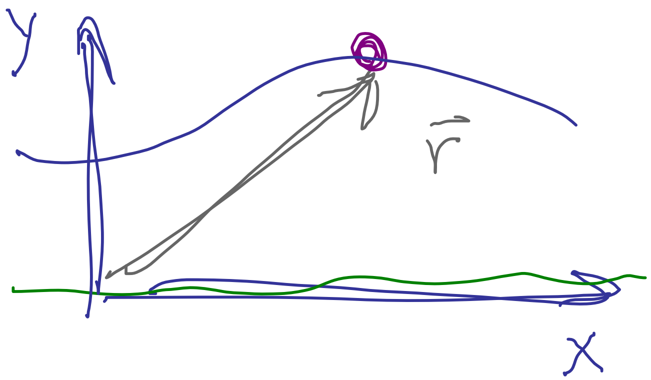
In 3-D x, y, z



$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

Coming up: Multiplication of vectors

$\vec{A} \cdot \vec{B} \Rightarrow$ scalar $\vec{A} \times \vec{B} \Rightarrow$ vector



$\vec{r} = x\hat{i} + y\hat{j}$

Location vector.

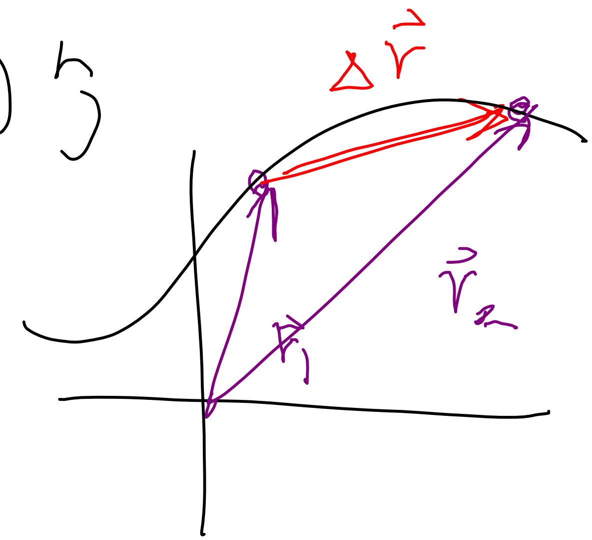
Units : $\underline{\underline{m}}$

Change in position

$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$= \Delta x\,\hat{i} + \Delta y\,\hat{j}$

$= \vec{r}_2 - \vec{r}_1$



How rapidly is position changing?

$$\vec{V}_{av} = \Delta\vec{r}/\Delta t = \frac{1}{\Delta t}\left(\Delta x\,\hat{i} + \Delta y\,\hat{j}\right)$$

$$= \left(\frac{\Delta x}{\Delta t}\,\hat{i} + \frac{\Delta y}{\Delta t}\,\hat{j}\right)$$

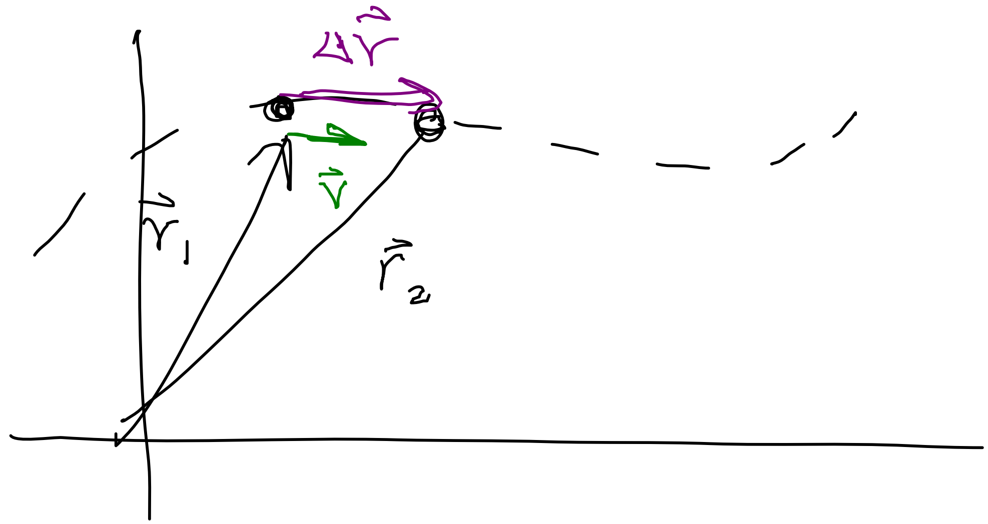
$$= \overline{V}_x\,\hat{i} + \overline{V}_y\,\hat{j}$$

<u>average</u> <u>velocity</u> m/s

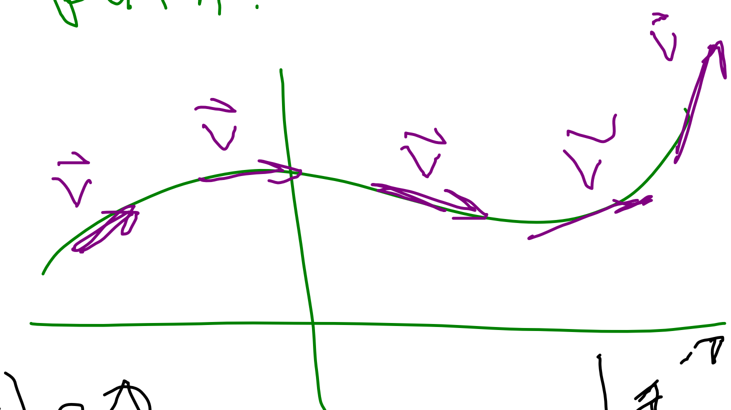
Instantaneous velocity.

Do this for $\Delta t \to 0$

$$\longrightarrow \vec{V} = dx/dt\,\hat{i} + \frac{dy}{dt}\,\hat{j} = V_x\,\hat{i} + V_y\,\hat{j}$$

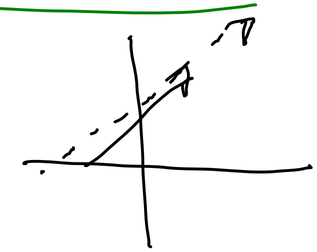


Velocity vector is tangent to path.



If velocity were constant

$$dx/dt = \text{constant} \qquad dy/dt = \text{constant}$$



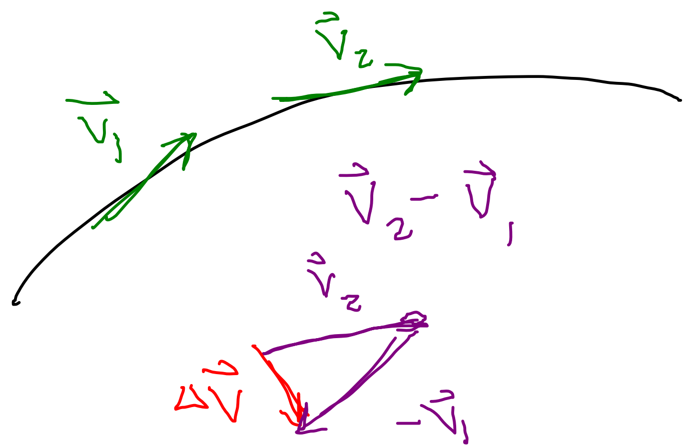
$$x = x_0 + v_x t \qquad y = y_0 + v_y t$$

initial x, y coords

How fast is velocity changing?

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

$$= \bar{a}_x \hat{i} + \bar{a}_y \hat{j}$$



What is instantaneous rate of change of velocity? $\Delta t \to 0$

$$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x \hat{i} + a_y \hat{j}$$

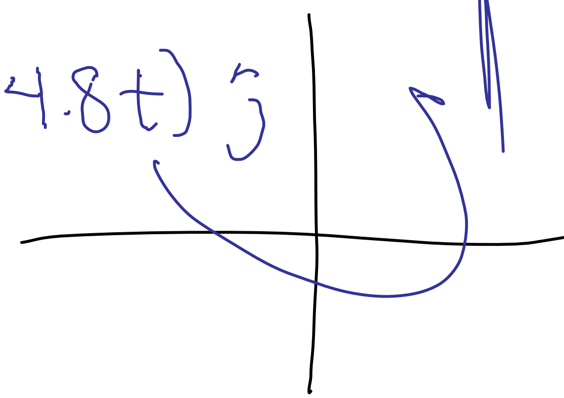
$$\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j}$$

3.30 Position of object as fn of time is

$$\vec{r} = (3.2t + 1.8t^2)\hat{i} + (1.7t - 2.4t^2)\hat{j} \quad \text{m}$$

Find object's accel. vector.

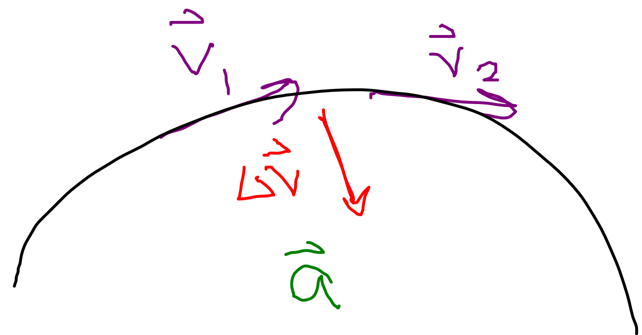
$$\vec{v} = (3.2 + 3.6t)\hat{i} + (1.7 - 4.8t)\hat{j} \quad \text{m/s}$$



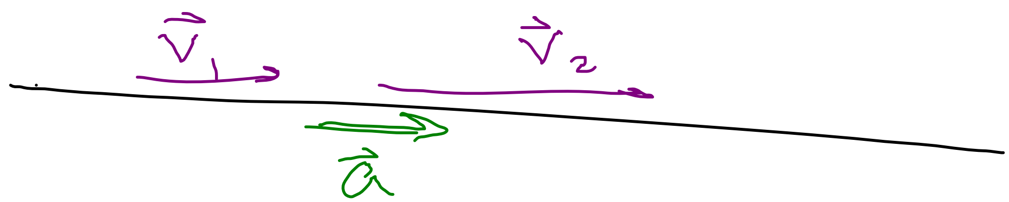
$$\vec{a} = 3.6\hat{i} - 4.8\hat{j} \quad \text{m/s}^2$$

Interesting case:

Constant acceleration.



Non-const accel



$a_x = \text{const}$

$a_y = \text{const}$

Free-fall

$a_x = 0$

$a_y = -g$

$a_x = \text{const} \qquad a_y = \text{const}$

Speed $= \|\vec{v}\|$
$= \sqrt{v_x^2 + v_y^2}$

$$\frac{dv_x}{dt} = a_x \longrightarrow \boxed{v_x = v_{0x} + a_x t}$$

Initial x-velocity

$$\frac{dv_y}{dt} = a_y \longrightarrow \boxed{v_y = v_{0y} + a_y t}$$

Initial y-velocity

$$v_x = \frac{dx}{dt} =$$

$$\boxed{x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2}$$

Initial x-coord

$$\boxed{y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2}$$

$$v_y = \frac{dy}{dt}$$