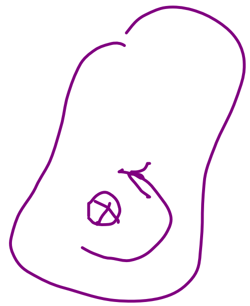
Phys 2110-4 4/5/13

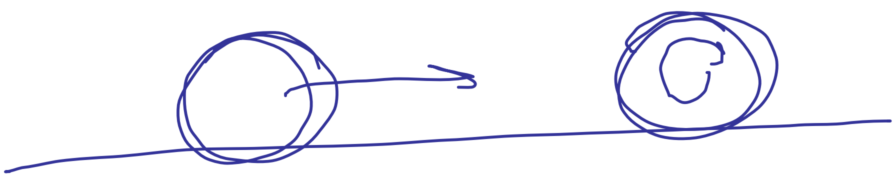
Rotations

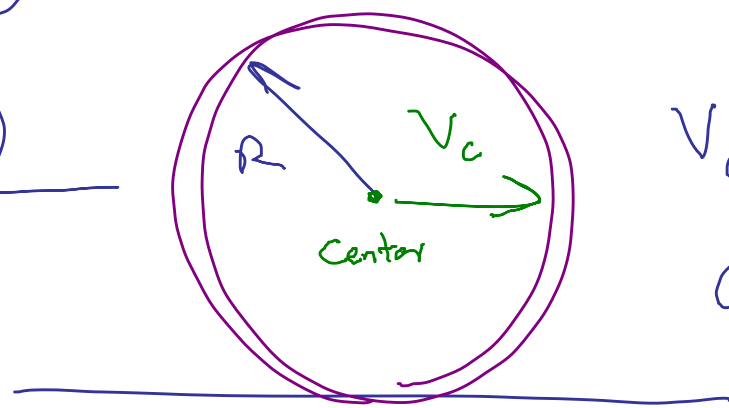


$$\tau_{net} = I\alpha$$

$$K = \frac{1}{2} I \omega^2$$

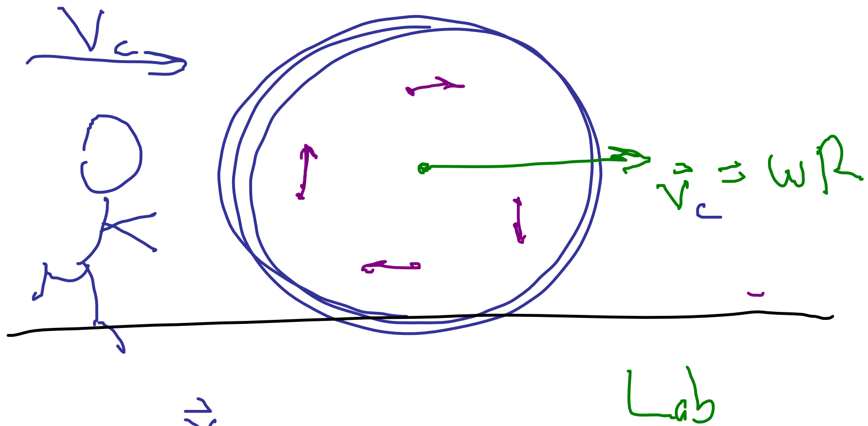
Rolling w/o slipping



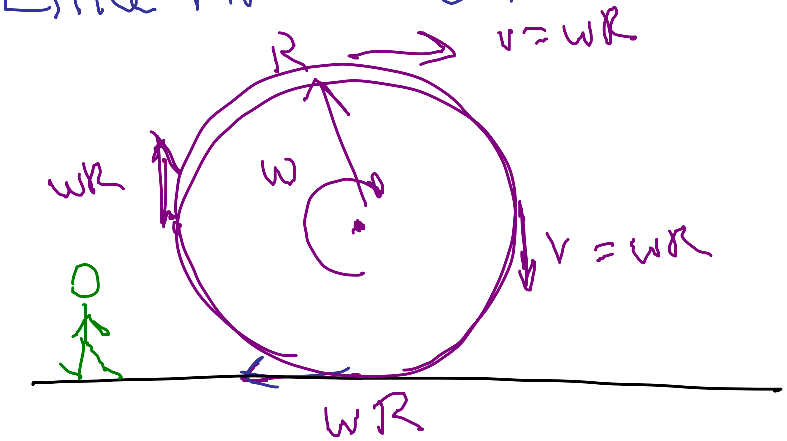


$$V_c = \omega R$$

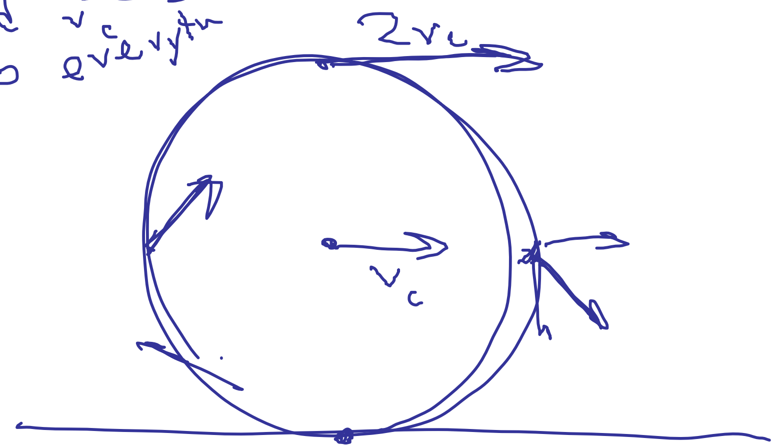
$$a_c = \alpha R$$



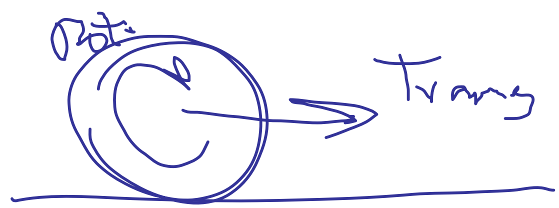
Little man sees:



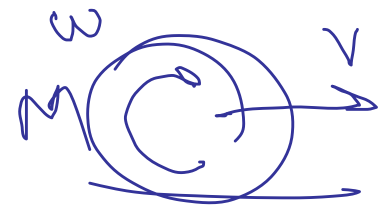
Add $\vec{v}_c$ onto everythn



KE of a rolling object



KE of rolling object



Translational Rotational.

$$KE = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = K_{tra} + K_{rot}$$

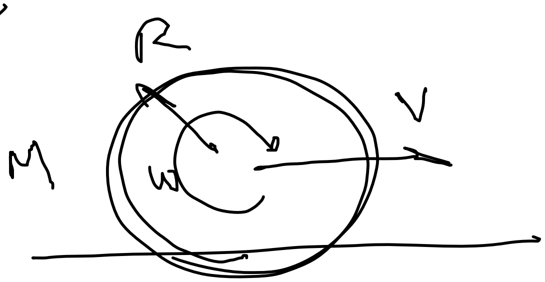
$v = \omega R$

p.167-168

10.39 What fraction of a solid disk's kinetic energy is rotational if it rolls w/o slipping.

$I = \frac{1}{2}MR^2$

$\omega = v/R$

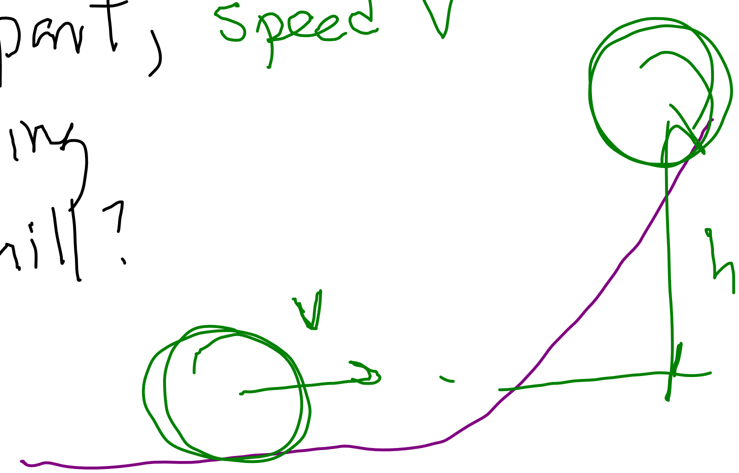


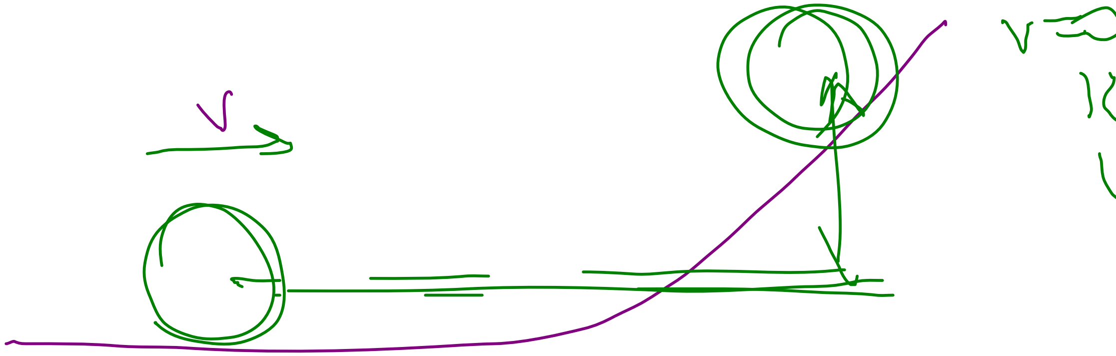
$$KE = K_{tra} + K_{rot}$$
$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}M\cancel{R^2}\right)\left(\frac{v}{\cancel{R}}\right)^2$$
$$= \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2$$

$$K_{Tot} = \underbrace{\tfrac{1}{2}Mv^2}_{trans} + \underbrace{\tfrac{1}{4}Mv^2}_{rot} = \tfrac{3}{4}Mv^2$$

$$\frac{K_{rot}}{K_{Total}} = \frac{\tfrac{1}{4}\cancel{Mv^2}}{\tfrac{3}{4}\cancel{Mv^2}} = \tfrac{1}{3}$$

Example Ball (solid sphere) rolls on flat part, speed V
rolls up hill, w/o slipping
How high does it go up hill?





Cons of Energy

Kin E

$$K_{tran} + K_{rot} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2$$

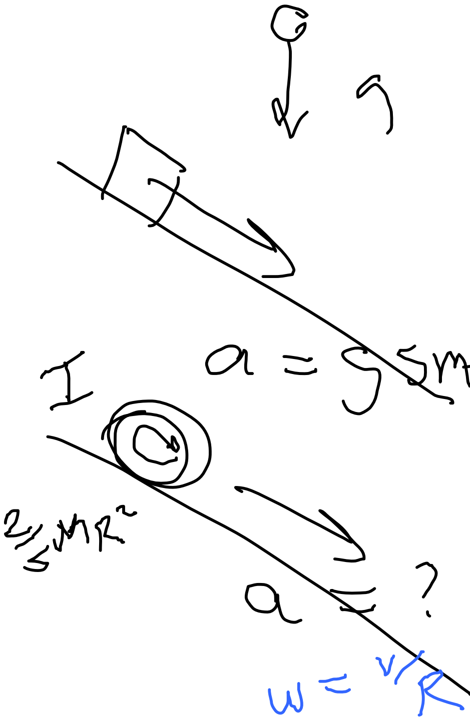
$$= \frac{7}{10}Mv^2 = Mgh$$

Solve for h

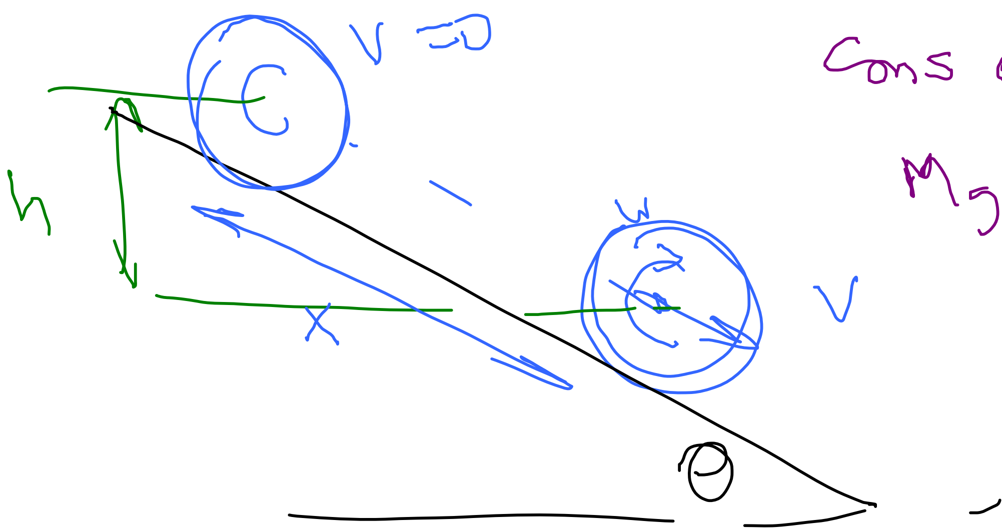
$$h = \frac{7}{10}\frac{v^2}{g}$$

Hollow sphere $I = \frac{2}{3}MR^2$

Ball rolls down hill, what's it's acceleration.



First, do it by cons of energy



Cons of E

$$Mgh = K$$
$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$
$$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2$$
$$= \frac{7}{10}Mv^2$$

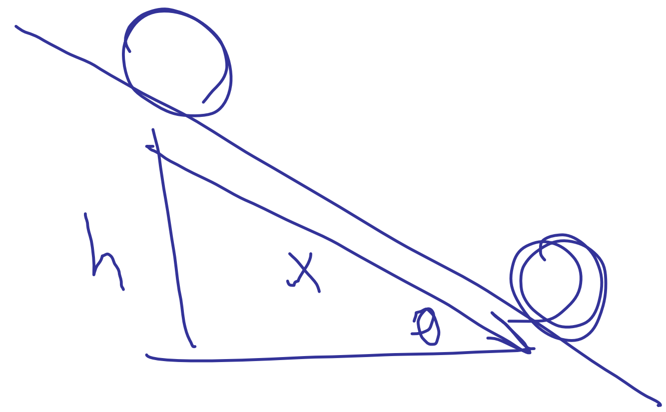
$$\cancel{M}gh = \frac{7}{10}\cancel{M}v^2$$

$$v^2 = \frac{10}{7}gh$$

$$= \frac{10}{7}g\,x\sin\theta$$

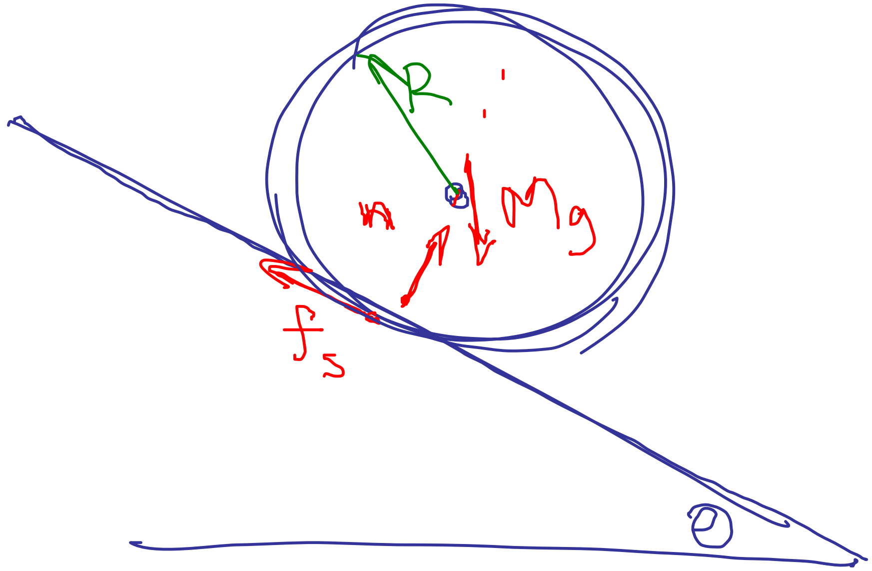
$$= (2x)\left(\frac{5g}{7}\sin\theta\right)$$

$$\underline{a = \frac{5}{7}g\sin\theta}$$



$$h = x\sin\theta$$

$$\boxed{\begin{aligned} v^2 &= 2aX \\ &= (2x)\,a \end{aligned}}$$



Net force down slope

$$F_{x\,tot} = \boxed{Mg\sin\theta - f_s = Ma}$$

Torque about center

Mg, n no torque

$$\tau_{net} = f_s R = I\alpha = I\,a/R$$

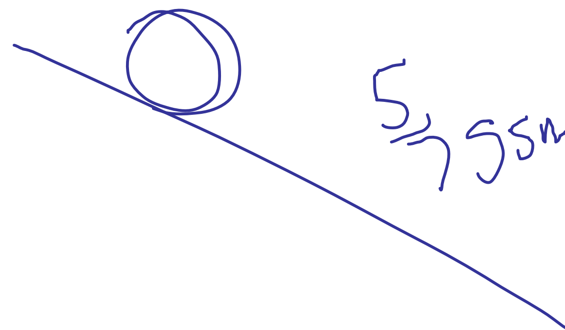
$$\alpha = a/R$$

$$f_s = I\,a/R^2 = \tfrac{2}{5}MR^2\,a/\cancel{R^2} = \tfrac{2}{5}Ma$$

$$Mg\sin\theta - \frac{2}{5}Ma = Ma$$

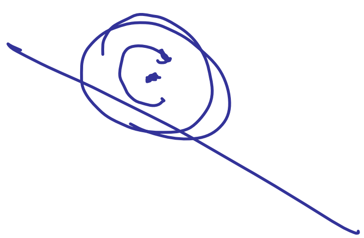
$$\cancel{M}g\sin\theta = \cancel{M}a\left(\frac{7}{5}\right)$$

$$\boxed{a = \frac{5}{7}g\sin\theta}$$



$\frac{5}{7}g\sin\theta$

Technic.



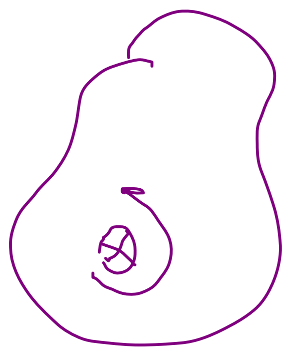
Ctr of ball is accel.

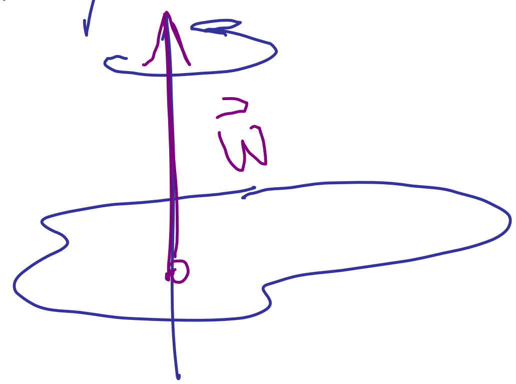
It's OK

Ch 11 More on rotations

Vector nature of our quantities.

Angular velocity





Essential: Cross product

$\vec{a}, \vec{b} \longrightarrow \vec{c}$