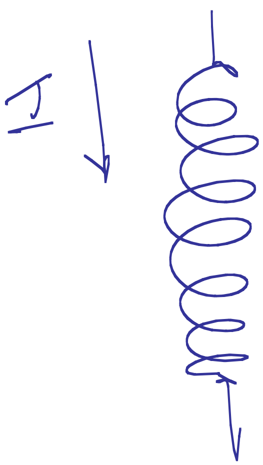
Phys 2120-4 10/22/12

Note Title 10/22/2012

Self-inductance



Inductor,

L

$$\mathcal{E} = -L\frac{dI}{dt}$$

$\frac{d\Phi_B}{dt} \Rightarrow \mathcal{E}$ $\quad \frac{dI}{dt}$

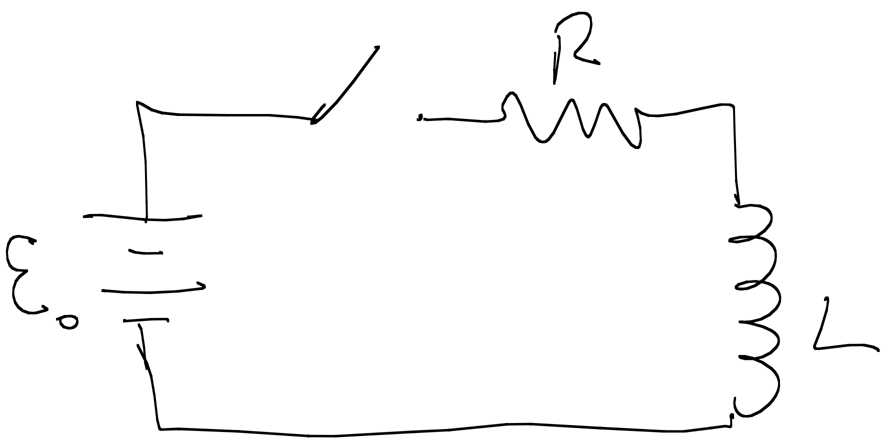
opposes it

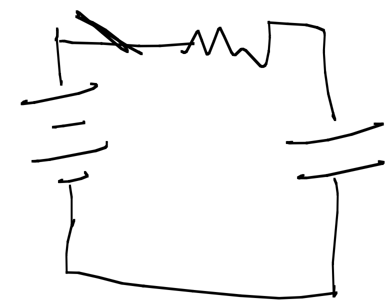
solenoid

$$L = \mu_0 n^2 A l \qquad n = \frac{N}{l}$$

Henry

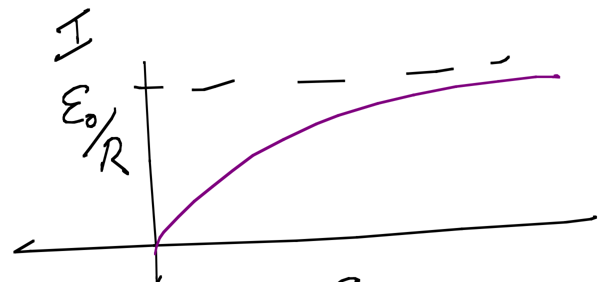
RL circuit

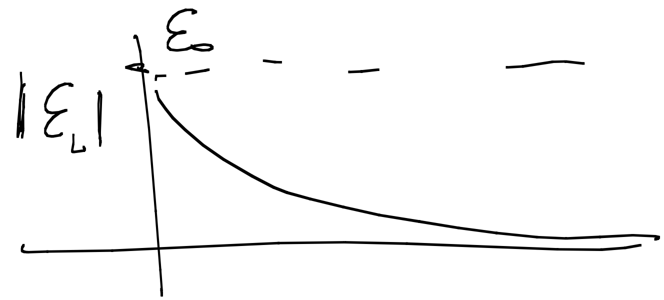


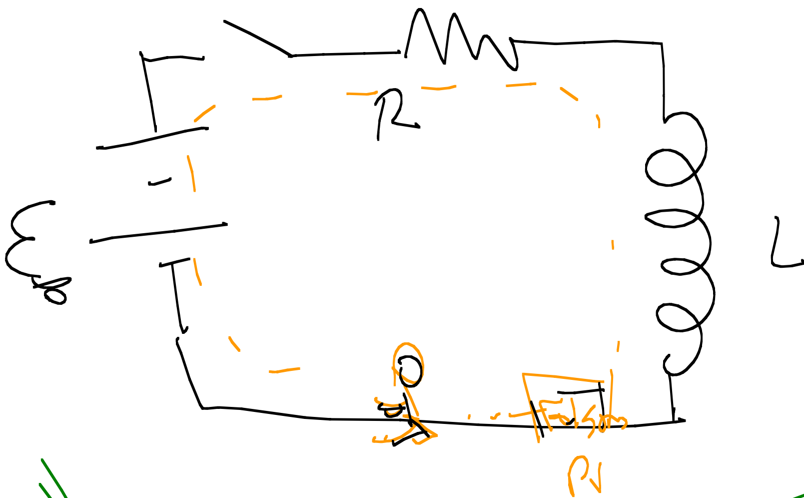


RC

Expect







$$\mathcal{E}_0 - IR - L\frac{dI}{dt} = 0 \quad (1)$$

Use $\mathcal{E}_L = -L\frac{dI}{dt}$

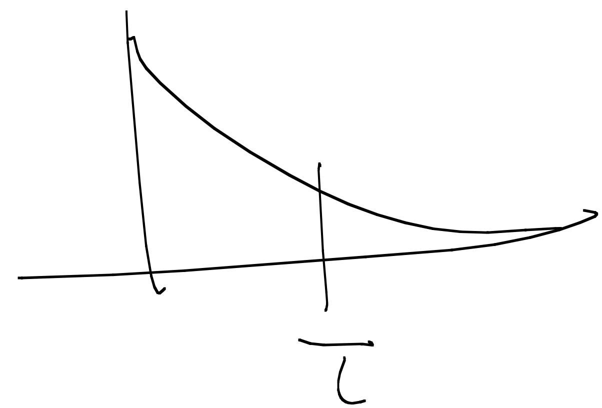
$$\frac{dI}{dt} = -\frac{1}{L}\mathcal{E}_L$$

Substitute in (1), take d/dt (1)

Eqn gives

$$\frac{R}{L}\mathcal{E}_L + \frac{d\mathcal{E}_L}{dt} = 0$$

$$\frac{d\mathcal{E}_L}{dt} = -\frac{R}{L}\mathcal{E}_L \qquad \mathcal{E}_L \propto e^{-Rt/L}$$

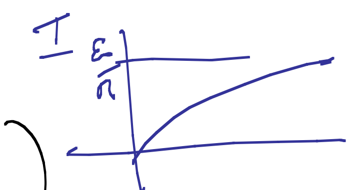


$$\mathcal{E}_L = (-\mathcal{E}_0)\, e^{-t/[L/R]}$$

Time constant $\tau_L = L/R$

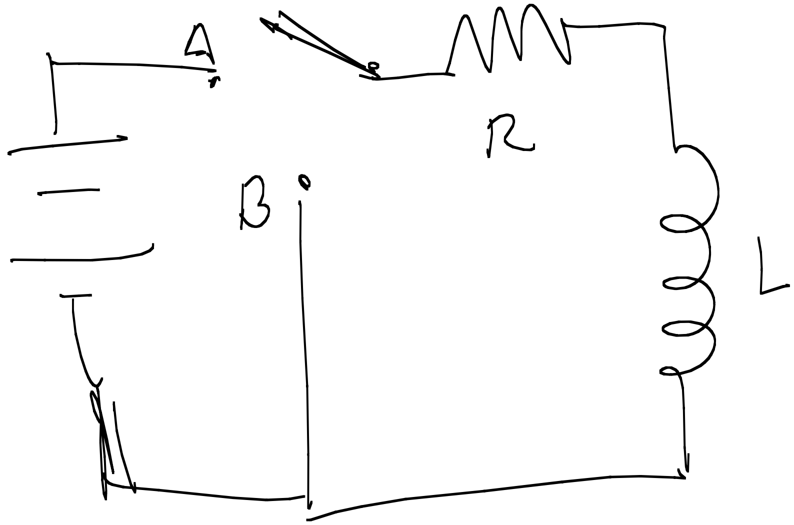
Current

$$-L\frac{dI}{dt} = -\mathcal{E}_0 e^{-t/\tau}$$

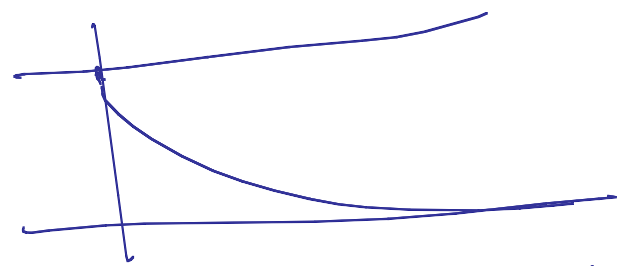


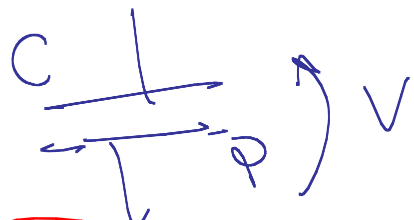
Substitute (1)

$$IR = \mathcal{E}_0\left(1 - e^{t/\tau}\right)$$



→ A current is constant.





$Q = CV$

Capacitors store energy

$U = \frac{1}{2} CV^2$

J. Cash's (1) mult by:

$$I\mathcal{E} - I^2R - LI\frac{dI}{dt} = 0$$

E supplied by battery

Loss of E in resistor

E "lost" to the inductor.

$$I\mathcal{E}_0 = I^2R + \frac{d}{dt}\left(\frac{1}{2}LI^2\right)$$

$\mathcal{E}$ battery

E lost to res

Rate at which energy goes into inductor

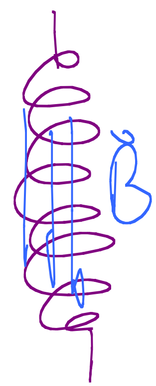
$$I\frac{dI}{dt}$$

$$= \frac{L}{2}\frac{d}{dt}\left(I^2\right)$$

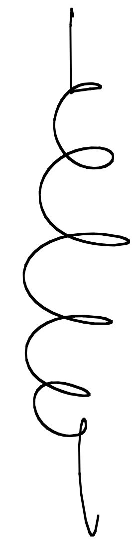
$$U = \frac{1}{2}LI^2$$

Energy stored in inductor

$$U = \frac{1}{2}CV^2$$



Solenoid: $U = \frac{1}{2} L I^2 = \frac{1}{2} \mu_0 n^2 A \ell I^2$



Use $B = \mu_0 n I$. Substitute

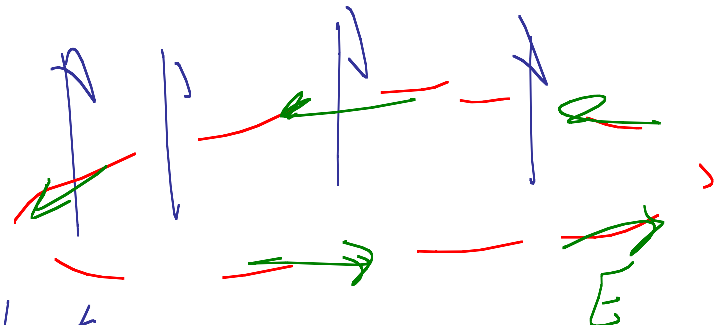
$$U = \frac{1}{2} \frac{1}{\mu_0} B^2 A \ell = \frac{B^2}{2\mu_0} (\text{Volume})$$

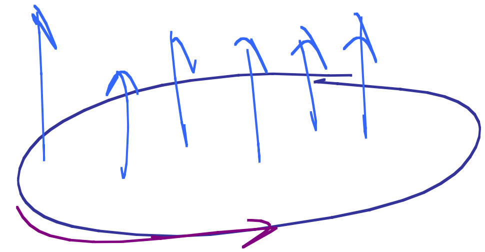
V

Energy density:

$$u_B = \frac{B^2}{2\mu_0} \qquad u_E = \frac{1}{2} \epsilon_0 E^2$$

Induced $\vec{E}$ fields





$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$= -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

Still true

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

$$= -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

even without wire.

Maxwell Eqns 4 of them

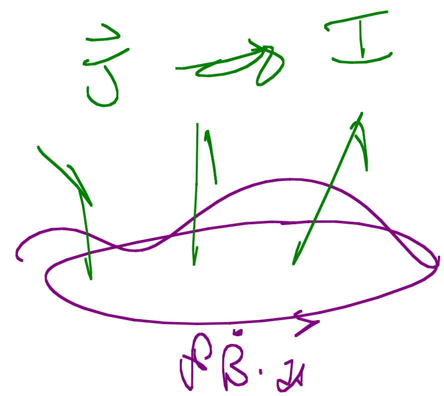
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} = \frac{1}{\epsilon_0}\int \rho \, dV$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} = \mu_0 \int \vec{J} \cdot d\vec{A}$$

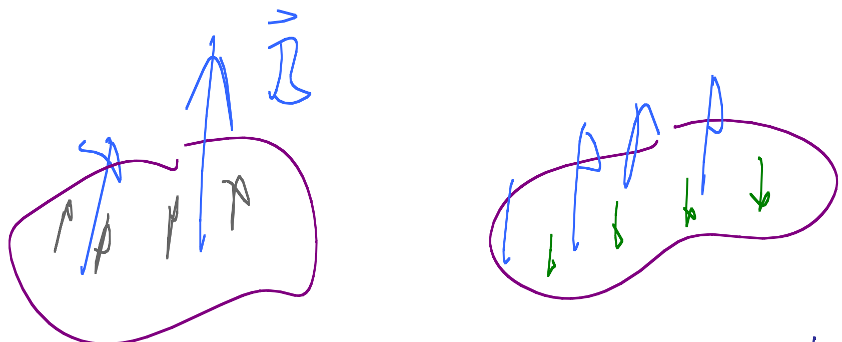
Ampere's Law not quite right.



Electrostatics: $\oint \vec{E}\cdot d\vec{r} = 0$

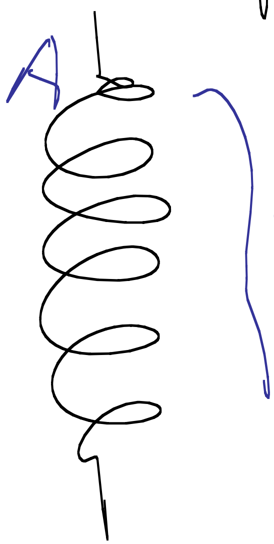
With currents $\oint \vec{E}\cdot d\vec{r} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$

Diamagnetism



Diamagnetic

27.17 Find self-inductance of a 1000 turn solenoid 50 cm long, and 4.0 cm in diameter.



$$n = \frac{N}{l}$$

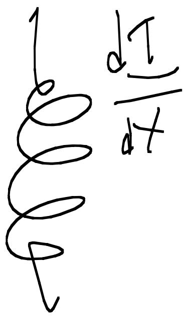
$$L = \mu_0 n^2 A l$$

$$= (4\pi \times 10^{-7}) \left(\frac{1000}{0.50}\right)^2 \left(\pi \left(2.0 \times 10^{-2}\right)^2\right) \cdot (0.50)$$

$$= 3.2 \times 10^{-3} \text{ Henry}$$

$$= 3.2 \text{ mH}$$

27.18 The current in an inductor is charging at 100 A/s & inductor emf is 40 V. What's self-inductance



$$\mathcal{E} = L \frac{dI}{dt}$$

(abs values)

$$L = \frac{\mathcal{E}}{\left(\frac{dI}{dt}\right)} = \frac{40\,V}{100\,A/s}$$

$$= 0.40\,H$$

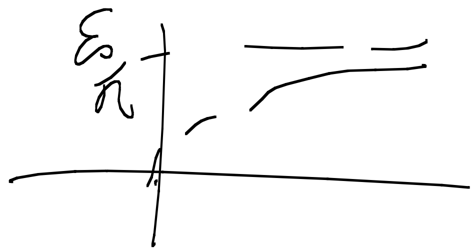
27.52 The current in a series RL circuit rises to half its final value in 7.6 s

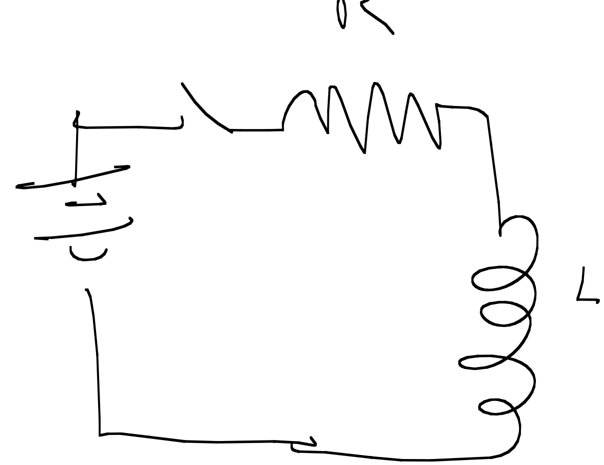
What's the time constant?

$$I = \frac{\mathcal{E}_0}{R}\left(1 - e^{-t/\tau}\right)$$

At $t = 7.6\,s$ $\quad \frac{\mathcal{E}_0}{R} = \frac{\text{final}}{\text{val}}$

$$\left(1 - e^{-t/\tau}\right) = \frac{1}{2} = 0.5$$





Math

$$e^{-t/\tau} = 0.5$$

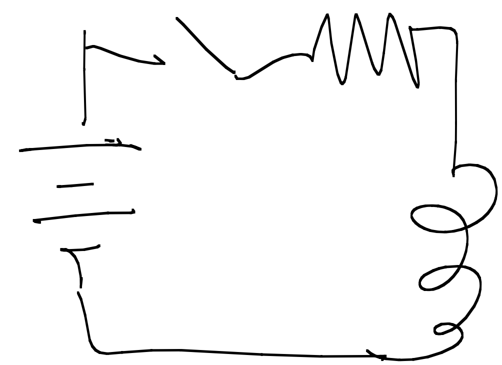
Find ln on calculator

$$t/\tau = 0.693$$

$$\Rightarrow \tau = 11.0\,s$$

27.53 In series RL circuit $\mathcal{E}_0 = 45V$
$R = 3.3\,\Omega$ $L = 2.1$ H. If current
is 9.5A how long has switch been closed

$$\frac{\mathcal{E}_0}{R} = 13.6A$$



$$9.5A = 13.6A\left(1 - e^{-t/\tau}\right)$$

→ Find t Do math → $\boxed{0.763\,s}$

$$\tau = \frac{L}{R} = 0.636\,s$$