Phys 4900, Fall 2011
Problem Set #6

1. *Griffiths EP*, **3.23** In the stationary target frame, total momentum is

$$p'_{\mathrm{Tot}}{}^{\mu} = \left(\frac{E' + mc^2}{c}, \mathbf{p}'\right)$$

In the CM frame it is

$$p_{\mathrm{Tot}}{}^{\mu} = \left(\frac{2E}{c}, \mathbf{0}\right)$$

Using invariance of the square of the total momentum one can show

$$E'mc^2 + m^2c^4 = 2E^2$$

and with

$$E' = \gamma' mc^2 \qquad \gamma' = \frac{1}{\sqrt{1 - u^2/c^2}}$$

$$E = \gamma mc^2 \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

one gets

$$(\gamma' + 1)m^2c^4 = 2\gamma^2 m^2c^4$$

from which eventually you can show

$$v = \frac{c^2}{u}\left(1 - \sqrt{1 - u^2/c^2}\right)$$

2. *Griffiths EP*, **3.25**
Mandelstam! For $A + B \to C + D$, define

$$s \equiv (p_A + p_B)^2/c^2 \qquad t \equiv (p_A - p_C)^2/c^2 \qquad u \equiv (p_A - p_D)^2/c^2$$

From momentum conservation, $p_A + p_B = p_C + p_D$ there will be relations between the three variables. You can make the algebra shorter by using

$$s = (p_A + p_B)^2/c^2 = \frac{(p_A + p_B + p_C + p_D)^2}{4c^2}$$

not to mrntion

$$\frac{(p_A + p_B - p_C - p_D)^2}{4c^2}$$

You'll notice that when adding these a lot of cross terms will cancel. Using $p_A^2 = m_A^2c^4$ etc. get the result

$$s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$$

4. *Griffiths EP*, **3.27** The Compton formula was probably given in the Modern Physics book, but let's derive it properly with relativstic conservation laws.

The energy of a photon is

$$E = h\nu = \frac{hc}{\lambda}$$

If the photon scattering angle is θ (new photon wavelength is λ') and the electron scattering angle is ϕ, then energy conservation gives

$$\frac{hc}{\lambda} + mc^2 = \frac{hc}{\lambda'} + \sqrt{p^2c^2 + M^2c^4}$$

Conservation of x-momentum gives

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + p\cos\phi$$

and conservation of transverse momentum gives

$$\frac{h}{\lambda'}\sin\theta = p\sin\phi$$

Use algebra to get the famous result (it isn't all that trivial!),

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$$

5. *Griffiths EP*, **4.8** Use the two expressions for angular momentum for a rotating solid sphere,

$$L = \frac{\hbar}{2} \qquad \text{and} \qquad L = I\omega = \tfrac{2}{5}Mr^2\omega\ .$$

Equate these, and use $v = r\omega$ for the speed of a point on the equator to solve for v in terms of r (it is inversely proportional). So a *maximum* value of r gives a *minimum* value of v and thus show that with $r < 10^{-18}$ cm the minimum v is much large than c.