Phys 2920, Spring 2012
Problem Set #6

1. Find a set of formulae which transforms cylindrical coordinates (ρ, ϕ, z) to spherical coordinates, (r, θ, ϕ).

2. Express the following (spherical) entities in terms of the Cartesian unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$:

a) $\hat{\mathbf{e}}_r$, $\hat{\mathbf{e}}_\theta$ and $\hat{\mathbf{e}}_\phi$ for $r = 1$, $\theta = \frac{\pi}{2}$, $\phi = 0$

b) $\hat{\mathbf{e}}_r$, $\hat{\mathbf{e}}_\theta$ and $\hat{\mathbf{e}}_\phi$ for $r = 1$, $\theta = \frac{\pi}{2}$, $\phi = \frac{\pi}{2}$

c) $\hat{\mathbf{e}}_r$ for $\theta = \pi$. (Do $\hat{\mathbf{e}}_\theta$ and $\hat{\mathbf{e}}_\phi$ have any meaning for this case?)

3. *(VA 7.38)* Express each of the following loci in spherical coordinates (be careful that you've handled the angles θ and ϕ correctly):

a) the sphere $x^2 + y^2 + z^2 = 9$

b) the cone $z^2 = 3(x^2 + y^2)$

c) the paraboloid $z = x^2 + y^2$

d) the plane $z = 0$

e) the plane $y = x$

4. *(VA 7.43)* Represent the vector $\mathbf{a} = 2y\ \hat{\mathbf{i}} - z\ \hat{\mathbf{j}} + 3x\ \hat{\mathbf{k}}$ in spherical coordinates and determine a_r, a_θ and a_ϕ.

5. If

$$\mathbf{A} = \frac{p_0 \omega^2}{4\pi\epsilon_0 c^2} \left(\frac{\cos\theta}{r} \right) \cos[\omega(t - r/c)]\hat{\mathbf{e}}_r$$

find $\nabla \times \mathbf{A}$. *(Note, even though there's a t in there, which does stand for time, the derivatives of the curl treat it as any other constant.)*

Here, p_0, ω are constants. The formula for $\mathbf{A}$ is the vector potential far from an electric dipole which has magnitude p_0 and oscillates with angular frequency ω. The curl of $\mathbf{A}$ gives the magnetic field $\mathbf{B}$.

6. If

$$V = \frac{\alpha}{r} + \frac{\beta}{r^2} \cos\theta$$

where α and β are constants, show that $\nabla^2 V = 0$.

7. Prove that for a function Φ given in cylindrical coordinates by

$$\Phi(\rho, \phi) = \ln\left(\frac{\rho}{a}\right) + \left(A\rho^n + \frac{B}{\rho^n}\right)(C \sin n\phi + D \cos n\phi) \quad ,$$

where A, B, C, D are all constants and n is an integer, we have $\nabla^2 \Phi = 0$.