Phys 2920, Spring 2010
Problem Set #3

1. Generally matrices do not commute (for multiplication). The extent to which they do not is given by the **commutator**, which for matrices A and B is given by

$$[\mathsf{A}, \mathsf{B}] \equiv \mathsf{AB} - \mathsf{BA}$$

The following matrices are very useful in physics:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

a) Evaluate each of the three following commutators, and for each express the result in terms of the σ matrices themselves.

$$[\sigma_x, \sigma_y] \qquad [\sigma_y, \sigma_z] \qquad [\sigma_z, \sigma_x]$$

b) Are the σ matrices symmetric? Orthogonal? Hermitian? Unitary?

2. Show that the matrix

$$\mathsf{R} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

is an orthogonal matrix.

3. Find the determinant of the matrix

$$\mathsf{A} = \begin{pmatrix} 5 & 4 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 9 \\ 1 & -2 & -1 & 4 \end{pmatrix}$$

any way you can!

4. For the following matrices, figure out if each has an inverse (give a good mathematical reason) and *then* use a computer to find the inverse.

$$\text{(a)} \qquad \mathsf{A} = \begin{pmatrix} 1 & 3 & -4 \\ 1 & 5 & -1 \\ 3 & 13 & -6 \end{pmatrix} \qquad\qquad \text{(b)} \qquad \mathsf{B} = \begin{pmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{pmatrix}$$

5. Solve the set of equations

$$\begin{aligned} x + 2y - 4z &= -4 \\ 2x + 5y - 9z &= -10 \\ 3x - 2y + 3z &= 11 \end{aligned}$$

by first writing it in matrix/vector form and then using matrix inversion to get (x, y, z). You can get help from Maple for the last step.

6. Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 2 & -3 \\ 2 & -5 \end{pmatrix}$$

Don't use a computer on this!!