Phys 3810, Spring 2012
Problem Set #7, Hint-o-licious Hints

1. *Griffiths*, **4.29** Show that the eigenvectors of S_y (well, one choice for them) are

$$\chi_+^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} \qquad \chi_-^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

2. *Griffiths*, **4.52** Follow the example of spin given out in class (G's problem 4.31). The eigenvectors of S_z are

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Using (4.136), find the action of the raising and lowering operators S_+ and S_- on all the eigenstates $|\frac{3}{2}\ m\rangle$ and then construct the matrices for these operators. Get S_x from $S_x = \frac{1}{2}(S_+ + S_-)$. You should get

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$

but *show* this!

You just need to get eigenvalues of S_x but it should be clear what they ought to be! For this you need to take the determinant of a 4×4 matrix which needs to be done by an expansion (not by zipping along all the diagonals as you can for 3×3).

3. *Griffiths*, **5.2** (a) Show that the fractional difference between m_{e} and μ_{H} is 5.4×10^{-4}. This is the same as the fractional change in the binding energy. (Show all of this!)

(b) It's same fractional correction to R; one finds that for the H atom

$$R_H = 1.096 \times 10^7\ \mathrm{m}^{-1}$$

The fractional difference between μ_H and μ_D (reduced masses for the H and D atoms) is 2.7×10^{-4}. Take differentials to get the fractional change in the Balmer wavelength; it comes out to about 17.9 nm.

(c) The reduced mass for positronium is half the electron mass!

(d) The reduced mass for muonium is 185.9 times the electron mass. That's the factor by which you need to fix R from the value given in the book. With this new value of R, get Lyman-α. It comes out to about 6.54×10^{-10} m.

4. *Griffiths*, **5.3** The energy of the photon emitted in the transition (always between adjacent HO states) is $\hbar\omega$. The *frequency* of the radiation is

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$$

where μ is the reduced mass of the oscillator system. Show that if μ changes, the change in frequency is related to the change in μ by

$$d\nu = -\frac{1}{2}\nu\frac{d\mu}{\mu}$$

What is the fractional difference in reduced mass between the two molecules?

5. *Griffiths*, **5.15** Well, he gives the answer. Get it dividing the E_{tot} from (5.45) by Nq.

6. *Griffiths*, **5.16** I get: (a) Fermi energy of copper is 7.05 eV. (b) Electron speed for this energy is $1.58 \times 10^6 \frac{\text{m}}{\text{s}}$. (c) Fermi temperature is 8.19×10^4 K. (d) Degeneracy pressure is 3.84×10^{10} Pa.

7. *Griffiths*, **5.19** I get (with the help of Maple's `fsolve` function) a root of $z = 2.628$), leading to an energy of

$$E = 0.345 \text{ eV}$$

But *show all of this*.

8. *Griffiths*, **5.35** **a)** Show that the total electron energy is

$$E_{\text{Tot}} = 3\left(\frac{9\pi}{4}\right)^{2/3}\frac{\hbar^2(Nq)^{5/3}}{10mR^2}$$

b) Any way you can, if only by analogy with the electrostatic result, show that

$$E_{\text{grav}} = -\frac{3}{5}\frac{GM_s^2}{R}$$

c) I agree with his R and numerical coefficient of $N^{-1/3}$.

d) I get $R = 7170$ km.

e) I get $E_F = 194$ keV. Note, the mass of an electron is 511 keV.

9. *Griffiths*, **5.36**

a) I find

$$E_{\text{tot}} = \frac{\hbar c V K_F^4}{4\pi^2}$$

b) Note, we still have $k_F = (3\rho\pi^2)^{1/3}$. This gives

$$E_{\text{tot}} = \frac{\hbar c\pi^{2/3}(3Nq)^{4/3}}{4V^{1/3}}$$

where of course $V = \frac{4}{3}\pi R^3$, so that $E \propto 1/R$. Eventually I deduce

$$N_c^{2/3} = \frac{5}{16}\frac{\hbar c}{GM^2}3^{2/3}\pi^{1/3}$$

which gave me

$$N_c = 2.06 \times 10^{57}$$

which comes out to 1.7 times the mass of the Sun.

c) Here we essentially re-do Prob 5.35 for neutrons. I get

$$R = \frac{\hbar^2}{GM^3 N^{1/3}}\left(\frac{9\pi}{4}\right)^{2/3}$$

which fora star the mass of the sun gives $R = 12.4$ km.

The neutron Fermi energy comes out to 56.2 meV which is failry small compared to the neutron mass 938 MeV.