Name________________________

Class Time: 9am 10am 11am

Feb 13, 2003

Phys 2120 — Spring 2003
Exam #1

1.	________	(6)
2.	________	(12)
3.	________	(12)
4.	________	(16)
5.	________	(4)
6.	________	(11)
7.	________	(16)
8.	________	(15)
9.	________	(8)
Total	________	(100)

You must show all your work and include the right units with your answers!

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \, \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \qquad \epsilon_0 = 8.85 \times 10^{-12} \, \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}} \qquad e = 1.602 \times 10^{-19}\,\mathrm{C}$$

-27..oops!

$$m_{\mathrm{elec}} = 9.1094 \times 10^{-31}\,\mathrm{kg} \qquad m_{\mathrm{prot}} = 1.673 \times 10^{\cancel{-26}}\,\mathrm{kg} \qquad 1\,\mathrm{eV} = 1.609 \times 10^{-19}\,\mathrm{J}$$

$$\mathbf{F} = m\mathbf{a} \qquad g = 9.80\,\frac{\mathrm{m}}{\mathrm{s^2}} \qquad F = k\frac{|q_1 q_2|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

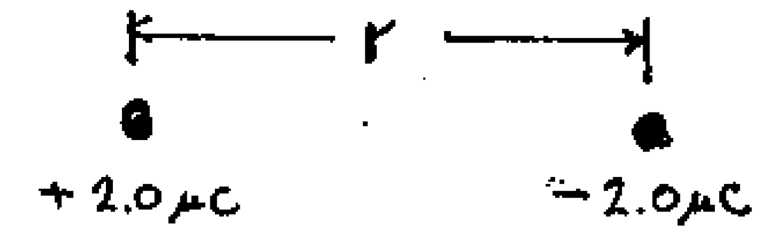
$$dq = \lambda dx \qquad dq = \sigma dA \qquad dq = \rho dV$$

$$\mathbf{F} = q\mathbf{E} \qquad E_{\mathrm{pt\ ch}} = k\frac{|q|}{r^2} \qquad E_{\mathrm{plane}} = \frac{\sigma}{2\epsilon_0} \qquad E_{\mathrm{cond\ surf}} = \frac{\sigma}{\epsilon_0}$$

$$\Phi = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\mathrm{encl}}}{\epsilon_0} \qquad \Delta U + \Delta K = 0 \qquad \Delta U = q\Delta V \qquad \Delta V = -\int_i^f \mathbf{E} \cdot d\mathbf{s}$$

$$V_{\mathrm{pt-ch}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \qquad E_x = -\frac{\partial V}{\partial x} \qquad E_{x,\mathrm{uniform}} = -\frac{\Delta V}{\Delta x}$$

1. Two point charges, $2.0\,\mu$C and $-2.0\,\mu$C, exert an attractive force of magnitude 0.10 N on one another.



By what distance are the charges separated? (6)

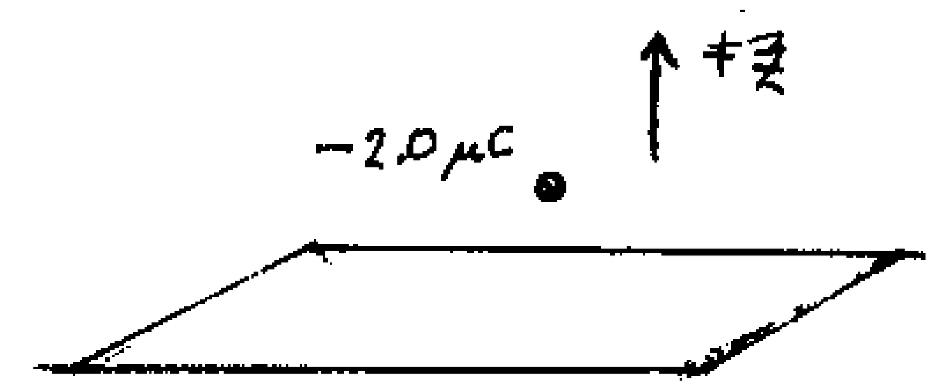
Using $F = k\dfrac{|q_1 q_2|}{r^2}$, solve for r^2:

$$r^2 = \frac{k|q_1 q_2|}{F} = \frac{(8.99\times10^9\ \frac{Nm^2}{C^2})(2.0\times10^{-6}C)(2.0\times10^{-6}C)}{(0.10\ N)} = 0.360\ m^2$$

Then:

$$\boxed{r = 0.60\ m}$$

2. A $-2.00\,\mu$C charge is near a very large uniformly-charged sheet. It experiences a force of 0.600 N *away from* the sheet (in the $+z$ direction).



a) What is the direction of the electric field at the location of the point charge? (2)

Since the charge q is negative here and $\vec{F} = q\vec{E}$, the direction of $\vec{E}$ is opposite that of the force. So $\vec{E}$ points in the...

$$\boxed{-z \text{ direction.}}$$

b) What is the magnitude of the E field which acts on the point charge? (5)

$$|\vec{E}| = |\vec{F}/q| = (0.600\,N)/(2.00\times10^{-6}\,C)$$

$$= \boxed{3.00\times10^5\ N/C}$$

c) What is the charge density of the sheet of charge? (5)

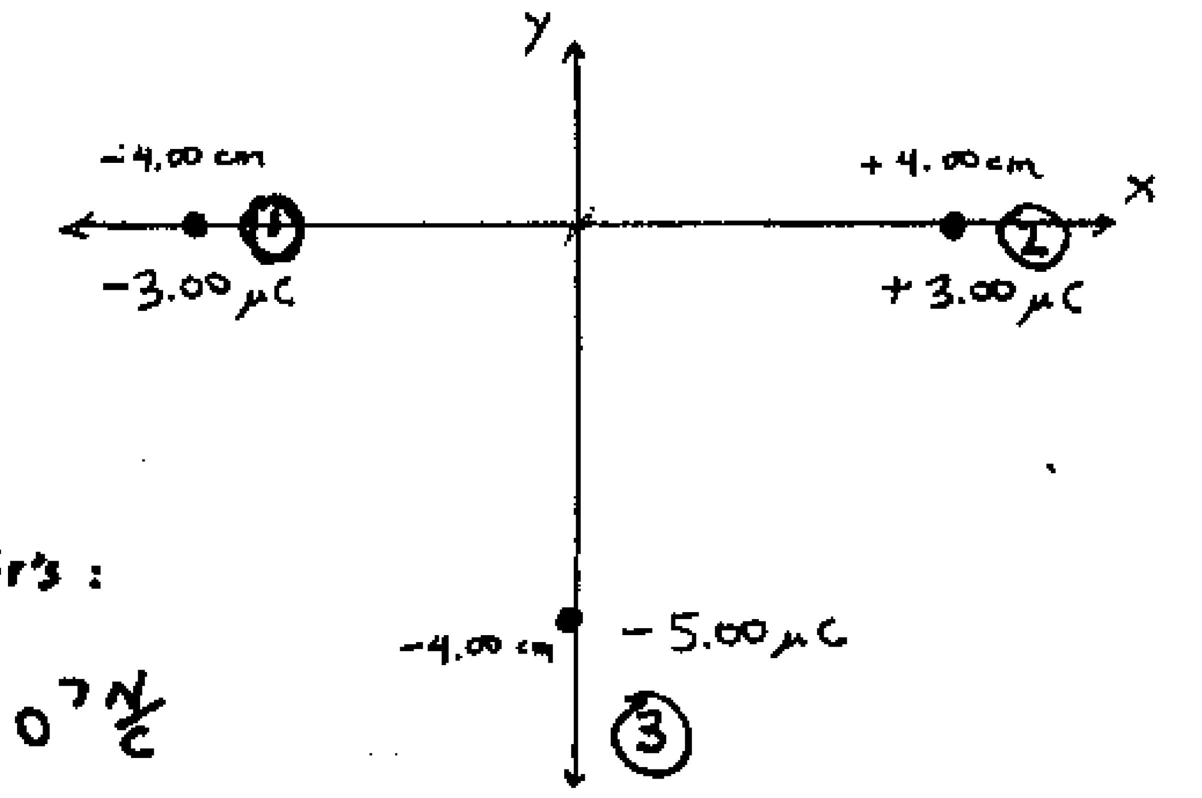
The (magnitude) of the $\vec{E}$ field from a plane of charge is given by $|\vec{E}| = \dfrac{|\sigma|}{2\epsilon_0}$. So:

$$|\sigma| = 2\epsilon_0|\vec{E}| = 2\,(8.85\times10^{-12}\,\tfrac{C^2}{Nm^2})\,(3.00\times10^5\,\tfrac{N}{C}) = 5.31\times10^{-6}\ \tfrac{C}{m^2}$$

However, the sheet has a negative charge (it repels the neg. pt charge; the $\vec{E}$ field points toward it) so σ is:

$$\boxed{\sigma = -5.31\times10^{-6}\ \tfrac{C}{m^2}}$$

3. Point charges $-3.00\,\mu\text{C}$ and $+3.00\,\mu\text{C}$ lie on the x axis at $x = -4.00\,\text{cm}$ and $x = +4.00\,\text{cm}$, respectively. A $-5.00\,\mu\text{C}$ charge lies on the y axis at $y = -4.00\,\text{cm}$.



Find the magnitude and direction of the electric field at the origin. (12)

$\vec{E}$ fields due to charges ①, ② and ③ have mag's & dir's:

① $E_1 = k\dfrac{|q_1|}{r_1^2} = (8.99\times10^9)\dfrac{(3.00\times10^{-6})}{(4.00\times10^{-2})^2}\,\frac{N}{C} = 1.69\times10^7\,\frac{N}{C}$

Points in the $-x$ direction

② $E_2 = k\dfrac{|q_2|}{r_2^2} = (8.99\times10^9)\dfrac{(3.00\times10^{-6})}{(4.00\times10^{-2})^2}\,\frac{N}{C} = 1.69\times10^7\,\frac{N}{C}$

Points in the $-x$ direction also!

③ $E_3 = k\dfrac{|q_3|}{r_3^2} = (8.99\times10^9)\dfrac{(5.00\times10^{-6})}{(4.00\times10^{-2})^2} = 2.81\times10^7\,\frac{N}{C}$

Points in the $-y$ direction

Total $\vec{E}$ field:

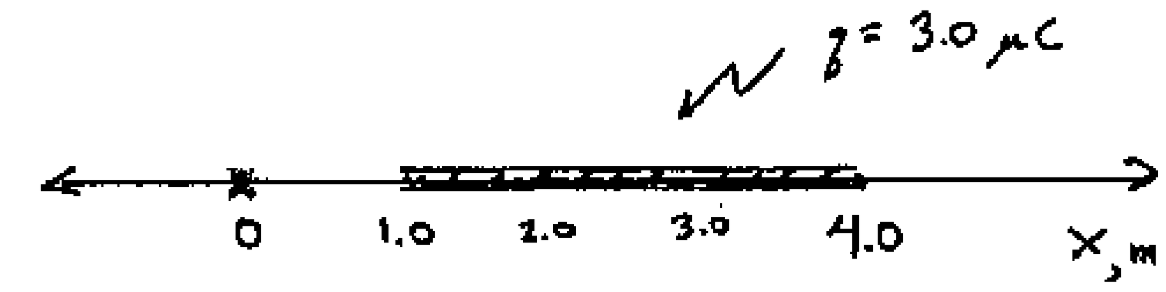
$E_x = -3.37\times10^7\,\frac{N}{C}$

$E_y = -2.81\times10^7\,\frac{N}{C}$

$|\vec{E}| = \sqrt{E_x^2+E_y^2} = \boxed{4.39\times10^7\,\frac{N}{C}}$

$\boxed{\theta_E = -140^\circ}$ (from +x axis)

4. A charge of $q = 3.0\,\mu\text{C}$ is evenly distributed along a rod which lies on the x axis, extending from $x = 1.0\,\text{m}$ to $x = 4.0\,\text{m}$.



a) What is the (linear) charge density λ of the rod. (3)

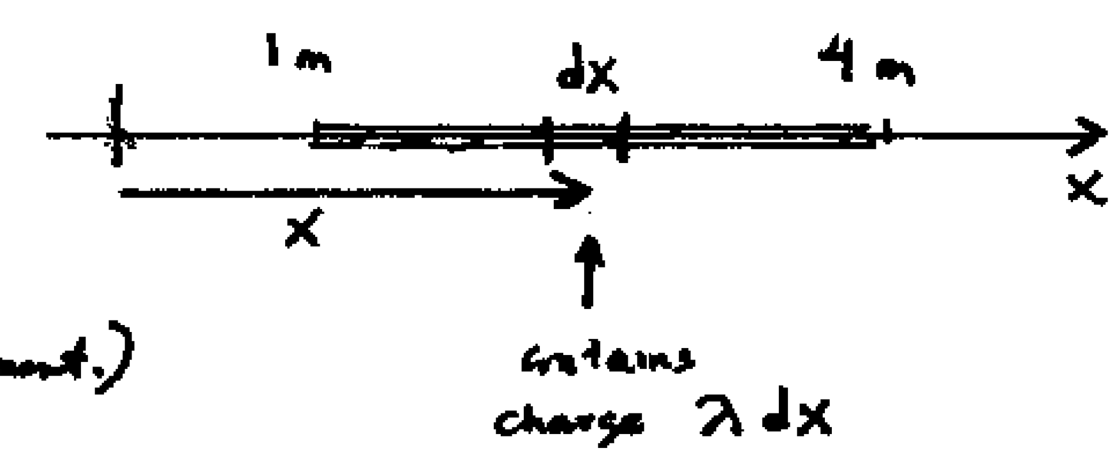
Length of rod is 3.0 m. Then:

$\lambda = \dfrac{q}{L} = \dfrac{3.0\,\mu\text{C}}{3.0\,\text{m}} = \boxed{1.0\times10^{-6}\,\frac{C}{m}}$

b) We want to find the electric field at the origin due to the charged rod.

Set up the integral which will give the value of E_x at the origin. You don't need to evaluate the integral or plug in any numbers (yet). (7)

Element dx of rod at coord x is a distance x away from the origin. The contribution to E_x is



$dE_x = -k\dfrac{\lambda\,dx}{x^2}$

(Minus sign is because field points away from charge element.)

Sum up:

$$E_x = \int_{1\,\text{m}}^{4\,\text{m}}\left(-\frac{k\lambda\,dx}{x^2}\right) = \boxed{-k\lambda\int_{1\,\text{m}}^{4\,\text{m}}\frac{dx}{x^2}}$$

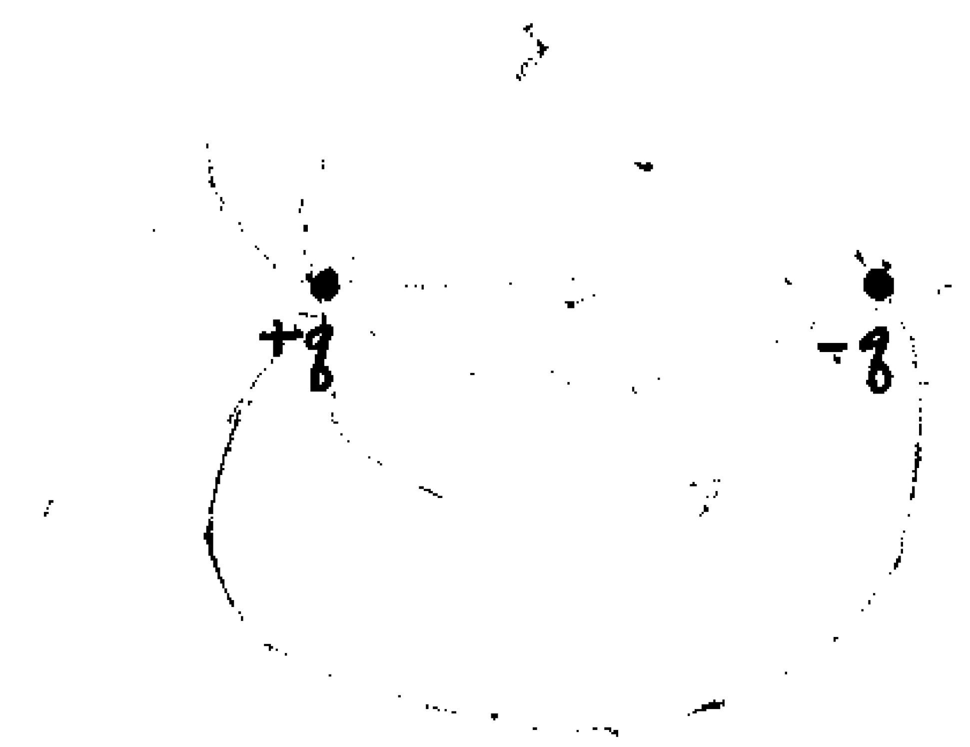
c) Now evaluate the integral from (b) and plug in the numbers to find E_x at the origin. (6)

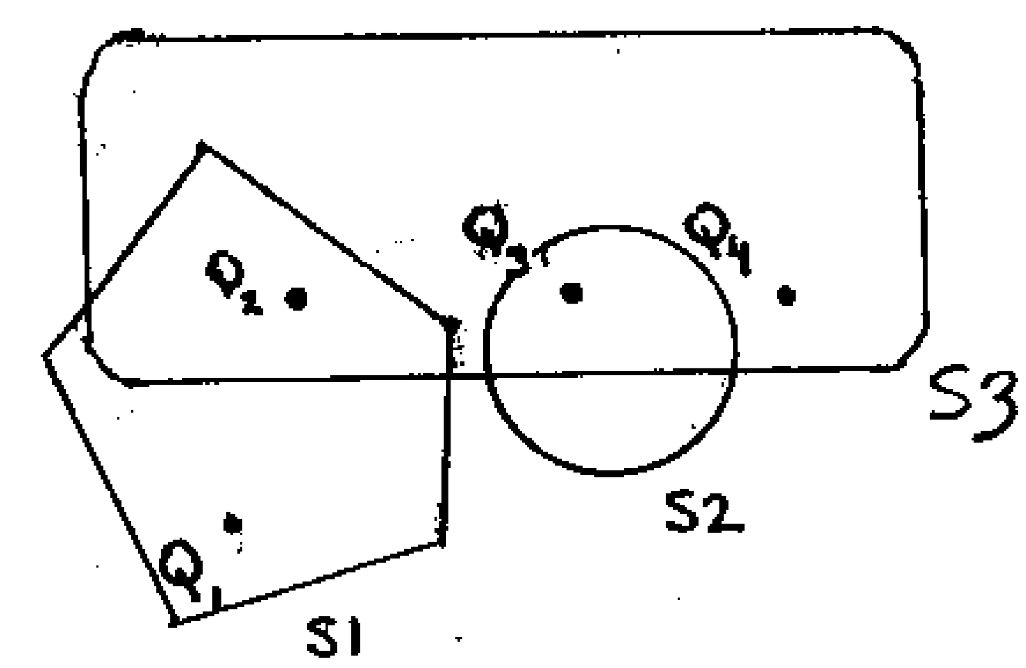
$$E_x = -k\lambda\left(-\frac{1}{x}\right)\Big|_{1\,\text{m}}^{4\,\text{m}} = k\lambda\left(\frac{1}{x}\right)\Big|_{1\,\text{m}}^{4\,\text{m}}$$

$$= \left(8.99\times10^9\,\frac{N\,m^2}{C^2}\right)\left(1.0\times10^{-6}\,\frac{C}{m}\right)\left(\frac{1}{4\,\text{m}}-\frac{1}{1\,\text{m}}\right)$$

$$= \boxed{-6.74\times10^3\,\frac{N}{C}}$$

5. In the space given here, give a sketch of the field lines for the electric field arising from two charges $-q$ and $+q$: (4)





6. Three closed surfaces are drawn in the vicinity of four charges $Q_1, \ldots Q_4$, with:

$Q_1 = 8.0\,\mu\text{C}$ $\quad Q_2 = 3.0\,\mu\text{C}$ $\quad Q_3 = 2.5\,\mu\text{C}$ and $Q_4 = 3.5\,\mu\text{C}$

a) Rank the electric fluxes through each of the surfaces from least to the greatest (6)

Same as the ranking of the net charge enclosed:

S1 : $11.0\,\mu C$ S2 : $2.5\,\mu C$ S3 : $9.0\,\mu C$

so ranking of fluxes is:

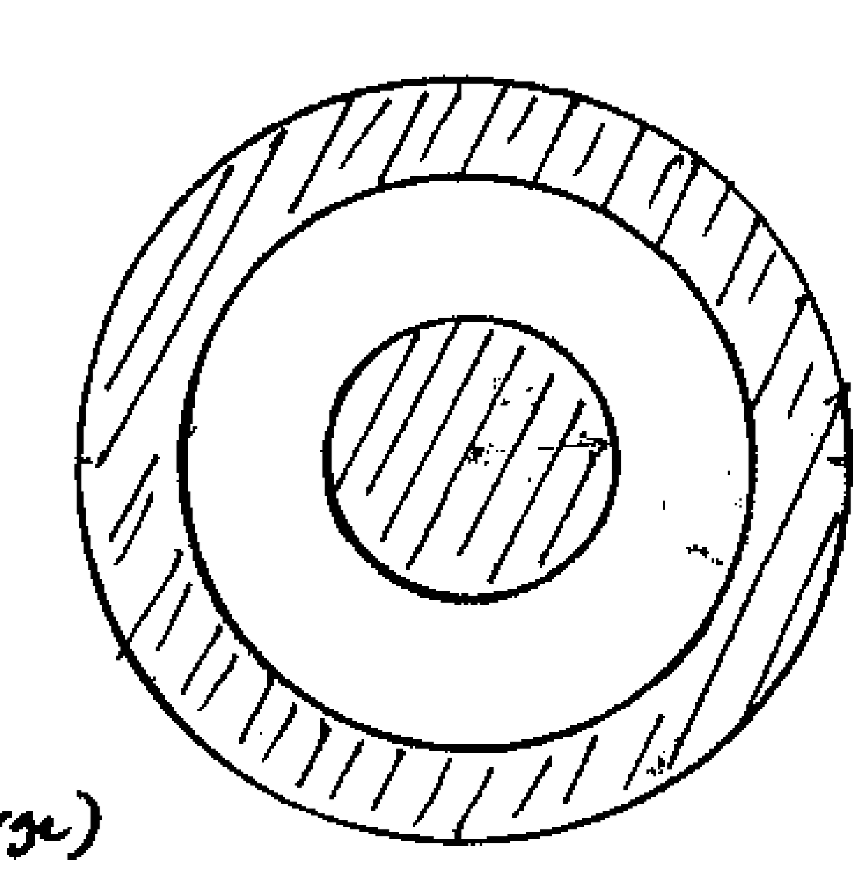
$$\boxed{S2 < S3 < S1}$$

b) Find the net electric flux through surface $S2$. (5)

For S_2, $q_{enclosed} = 2.5 \times 10^{-6} C$ so by Gauss's Law,

$$\Phi_2 = \frac{q_{enc}}{\epsilon_0} = \frac{2.5 \times 10^{-6} C}{\left(8.85 \times 10^{-12} \frac{C^2}{Nm^2}\right)} = \boxed{2.8 \times 10^5 \frac{N m^2}{C}}$$

7. A solid conducting sphere of radius 2.0 cm has a positive charge of $+8.0\,\mu$C. A conducting spherical shell of inner radius 4.0 cm and outer radius 5.0 cm is concentric with the solid sphere and has a net charge of $-4.0\,\mu$C.



a)Find the radial component of the electric field at the following distances from the center of the spheres: (10)

i) $r = 1.0$ cm

$r = 1.0$ cm is within the conducting sphere. Using a Gaussian surface of radius 1.0 cm (which encloses no charge) Gauss' Law gives:

$$\Phi = E_r 4\pi r^2 = q_{enc}/\epsilon_0 = 0 \quad \Rightarrow \quad \boxed{E_r = 0}$$

All of sphere's charge is on its surface.

ii) $r = 3.0$ cm

$r = 3.0$ cm is between the sphere & the shell. Gaussian surface with $r = 3.0$ cm encloses $+8.0\,\mu C$ of charge. As in i, Gauss's Law gives

$$E_r = \frac{q_{enc}}{4\pi\epsilon_0 r^2} = \frac{(8.0\times10^{-6}\,C)}{4\pi(8.85\times10^{-12}\,\frac{C^2}{Nm^2})(3.0\times10^{-2}\,m)^2} = \boxed{8.0\times10^{7}\,N/C}$$

iii) $r = 4.5$ cm

$r = 4.5$ cm is within the metal of the conducting shell. Generally, $\vec{E} = 0$ within a conductor (for electrostatics) so

$$\longrightarrow \boxed{E_r = 0}$$

iv) $r = 7.0$ cm

$r = 7.0$ cm is outside the shell. Gaussian surface with $r = 7.0$ cm encl's $+4.0\,\mu C$ of charge. As in i, Gauss Law gives

$$E_r = \frac{q_{encl}}{4\pi\epsilon_0 r^2} = \frac{(4.0\times10^{-6}\,C)}{4\pi(8.85\times10^{-12}\,\frac{C^2}{Nm^2})(7.0\times10^{-2}\,m)^2} = \boxed{7.3\times10^{6}\,\frac{N}{C}}$$

b) What is the charge on the inner surface on the spherical shell? (3)

Consider Gaussian surface drawn within metal of conducting shell. Has no flux so total enclosed charge is zero. Sphere has charge $+8.0\,\mu C$, the rest is on inner surface of shell so:

$$\boxed{q_{inner} = -8.0\,\mu C}$$

c) What is the charge on the outer surface on the spherical shell? (3)

The rest of shell's charge is on outer surface, with

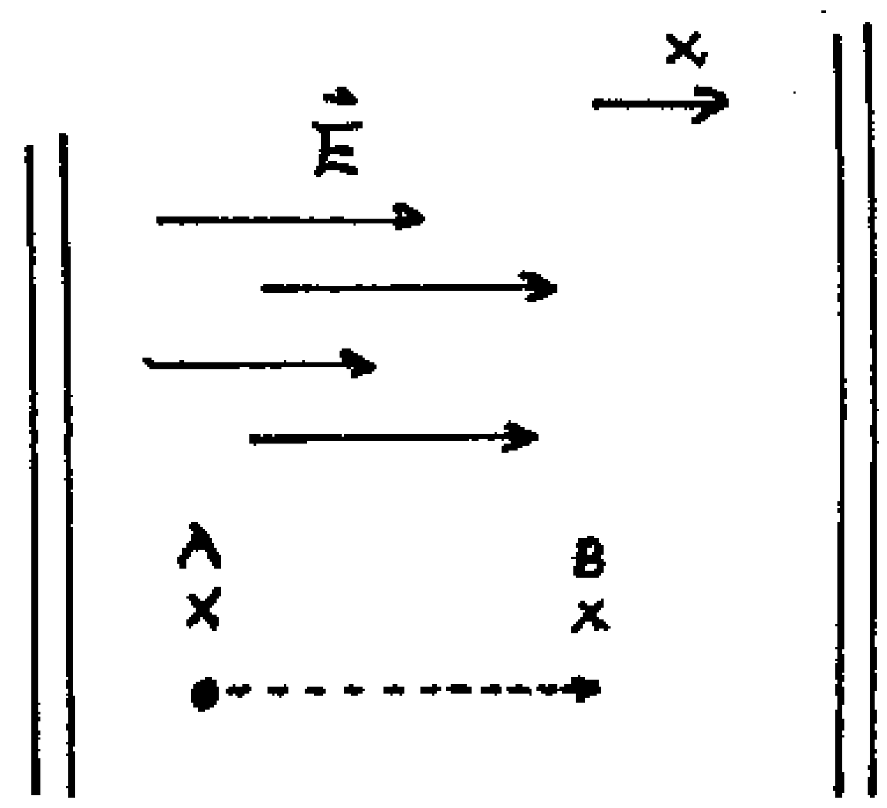
$$q_{inner} + q_{outer} = -4.0\,\mu C.$$

Using answer to (b),

$$q_{outer} = \boxed{+4.0\,\mu C}$$

8. A proton is released from rest in a uniform electric field of 8.0×10^4 V/m directed along the positive x axis as shown here.

The proton moves a distance of 0.50 m as it moves from A to B in the direction of the electric field.



a) What is the change in electric potential as we go from A to B ? (5)

Here, $E_x = +8.0 \times 10^4 \frac{V}{m}$. Then, from $E_x = -\frac{\Delta V}{\Delta x}$

$$\Delta V = -E_x \Delta x = -(8.0 \times 10^4 \tfrac{V}{m})(0.50\,m)$$
$$= -4.0 \times 10^4\ V$$

b) Find the change in the potential energy of the proton as it moves from A to B. (5) Charge of proton is $+e$

$$\Delta U = q \Delta V = (1.602 \times 10^{-19} C)(-4.0 \times 10^4\ V)$$
$$= -6.4 \times 10^{-15} J$$
$$= -4.0 \times 10^4 eV$$

c) Find the speed of the proton when it arrives at B. (5)

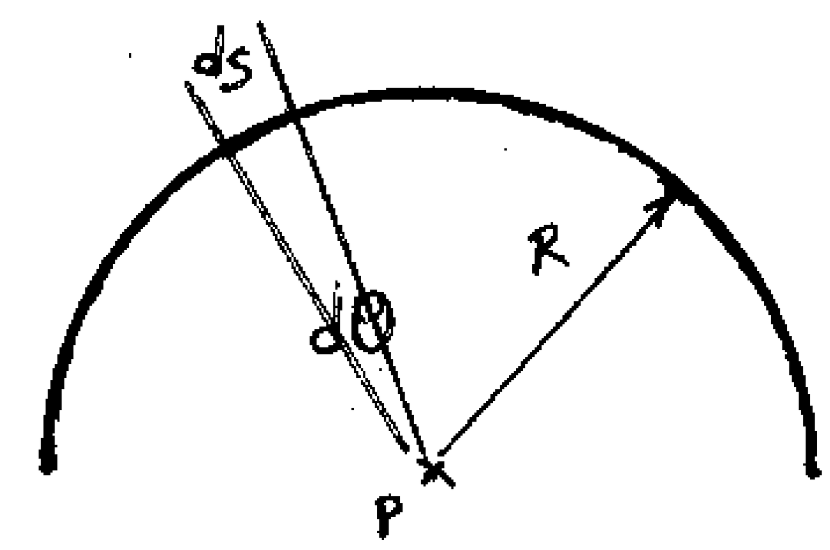
Proton starts from rest so that its final KE is the amt of pot'l energy it loses. So:

$\frac{1}{2} m v^2 = 6.4 \times 10^{-15} J \qquad v^2 = \frac{2(6.4 \times 10^{-15} J)}{(1.67 \times 10^{-27} kg)} = 7.66 \times 10^{12} \frac{m^2}{s^2}$

$$v = 2.8 \times 10^6 \frac{m}{s}$$

Typo on p. 2 for M_{proton}. Sorry! -DPM

9. A rod is bent into a semi-circular arc of radius R. The rod has a uniform charge distribution λ. Find the electric potential at the center of the arc (point P). (8)



Charge contained in section $d\theta$ is

$$dq = \lambda\, ds = \lambda R\, d\theta$$

Evaluate $V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$

For each charge element dq the distance r from P is R.

Integral goes from $\theta = 0$ to $\theta = \pi$. Then:

$$V = \frac{1}{4\pi\epsilon_0} \int_0^{\pi} \frac{\lambda R\, d\theta}{R} = \frac{1}{4\pi\epsilon_0} \lambda \int_0^{\pi} d\theta = \frac{\lambda \pi}{4\pi\epsilon_0} = \boxed{\frac{\lambda}{4\epsilon_0}}$$

Can also be expressed as:

$$V = k\lambda \int_0^{\pi} d\theta = k\lambda\pi$$