Astr 1010
Problem Set #5, Solutions

1. For Mercury we have $P_{\rm yr} = 0.2409$ and $a_{\rm AU} = 0.387$ (data is from Table 4 in *Explorations*). Comparing the two sides of Kepler's Third Law, we find:

$$P_{\rm yr}^2 = 5.803 \times 10^{-2} \qquad a_{\rm AU}^3 = 5.80 \times 10^{-2}$$

which agree to three figures.

For the four given planets, we find:

Planet	$P_{\rm yr}$	$a_{\rm AU}$	$P_{\rm yr}^2$	$a_{\rm AU}^3$
Mercury	0.2409	0.387	5.803×10^{-2}	5.80×10^{-2}
Mars	1.8809	1.524	3.5377	3.540
Saturn	29.4577	9.539	8.67756×10^{2}	8.680×10^{2}
Neptune	164.793	30.06	2.71567×10^{4}	2.716×10^{4}

The results agree (out to the permitted number of significant digits).

2. Here, $a_{\rm AU} = 3.3$, so

$$a_{\rm AU}^3 = (3.3)^3 = 35.9$$

By Kepler's Third Law, we also have

$$P_{\rm yr}^2 = a_{\rm AU}^3 = 35.9$$

and solving for $P_{\rm yr}$ gives

$$P_{\rm yr} = \sqrt{35.9} = 6.0$$

The period of the planet is 6.0 yr.

3. Here, $P_{\rm yr} = 11.18$, so

$$P_{\rm yr}^2 = (11.18)^2 = 125.0$$

By Kepler's Third Law, we also have

$$a_{\rm AU}^3 = P_{\rm yr}^2 = 125.0$$

and solving for $a_{\rm AU}$ gives

$$a_{\rm AU} = (125.0)^{1/3} = 5.0$$

The semi–major axis of the planet is 5.0 AU.

4. We are given the period of Halley's Comet, so Kepler's 3rd Law will give us the semi–major axis of the orbit. We get:

$$P_{\rm yr}^2 = (76.1)^2 = 5.79 \times 10^3 = a_{\rm AU}^3$$

so that

$$a_{\rm AU} = (5.79 \times 10^3)^{1/3} = 18.0$$

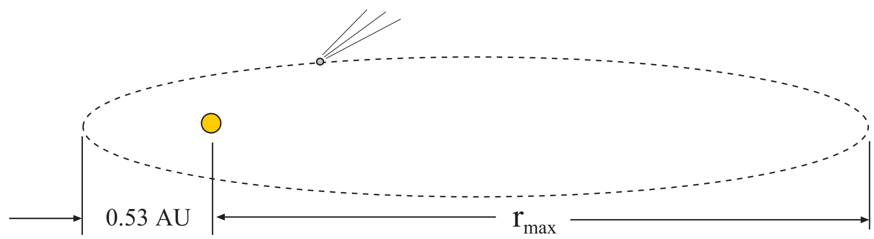
so the semi–major axis is $a = 18.0\,\mathrm{AU}$.

Now, the full long dimension of the ellipse is equal to $2a$ but it is also equal to $r_{\mathrm{min}} + r_{\mathrm{max}}$, and we already know r_{min}. So we can find r_{max}:

$$r_{\mathrm{min}} + r_{\mathrm{max}} = 2a = 2(18.0\,\mathrm{AU}) = 36.0\,\mathrm{AU}$$

$$r_{\mathrm{max}} = 36.0\,\mathrm{AU} - r_{\mathrm{min}} = 36.0\,\mathrm{AU} - 0.53\,\mathrm{AU} = 35.5\,\mathrm{AU}$$

So Halley's Comet gets as far as 35.5 AU from the sun. (This is beyond the orbit of Neptune.)



5. If an object falls from rest in 5.0 s it has fallen a distance

$$d = \tfrac{1}{2}gt^2 = \tfrac{1}{2}9.8(\tfrac{\mathrm{m}}{\mathrm{s}^2})(5.0\,\mathrm{s})^2 = 122.5\,\mathrm{m}$$

and its speed is

$$v = gt = (9.8\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(5.0\,\mathrm{s}) = 49\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

Now suppose we are given that the distance the object has fallen is 300 m. What is the time t? We find:

$$d = \tfrac{1}{2}gt^2 \qquad \Longrightarrow \qquad t^2 = \frac{2d}{g} = \frac{2(300\,\mathrm{m})}{(9.8\,\frac{\mathrm{m}}{\mathrm{s}^2})} = 61.2\,\mathrm{s}^2$$

and this gives:

$$t = \sqrt{61.2\,\mathrm{s}^2} = 7.8\,\mathrm{s}$$

At this time, the speed of the object is

$$v = gt = (9.8\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(7.8\,\mathrm{s}) = 7.7\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

6. On the moon, the acceleration of gravity is $g_{\mathrm{Moon}} = 5.3\,\frac{\mathrm{m}}{\mathrm{s}^2}$, so in 1 s, a rock will fall a distance

$$d = \tfrac{1}{2}gt^2 = \tfrac{1}{2}(1.6\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(1.0\,\mathrm{s})^2 = 0.80\,\mathrm{m}$$

and in 3.0 s it will fall

$$d = \tfrac{1}{2}gt^2 = \tfrac{1}{2}(1.6\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(3.0\,\mathrm{s})^2 = 7.2\,\mathrm{m}$$

To find the time to fall 300 m, use $d = 300\,\mathrm{m}$ and find t:

$$d = \tfrac{1}{2}g_{\mathrm{Moon}}t^2 \quad \Longrightarrow \quad t^2 = \frac{2d}{g_{\mathrm{Moon}}} = \frac{2(300\,\mathrm{m})}{(1.6\,\frac{\mathrm{m}}{\mathrm{s}^2})} = 375\,\mathrm{s}^2$$

which gives

$$t = \sqrt{375\,\mathrm{s}^2} = 19.4\,\mathrm{s}$$

At that time, its speed is

$$v = gt = (1.6\,\tfrac{\mathrm{m}}{\mathrm{s}^2})(19.4\,\mathrm{s}) = 31\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

$P_{\mathrm{yr}}^2 = a_{\mathrm{AU}}^3$ Object falling from rest: $d = \frac{1}{2}gt^2$ $v = gt$ $g = 9.8\frac{\mathrm{m}}{\mathrm{s}^2}$