Phys 2920, Spring 2013
Problem Set #7

1. *(VA 5.28)* If $\mathbf{R}(t) = (3t^2 - t)\,\hat{\mathbf{i}} + (2 - 6t)\,\hat{\mathbf{j}} - 4t\,\hat{\mathbf{k}}$, find (a) $\int \mathbf{R}(t)\,dt$, and (b) $\int_2^4 \mathbf{R}(t)\,dt$

2. *(VA 5.34)* Evaluate $\int_2^3 \mathbf{A} \cdot \frac{d\mathbf{A}}{dt}\,dt$ if $\mathbf{A}(2) = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\,\hat{\mathbf{k}}$ and $\mathbf{A}(3) = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\,\hat{\mathbf{k}}$.

3. *(VA 5.38)* If $\mathbf{F} = (5xy - 6x^2)\,\hat{\mathbf{i}} + (2y - 4x)\,\hat{\mathbf{j}}$, evaluate $\int_C \mathbf{A} \cdot d\mathbf{r}$ along the curve C in the xy plane, $y = x^3$ from the point $(1, 1)$ to $(2, 8)$.

4. *(VA 5.37)* If $\mathbf{A} = (2y+3)\,\hat{\mathbf{i}} + xz\,\hat{\mathbf{j}} + (yz - x)\,\hat{\mathbf{k}}$, evaluate $\int_C \mathbf{A} \cdot d\mathbf{r}$ along the following paths C:

(a) $x = 2t^2$, $y = t$, $z = t^3$ from $t = 0$ to $t = 1$.

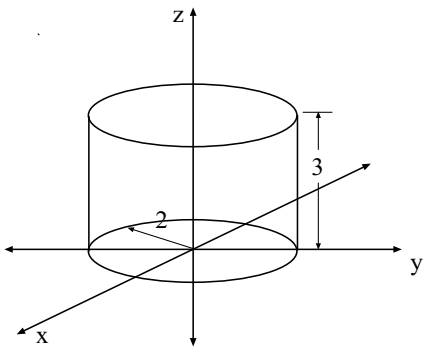
(b) the straight lines from $(0, 0, 0)$ to $(0, 0, 1)$, then to $(0, 1, 1)$, and then to $(2, 1, 1)$.

(c) the straight lines joining $(0, 0, 0)$ and $(2, 1, 1)$.

5. Evaluate the surface integral $\oint_S \mathbf{a} \cdot d\mathbf{S}$, where the vector field is given (in cylindrical coordinates) by

$$\mathbf{a} = \rho^2 \cos^2\phi\,\hat{\mathbf{e}}_\rho + \rho \sin\phi\,\hat{\mathbf{e}}_\phi + \rho z^3\,\hat{\mathbf{e}}_z$$

and the closed surface is a circular cylinder of radius 2 whose axis is the z axis; it has height 3 and extends from $z = 0$ to $z = 3$.



6. Find the moment of inertia (about the z axis) of the "ice cream cone" volume which was used in another example in class and which is shown here. (It is a sector of a solid sphere of radius R, out to an angle $\theta = \pi/6$ out from the z axis)

Assume its mass density ρ_{mass} is uniform. Express the answer in terms of the total mass M of the object.