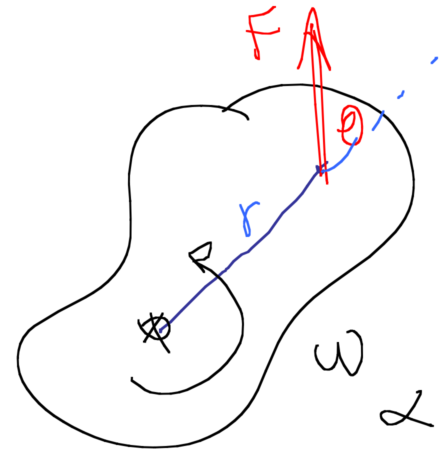
Phys 2110-3 11/3/10

Note Title 11/3/2010

Rotations

$$\tau_{\substack{net \\ (ext)}} = \underbrace{\left(\sum_i m_i r_i^2\right)}_{I}\alpha$$



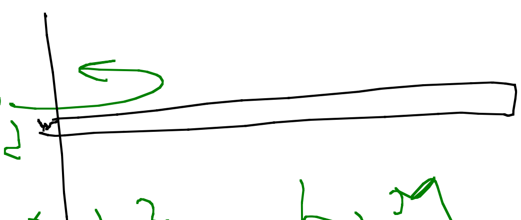
$$F = m a$$

$\tau = r F \sin\theta$

(+ or −) dep on CW or CCW

Calc I. $\sum m_i r_i^2$

Stick rotates about end

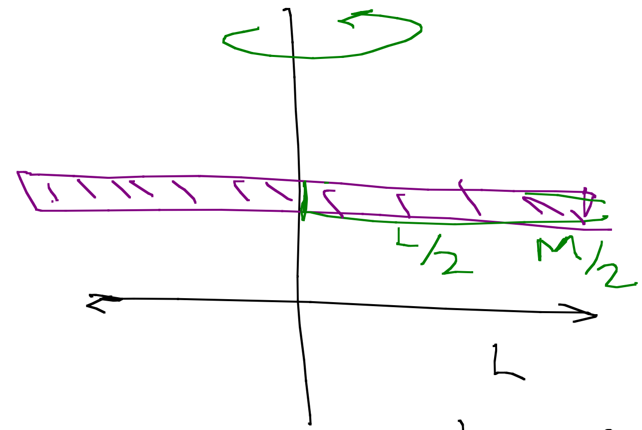


$$I = \frac{1}{3} M L^2$$

L, M

$$I_{\text{whole thing}} = 2\left(\frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2\right)$$

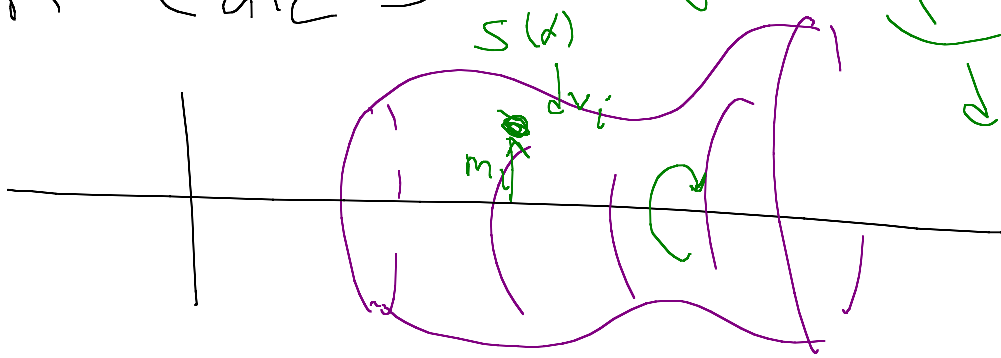
2 little sticks



$$= \frac{1}{12} M L^2$$

In Calc 3

$$\int \underbrace{s^2 \, \rho(x)\, dV}_{dm}$$



$$I = \frac{M}{L}\int_0^L x^2 \, dx$$

$$= \frac{M}{L}\frac{L^3}{3}$$

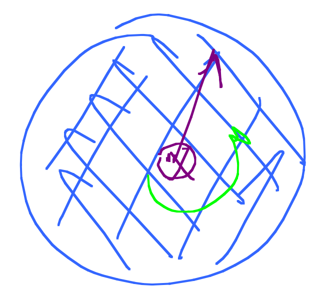
$$= \frac{1}{3} M L^2$$

Results for simple shape

$$I_{hoop} = MR^2$$

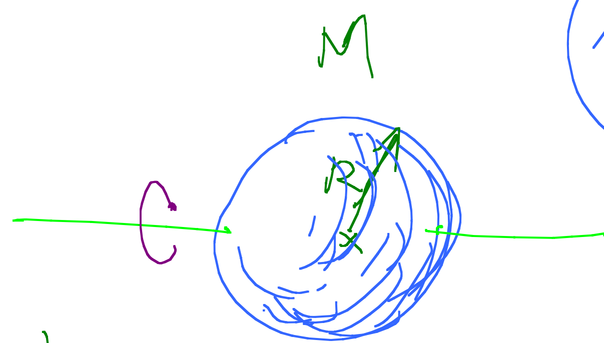
Hoop

$$I_{disk} = \frac{1}{2}MR^2$$



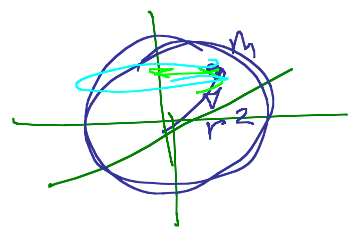
Disk

$$I_{\text{solid sphere}} = \frac{2}{5}MR^2$$

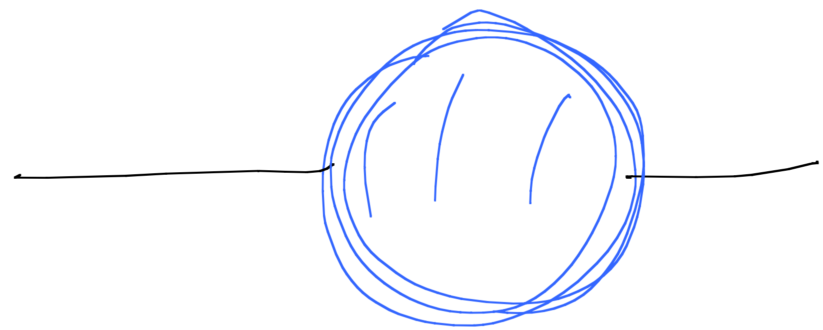


Solid Sphere
Uniform

How to do it wrong



$I_{\text{hollow sphere}} = \frac{2}{3} M R^2$



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M, R Hollow Sphere

Always of form

$I = \sum m r^2 = M l^2$

Rad of gyration

$\tau = I \alpha$

Examples

10.27 The shaft connecting a turbine and generator is a solid cylinder of mass 6.8×10^3 kg and diameter 85 cm. Find its rotational inertia

$$I = \frac{1}{2} M R^2 = \frac{1}{2} (6.8 \times 10^3 \text{ kg})(0.425 \text{ m})^2$$

$$= 6.14 \times 10^2 \text{ kg m}^2$$

10.33 A 108 g Frisbee is 24 cm in diameter & has $\frac{1}{2}$ mass spread uniformly in a disk & other $\frac{1}{2}$ is in the rim.

With a quarter-turn flick of wrist student sets Frisbee rotating at 550 rpm.

a) what is rot. inertia of Frisbee

b) Magnitude of torque?

a) $I = I_{dish} + I_{rim}$

$$= \frac{1}{2}(0.054\ kg)(0.12\ m)^2 + (0.054\ kg)(0.12\ m)^2$$

$$= 1.17 \times 10^{-3}\ kg\ m^2$$

$m = 108\ g$

$0.054\ kg$ is dish,

$0.054\ kg$ is rim

b) $\tau = I\alpha$

$\omega_0 = 0$ $\quad$ $\omega = 550\ rpm \simeq 55\ \frac{rad}{s}$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\alpha = \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)} = \frac{\left(55\,\frac{\text{rad}}{s}\right)^2 - 0^2}{2\left(\pi/2\ \text{rad}\right)}$$

$$= 1056\ \text{rad}/s^2$$

$$\tau = I\alpha = (1.17\times10^{-3}\ \text{kg m}^2)\left(1056\ \frac{\cancel{\text{rad}}}{s^2}\right)$$

$$= 1.24\ \text{N}\cdot\text{m}$$

$$\frac{\text{kg m}^2}{s^2} = \text{N}\cdot\text{m}$$

10.34 At MIT Magnet Labratory energy stored fly wheel

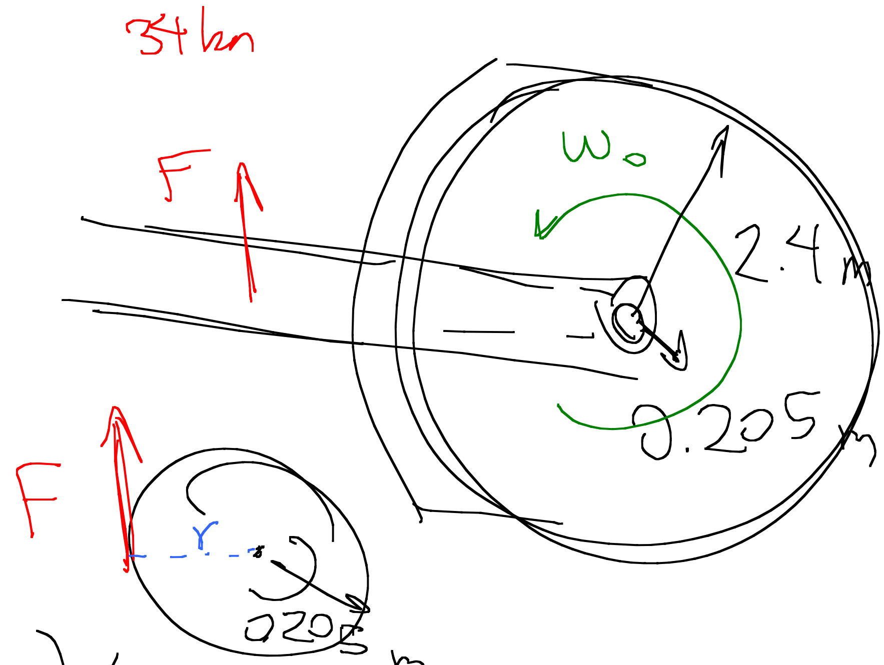
mass 7.7×10^4 kg radius 2.4 m.

Flywheel rides on shaft 41 cm dian

→ 20.5 cm radius

Friction force of 34 kN acts tangentially on the shaft, how long will it take flywheel to stop from rot'n rate of 360 rpm $= 37.68 \frac{rad}{s}$

$$\tau = -Fr$$



$$= -(34 \times 10^3 \, N)(0.205 \, m)$$

$$= -6.97 \times 10^3 \, N \cdot m \quad =$$

$$I_{total} \approx I_{flywheel} = \frac{1}{2}(7.7 \times 10^4 \, kg)(2.4 \, m)^2$$

$\alpha = \tau / I$ gives α (neg)

$\omega = \omega_0 + \alpha t$

$\longrightarrow t$

$t = 20.0$ min