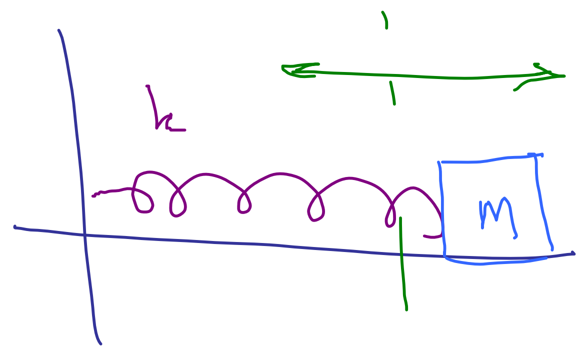
Phys 2110-4 4/9/12

Note Title 4/9/2012

Oscillations



$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$$

$$x = A\cos(\omega t + \phi)$$

ω = angular frequency

$$f = \frac{\omega}{2\pi} \qquad T = \frac{1}{f}$$

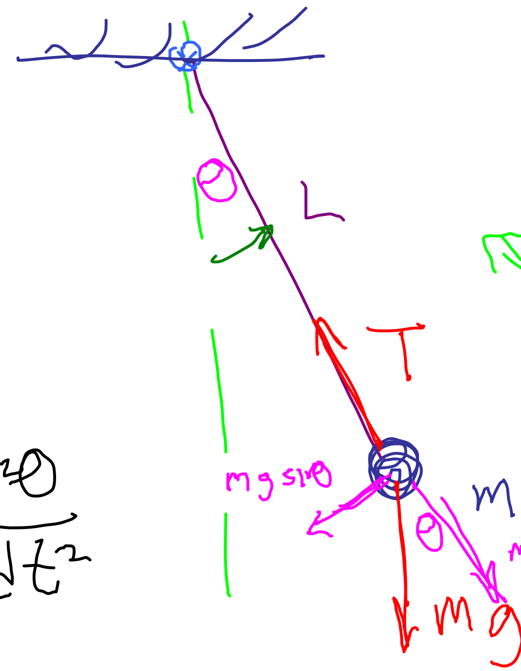
Simple pendulum.

Torques ~~exerted~~ by these forces

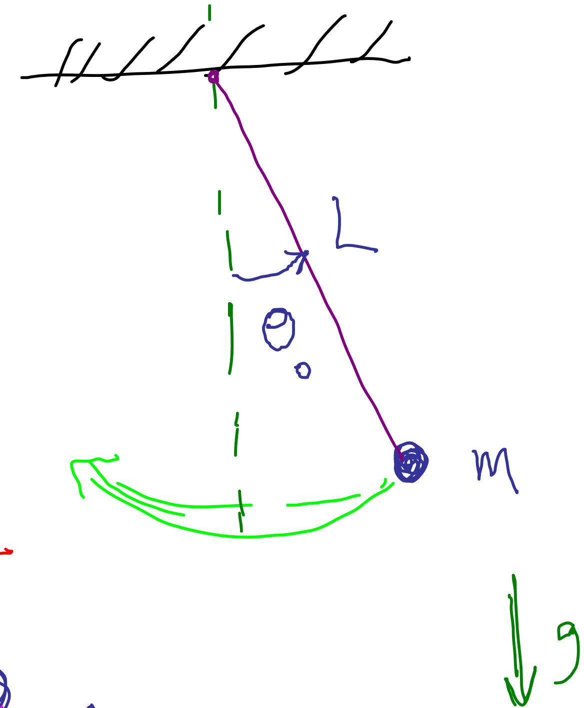
$$\tau = -Lmg\sin\theta$$

$$= I\alpha = (mL^2)\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$



$\theta(t)$



$$\frac{d^2x}{dx^2} = -w^2 X$$

We don't have the nice eq'n $\frac{d^2\theta}{dt^2} = -\omega^2\theta$

Cheat: θ is in radians.

When θ is small $\theta \approx \sin\theta$

θ, deg	θ	$\sin\theta$
0.57°	0.01	0.0099998
5.73°	0.1	0.09983
11.45°	0.2	0.19866

$\sin\theta \to \theta$

You have seen this Taylor series

$$\sin X = X - \frac{X^3}{3!} + \frac{X^5}{5!} - \ldots$$

$$\sin X \approx X$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \qquad \frac{g}{L} \rightarrow \omega^2$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\omega = \sqrt{\frac{g}{L}} \qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$

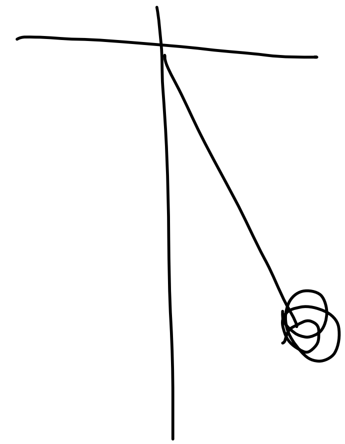
$$\theta(t) = \theta_0 \cos(\omega t + \phi)$$

amplitude

Depends on

L, g Not on m

$$T = 2\pi\sqrt{\frac{L}{g}}$$

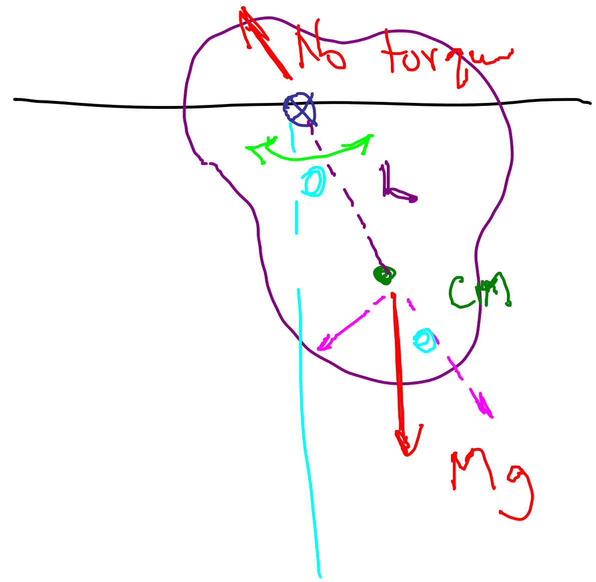


To the extent our approx is good, does not depend on θ.

Plug in numbers, $L = 1\,\text{m}$

$$T = 2\pi\sqrt{\frac{L}{g}} = 2.01\,\text{s}$$

Physical Pendulum



$$\tau = -MgL\sin\theta$$

$$= I\alpha = I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{MgL}{I}\sin\theta = -\underbrace{\frac{MgL}{I}}_{\omega^2}\theta$$

$$\omega = \sqrt{\frac{MgL}{I}} \qquad f = \frac{\omega}{2\pi} \qquad T = \frac{1}{f}$$

Torsional pendulum

Fiber: Gives a torque

$$\tau = -K\theta$$

torsional constant

$N\cdot m = \frac{N\cdot m}{rad.}$

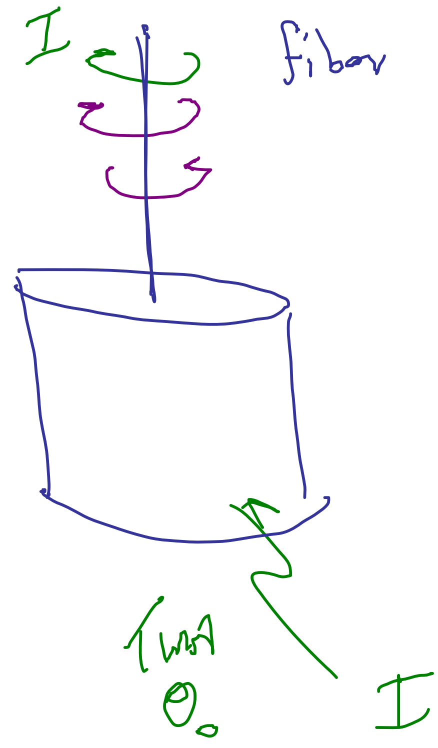
$$= I\alpha = I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{K}{I}\theta$$

ω^2

$$\omega = \sqrt{K/I}$$

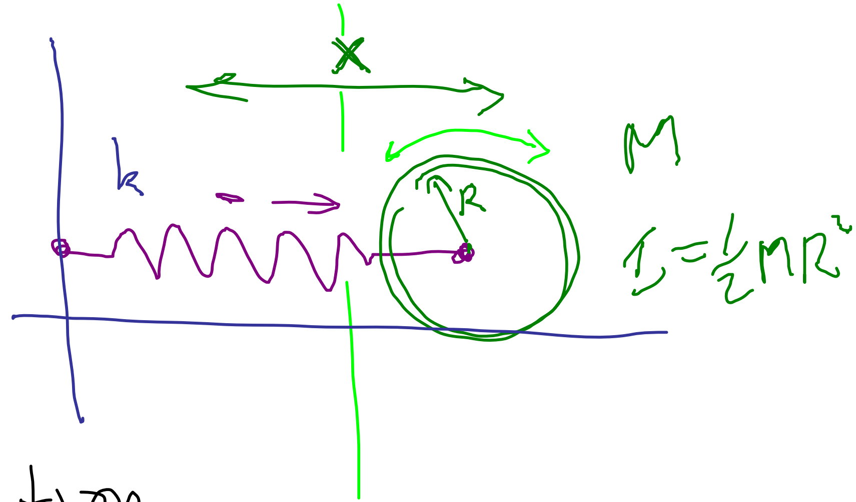
$$f = \omega/2\pi$$



13.63 (Use energy techniques)

Spring attached to rolling cylinder. ...

Find angular freq. of motion.



$v = \frac{dx}{dt}$

$$E = \frac{1}{2}kx^2 + K_{rot} + K_{trans}$$

$$= \frac{1}{2}kx^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 + \frac{1}{2}Mv^2$$

$$= \frac{1}{2}kx^2 + \frac{3}{4}Mv^2 = \text{const}$$

$$E = \frac{1}{2}kx^2 + \frac{3}{4}Mv^2 = \text{const}$$

Take d/dt of both sides, $\frac{dv}{dt}$

$$\frac{1}{2}k(2x)\cancel{\frac{dx}{dt}} + \frac{3}{\cancel{4}_{2}}M(\cancel{2v})a = 0$$

$= v$

$$kx + \frac{3}{2}Ma = 0$$

$$\frac{d^2x}{dt^2} = a = -\frac{2k}{3M}x$$

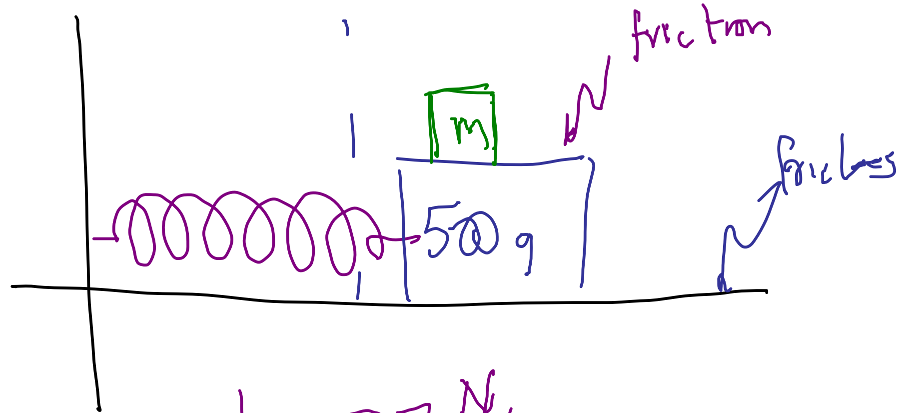
$$\frac{d^2x}{dt^2} = -\frac{2k}{3M}x$$

ω^2

$$\omega = \sqrt{\frac{2k}{3M}}$$

13.74

A is limited in size so that little mass stays on.



$k = 8.7\ N/m$

$T = 1.8\ s$