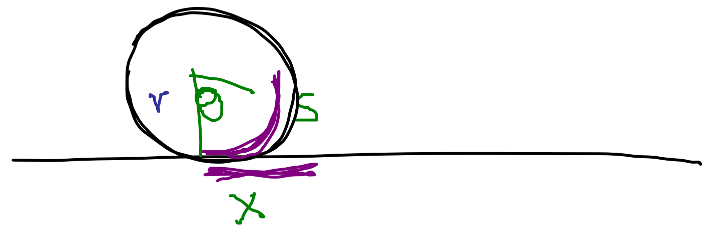
Phys 2110-5 11/7/12

Note Title 11/7/2012

Rolling motion

Rolling w/o slipping

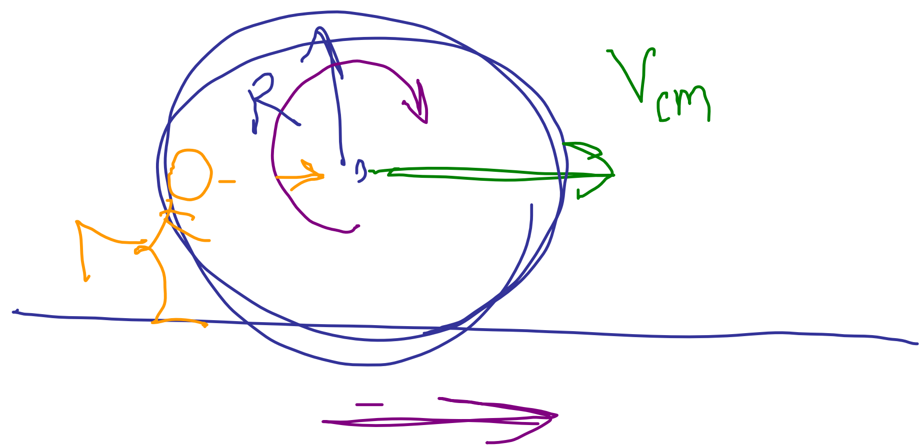


$$s = x = r\theta$$

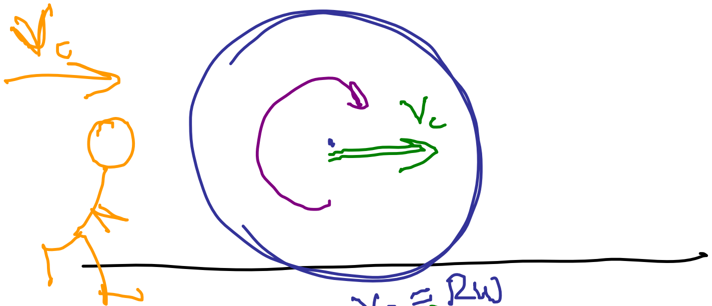
$$\frac{dx}{dt} = r\frac{d\theta}{dt}$$

$$V_{cm} = r\omega$$

$$a_{cm} = r\alpha$$

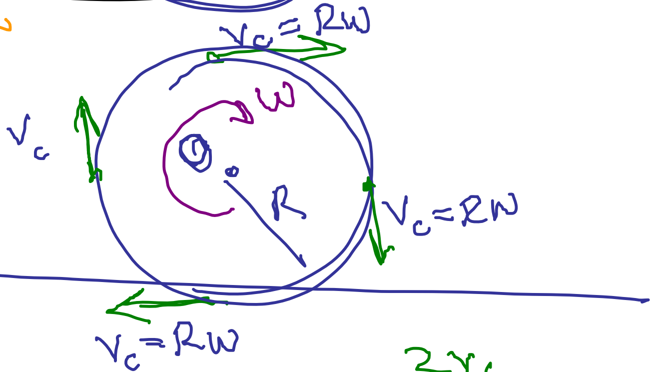


Parts of wheel have different velocities:



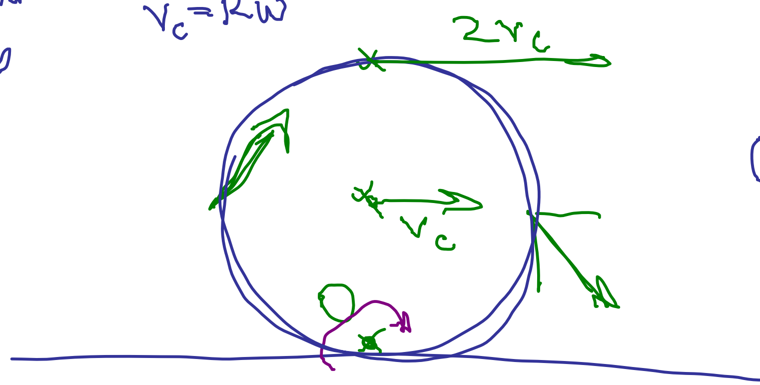
(1)

He sees:



(2) little man sees

Go back to lab



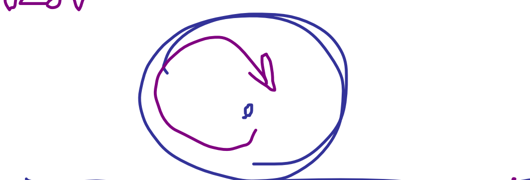
(3)

Rolling wheel has KE

slipping

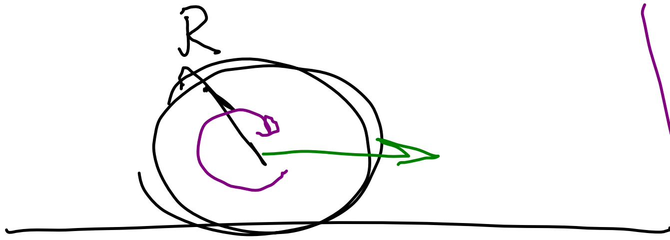
$KE = \frac{1}{2} M v^2$

Rot'n



$KE = \frac{1}{2} I \omega^2$

Rolling obj is doing **both**

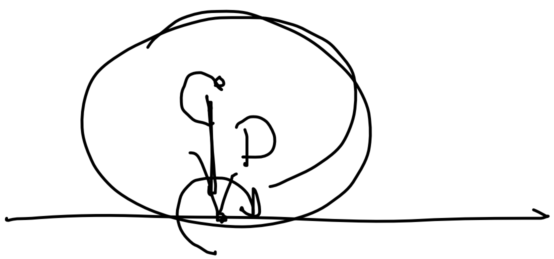


$$\boxed{K_{roll} = \frac{1}{2}Mv_c^2 + \frac{1}{2}I\omega^2}$$

where $\omega = v_c/R$

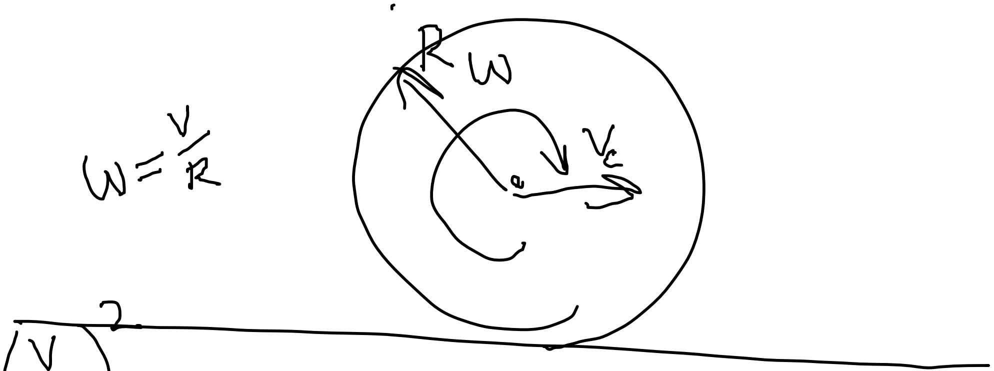
See both can show why this is true w/ parallel axis theorem

$$K_{roll} = K_{trans} + K_{rot}$$



$$I = I_{cm} + MD^2$$

10.39 What fraction of a solid disk's kinetic energy is rot'l if it's rolling w/o slipping?



$\omega = \frac{v}{R}$

$I = \frac{1}{2}MR^2$

$$KE = K_{trans} + K_{rot}$$
$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2$$
$$= \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}Mv^2$$

$K_{rot} = \frac{1}{4}Mv^2$

frac which is rot'l $\dfrac{\frac{1}{4}Mv^2}{\frac{3}{4}Mv^2} = \boxed{\frac{1}{3}}$

10.62 A hollow ball rolls along a horiz surface at 3.7 $\frac{m}{s}$ when it encounters upward incline. If it rolls w/o slipping up the incline, what max height it reach?

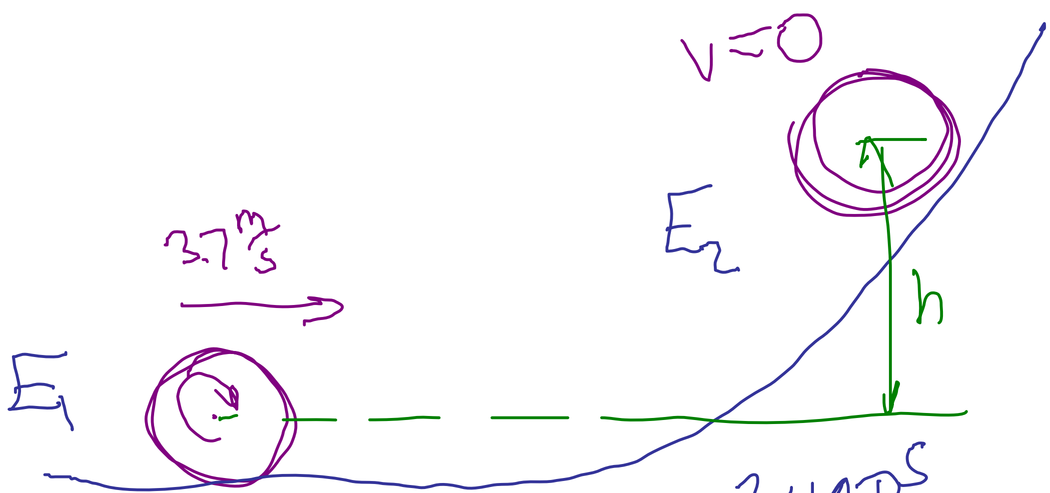
Cons of energy

$$E_1 = E_2$$

$$K_{rolling} = U_{grav}$$

$$K_{trans} + K_{rot} = Mgh$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh$$



$$I = \frac{2}{3}MR^2$$

$$\omega = \frac{v}{R}$$

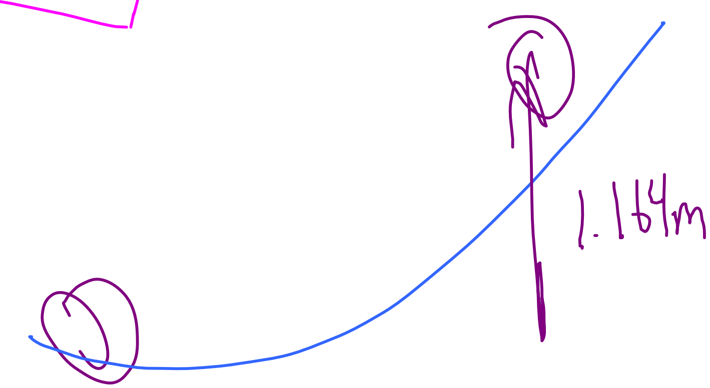
$$\frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{3}MR^2\right)\left(\frac{v}{R}\right)^2 = Mgh$$

$$\frac{1}{2}\cancel{M}v^2 + \frac{1}{3}\cancel{M}v^2 = \cancel{M}gh$$

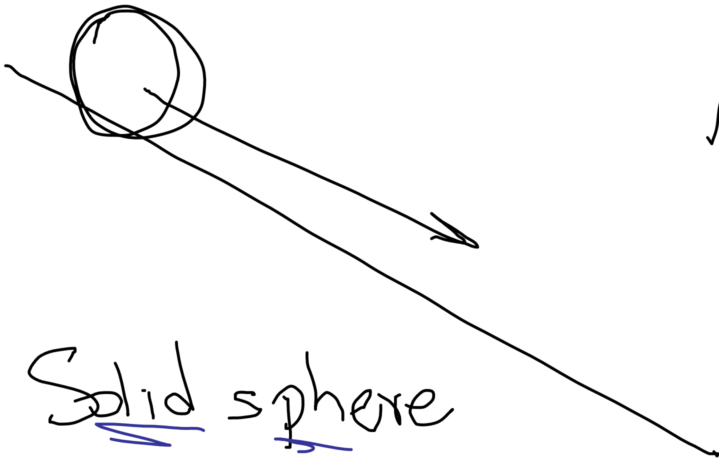
$$\frac{5}{6}v^2 = gh$$

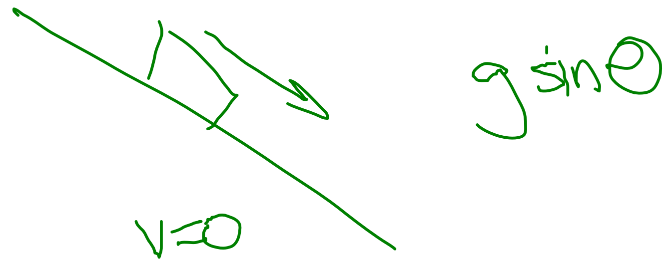
$$h = \frac{5}{6}\frac{v^2}{g} = \frac{5}{6}\frac{(3.7\,\frac{m}{s})^2}{(9.8\,\frac{m}{s^2})}$$

$$\boxed{= 1.164\ m}$$



Accel. of ball rolling down hill.





Solid sphere

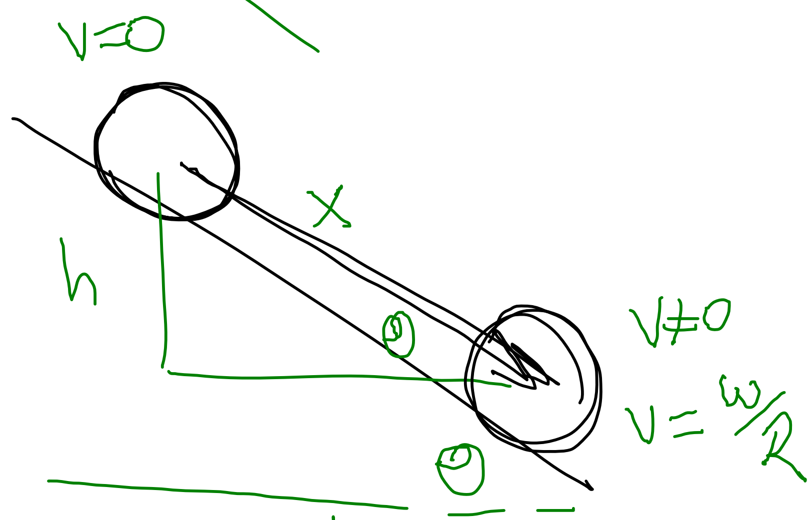
Cons of E

$$\cancel{M}gh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2$$

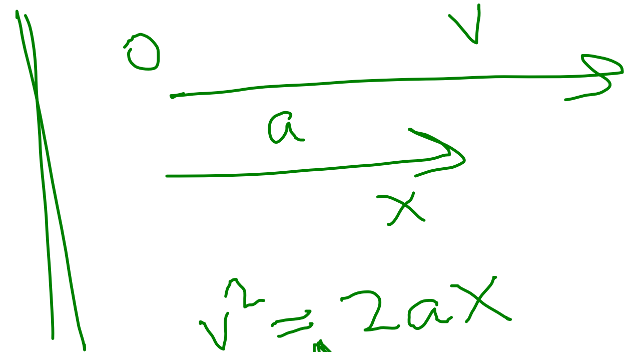
$$= \frac{7}{10}\cancel{M}v^2$$



$\sin\theta = h/x$

$$gh = \frac{7}{10} v^2 \qquad g x \sin\theta = \frac{7}{10} v^2$$

$$v^2 = \frac{10}{7} g \sin\theta \, x$$



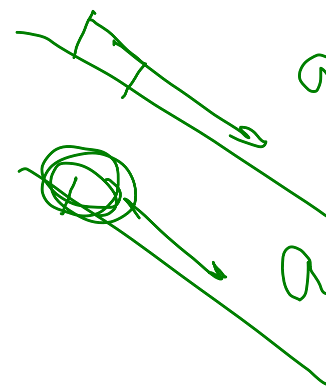
$$v^2 = 2\left(\frac{5}{7} g \sin\theta\right) x$$

$$v^2 = 2ax$$

a

$$\boxed{a = \frac{5}{7} g \sin\theta}$$

solid sphere



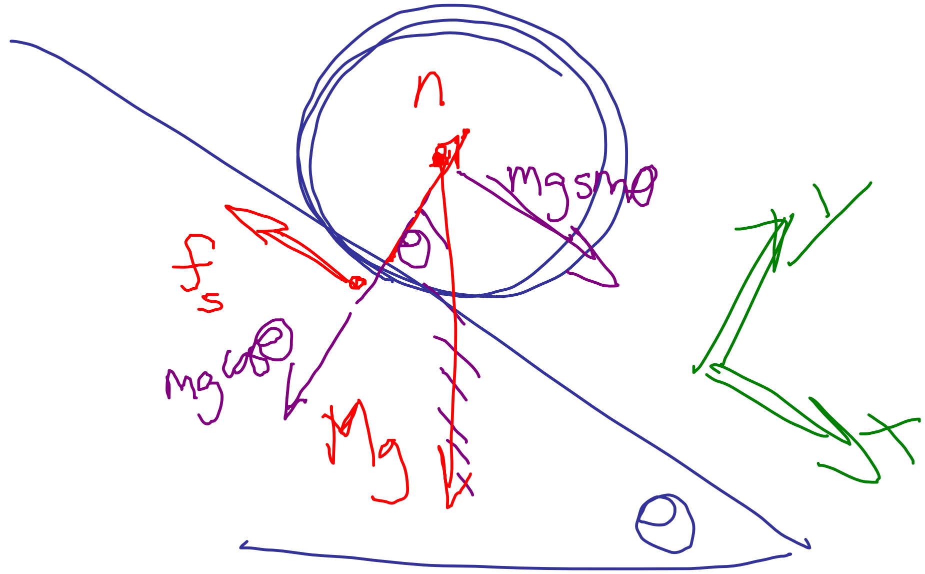
$$a = g \sin\theta$$

$$a = \frac{5}{7} g \sin\theta$$

Another way::

Forces & torques

$$\boxed{F_x = Mg\sin\theta - f_s = Ma}$$



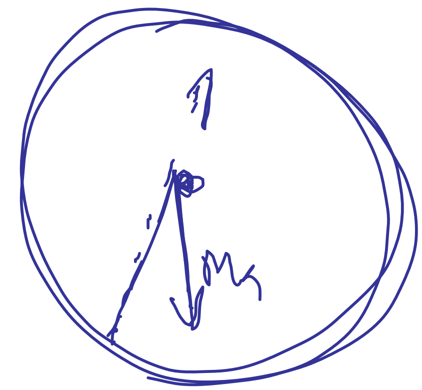
Torques

$\vec{n}$ no torque
$M\vec{g}$ no torque

clockwise

$$\tau = f_s \cancel{R} = I\alpha = \frac{2}{5}M\cancel{R^2}\,\frac{a}{\cancel{R}}$$

$$\boxed{f_s = \frac{2}{5}Ma}$$



Substitute

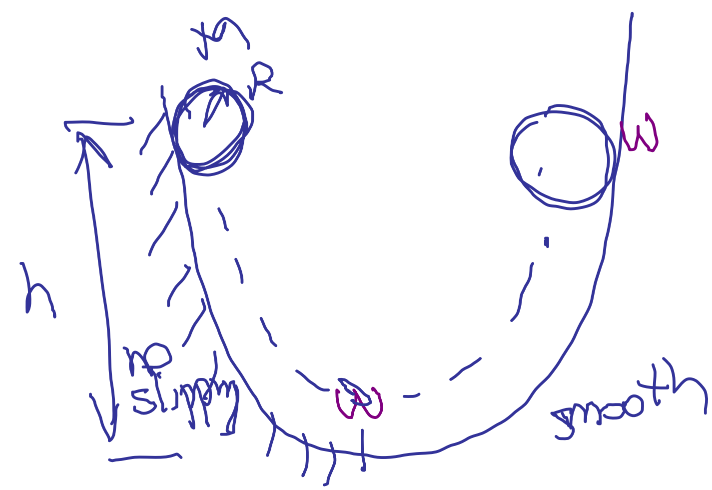
$$\cancel{M} g \sin\theta - \frac{2}{5}\cancel{M} a = \cancel{M} a$$

$$g \sin\theta = \frac{7}{5} a$$

$$\boxed{a = \frac{5}{7} g \sin\theta}$$

solid sphere

10.64



On the way to bottom E consd

At bottom ω.

Same ω on other side.