Phys 3810, Spring 2012
Problem Set #6, Hint-o-licious Hints

1. *Griffiths*, **4.14** The trick here is that the probability to find the electron at a certain radius r (within dr) is *not* $R(r)^2\,dr$. A hint comes from how $R(r)$ is normalized (as used in Prob 6, Griff 4.11); the integral of any probability distribution must give 1.

When you do get the right probability function, find its maximum to get the most probable value.

2. *Griffiths*, **4.16** Note that *only* place in the whole derivation where the electric charges showed up was in the potential

$$V(r) = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

Obviously if the central charge is replaced by $+Ze$, then the e^2 in the potential gets replaced by Ze^2, but then e^2 should be replaced *everywhere* by this. See how the result for the energy levels changes. (Is it proportional to Z? To Z^2?)

3. *Griffiths*, **4.20** You need to use

$$\frac{d}{dt}\langle \mathbf{L}\rangle = \frac{i}{\hbar}\langle [H, \mathbf{L}]\rangle$$

where

$$H = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + V(\mathbf{r})$$

to get the result. Just choose any component of this equation, say z. Show that

$$\frac{1}{2m}[p_x^2, xp_y - yp_x] = \frac{-i\hbar}{m}p_x p_y \qquad \frac{1}{2m}[p_y^2, xp_y - yp_x] = \frac{+i\hbar}{m}p_x p_y$$

so these terms will cancel. Then show

$$[V(\mathbf{r}), yp_y - yp_x] = i\hbar\left(x\frac{\partial V}{\partial y} - y\frac{\partial V}{\partial x}\right) = i\hbar(\mathbf{r}\times\nabla V)_z$$

This will give

$$\frac{d}{dt}\langle L_z\rangle = \langle N_z\rangle$$

4. *Griffiths*, **4.23** Apply the (analytic) raising operator to $Y_2^1(\theta,\phi)$ but also show from (4.120) and (4.121)

$$L_+ Y_2^1 = 2\hbar Y_2^2$$

5. *Griffiths*, **4.26** Multiply a lot of little matrices.

Show that if $j \neq k$ then

$$\sigma_j\sigma_k = i\sum_l \epsilon_{jkl}\sigma_l$$

and if $j = k$ then $\sigma_j \sigma_k = \sigma_j^2 = \mathbf{1}$. But results are contained in

$$\sigma_j \sigma_k = \delta_{jk} + i \sum_l \epsilon_{jkl} \sigma_l$$

6. *Griffiths*, **4.27** Normalizaton should be easy. Remember that the condition is $\chi^\dagger \chi = 1$ and for $\chi^\dagger$ you have to do a complex conjugation. In part (b) you should get

$$\langle S_x \rangle = 0 \qquad \langle S_y \rangle = -\frac{12}{25}\hbar \qquad \langle S_z \rangle = -\frac{7}{50}\hbar$$

On (c) you should get

$$\sigma_{S_z} = \frac{12}{25}\hbar$$