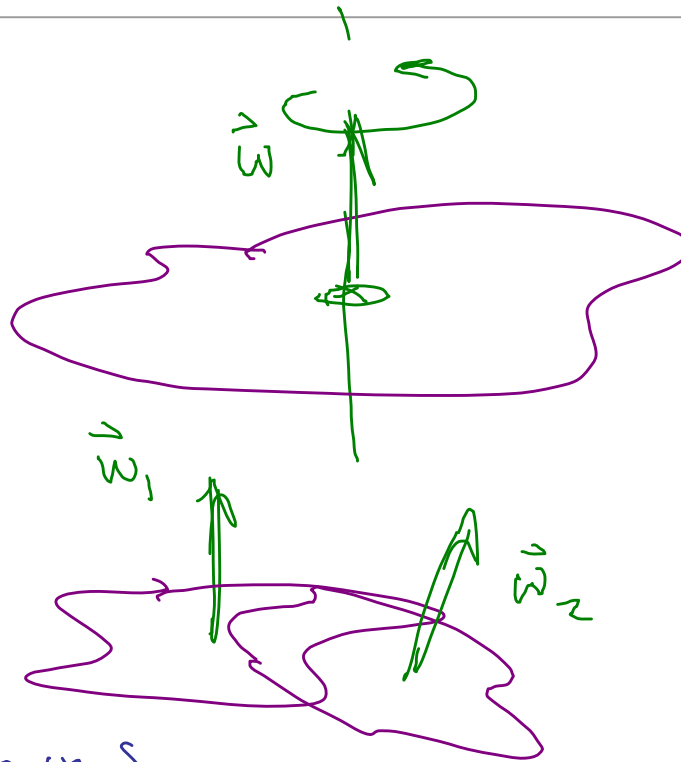
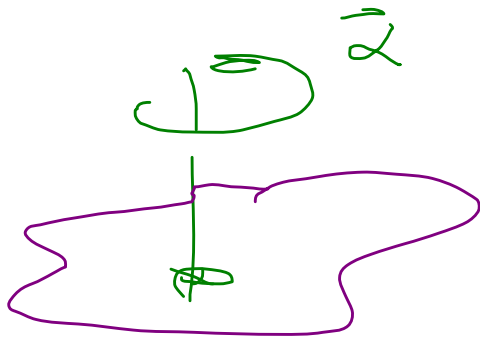


# Rotations



Right-handed sense

$$\Delta \vec{\omega} = \vec{\omega}_2 - \vec{\omega}_1$$

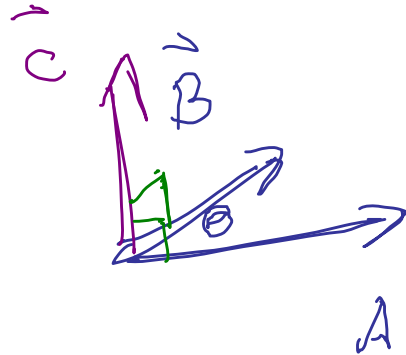
$$\vec{\omega} = \frac{\Delta \vec{\omega}}{\Delta t} \quad \text{vector.}$$

convert def's in terms  
of vectors

Need vector operation called cross-product.

$\vec{A} \cdot \vec{B}$ , dot product  $\Rightarrow$  scalar.

$$\vec{A} \times \vec{B} = \vec{C}$$

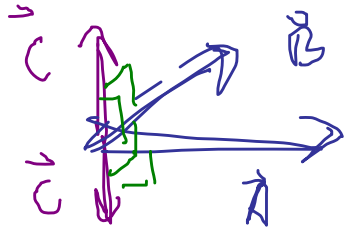


What is vector  
 $\theta$  is angle between  
 $\vec{A}$  &  $\vec{B}$ .

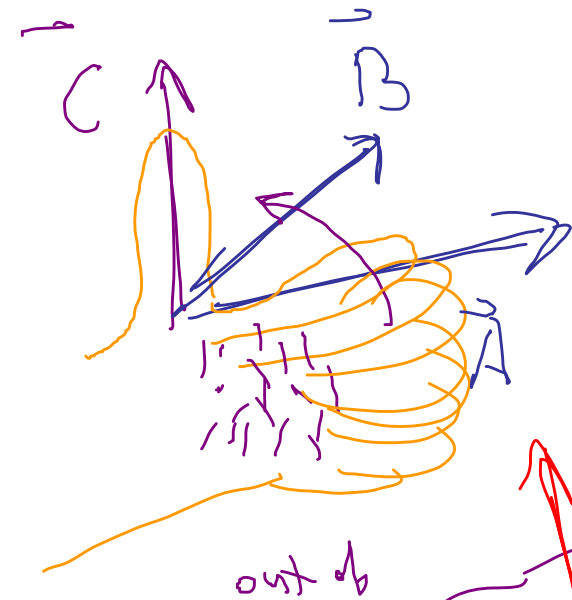
$\vec{C}$  is perp. to  
both  $\vec{A}$  &  $\vec{B}$

Magnitude of  $C$  is

$$|\vec{C}| = AB \sin \theta$$



Dir. of  $\vec{C}$  is det'd by  
right-hand rule.



out of



Thumb goes in dir of  $\vec{C}$

Def of torque really involves  
cross product

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j}$$

mnemonic

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Diagram illustrating the mnemonic for the cross product using a determinant. The first row contains the unit vectors  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$ . The second row contains the components  $A_x$ ,  $A_y$ , and  $A_z$ . The third row contains the components  $B_x$ ,  $B_y$ , and  $B_z$ . A pink circle highlights the  $\hat{i}$  term, and a pink arrow points from it to the  $A_y B_z - A_z B_y$  term in the equation above. A pink circle highlights the  $\hat{j}$  term, and a pink arrow points from it to the  $A_z B_x - A_x B_z$  term. A pink circle highlights the  $\hat{k}$  term, and a pink arrow points from it to the  $A_x B_y - A_y B_x$  term. The signs  $+$ ,  $-$ , and  $-$  are written above the  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$  terms respectively.

$$\underline{\underline{\tau = I \alpha}}$$

$$\alpha = a/r$$

$$\tau = I\alpha$$

$$F = ma$$

$$K = \frac{1}{2} I \omega^2$$

$$K = \frac{1}{2} m v^2$$

$$L = I\omega$$

Angular  
momentum

$$p_x = m v_x$$

$$L = I\omega$$

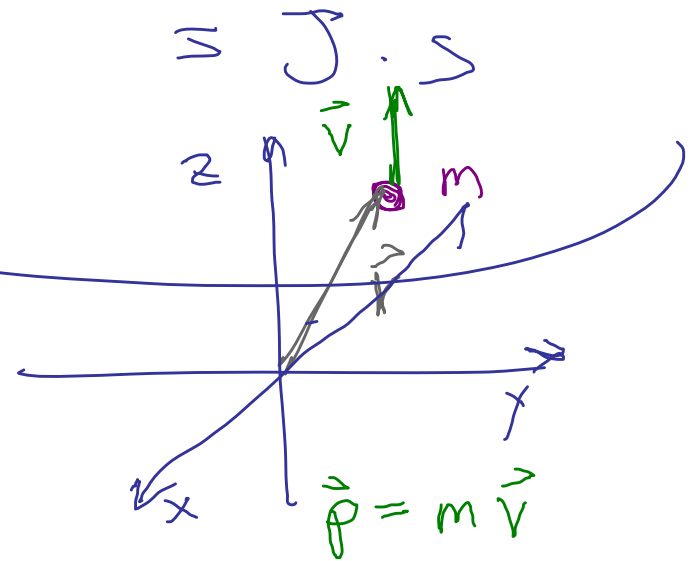
units  $\text{kg m}^2 / \text{s} = \frac{\text{kg m}^2}{\text{s}}$  "action"

angular  
momentum

(Really a vector,  $\vec{L} = I \vec{\omega}$ )

Real def of angular mom.

$$\vec{L} = \vec{r} \times \vec{p}$$



Before disc. objects, etc.)

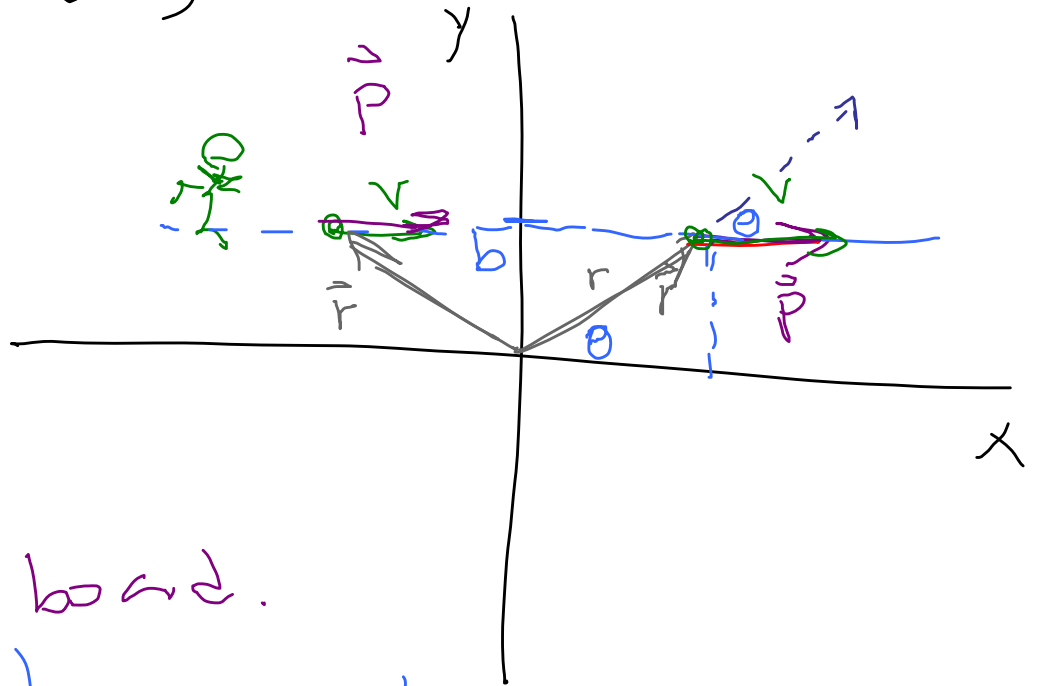
In this pr, find  $\vec{L}$  for little man.

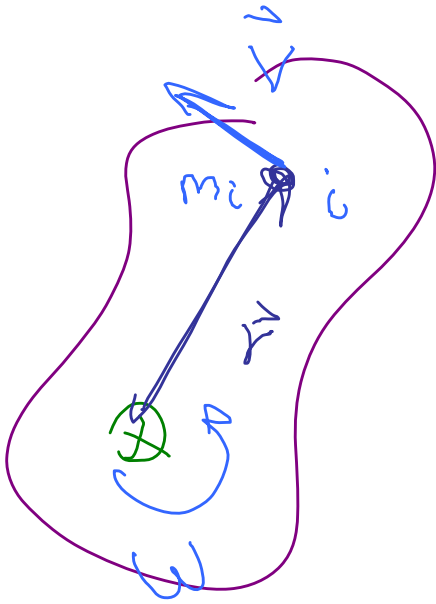
What is  $\vec{r} \times \vec{p}$ ?

Vector points into board.

$$|\vec{L}| = \underbrace{r p}_{mv} |\sin \theta| = m v b$$

then move on to rotating rigid body.





$$\begin{aligned}
 \vec{L} &= (m_i v_i) \vec{r}_i \\
 &= (m_i r_i \omega) \vec{r}_i \\
 &= (m_i r_i^2) \omega
 \end{aligned}$$