Notes for Phys 2020

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These notes are just intended to give an overview of the topics and major equations covered in class. At present they lack illustrations, which can be important for some topics.

1 Electric Forces and Electric Fields

1.1 Electric Charge

All the elementary particles in nature have a property (a quantity) known as **electric charge**. Electric charge can be positive or negative; in the SI system electric charge is measured in **coulombs**. Often we will let the letter Q stand for charge.

The electron carries a charge $-e$ and the proton carries a charge$+e$ where

$$e = 1.6022 \times 10^{-19}\ \mathrm{C}$$

(The nucleus of the atom contains the protons and also neutrons, which have zero electric charge.) Usually an atom contains the same number of electrons as protons and so has a net charge of zero, i.e. it is electrically neutral. To see the effects of electric charge we need to separate the charges; generally, this means moving electrons from one place to another.

It turns out that for reasons not completely understood, the *free* charges in nature must be some integer multiple of e, that is $q = Ne$ where N is some integer. Quarks can have charges of $\pm\frac{e}{3}$ and $\pm\frac{2e}{3}$ but as far as anyone knows you can't isolate a single quark.

1.2 Behavior of Charges

Conductors are materials for which any excess charge is free to move through the material in response to electrical forces. **Insulators** are materials for which an excess charge will stay where it is put.

1.3 Coulomb's Law

Electrical charges which have the same sign will *repel* one another; by Newton's Third Law they exert equal and opposite forces on one another which are directed *away from* the other charge.

Charges with opposite signs *attract* one another. The forces are equal in magnitude and directed *toward* the other charge.

Suppose we have two charges with values q_1 and q_2. (They can be positive or negative charges.) Suppose the distance between the charges is r. Then the *magnitude* of the force between them is given by **Coulomb's law**:

$$F = k\frac{|q_1|\,|q_2|}{r^2} \tag{1}$$

where $k = 8.99 \times 10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}$. To get the *direction* of the electric force on a given charge we need to think about the locations and the *signs* of the other charges.

When there are several other charges around a charge q_1, add up the individual force (vectors) to get the total electrical force on q_1.

While the number k is useful in Coulomb's law we will later find it more useful to use a constant denoted as ϵ_0. The numbers are related by

$$k = \frac{1}{4\pi\epsilon_0} \qquad \text{which gives} \qquad \epsilon_0 = 8.854 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}$$

1.4 The Electric Field

Since by Coulomb's law the force on any one charge is the sum of forces each of which is *proportional* to the given charge, we will get a more useful and general quantity if we divide out the charge from that force; when we do, we get a quantity called the **electric field**:

$$\mathbf{E} = \mathbf{F}/q_0$$

The electric field is a vector and has units of N/C.

For a given configuration of charges we can (in principle) find the electric field at any point. When we do, if we place a charge q_0 at that point, the force on that charge is

$$\mathbf{F} = q_0\mathbf{E}$$

The electric field form a point charge q at a distance of r from that charge has a magnitude $E = k\frac{|q|}{r^2}$. If q is positive, $\mathbf{E}$ points away from the charge. If q is negative, it points toward the charge. This gives a force which is consistent with Coulomb's law.

The force due to an infinite, uniform planar sheet of charge has magnitude $E = \frac{\sigma}{\epsilon_0}$, where σ is the surface charge density on the sheet. This field is directed away from the sheet if σ is positive.)

The electric field in the region between two such uniform infinite sheets of opposite charge densities $+\sigma$ and $-\sigma$ has magnitude $E = \frac{\sigma}{\epsilon_0}$ (This field is directed from the positive plate to the negative plate.)

Often it is useful to make **field line** diagrams to show the direction of the electric field at all points in space.

When excess charge is placed on a conductor or when a conductor is placed in an external magnetic field so that it becomes polarized, the unbalanced charges all reside on the *surface* of the conductor. Furthermore, the electric field is zero in the interior of the conductor. Since the charges all move freely they will arrange themselves so that we have zero field inside the interior.

2 Electric Potential Energy and the Electric Potential

When an electric charge moves through an electric field the electric force does work on the charge. The electric force is a conservative force so that we can talk about a potential energy due to electricity just as we did with gravity first semester. An electric charge will thus tend to "prefer" locations where there is a low electric potential energy. We will use "EPE" to denote this **electric potential energy**. In fact, for many of our problems we ill be able to ignore gravity and we will *only* talk about this kind of potential energy. Electric potential energy is a scalar and is measured in joules.

Potential energy is a useful concept because we can use the principle of conservation of energy to solve many problems: The total energy (kinetic plus potential) stays the same.

2.1 The Electric Potential

Since the force and the work done on a charge in moving it through an E field are proportional to the charge, it is useful to find the electric potential energy *per unit charge*. That is, divide the charge in EPE by the charge to get a more useful quantity. This quantity will be denoted by V and is called the **electric potential**. So ΔV is given by:

$$\Delta V = \frac{\Delta \text{EPE}}{q_0}$$

The electric potential is a scalar and its units are J/C. We call this combination a "**Volt**":

$$1 \text{ V} = 1 \tfrac{\text{J}}{\text{C}}$$

A useful unit of energy is the **electron–Volt** (eV), which is the change in EPE when a charge of magnitude e moves through a potential idfference of 1 Volt. Specifically,

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

For a point charge q, when we are at a distance of r the electrical potential is

$$V = k\frac{q}{r}$$

Don't confuse this with a similar–looking expression for the electric *field* from a point charge. Note that there is no absolute value in this expression; it gives the (scalar) value of V.

To find the electric potential (at a particular location) due to a group of point charges, just add up the potentials due to the individual charges.

An equipotential surface is the set of points on which the electric potential has a certain value (i.e. it is "constant"). Electric field lines are perpendicular to equipotential surfaces.

One can show that for a uniform electric field, the field and potential are related by

$$E_x = -\frac{\Delta V}{\Delta x}$$

that is, the electric field component in a certain direction is what you get by dividing the charge in V by the change in x for that direction (with a minus sign out in front). From this it is clear that the units of the electric field can also be expressed as V/m.

2.2 Capacitors

A capacitor, as we'll use the term, is a pair of conducting planes which are close together as compared with their dimensions. We'll consider charges on these plates but only the case where there are opposite charges on the two plates: q on one of them and $-q$ on the other.

The charge stored on the plates is proportional to the potential difference across the plates:

$$q = CV$$

where C is the **capacitance** of the device, measured in Farads. A Farad is equal to C/V (Coulomb per Volt). A Farad is actually a huge amount of capacitance.

The capacitance of a pair of parallel plates of area A and separation d is

$$C = \epsilon_0 \frac{A}{d}$$

When we fill the area between the plates of a capacitor with an insulator we get a device with a capacitance which is larger by a factor of κ, where κ is called the **dielectric constant** of the insulating material. Thus:

$$C = \kappa C_{\rm air}$$

The dielectric becomes polarized when it is inside a charged capacitor, that it, the positive and negative charges have small displacements in opposite directions. The effect of this polarization is (for a fixed charge on the plates q) to decrease the electric field inside the capacitor and also the voltage V. This increases the capacitance, by the factor κ.

A capacitor stores electrical energy, since it takes work to separate the charges. If q and V are the charge and voltage on a capacitor C, the energy stored is

$$E = \tfrac{1}{2}CV^2 = \frac{q^2}{2C}$$

3 Electric Circuits

3.1 Electric Current

Electric current (for our purposes) is the motion of charge within a conductor. In reality it is the electrons which move in the conductor but because of a historical accident we will use the common convention that there are an equal number of *positive* charges moving in the opposite direction. For all of our work it won't make any difference.

Electric cuurent can be make to flow in a wire by setting up an **electromotive force** (emf) across its ends. One can do this with a common battery for which there is a decrease in chemical energy is charge is delivered from one terminal of the battery to the other. This causes the charges in the wire to do a slow drift so that the charge is transferred. (We observe the effects of electric current so quickly because the conductor is already filled with free charges.)

For our purposes we will use "emf" "voltage" and "potential difference" interchangeably. All of them are measured in volts.

Electric current is measured by counting the amount of charge that passes by any cross-sectional area of the wire in a given amount of time. If a charge q passes by some placein the wire in a time interval t then the current I is given by

$$I = \frac{q}{t}$$

Current is measured C/s (coulombs per second), which is abbreviated as the **ampere**:

$$1\ \mathrm{A} = 1\,\tfrac{\mathrm{C}}{\mathrm{s}}$$

3.2 Ohm's Law

We find that for most conductors the potential difference is proportional to the current passing through the conductor. The constant of proportionality is the **resistance** of the conductor:

$$V = IR$$

This relation is known as **Ohm's law**. Resistance has units of V/A, which is called an **ohm** and is abbreviated as Ω.

For a conductor with cross-sectional A and length L, the resistance is

$$R = \rho\frac{L}{A}$$

The constant ρ is the **resistivity** of the material of which the resistor is made. Resistivity has units of $\Omega \cdot \mathrm{m}$.

3.3 Alternating Current

In fact the potential difference that is used to run household appliances (i.e. the plug in the wall!) is not a constant value and if it is connected to a resistance the current is not constant either. The wall voltage oscillates sinusoidally with an amplitude of about 165 V and a frequency of 60 cycles per second. (In the USA, that is).

I will skip all the material on alternating current in lecture, but since we will be solving a few problems which use household voltages it is worth mentioning how it is that we can still do this. We first define the **root–mean–square** values of the voltage and current, which are the peak values divided by the square root of two:

$$V_{\mathrm{rms}} = V_0/\sqrt{2} \qquad I_{\mathrm{rms}} = I_0/\sqrt{2}$$

When we use the rms values it turns out that we can use the DC circuit equations $V = IR$ and $P = IV$ with correct results.

3.4 Resistors in Series and in Parallel

When resistors are combined in **series** we mean that they are joined end–to–end without any junctions occuring between the resistors. In such a configuration the *current* in all the resistors is the same though in general the potential difference acroos each will be different. If the individual values of the resistors are R_1, R_2, $R_3 \ldots$ then the set can be replaced by an **equivalent resistance** which si just equal to the sum:

$$R_{\rm eq} = R_1 + R_2 + R_3 + \cdots$$

By this we mean that with this replacement you get get the same current if the same voltage is applied.

Whe resistors are connected in **parallel** we mean that their ends are joined together. In such a configuration the *potential difference* across each of the resistors is the same though in general the currents in each will be different. If the individual values of the resistors are R_1, R_2, R_3, $\ldots$ then the equivalent resistance of the set is given by the relation

$$\frac{1}{R_{\rm eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

With this replacement, we get the same *total* current going into the resistor combination if the same voltage is applied across it.

3.5 Kirchhoff's Rules

To solve the really complicated (DC) circuits we need more than the rules for reducing sets of resistors. By using two rules and lots o' math one can solve for the currents in all the branches of a complicated circuit:

• At any junction, the sum of currents entering the junction equals the sum of currents coming out of the junction.

• Around any loop the sum of voltage rises equals the sum of the voltage drops, i.e. the sum of all the potential differences is zero.

4 Magnetic Forces and Magnetic Fields

4.1 The Magnetic Field

Permanent magnets have been known for hundreds of years, ever since the ancient Greeks used them as a way to make their refrigerator doors more cheerful. When a small magnet is free to rotate we find that one end of it wants to point toward the geopgraphic North direction (and the other to the South). Appropriately these are called the North and South ends of the magnet.

One finds that the North ends of magnets (or their South ends) will repel one another. It seems that *every magnet* has a North and South pole; it is impossible to isolate "Northness" from "Southness" as one can do with positive and negative electric charges.

4.2 Magnetic Forces

A magnetic field exerts a force on a particle of charge q but only if the charge is in motion and then *only* if the velocity of the particle has a component *perpendicular* to the direction of B.

If the velocity of the particle is (completely) perpendicular to $\mathbf{B}$ then the force on the particle has magnitude $F = qvB$ and the direction of the force is perpendicular to both $\mathbf{V}$ and $\mathbf{B}$. Even having said this there is an ambiguity in the direction of the force, but it is decided by the right–hand–rule which, as we'll use it in this class, goes as follows (using your right hand):

- Thumb goes in direction of velocity
- Fingers go in direction of magnetic field.
- Palm faces in the direction of the force on the particle.

This gives the direction of the force on a moving *positive* charge. If the charge is negative, take the opposite direction from what this rule gives.

If the velocity of the particle is not perpendicular to the B field and the angle between the velocity vector and the B field is θ then the magnitude of the force is given by

$$F = |qvB\sin\theta|$$

4.3 Motion of Particles in a Magnetic Field

The fact that the force is *perpendicular* to the direction of motion of the charge has some interesting consequences.

When we have and E field and a B field at right angles to one another a charged particle passing through this region with a velocity perpendicular to both fields will experience no net force only if the speed of particle has the value $v = E/B$.

When a charged particle enters a region of a uniform magnetic field with a velocity perpendicular to the field direction, the particle will move in a circular path. The relation between the parameters of the motion is

$$r = \frac{mv}{qB}$$

where r is the radius of the orbit, m and q are the mass and charge of the particle, v is the particle's speed (which does not change) and b is the magnitude of the B field.

Such a particle motion occurs in the accelerators of physics and in the mass spectromemter (a device used in chemical analysis). For these applications we are generally given the potential V through which the charge q is accelerated (instead of its speed v). Using $qV = \frac{1}{2}mv^2$, we find that the relation between the parameters is

$$Br = \sqrt{\frac{2mV}{q}}$$

4.4 Force On a Current–Carrying Wire

If a length L of a straight wire carries a current I in a direction which is not parallel to the B field it will experience a force which is perpendicular to both the direction of the current and the B field. The magnitude of this force is

$$F = ILB\sin\theta$$

where θ is the angle between the current direction and that of the B field. Again there is an ambiguity in the direction of the force which is resolved by the right–hand–rule as applied to currents,

- Thumb goes in direction of current
- Fingers go in direction of magnetic field.
- Palm faces in the direction of the force on the wire.

4.5 Magnetic Fields from Currents

The magnetic field around a long straight wire is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

where I is the current in the wire, r is the distance from the wire and μ_0 is a new constant which appears in expressions where we find the magnetic field due to a current, namely

$$\mu_0 = 4\pi \times 10^{-7}\,\frac{\mathrm{T{\cdot}m}}{\mathrm{A}}$$

The direction of the B field is found from the second right–hand–rule, in which the thumb of the right hand points in the direction of the current and the fingers wrap around the wire to give the direction of the B field.

We can now find the force between two very long straight wires which are parallel to one another. If one wire carries a current I_1 and the other a current I_2 and they are separated by a distance r, the force felt by a length L of one of the wires is

$$F = L\frac{\mu_0 I_1 I_2}{2\pi r}$$

We find that if the currents go in the same direction there is an *attractive* force and if the currents go in opposite directions the force is repulsive (contrary to what one might think!).

Two other current configurations are of interest. At the center of a circular loop of radius R which carries a current I, the magnitude of the magnetic field is

$$B = \frac{\mu_0 I}{2R}$$

A solenoid is a helix formed from a current—carrying wire. For a "long" solenoid (the only kind we'll consider), the magnetic field inside the helix is very nearly uniform, and the B field outside is weak. If the number of turns per unit length is n and the current in the wire is I, the magnitude of the B field inside the solenoid is

$$B = \mu_0 n I$$

5 Electromagnetic Induction

5.1 Changing Magnetic Field Makes an Emf

We can induce an electric current to flow through the wire in a coil if we change the magnetic field passing "through" the coil. This will also happen if the magnetic field through the coil stays the same but the shape (area) of the coil changes.

The current flows only while a *change* in the B field occurs. When the B field through the coil is steady there is no current. This is shown with a coil connect to an ammeter around which we make rapid changes in the magnetic field using a cow magnet. Yes, they really put those tings inside cows.

5.2 Example: A Simple Circuit

We do a "thought experiment" where a simple circuit with a single resistance (like a alight bulb) has two long leads which maintain contact with a sliding segment segment of length L. We will use this simple circuit to develop ideas about EM induction.

When the segment slides on the rails the bulb lights up! This is because an emf $\mathcal{E}$ is developed in the loop, so that if the bulb has resistance R there is a current

$$I = \mathcal{E}/R$$

in the loop.

To clarify matters, we consider the motion of the conducting bar through the magnetic field by itself. Noting that the free charges now have a velocity perpendicular to the magnetic field, there must be magnetic force on them along the length of the bar.

5.3 Magnetic Flux

Magnetic flux through a surface bounded by a conducting loop (for a uniform magnetic field) is defined as the *normal component* of the magnetic field times the area of the loop:

$$\Phi = BA\cos\phi$$

where ϕ is the angle between the B field and the normal to the loop.

5.4 Faraday's Law

When the magnetic flux through a loop is changing — and only while it is changing — there is an induced emf in the conducting loop. For a single loop the magnitude of the induced emf is given my

$$|\mathcal{E}| = \left|\frac{\Delta\Phi}{\Delta t}\right|$$

But the complete law needs a couple extras: We can have a coil of N loop (all with same shape and area) which enhances the emf. Also, as we will see, the induced emf has a sense which is *opposite* that of the changing magnetic flux. With this in mind, Faraday's law is usually written as

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$$

While the above formula gives the *magnitude* of the (average) emf induced over the time of the changing flux, the direction of the induced current is not so clear. It is much clear when we use **Lenz's law**, which states:

The polarity of the induced current is such that the induced magnetic flux opposes the original change in flux.

5.5 Electric Generators

And thus we arrive at the means by which electric power is made. If we rotate a coil within a magnetic field, the magnetic flux is continually changing because of the changing *direction* of the B field.

5.6 Mutual Inductance and Self–Inductance

The changing flux within one circuit can arise from the fact that a current in a nearly circuit is changing, since that will give rise to a changing magnetic field which is "felt" by the former. We will say that a changing current is a **primary** circuit (1) gives rise to an induced emf in the **secondary** circuit (2). More specifically, the induced emf in the secondary is *proportional to* the rate of change of the current in the primary circuit.

Since the flux in circuit 2 is proportional the to current in 1, we can write

$$N_2\Phi_2 = MI_2$$

where N_2 is the number of turns in the secondary circuit (coil). The constant M depends on the geometry of the two circuits; it is called the **mutual inductance** of the two circuits. Then we can combine the two equations

$$\mathcal{E}_2 = -N_2\frac{\Delta\Phi_2}{\Delta t} \qquad \text{and} \qquad N_2\Delta\Phi_2 = M\Delta I_1$$

to get

$$\mathcal{E}_2 = -M\frac{\Delta I_1}{\Delta t}$$

In the same way, a current in a coil gives a magnetic flux *through the coil itself*, and so a changing current through a coil generates a (backwards) emf. Defining the **self–inductance** L by:

$$N\Phi = LI\ ,$$

where N is the number of turns in the coil we can then write

$$\mathcal{E} = -L\frac{\Delta I}{\Delta t}$$

We can give a formula for the self–inductance of a solenoid. It is

$$L = \mu_0 n^2 A\ell$$

where as before n is the number of turns per unit length, A is the cross-sectional area of the coils and ℓ is the length of the solenoid. A solenoid used in this fashion is often just referred to as an "inductor", with its "inductance" measured in Henrys.

Whe current is flowing through an inductor, there is energy stored in the magnetic field that is set up inside the coil. The reason that energy is stored is that when the current is building up from zero there is a backwards voltage resulting from the changes in the current. So the current must go through a rise in potential and so energy is expended. When a current I is flowing through an inductor with inductance L, the energy stored is

$$\tfrac{1}{2}LI^2$$

6 Electromagnetic Waves

6.1 Waves Formed From the E and B Fields.

When Maxwell found the equations that fully unified electricity and magnetism he was able to predict that there would be **wave solutions** to the equations; this means that oscillating electric and magnetic fields can propagate over long distance in space just as oscillations propagate along a string or a sound wave propagates through air.

In an electromagnetic wave travelling through a vacuum the E and B fields are both perpendicular to the direction of propagation. Furthermore these fields are perpendicular to one another and instantaneously have magnitudes related by

$$E = cB$$

Since the fields are perpendicular to the direction of propagation, the EM wave is a transverse wave.

Maxwell was able to predict the speed of these waves from the equations, and he found that the speed of EM waves (in a vacuum) is

$$v_{\text{EM rad}} = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 3.00 \times 10^8\ \tfrac{\text{m}}{\text{s}}$$

This was well-known to be the speed of light, so that Maxwell could speculate that light was a particular kind of EM radiation and that there could be other types, having longer and shorter wavelengths.

Nowadays the speed of light in vacuum (which is not much different from the sped of light in air, as we'll see) is denoted by c and to a better approximation is

$$c = 2.99792 \times 10^8\ \tfrac{\text{m}}{\text{s}}$$

In fact, the speed of light is *defined* to be a certain number of meters per second and since the "second" has a precise definition in terms of atomic properties, this provides the working definition of the *meter*

EM waves differ from the waves studied in Phys 2010 in that they do not travel through any kind of elastic medium which distorts to form the wave, though at one time it was thought that this was the case. The fact that EM waves don't need such a medium led to the development of the Theory of Relativity.

Wavelength, frequency and wave speed for electromagnetic waves have the same relation we had for waves through an elastic medium:

$$\lambda f = c$$

EM waves carry energy. The **intensity** of an EM wave is the amount of energy which passes through a given cross-sectional area in a given amount of time. The intensity is the energy per time, per area, and is measured in units of $\frac{\mathrm{J}}{\mathrm{s}}/\mathrm{m}^2 = \mathrm{W}/\mathrm{m}^2$ For an EM wave for which the amplitude of the electric field is E_0 and the amplitude of the magnietic field is B_0 (with $E_0 = cB_0$) is

$$\overline{S} = \tfrac{1}{2} c\epsilon_0 E_0^2 = \frac{c}{2\mu_0} B_0^2 \ .$$

6.2 Polarization

Given the direction of propagation of an EM wave, the electric field can point in any of the directions in the perpendicular plane. (The magnetic field is there too, at right angles to E but we will just discuss the direction of E, for simplicity.) We'll refer the the direction of E as the **polarization** of the EM wave.

Now in fact the light that we get from natural sources like the sun or lamps does *not* have a definite polarization; it is a combination of the light of all polarizations (it is **unpolarized**). But it was found that some crystals can force the light they transmit to have a definite polarization. The sheets made of these substances are called **polaroids**.

When unpolarized light is incident on a polaroid, the transmitted light has *half* of its original intensity:

$$\overline{S} = \tfrac{1}{2}\overline{S}_0$$

and is now polarized in the direction of the axis of the polaroid. But if light which is *already polarized* is incident on a polaroid whose axis has a direction which differ by and angle θ from the polarization direction of the incident light, the transmitted light is polarized in the new direction and has an intensity which is $\cos^2\theta$ times that of the incident light:

$$\overline{S} = \overline{S}_0 \cos^2\theta$$

This is usually called the **Law of Malus**.

7 Reflection and Mirrors

In this chapter and the next we will analyze the formation of images by optical devices: mirrors and lenses. To get our results we will find what happens to **light rays** which are incident on these objects.

When light rays encounter most surfaces they are reflected back, but at essentially random angles. This is known as **diffuse reflection**. When light encounters a very smooth metallic surface then **specular reflection** takes place. When we measure angles from the normal to the surface, then for specular reflection a ray incident at θ_i goes out at θ_r, with $\theta_i = \theta_r$.

7.1 Plane Mirrors

A flat (planar) mirror takes the light rays coming from an object and reflects them back so that when traced backwards, they *seem* to be coming from a place behind the mirror. They come back at the view just as if there was something of the same size and at an equal distance behind the mirror. The fictional source of the light is called the **image**.

7.2 Spherical Mirrors

If we take a spherical shell of radius R with a very smooth surface and coat the outside or inside with a relective material then take a piece of the sphere, we have a **spherical mirror**.

Such a piece of the sphere has a center of curvature, C, which is at a distance R from all points of the mirror. We will consider a particular line along a radius to use in analyzing the location of objects and images. This line is called the **principal axis**.

The point on the axis midway between C and the mirror if the **focal point** of the mirror for this axis. We denote it by F and we say that its distance from the mirror is f, with $f = R/2$.

For an object near the principal axis we can locate the image by tracing several rays for which the geometry is simple. Starting with some point of the object not on the axis, draw the rays:

• A ray which comes in parallel to the axis goes out along a line which passes through the focal point.
• A ray which comes in on a line through the focal point goes out parallel to the axis.
• A ray which comes in on a line through the center of curvature goes out along the *same line*.

Using these three rays (actually 2 are sufficient) we can locate the images and say whether it is upright or inverted.

We find that for a concave mirror when the object is between the focal point and the mirror, the object is in back of the mirror and upright. If the object is more distant than the focal point the image is *in front* of the mirror and is inverted.

For a convex mirror we find that the image is always behind the mirror and is upright.

If the image is in front of the mirror it is a **real image**. If it is behind the mirror it is a **virtual image**.

The image will have its own size as well. If the object has a height (measured perpendicular to the principal axis) h_o and the image has height h_i then the **magnification** by the mirror is $m = \frac{h_i}{h_o}$. Note that when the image is inverted, h_o and h_i have opposite signs.

7.3 The Mirror Equations

To use the equation which gives the location we must be very careful with the *signs* of the distances involved. Generally a length measured *in front* of the mirror is positive and one measured *behind* the mirror is negative. In particular:

• The object distance d_o is positive when the object is in front of the mirror. We won't discuss the case when the object is behind the mirror.

• The image distance d_i is positive when the image is in front of the mirror (real) and negative when it is behind the mirror (virtual).

• Focal length f is positive when the focal point is in front of the mirror (as with a concave mirror) and negative when it is behind the mirror (as with a convex mirror).

With these in mind, the mirror equations are:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad\qquad m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

8 Refraction and Lenses

8.1 The Index of Refraction

Light will propagate through transparent materials like water or glass much as it does through a vacuum, but the speed of light is significantly less. If the speed of light through the material is v then we define the **index of refraction** for the material as

$$n = \frac{c}{v} \ .$$

Since $v < c$, we must have $n > 1$. For example, for water, $n = 1.33$.

8.2 Snell's Law

When a beam of light strikes an interface between two transparent media, it is bent, or **refracted**. The angles of incidence and refraction, both measure rom the normal to the plane surface between the two media are not equal but are realted by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \ .$$

Here, θ_1 and θ_2 are the angles in media 1 and 2 and n_1 and n_2 are the respective indices of refraction of the two media.

This relation is known as **Snell's Law**.

If a ray passes from a material which has a larger n to one which has a smaller n, the ray bends *away* from the normal. Eventually you will reach an angle of incidence (within the "optically thick" material) for which the refracted ray goes at 90° to the normal. It can't go any more than that! The angle of incidence here is called the **critical angle** for the two media and one can show that it is given by the formula

$$\sin\theta_{\rm crit} = \frac{n_2}{n_1}$$

where the "incident" and "refracted" media are 1 and 2, respectively.

8.3 Dispersion of Light

It turns out that the index of refraction in most materials is slightly different for different colors of light. So when a beam of white light strikes an interface the different colors will bend at slightly different angles. We call this phenomenon the **dispersion** of light.

We can see this effect with a piece of glass which has non-parallel edges. The light which emerges has rays for different colors going off at different angles.

The phenomenon is perhaps most well–known from how it causes a rainbow to appear when light from the sun goes into a region of suspended water droplets and then refracts and reflects within the droplet back to the viewers. The rays that emerge are dispersed because of the the different refraction angles for the different colors.

8.4 Lenses

One can fashion pieces of clear glass or plastic sysmmetric about an axis such that rays which arrive at the device parallel to the axis are all bent to meet at the same point. Such a device is a **converging lens**.

Similarly, one can make a device such that rays arriving parallel to the axis seem to be diverging from the same point on the "arrival" side. Such a device is a **diverging lens**.

For the devices the distance from the lens from the points of convergence or divergence (the **focal points**) is the **focal length** of the lens.

In discussing the formation images by lens I will assume that light is coming in from the left; it passes thru the lens and then is detected (seen) by someone on the right side of the lens.

For a converging lens, if a ray comes in parallel to the axis, it exits along a line which passes through the right focal point. If it comes in along a line passing through the left focal point it goes out parallel to the axis.

For a diverging lens if a ray comes parallel to the ais, it exits along a line passing through the left focal point. If the ray comes in along a line which passes through the right focal point, the ray goes out parallel to the axis.

For both type of lenses, ray which passes through the center of the lens keeps going in the same direction.

We can find the location of an image by tracing some rays that come a particular point on the object. Using the behavior of the lenses we draw the following rays:

• A ray which comes in parallel to the axis goes out along a line which passes through a focal point.
• A ray which comes in on a line through a focal point goes out parallel to the axis.
• A ray which comes in on a line through the center of lens continues through the lens along the *same line*.

If these rays meet at a point on the right side than a **real image** is formed. If when tracee backwards they seem to come from a point on the left side, then that fictional place of origin is the **virtual image**.

The image may be upright or inverted. The magnification m is the ratio of the the height of the image to that of the object: $m = h_i/h_o$. If m is negative then the image is inverted.

8.5 The Lens Equation

As with mirrors, we can solve a simple equation to relate the distances to the object and image and the focal length. In fact the formula has the same form as the mirror equation, but we must interpret the symbols correctly. Our conventions are:

• The object distance d_o is positive when the object on the left side of the lens. We won't discuss the case when the object is on the right (?!?)

• The image distance d_i is positive when the image is on the right side (the opposite side from the object) and negative when it is on the left (the same side as the object). (Be careful with this convention; it would seem to be the opposite of what we use for a *mirror*.

• Focal length f is positive for a converging lens, and negative for a diverging lens.

With these in mind, the lens equations are:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad\qquad m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

8.6 Fixing Human Vision

The human eye contains a lens which can change its focal length so as to focus the images of object on the back of the eye. The lens *should* be deformable enough so that objects *very* far away can be focused and objects as close to your eye as, say, 25 cm can be focused. But there are many among us whose eye muscles don't work so well and these limits are not attainable, at least not without help.

People who have trouble getting distant objects to focus are **near-sighted**, or **myopic**. They can focus objects only out to a particular (finite) distance; this is the distance to what is called the person's **far point**. The person's vision can be corrected with a lens; the idea is that this lens takes an object at infinity and makes an image at the person's far point. With such a lens the person can focus all closer objects so the vision problem is solved — artificially.

If the distance to the far point is $D_{\rm far}$ then we have $d_o = \infty$ and $d_i = -D_{\rm far}$ (minus sign because image is on same side as object!) and then the lens equation gives

$$\frac{1}{\infty} + \frac{1}{(-D_{\rm far})} = \frac{1}{f}$$

so that $f = -D_{\rm far}$. Thus the problem of myopia is solved with a diverging lens.

People who have trouble seeing close objects are **far-sighted**, or **hyperopic**. These people would *like* to focus objects which are as close as, say, 25 cm but they can only focus on objects as close as a place which called the **near point**, which is farther than 25 cm.

This vision defect can be corrected with a lens which takes an object at 25 cm and makes an image at the near point, where the eye can deal with it properly.

Because of the fact that $1/f$ shows up in the lens equations, opticians prefer to use it as a measure of the focusing power of the lens, and refer to it as the **refractive index**. It has units of m^{-1}, which they refer to as a **diopter**. So they tell me.

9 Interference and Diffraction

Waves are different from particles in that combining separate waves doesn't necessarily make a bigger wave. We get a wave of greater amplitude if the separate waves are mostly *in phase* and we get a wave of smaller amplitude if the separate waves are mostly *out of phase*.

We can demonstrate the wave nature of light by passing a beam through a small opening or a pair of openings close together. The beam will spread out from the opening(s) in such a way that some locations will receive a lot of light and some very little. If the light goes on to strike a flat screen there will bright and dark spots on the screen, often called **fringes**.

To do these experiments it is necessary to have **coherent** light. This means that the light behaves like a wave with a definite phase, with well-defined maxima and minima in the amlitudes of the fields. The natural light around us is *not* coherent, but we can get a coherent beam from it if the light passes through a narrow opening. Otherwise (such as in our lab exercises on this subject) we can use the light from a laser which is coherent.

9.1 The Two-Slit Experiment

The clearest demonstration of the wae nature of light is the **two–slit experiment**, where coherent falls on two very narrow openings in a barrier. We need to assume that the *size* of the openings is much smaller than their separation (which is also pretty "small"). The light which comes from the two holes goes on the strike a screen where a pattern can be observed.

The pattern of bright and dark "fringes" occurs because at some points on the screen there is constructive interferece from the light which originates at the two holes; at other places there is destructive interference. When the difference in path lengths to the two holes is an integer number of wavelengths then the former occurs, and when the difference in path lengths is $\frac{1}{2}, \frac{3}{2}, \frac{5}{2} \dots$ wavelengths then the latter occurs. If θ is the angle separating a point on the screen from the center of the pattern, then we have:

$$\text{Bright fringes:} \qquad \sin\theta_{\text{bright}} = m\frac{\lambda}{d} \qquad m = 0, 1, 2, 3, \dots$$

$$\text{Dark fringes:} \qquad \sin\theta_{\text{dark}} = (m + \tfrac{1}{2})\frac{\lambda}{d} \qquad m = 0, 1, 2, 3, \dots$$

9.2 Diffraction; The Single–Slit Experiement

It is also true that coherent light which passes through a *single* slit in a barrier will make a bright/dark pattern on a distant screen.

$$\text{Dark fringes:} \qquad \sin\theta_{\text{dark}} = m\frac{\lambda}{w} \qquad m = 1, 2, 3, \dots$$

10 "Modern" Physics

The laws of physics as written around 1900 were apparently complete and successful. These laws included Newton's law of motion and the laws of electricity which we saw this semester.

Around the previous turn of the century some sticky points in a few experimental results found simple solutions which involved some major changes in the pictures we must form of the microscipic world.

10.1 Photons: Waves (Fields) have Particle Properties

In a result known as the "blackbody radiation" experiment, the relation between the temperature of a hot object and the amount of radiation it emits at various frequencies was predicted using pre–1900 physics, but the predictions failed. It was found that it *could* be understood if the energy contained in the EM radiation came in identical pieces. For a frequency of radiation f, the energy contained in the field can have the values

$$E = nhf \qquad n = 0, 1, 2, \ldots \qquad h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$$

To express this idea we often say that the energy values of the radiation are **quantized**, or in contrast to taking on continuous values, the values are **discrete**.

Another bit of evidence came from the **photoelectric effect**. In this experiment, light strikes an electrode in an evacuated tube; under the right conditions the light can cause electrons to be ejected from the metal surface and in addition to give them some kinetic energy. (The electrons fly to another electrode and then a current is detected in the circuit containing these electrodes.)

It was found that, contrary to the expectations of pre-1900 physics, if the wavelength of the light is too big, *no* photoelectrons will be ejcted regardless of the intensity of the light. But once the wavelength is small enough, electrons will be ejected and their number will increase with the intensity of the light. Also, as the wavelength is decreased past this critical value, the electrons will have more kinetic energy after leaving the metal.

An simple explanation for this was divised by Einstein in 1905. We use the fact that radiation comes in bundles of energy with $E = hf$. An electron is ejected by the interaction of a single photon with the atomic electron. There is some energy required to remove the electron (and leave it with no extra energy) and that is the **work function**, W_0 of the metal. If the photon has sufficient energy the electron may be removed, but if the frequency of the light is too small then the electron can't be removed no matter how many photons are hitting the metal. By energy conservation,

$$E_{\mathrm{photon}} = W_0 + \mathrm{KE}$$

where the KE here is the kinetic enery the ejected electron has.