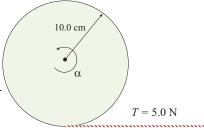
Name_____

Apr. 18, 2007

- 1. A uniform cylinder of mass $3.0~\rm kg$ and radius $10.0~\rm cm$ has a rope wrapped around its edge; a tension of $5.0~\rm N$ is exerted on the rope. The cylinder rotates at a constantly increasing rate, starting from rest.
- **a)** What is the magnitude of the torque exterted on the cylinder?



Force is exerted at the edge of the cylinder, perpendicular to the line joining the application point to the axis, so

$$\tau = rF = (0.100 \text{ m})(5.0 \text{ N}) = 0.50 \text{ N} \cdot \text{m}$$

b) What is the angular acceleration of the cylinder?

The moment of inertia of the wheel is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(3.0 \text{ kg})(0.100 \text{ m})^2 = 0.015 \text{ kg} \cdot \text{m}^2$$

So using au=Ilpha the angular acceleration of the wheel is

$$\alpha = \frac{\tau}{I} = \frac{(0.50 \,\mathrm{N} \cdot \mathrm{m})}{(0.015 \,\mathrm{kg} \cdot \mathrm{m}^2)} = 33.3 \frac{\mathrm{rad}}{\mathrm{s}^2}$$

c) What is the angular velocity of the cylinder 2.0 s after the rope starts pulling? Express this answer in $\frac{\text{rev}}{\text{s}}$.

$$\omega = \omega_0 + \alpha t = 0 + (33.3 \frac{\text{rad}}{\text{s}^2})(2.0 \text{ s}) = 66.7 \frac{\text{rad}}{\text{s}} = (66.7 \frac{\text{rad}}{\text{s}}) \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 10.6 \frac{\text{rev}}{\text{s}}$$

d) What is the kinetic energy of the cylinder at this time?

$$KE = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.015 \,\mathrm{kg \cdot m^2})(66.7 \frac{\mathrm{rad}}{\mathrm{s}})^2 = 33.4 \,\mathrm{J}$$

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2. A uniform beam of length 5.00 m has a mass of 1500 N; it is supported on its two ends (the supports exert only upward forces). A man with a weight of 920 N stands on the beam 3.00 m from the left end.

Find the forces exerted by the left and right supports.

The forces acting on the beam have been added to the figure. We want to solve for F_1 and F_2 . (The force of gravity on the beam acts at its center, $2.5~\mathrm{m}$ from either end.)

The total force on the beam is zero, which gives:

$$F_1 + F_2 - 1500 \text{ N} - 920 \text{ N} = 0$$

The total torque on the beam is zero; it is convenient to choose the left end of the beam as the location of the axis. This gives:

$$-(1500 \text{ N})(2.5 \text{ m}) - (920 \text{ N})(3.0 \text{ m}) + F_2(5.0 \text{ m}) = 0$$

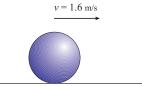
Solve for F_2 , get $F_2=1302\,\,\mathrm{N}$. Put this result into the first equation and get

$$F_1 = 2420 \text{ N} - F_2 = 1118 \text{ N}$$

So the forces are

$$F_1 = 1120 \text{ N}, \qquad F_2 = 1300 \text{ N}$$

3. A solid spherical ball of mass 2.00 kg and radius 10.0 cm rolls without slipping on a flat surface at a speed of $1.6\frac{m}{s}$. What is its (total) kinetic energy?



3.0 m -

1500 N

920 N

Use
$$v=\omega r$$
 , and $I=\frac{2}{5}mr^2$,

$$KE_{roll} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mR^2\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

Plug in:

$$KE = (0.70)(2.00 \text{ kg})(1.6\frac{\text{m}}{\text{s}})^2 = 3.58 \text{ J}$$

You must show all your work and include the right units with your answers!

$$A_x = A\cos\theta \qquad A_y = A\sin\theta \qquad A = \sqrt{A_x^2 + A_y^2} \qquad \tan\theta = A_y/A_x$$

$$g = 9.80_{s^2}^{\frac{m}{2}} \quad \mathbf{F}_{\mathrm{net}} = m\mathbf{a} \qquad f_s^{\mathrm{Max}} = \mu_s F_N \qquad f_k = \mu_k F_N \qquad a_c = \frac{v^2}{r} \qquad F_c = \frac{mv^2}{r}$$

$$W = Fs\cos\theta \qquad \mathrm{KE} = \frac{1}{2}mv^2 \qquad \mathrm{PE}_{\mathrm{grav}} = mgy \qquad \Delta E = \Delta \mathrm{KE} + \Delta \mathrm{PE} = W_{\mathrm{nc}}$$

$$2\pi \ \mathrm{rad} = 360 \ \mathrm{deg} = 1 \ \mathrm{rev} \qquad s = \theta r \qquad \omega = \omega_0 + \alpha t \quad \theta = \omega_0 t + \frac{1}{2}\alpha t^2 \quad \omega^2 = \omega_0^2 + 2\alpha\theta \quad \theta = \frac{1}{2}(\omega_0 + \omega)t$$

$$v = v_t = \omega r \qquad a_c = \frac{v^2}{r} = \omega^2 r \qquad a_t = r\alpha \qquad \tau = rF \sin\phi \qquad \tau = I\alpha \qquad I = \sum_i m_i r_i^2$$

$$I_{\mathrm{cyl}} = \frac{1}{2}MR^2 \qquad I_{\mathrm{rod, ctr}} = \frac{1}{12}ML^2 \qquad I_{\mathrm{rod, end}} = \frac{1}{3}ML^2 \qquad I_{\mathrm{sph, sol}} = \frac{2}{5}MR^2 \qquad I_{\mathrm{sph, sh}} = \frac{2}{3}MR^2$$

$$\mathrm{Statics:} \qquad \sum \mathbf{F} = 0 \qquad \sum \tau = 0 \qquad \mathrm{KE}_{\mathrm{rot}} = \frac{1}{2}I\omega^2 \qquad \mathrm{KE}_{\mathrm{roll}} = \mathrm{KE}_{\mathrm{trans}} + \mathrm{KE}_{\mathrm{rot}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$v_c = r\omega \qquad a_c = r\alpha \qquad L = I\omega \qquad f = \frac{1}{T} = \frac{\omega}{2\pi} \qquad \omega = \sqrt{\frac{k}{m}} \qquad \omega = \sqrt{\frac{g}{L}}$$