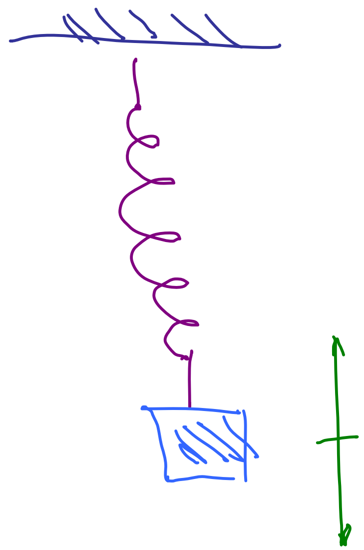
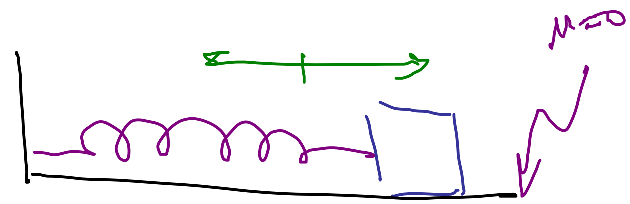
Phys 2110-4 4/4/12

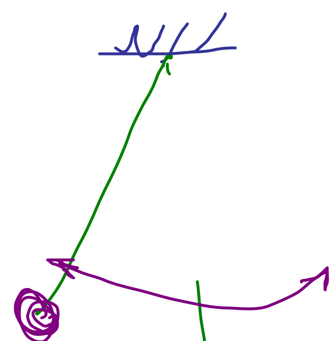
Note Title 4/4/2012

Ch 13 Oscillatory Motion

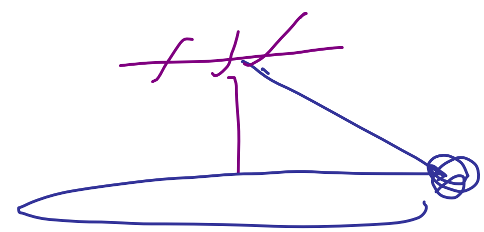




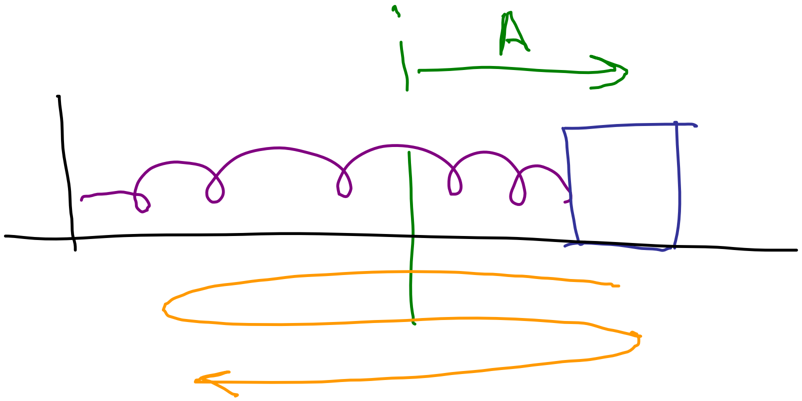
$$|F_{spr}| = |kx|$$



Pendulum

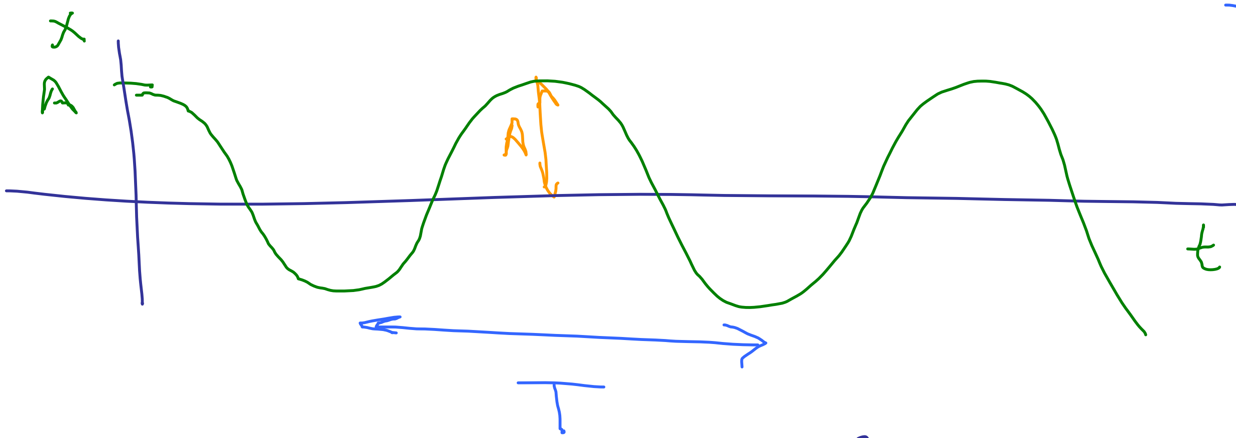


Simple Harmonic Motion.



$$E = \frac{1}{2} k A^2$$

A = Amplitude

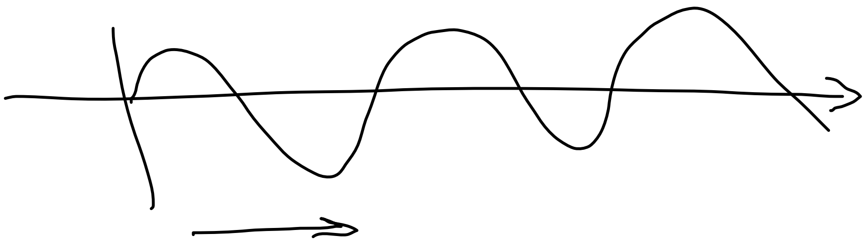


T = period of motion

$v = 2\pi R / T$

$\#$ oscillations / time = frequency = f

= osc / sec = Hertz



ω = angular frequency

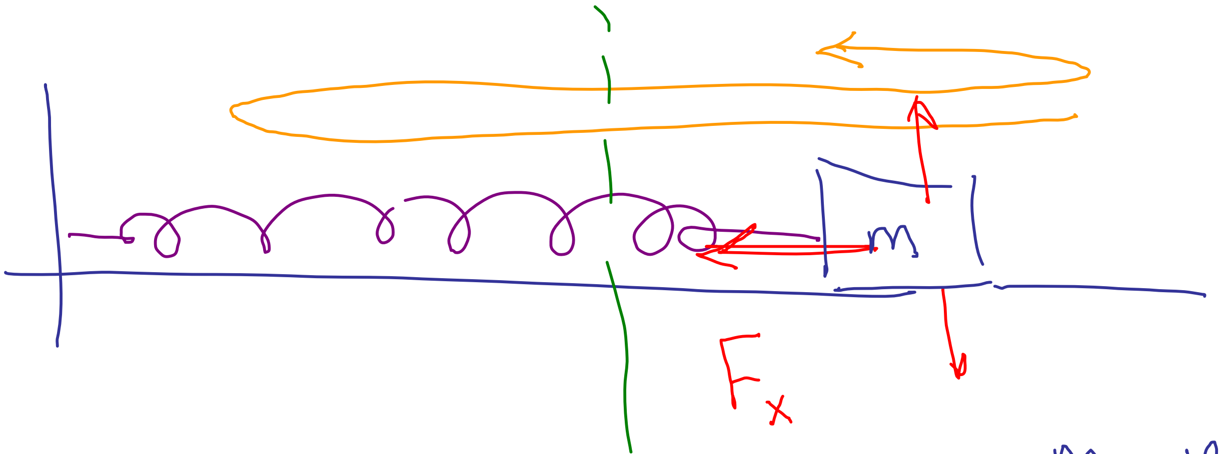
$= 2\pi f$

$= \text{rad}/\text{sec} = 1/\text{sec}$

$$f = \frac{1}{T} = \frac{1 \text{ osc}}{T}$$

$$= \text{osc}/\text{sec}$$

Solve the horizontal mass/spring system.



Max force at ends

(Max accel too.)

Middle : No force

$a = 0$

mass at x

$$F_x = -kx = m a_x = m \frac{d^2x}{dt^2}$$

$$d^2x/dt^2 = -k/m \, x = -\omega^2 x$$

$x(t)$

Define

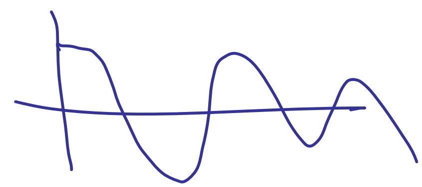
$k/m \equiv \omega^2$

$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad x(t)$$

Take 2nd deriv. of fn = neg. number
orig. function. Trig fns.

$$x(t) = A_1 \sin(\omega t) + A_2 \cos(\omega t)$$

Differential Equation.

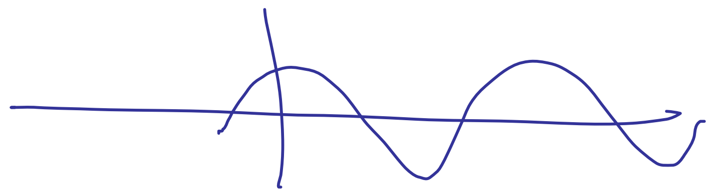


For us, $x(t) = A \cos(\omega t)$

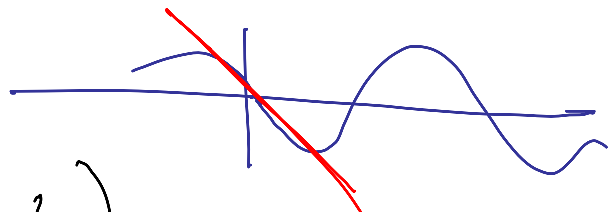
More generally,

$$x(t) = A\cos(\omega t + \phi)$$

constant



phase constant



$$\omega^2 = k/m$$

$$\omega = \sqrt{k/m}$$

$$x(t) = A\cos(\omega t)$$

$$\omega(t+T) = \omega t + 2\pi$$

If $t \to t+T$

$\omega t \to \omega t + 2\pi$

$$\omega T = 2\pi \qquad \frac{1}{T} = f$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Angular frequency

$$x = A\cos(\omega t)$$

$$= A\cos(2\pi f t)$$

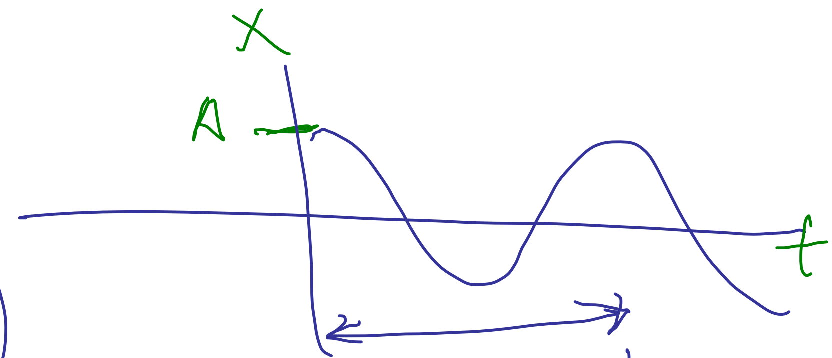
$$\omega = \sqrt{\frac{k}{m}} \qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} \qquad \frac{d^2x}{dt^2} = -\omega^2 x$$

$$x(t) = A\cos(\omega t)$$

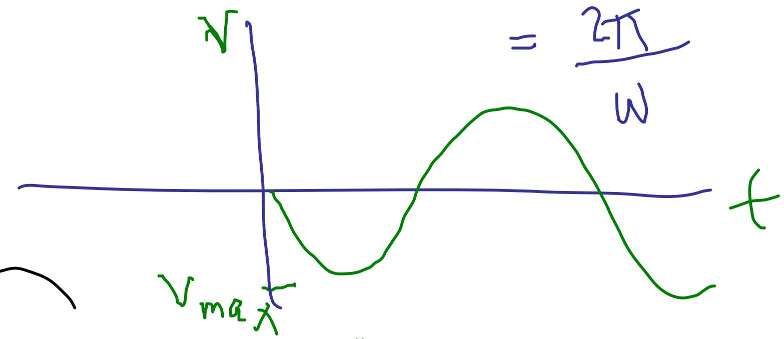
$$v(t) = -\omega A \sin(\omega t)$$

$$a(t) = -\omega^2 A \cos(\omega t)$$

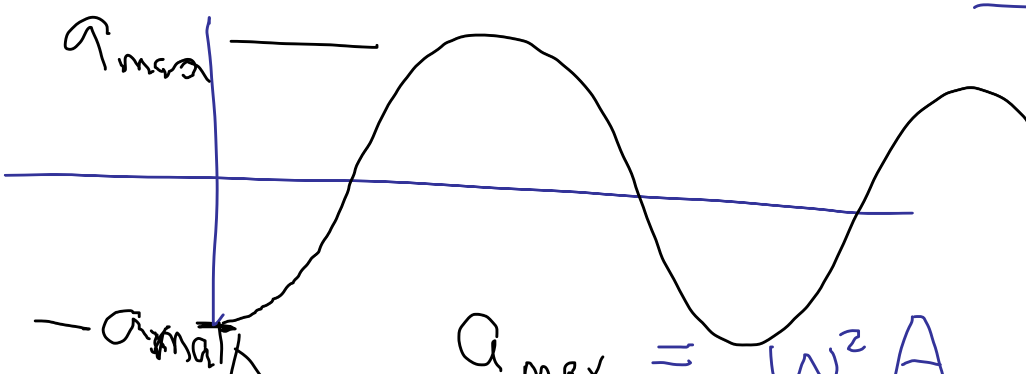


$T = 1/f$

$= \frac{2\pi}{\omega}$



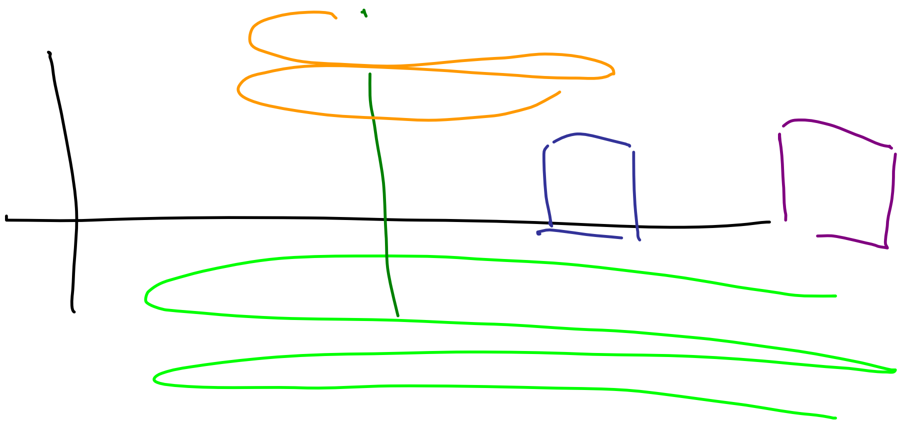
$= \omega A$



$$a_{max} = \omega^2 A$$

$$T = \frac{2\pi}{\omega} \qquad \omega = \sqrt{\frac{k}{m}}$$

Does not depend on A



Energy = K + U

$$K = \frac{1}{2}mv^2$$

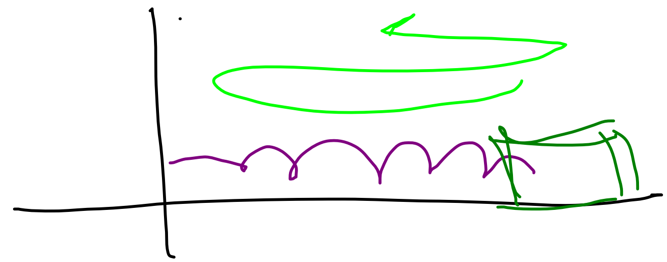
$$= \frac{1}{2}m(\omega A)^2\sin^2(\omega t)$$

$\omega^2 = k/m$

$$= \frac{1}{2}kA^2\sin^2(\omega t)$$

$$U = \frac{1}{2}kx^2$$

$$= \frac{1}{2}kA^2\cos^2(\omega t)$$



Ends: $K = 0$

$U = \text{Max}$

$= \frac{1}{2}kA^2$

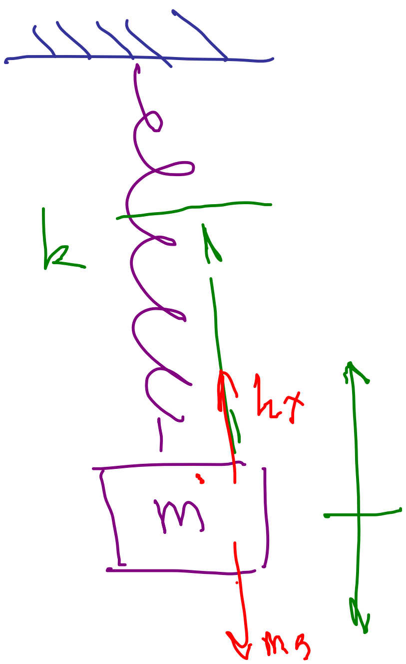
Middle: $K = \text{max}$

$= \frac{1}{2}mv_{max}^2$

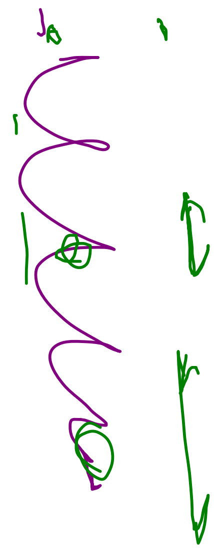
$U = 0$

$$E = \tfrac{1}{2}kA^2\big(\underbrace{\sin^2(\omega t) + \cos^2(\omega t)}_{=1}\big)$$

$$= \tfrac{1}{2}kA^2$$



$$\omega = \sqrt{k/m}$$



Spring has mass

$$m \rightarrow m_{hang} + \frac{m_s}{3}$$

13.25 A 50-g mass is attached to spring & undergoes H.M. Max accel is 15 $\frac{m}{s^2}$ and max speed is 3.5 $\frac{m}{s}$. Determine

a) Angular frequency ω

b) Spring constant k

c) Amplitude A

$a_{max} = A\omega^2$ $\quad v_{max} = A\omega$ $\quad \omega = \sqrt{\frac{k}{m}}$

$$\frac{a_{max}}{v_{max}} = \frac{A\omega^2}{A\omega} = \omega = \frac{15\ m/s^2}{3.5\ m/s^2}$$

$$= 4.29\ s^{-1}$$

$$\omega A = 3.5\ m/s^2 \longrightarrow A = 0.817\ m$$

$$= \frac{3.5\ m/s}{4.29\ s^{-1}}$$

$$\omega = \sqrt{k/m} \qquad m = 0.050\ kg$$

$$\Longrightarrow 0.920\ N/m$$

13.34 A 450-g mass on a spring is osc at 1.2 Hz = f w/ total energy 0.51 J. What's the osc amplitude.

$$\omega = \sqrt{\frac{k}{m}} = 7.54\ s^{-1} \qquad k = 25.6\ \frac{N}{m}$$

$$E = 0.51\ J = \frac{1}{2}kA^2$$

Do it.

$$A = 20\ cm$$

The pendulum

$\overline{T}$, f, ω