Phys 2110-3 10/8/10

Note Title 10/8/2010

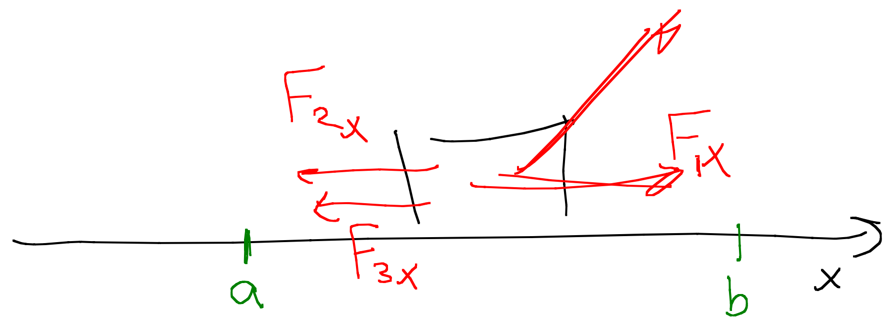
Chap 6 Work Kinetic Energy

$$W = \int F_x \, dx \qquad W = \int \vec{F} \cdot d\vec{r}$$

$$K = \frac{1}{2} m v^2$$

Work - Energy Theorem

$$W_{net} = \int_a^b F_{net,x} \, dx$$



$$F_{net,x} = m a_x$$

$$W_{net} = m\int_a^b a_x \, dx$$

$\vec{F}_{net}$

Here we regard a_x as a function of x.

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\underbrace{\frac{dx}{dt}}_{v_x} = v_x \frac{dv_x}{dx}$$

v_x as a fn of x

sub

$$W_{net} = \frac{m}{2}\int_a^b 2 v_x \frac{dv_x}{dx} \, dx$$

$$= \frac{m}{2}\int_a^b \frac{d}{dx}\left(v_x^2\right) dx = \frac{m}{2} v_x^2 \Big|_a^b$$

$$= \frac{1}{2} m v_x^2 \Big|_b - \frac{1}{2} m v_x^2 \Big|_a = K_b - K_a = \Delta K$$

Net work $= \Delta K$

work done by
<u>all</u> forces

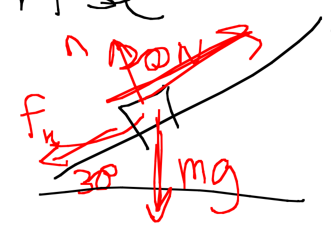
True in <u>all</u> cases

$$W = \int \vec{F} \cdot d\vec{r}$$

<u>6.43</u> You slide a box of books
at constant speed up a 30° ramp
applying force of 200 N <u>up</u> the ramp
Coeff. of fric is 0.18

a) How much work you do when box rise
1m vertically?

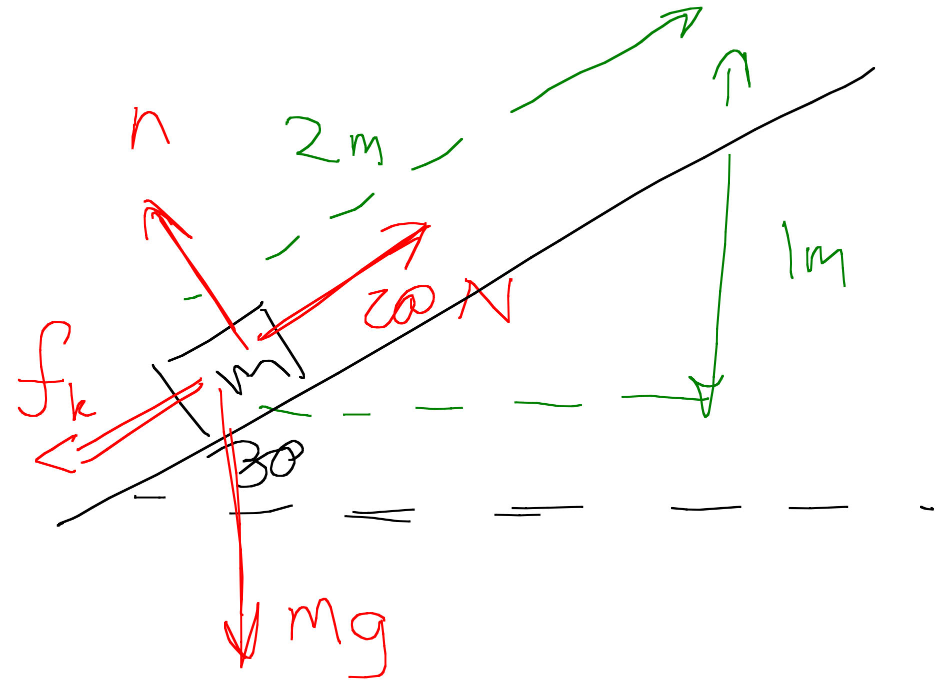
b) What is mass of box ?



constant speed

$W_{net} = \Delta K = 0$

$f_k = \mu_k mg\cos 30^\circ$



a) $W_{you} = (200N)(2m)(1) = 400J$

$W_{grav} = (mg)(2m)(\cos 120^\circ) = -mg\,(2m)\sin 30^\circ$

$W_{fric} = (\mu_k mg\cos 30^\circ)(2m)(-1)$

$W_{normal} = 0$

Add 'em up, $= 0$

$$W_{net} = (200N)(2m) - mg\sin\theta\,(2m) - \mu_k mg\cos\theta\,(2m) = 0$$

$\longrightarrow$ $m = 31.1$ kg

6.49 Find the angle between

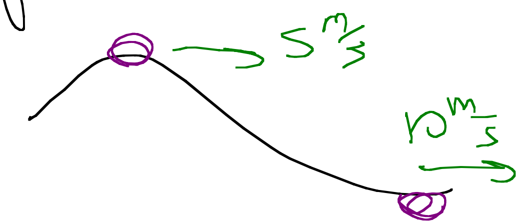
$\vec{A} = 3\hat{i} + 2\hat{j}$ $\quad$ $\vec{B} = -\hat{i} + 6\hat{j}$

$A = \sqrt{13}$ $\quad$ $B = \sqrt{37}$

$$\vec{A}\cdot\vec{B} = \underbrace{A_x B_x + A_y B_y}_{-3+12\,=\,9} = \underline{AB\cos\theta}$$

$$\cos\theta = \frac{9}{AB} = \frac{9}{\sqrt{13}\sqrt{37}} \Rightarrow 65.8$$

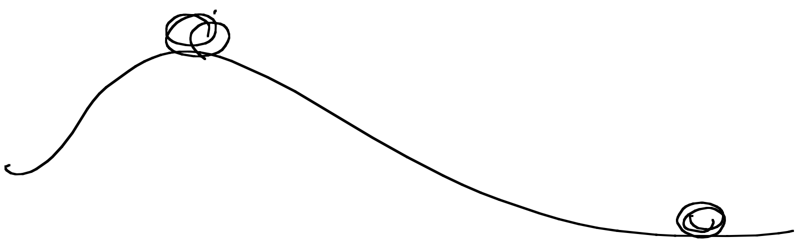
6.29 A 60-kg skateboarder comes over top of hill at $5.0\frac{m}{s}$ and reaches $10\frac{m}{s}$ at bottom. Find total work done on skateboarder between top & bottom.

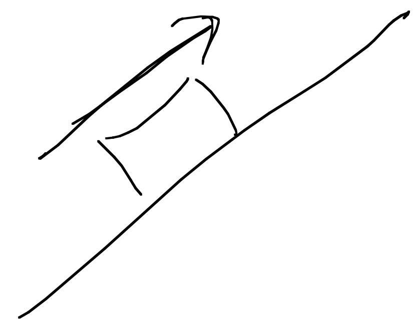


$$W_{net} = \Delta K = \Delta\left(\tfrac{1}{2} m v^2\right)$$

$$= \tfrac{1}{2}(60\,\mathrm{kg})\left(\left(10\tfrac{m}{s}\right)^2 - \left(5\tfrac{m}{s}\right)^2\right)$$

$$= 2250\ \mathrm{J} = 2.25\ \mathrm{kJ}$$





Power Rate at which work is done.

$$\bar{P} = \frac{W}{\Delta t} \qquad \text{“}\frac{\Delta W}{\Delta t}\text{”} = \frac{W}{\Delta t} \qquad P = \frac{dW}{dt}$$

Power : Scalar

Units? Units?

$\frac{W}{t}$

$\frac{J}{s} = \frac{kg\,m^2}{s^3} = 1 \text{ watt} = 1\,W$

1 horsepower = 746 W

$$P = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

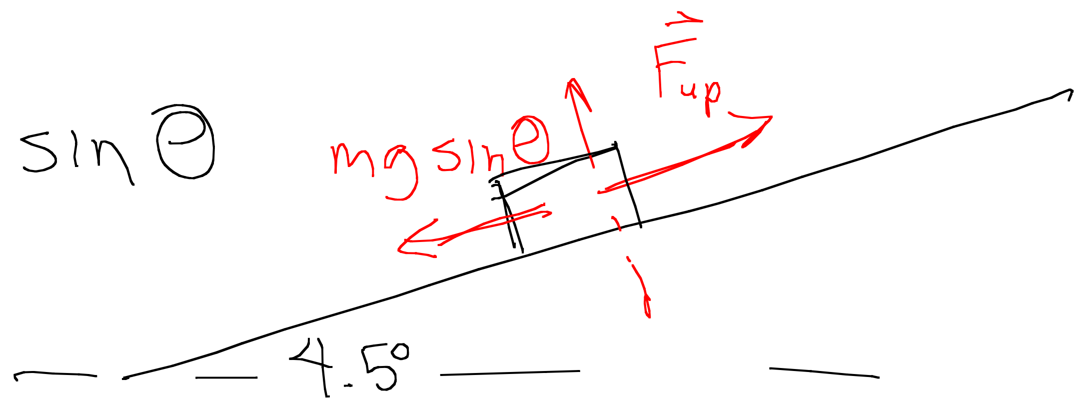
6.61 A 1750-kg car delivers energy to drive wheels at a rate of 35 kW. (Negl. air resistance.) What is greatest speed with which it can climb 4.5° slope?

$$F_{up} = mg\sin\theta$$

35 kW

$$P = F_{up}V = mgv\sin\theta$$

$$V = \frac{P}{mg\sin\theta}$$

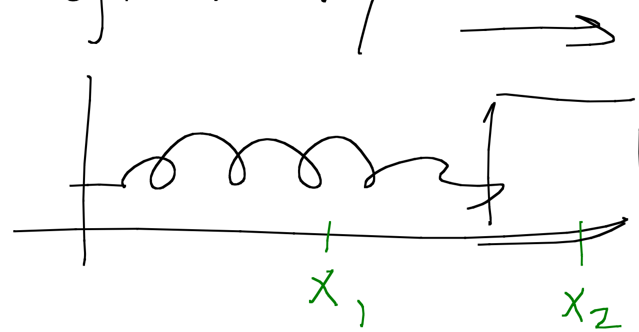


$$\boxed{26\,\frac{m}{s}}$$

Chap Conservation of Energy

For some forces, we get back the work done on them.

Calc work done by spring, gravity



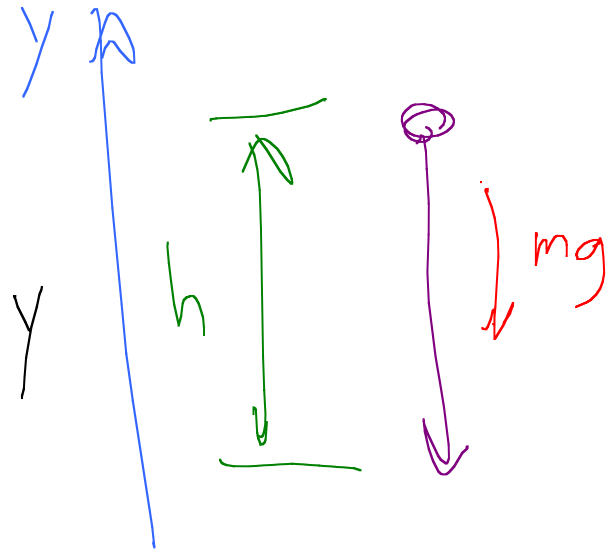
$$F_{spr} = -kx$$

$$W_{spr} = \int F_x \, dx$$

$$= \int_{x_1}^{x_2} -kx \, dx = -\frac{k}{2} x^2 \Big|_{x_1}^{x_2}$$

$$= \frac{k}{2}\left(x_2^2 - x_1^2\right)$$

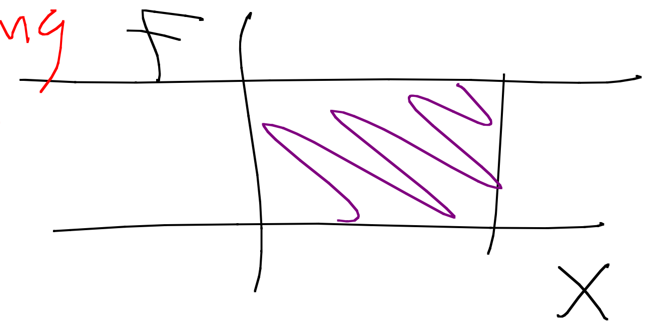
Work done by gravity



Falls: $W = mgh = -mg\Delta y$

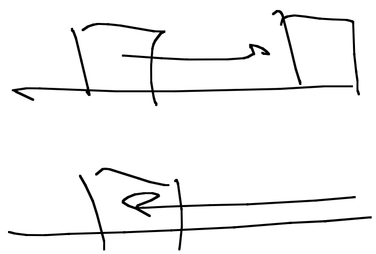
Rises: $W = -mgh$

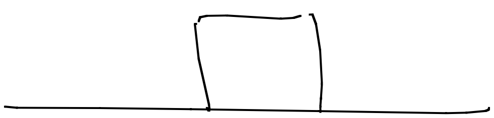
$= -mg\Delta y$

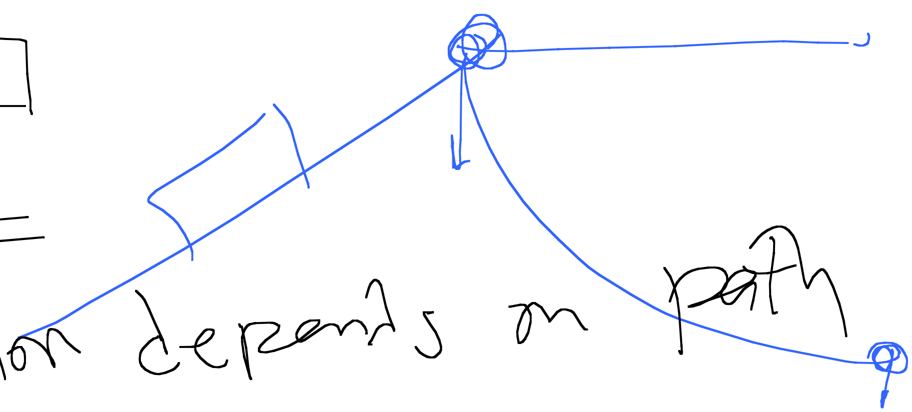


$W = -mg\Delta y$

Friction







Work done by Friction depends on path

$$W_{total} = -\sum mg\Delta y$$

$$-mg(y_b - y_a)$$

In all cases

$$W_{grav} = -mg\Delta y$$