

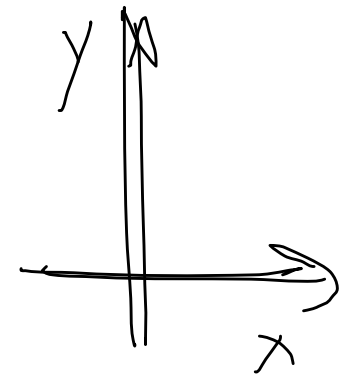
2D motion (ch 3)

Constant acceleration

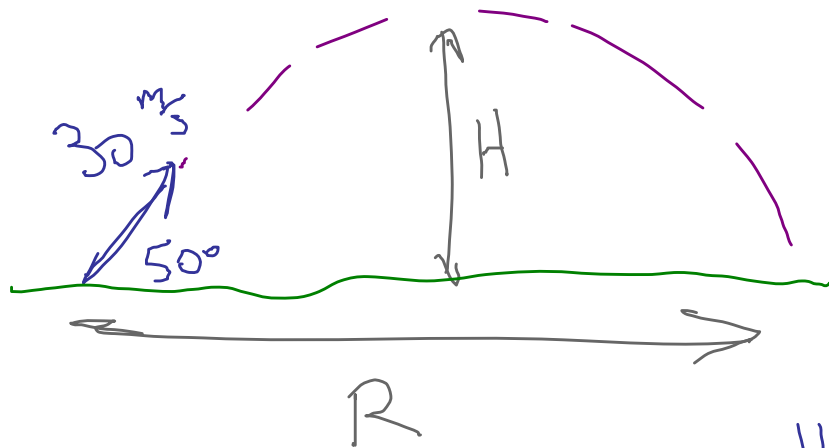
Projectile (free fall).

$$a_x = 0 \quad a_y = -g$$

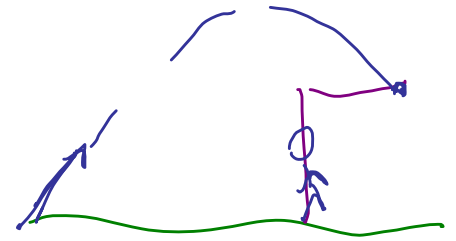
$$g = 9.8 \frac{\text{m}}{\text{s}^2}$$



## Example



Ground  
is flat



$$V_{x0} = 19.28 \text{ m/s}$$

$$V_{y0} = 22.98 \text{ m/s}$$

How long in flight?

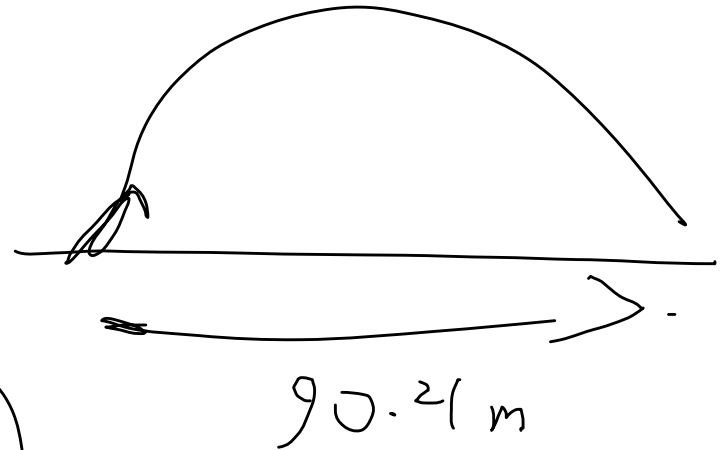
when does  $y = 0$

$$0 = (22.98 \text{ m/s})t - \frac{1}{2}gt^2$$

$$t = \begin{cases} 0 \text{ s} \\ 4.7 \text{ s} \end{cases}$$

what is  $x$  at this time

$$x = (19.28)(4.7) \\ = 90.4 \text{ m}$$



Find max ht.

$$v_y = 0 \quad (v_x = 19.28 \frac{\text{m}}{\text{s}}) \\ v_{y0} + a_y t \\ = (22.98 \frac{\text{m}}{\text{s}}) - g t \\ t = 2.34 \text{ s}$$

( $\frac{1}{2}$  total time of flight)

What is value of  $y$  at this time?

$$y = (22.98 \frac{\text{m}}{\text{s}}) t - \frac{1}{2} g t^2 \quad \text{Plum} \rightarrow 26.9 \text{ m}$$

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$$

Diagram showing a right triangle with a green hypotenuse. Red arrows point from the equation terms to the triangle:  $0$  points to the vertical side,  $v_{y0}$  points to the horizontal side,  $-g$  points to the vertical side, and  $h$  (circled) points to the vertical side.

Do it again with  $V_0$  at angle  $\theta$

$$V_{x0} = V_0 \cos \theta$$

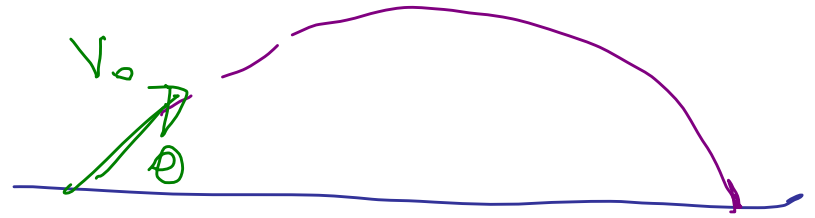
$$V_{y0} = V_0 \sin \theta$$

$$x = (V_0 \cos \theta) t$$

$$y = (V_0 \sin \theta) t - \frac{1}{2} g t^2$$

$$V_x = (V_0 \cos \theta)$$

$$V_y = (V_0 \sin \theta) - g t$$



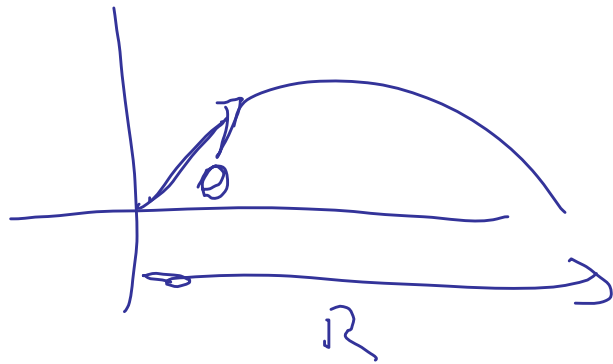
Time of flight  
when does  $y = 0$

$$(V_0 \sin \theta) t - \frac{1}{2} g t^2 = 0$$
$$t [V_0 \sin \theta - \frac{1}{2} g t] = 0$$
$$t = \frac{2 V_0 \sin \theta}{g}$$

What is  $x$  at this time?

$$X = V_0 \cos \theta \cdot \frac{2 V_0 \sin \theta}{g}$$

$$X = R = \frac{2 V_0^2 \sin \theta \cos \theta}{g} = \frac{V_0^2 \sin 2\theta}{g}$$

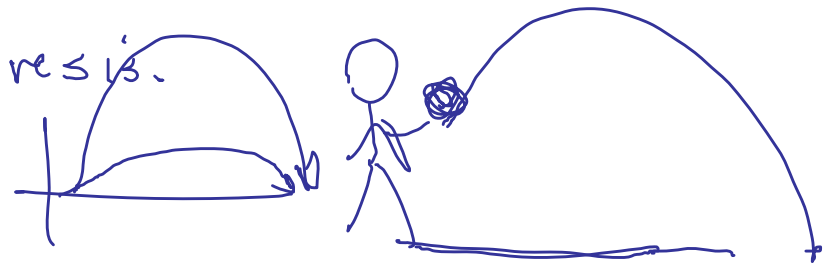


$\sin 2\theta$  is same

$$2\theta = 90^\circ \quad \theta = 45^\circ$$

$0^\circ$	$90^\circ$
$20^\circ$	$60^\circ$
$40^\circ$	$80^\circ$
$20^\circ$	$70^\circ$

No air resist.



$$\text{Max } h, v_y = 0 = v_0 \sin \theta - g t$$

$$v_y^2 = v_{\infty}^2 + 2a(y) t = \frac{v_0 \sin \theta}{g} = \frac{1}{2} \frac{2v_0 \sin \theta}{g}$$

$$0 = (v_0 \sin \theta)^2 - 2g H$$

$$H = \frac{v_0^2 \sin^2 \theta}{2g}$$

$$x = v_0 \cos \theta t$$

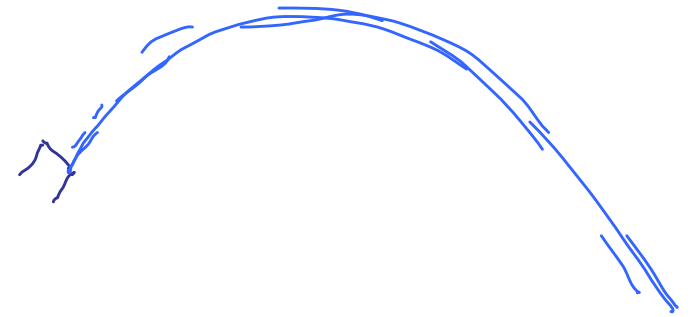
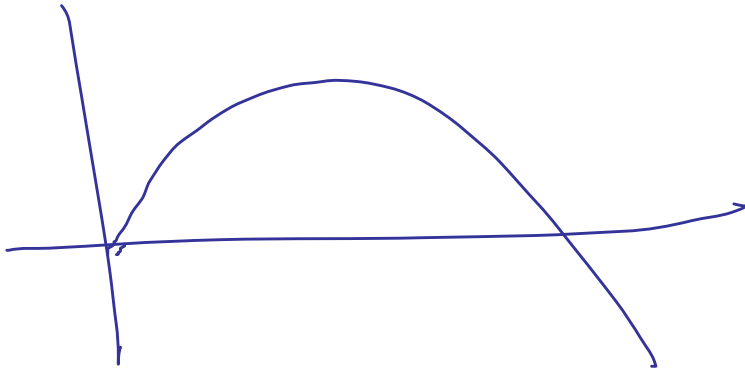
$$t = \frac{x}{v_0 \cos \theta}$$

plug in  $y(t)$   
equation.

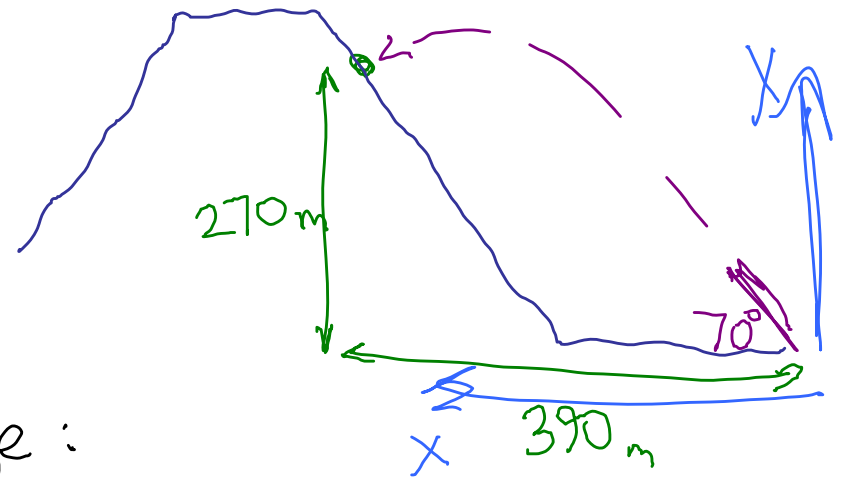
$$y = v_0 \sin \theta \left( \frac{x}{v_0 \cos \theta} \right) - \frac{1}{2} g \left( \frac{x}{v_0 \cos \theta} \right)^2$$

$$y = (\tan \theta) x - \frac{g x^2}{2 v_0^2 \cos^2 \theta}$$

Shape of trajectory is parabola



3.70 Want to fire projectile so that it reaches given point.



Find  $V_0$ . From prev. page:

$$y = (\tan \theta) x - \frac{g x^2}{2 V_0^2 \cos^2 \theta}$$

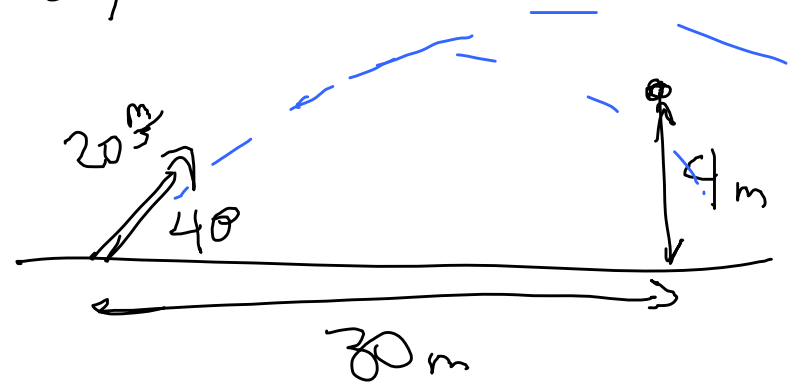
Given  $y = 270 \text{ m}$   $x = 390 \text{ m}$  Solve for  $V_0$

Do algebra  $\Rightarrow V_0 = 89.1 \frac{\text{m}}{\text{s}}$



## Example

Football thrown at  $40^\circ$ ,  $20 \text{ m/s}$   
toward goal; goal is  $30 \text{ m}$  away  
 $4 \text{ m}$  high. Does ball go over/under  
goal?

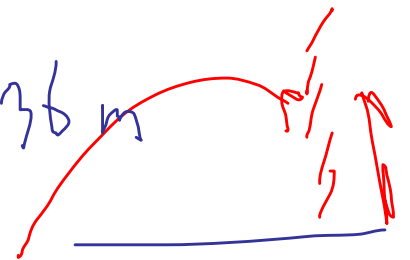


What is  $y$  when  $x = 30 \text{ m}$ ?

Solve for  $t$  when  $x = 30 \text{ m}$   
Get  $t = 1.96$  (solve  $x(t)$ )

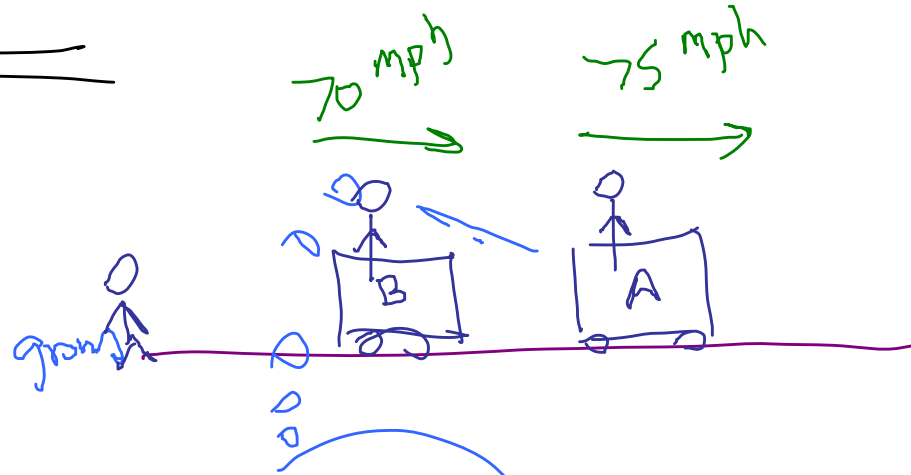
Put this  $t$  into  $y$  eqn  $\rightarrow$

$$y = 6.36 \text{ m}$$



# Relative motion

Easy examples



p. 34  
According  
to guy  
on  
ground

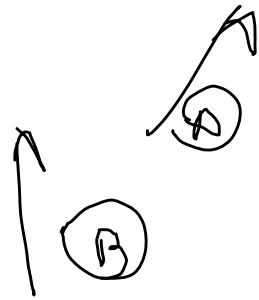
$$5 \text{ mph} = V_{A \text{ rel to } B}$$

$$V_{A \text{ rel gr}} = V_{A \text{ rel to } B} = V_{B \text{ rel gr}}$$

$$75 = 5 \quad 70$$

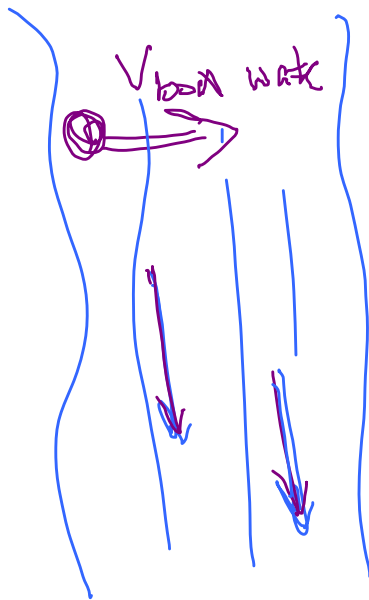
$$V_{A/gr} = V_{A/B} + V_{B/gr}$$

Deal with 2 dim, vector equation



$$\vec{V}_{A/gr} = \vec{V}_{A/B} + \vec{V}_{B/gr}$$

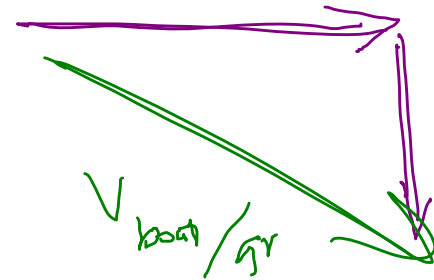
(gr)



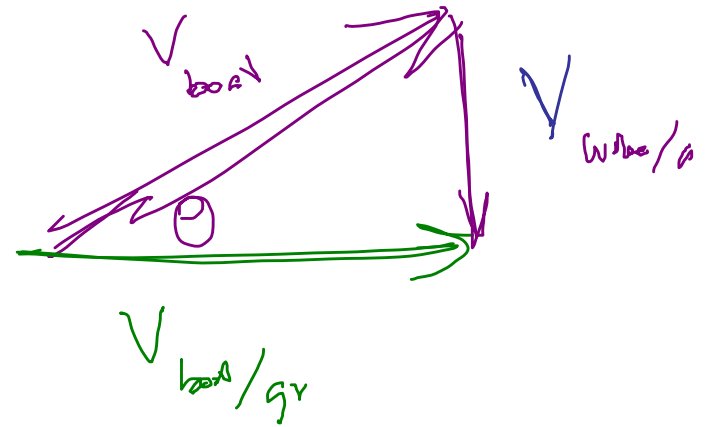
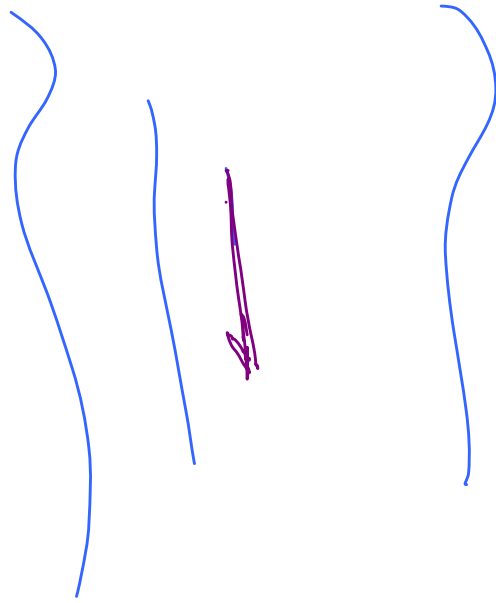
$$V_{water} = 10 \frac{m}{s}$$

$V_{water/gr}$

$$\vec{V}_{Boat/gr} = \vec{V}_{boat/water} + \vec{V}_{water/gr}$$



How can he end up opp the starting point?



must row w/ upstream component.

3.27

You wish to row across stream.  
You row at  $1.3 \frac{m}{s}$ , river flows at  
 $0.57 \frac{m}{s}$  - - - angle.