Name________________________

Class Time: 9am 10am 11am

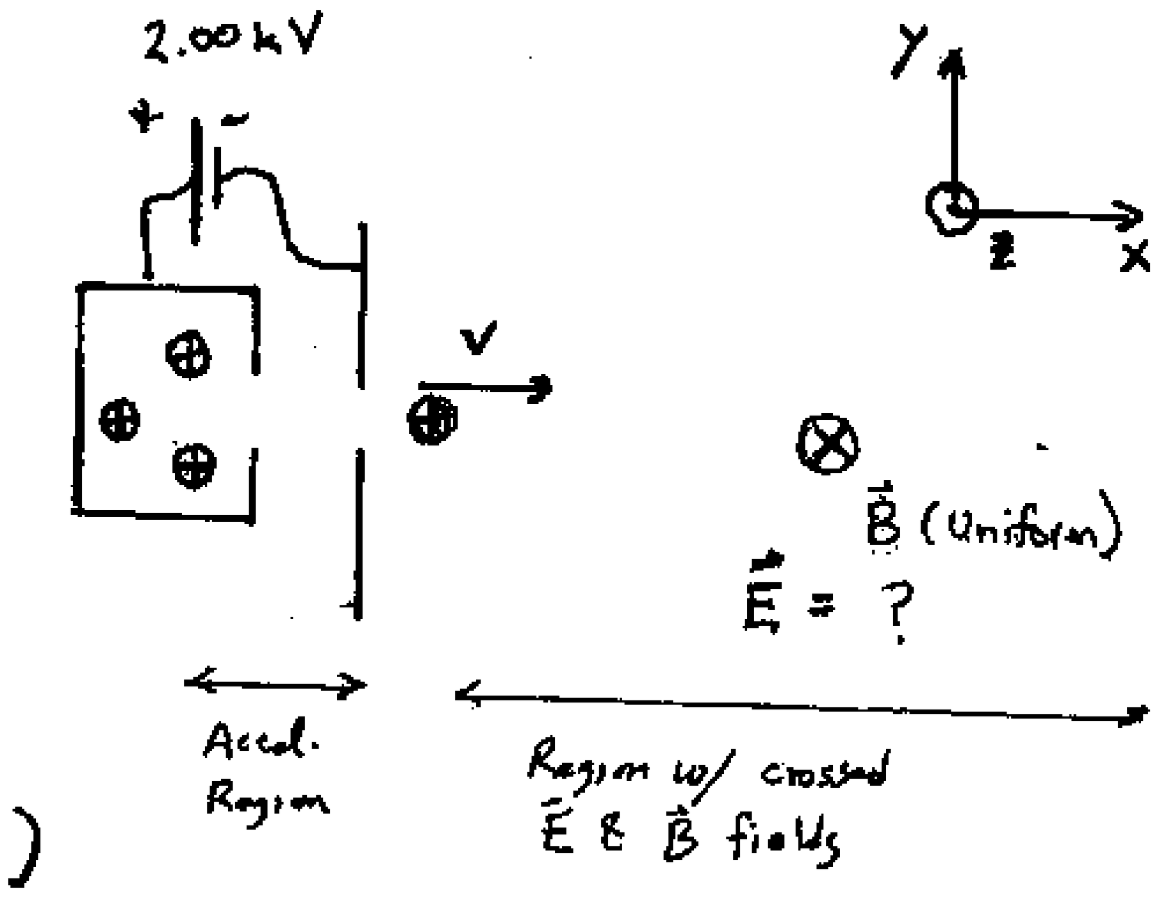
May 6, 2003

Phys 2120 — Spring 2003
Final Exam

1.	____________	(16)
2.	____________	(12)
3.	____________	(6)
4.	____________	(20)
5.	____________	(13)
6.	____________	(8)
7.	____________	(19)
8.	____________	(6)
Total	____________	(100)

You must show all your work and include the right units with your answers!

If one part of a problem requires a result from a previous part (which you can't get), you can explain what you *would* do if you had the previous answer.



1. Ions with charge $+e$ and mass 6.68×10^{-27} kg are accelerated from rest through a potential of 2.00 kV.

a) Find the speed of the ions as they leave the acceleration region. (6)

Kinetic energy gained is potential energy lost, so:

$$|q\Delta V| = \tfrac{1}{2}mv^2$$

$$\rightarrow \quad v^2 = \frac{2|q\Delta V|}{m} = \frac{2(1.602\times10^{-19}\,C)(2.00\times10^3\,V)}{(6.68\times10^{-27}\,kg)}$$

$$= 9.59\times10^{10}\,m^2/s^2$$

$$\rightarrow \quad v = 3.10\times10^5\,m/s$$

b) The ions enter a region a region where there are "crossed' E and B fields. As shown, the B field goes into the page and has magnitude 0.300 T. What is the *direction* and *magnitude* of the magnetic force on the ions? (5)

By right-hand rule, force on ions goes up (i.e. in $+y$ dir.)
Magnitude is

$$|\vec{F}| = qvB\sin\theta = (1.602\times10^{-19}C)(3.10\times10^5\,\tfrac{m}{s})(0.300T)\cdot 1$$

$$= 1.49\times10^{-14}\,N$$

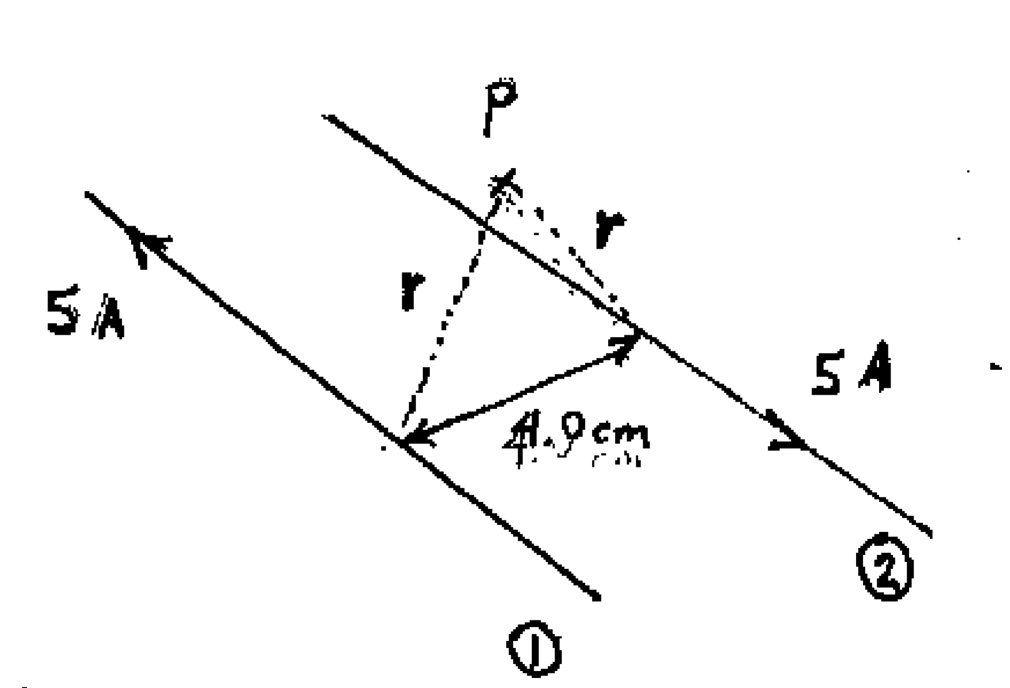
c) The E field is such that there is no net force on the ions. Find the direction and magnitude of the E field. (5)

Charge is positive so electric force points in same dir as $\vec{E}$ field. So we want the $\vec{E}$ field to point <u>down</u> (i.e. in $-y$ dir)
Magnitude of elec. force must be same as magnetic force, found in (b)
Since $F_{el} = qE$ we have:

$$E = \frac{F_{el}}{q} = \frac{1.49\times10^{-14}\,N}{1.602\times10^{-19}\,C} = 9.29\times10^4\,\tfrac{N}{C}$$

2. Consider two very long parallel wires separated by 4.00 cm. The wires carry currents of 5.00 A in opposite directions.

The point P is equidistant from both wires; note the angles in the cross-section view given at the right. The geometry is simple!



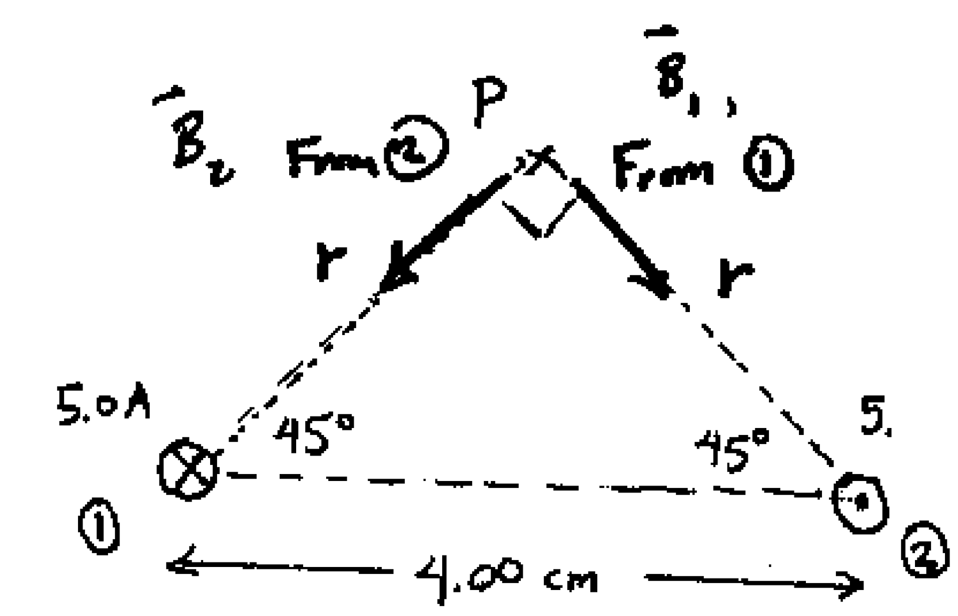
a) How far is P from each wire? (1)

Simple geometry sez

$$\sin 45° = \frac{2.00\,\text{cm}}{r} \longrightarrow r = \frac{2.00\,\text{cm}}{\sin 45°} = 2.83\ \text{cm}$$

b) On the second figure, draw the directions of the magnetic fields at P due to each wire. (4)

R-H rule for $\vec{B}$ field around wire and geometry ($\vec{B}$ field tangential around wire) gives the directions of fields $\vec{B}_1$ and $\vec{B}_2$ shown here.



c) What is the *magnitude* of the magnetic field due to *each* wire at P? (4)

For either $\vec{B}$ vector,

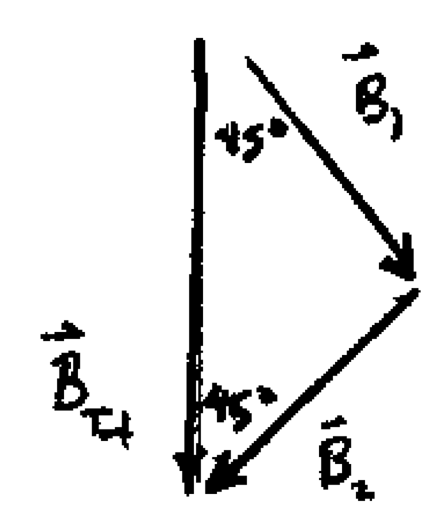
$$B = \frac{\mu_0 i}{2\pi r} = \frac{(4\pi\times10^{-7}\,\text{T·m/A})(5.00\,\text{A})}{2\pi\,(2.83\times10^{-2}\,\text{m})} = 3.53\times10^{-5}\ \text{T}$$

d) What is the magnitude and direction of the total B field at P? (3)

Adding the two vectors with mag's & dir's given in (b) & (c),

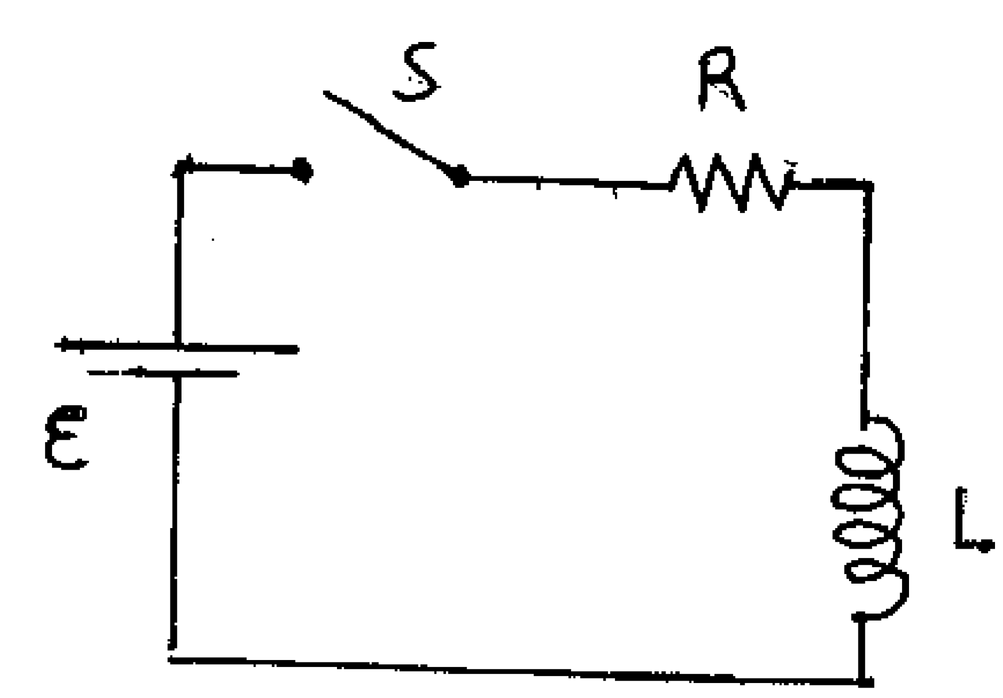
$$|\vec{B}_{Tot}| = \frac{B}{\sin 45°} = \frac{(3.53\times10^{-5}\,\text{T})}{\sin 45°} = 5.00\times10^{-5}\ \text{T}$$

and its direction is "downward" (as shown).



3. The simple RL circuit shown has a 10.0 Ω resistor and a 800. mH inductor.

After switch S is closed, how long does take for the current through L to reach 90.0% of its "maximum" value? (6)



$\mathcal{E}$ = 20.0 V
R = 10.0 Ω
~~L = 3.00 H~~
Use L = 800. mH !

From $i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$, with

$$\tau_L = \frac{L}{R} = \frac{(0.800\,\text{H})}{(10.0\,\Omega)} = 8.00\times10^{-2}\ \text{s}$$

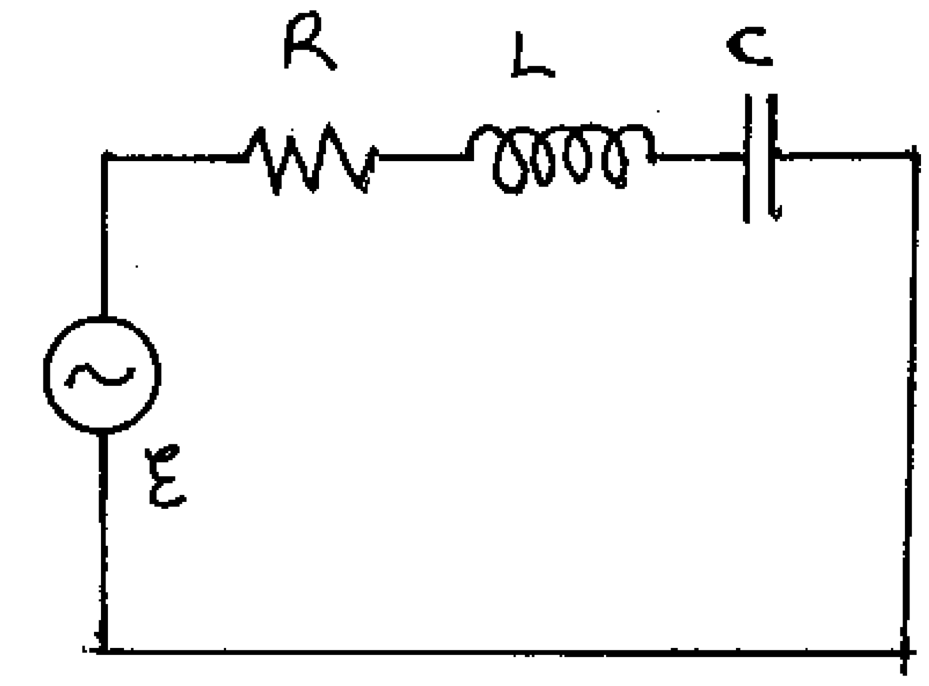
The "max" (asymptotic) value is $\mathcal{E}/R$, so

$\left(1 - e^{-t/\tau_L}\right) = 0.90$, and solve for t.

$$\Rightarrow e^{-t/\tau_L} = 0.10 \quad \Rightarrow -\frac{t}{\tau_L} = \ln(0.10)$$

$$t = -\tau_L \ln(0.10) = -(8.00\times10^{-2}\,\text{s})(\ln 0.10)$$

$$= 0.184\ \text{s}$$



4. An alternating–current voltage source with a frequency of 60.0 Hz and maximum voltage 150 V is connected in series with a 90.0 Ω resistor, a 250. mH inductor and a 15.0 μF capacitor.

a) What is the amplitude of the current in the circuit? (10)

$$\omega = 2\pi f = 2\pi(60.0\ \mathrm{s^{-1}}) = 377\ \mathrm{s^{-1}}$$

$$X_L = \omega L = (377\ \mathrm{s^{-1}})(0.250\ \mathrm{H}) = 94.2\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\ \mathrm{s^{-1}})(15.0\times10^{-6}\ \mathrm{F})} = 177\ \Omega$$

For series RLC,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 122\ \Omega$$

$$I = \mathcal{E}_m / Z = \frac{150.\ \mathrm{V}}{122\ \Omega} = 1.23\ \mathrm{A}$$

b) What is the amplitude of the voltage across the resistor? (2)

$$V_R = IR = (1.23\ \mathrm{A})(90.0\ \Omega) = 111\ \mathrm{V}$$

c) What is the amplitude of the voltage across the capacitor? (2)

$$V_C = I X_C = (1.23\ \mathrm{A})(177\ \Omega) = 218\ \mathrm{V}$$

d) What is the amplitude of the voltage across the ~~capacitor~~ inductor? (2)

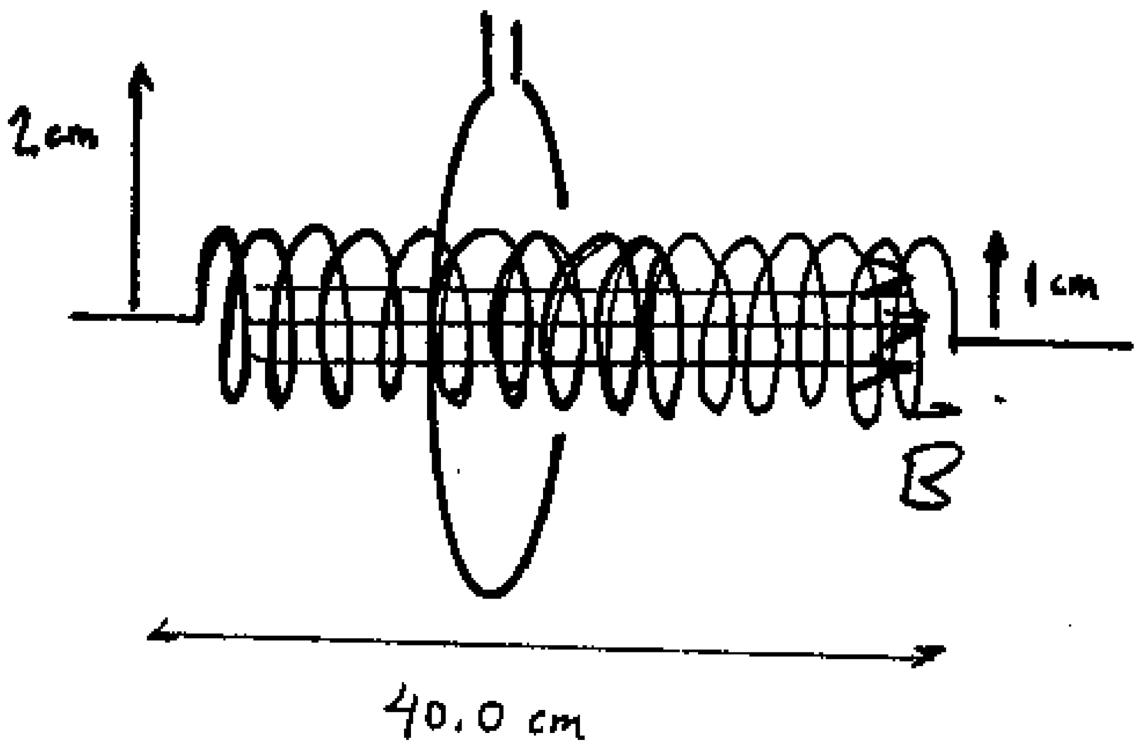
$$V_L = I X_L = 116\ \mathrm{V}$$

e) Do your answers to (b), (c) and (d) add up to give 150. V (the amplitude of the driving voltage)? If they do/don't, give an explanation as to why they should/shouldn't! (4)

Answers add to give a hell of a lot more than 150 V !
But these represent amplitudes only; the voltages themselves do not oscillate with the same phase. So the sum of the amplitudes is not especially meaningful. The sum of the instantaneous voltages across R, L & C does indeed oscillate with amplitude 150V (and freq. 60 Hz).

5. A solenoid of length 40.0 cm and with 4000 turns has a circular cross–section of radius 1.00 cm.

A single loop of radius 2.00 cm is concentric with the solenoid and encircles the solenoid at the middle.



a) If the current in the solenoid is 2.00 A, what is the magnitude of the B field inside the solenoid? (3)

$$n = \frac{4000 \text{ turns}}{40.0\times10^{-2}\,\text{m}} = 1.00\times10^{4}/\text{m}$$

$$B_{\text{solenoid}} = \mu_0 n i = (4\pi\times10^{-7}\,\tfrac{\text{T}\cdot\text{m}}{\text{A}})(1.00\times10^{4}/\text{m})(2.00\text{ A}) = 2.51\times10^{-2}\,\text{T}$$

b) When the 2.00 A current in flowing through the solenoid, what is the magnetic flux through the big loop? (3)

B field is essentially zero outside the solenoid so area covered by the field is just the x-sec area of the solenoid. Then:

$$\Phi_B = B\cdot A\cdot 1 = (2.51\times10^{-2}\text{T})\,\pi\,(1.00\times10^{-2}\text{m})^2 = 7.90\times10^{-6}\,\text{T}\cdot\text{m}^2$$

c) Using the definition of mutual induction: $M = N_2\Phi_{(2\,\text{from}\,1)}/i_1$, what is the mutual inductance M of the system? (3)

With 2 = single loop and 1 = solenoid we have

$$M = 1\cdot\frac{(7.90\times10^{-6}\,\text{T}\cdot\text{m}^2)}{(2.00\text{ A})} = 3.95\times10^{-6}\,\text{H} = 3.95\,\mu\text{H}$$

d) If the current in the solenoid goes from 2.00 A to 0.00 A in 0.200 s, what is the average emf induced in the the big loop over this period? (4)

From Faraday's law,

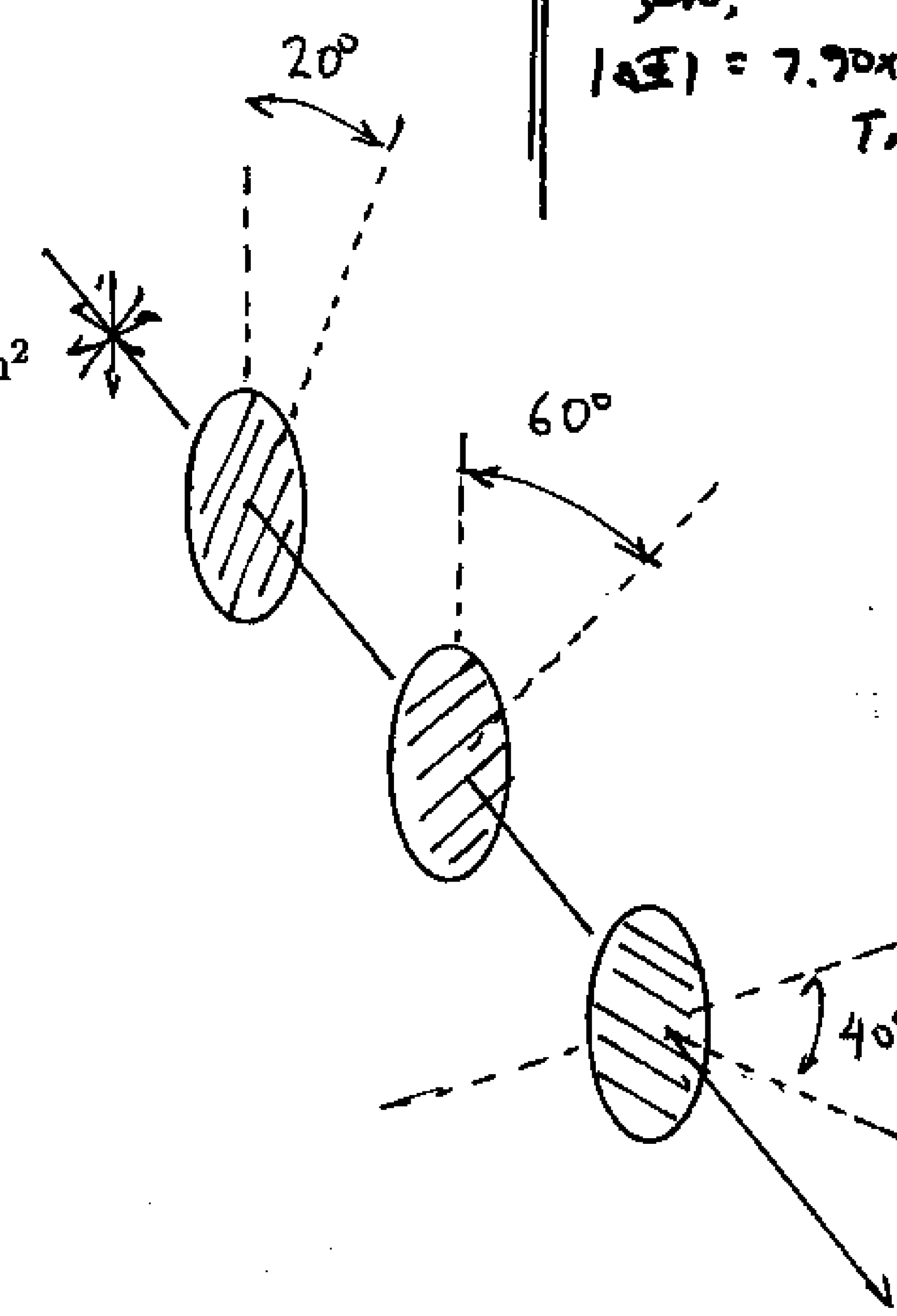
$$|\mathcal{E}_{avg}| = N\cdot\frac{|\Delta\Phi|}{\Delta t} = 1\cdot\frac{(7.90\times10^{-6}\,\text{T}\cdot\text{m}^2)}{(0.200\text{ s})}$$

$$= 3.95\times10^{-5}\,\text{V}$$

Since final flux is zero, $|\Delta\Phi| = 7.90\times10^{-6}$ Tm²

6. In the figure, initially unpolarized light of intensity 45.0 W/m² is sent through three polarizing sheets whose polarizing directions are as follows: The first is at 20.0° from the vertical; the second is at 60.0° from the vertical and the last one is at 40.0° from the horizontal!

What is the intensity of the light which is transmitted by the system? (8)



First polarizer mult's intensity by $\frac{1}{2}$ & pol's light at 20° from vertical.

Second polarizer mult's that intensity by $\cos^2(60°-20°) = \cos^2 40°$ and pol's light at 60° from vertical (30° from horizontal).

Last polarizer mult's that intens. by $\cos^2(30°+40°) = \cos^2 70°$. Final intensity is

$$I = (45.0\,\tfrac{\text{W}}{\text{m}^2})\cdot\tfrac{1}{2}\cdot\cos^2 40°\cdot\cos^2 70° = 1.54\,\tfrac{\text{W}}{\text{m}^2}$$

7. A certain type of glass has an index of refraction of 1.60 for red light ($\lambda = 700.\,\text{nm}$ in vacuum). calculate:

a) The frequency of this electromagnetic wave. (3)

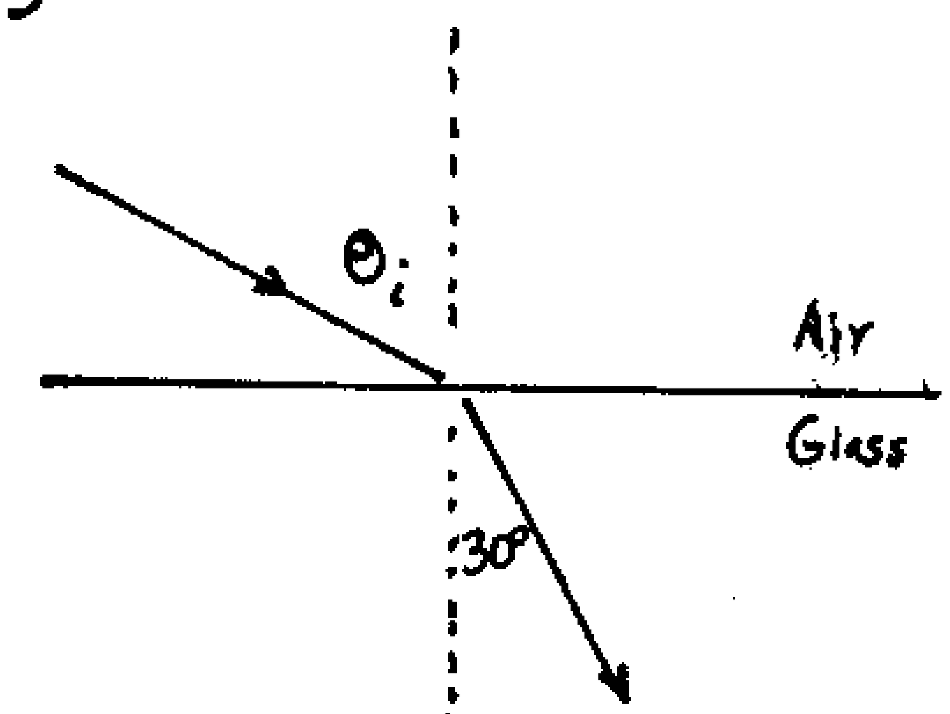
$$\lambda f = c \;\rightarrow\; f = c/\lambda = \frac{(2.998\times10^{8}\,\text{m/s})}{(700.\times10^{-9}\,\text{m})} = 4.28\times10^{14}\ \text{Hz}$$

b) Its speed of propagation in the glass. (2)

Use: $v_{medium} = c/n = \frac{(2.998\times10^{8}\,\text{m/s})}{(1.60)} = 1.87\times10^{8}\,\text{m/s}$

c) Its wavelength in the glass. (2)

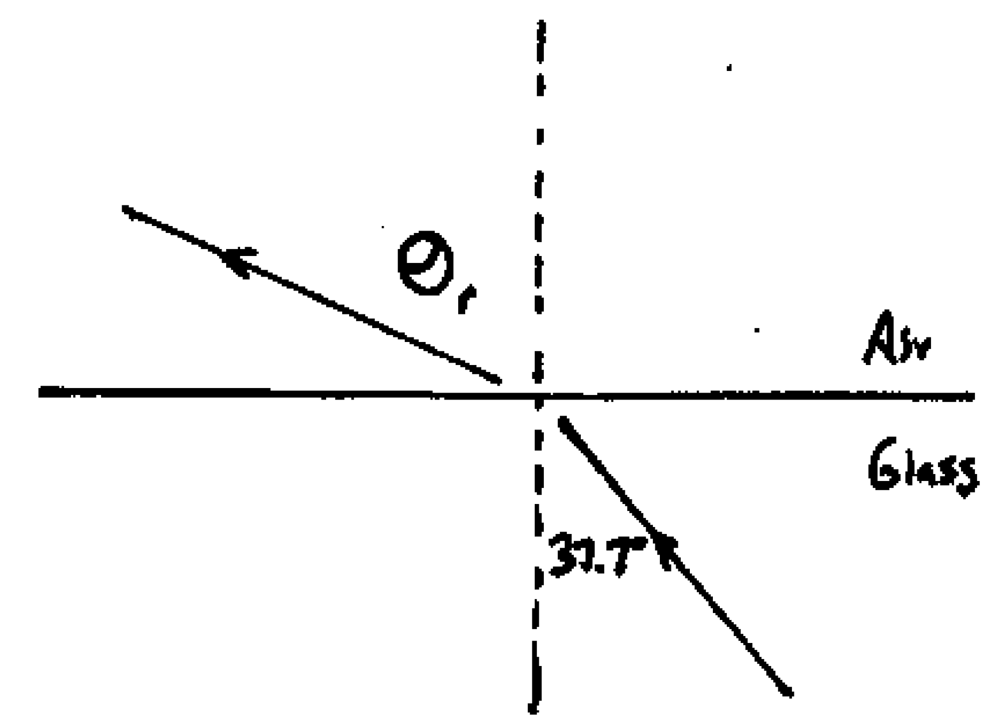
Use: $\lambda_{medium} = \frac{\lambda_{vac}}{n} = \frac{(700.\,\text{nm})}{(1.60)} = 438\,\text{nm}$



d) The angle of incidence θ_i of the ray of red light whose angle of refraction in the glass in 30.0°, as shown at the right. (Use $n_{\text{air}} = 1.00$.) (3)

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ w/ 1 = air, 2 = glass,

$\sin\theta_1 = \frac{n_2}{n_1}\sin\theta_2 = \frac{1.60}{1}\sin(30°) = 0.80$ $\quad \theta_i = 53.1°$



e) Now suppose that the ray of red light travelling in glass is reversed so that it is incident on the glass–air interface from below at an angle of 37.7° with the normal as shown here. What will be the angle of refraction when this reversed ray enters the air? (3)

w/ 1 = glass, 2 = air,

$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1.60}{1.00}\sin 37.7° = 0.978$

$\rightarrow \theta_r = \theta_2 = 78.1°$

f) What is the critical angle for the glass–air interface? (2)

Solve (e) w/ $\theta_2 = 90°$ $n_2 = 1.00$ $n_1 = 1.60$, or use:

$\sin\theta_c = \frac{1.00}{1.60} = 0.625 \rightarrow \theta_c = 38.7°$

g) If red light travelling in glass ($n = 1.60$) is incident on a glass–air interface at an angle of 69° with the normal, what will happen? (4)

This angle is way bigger than the critical angle found in (f)! The ray will be completely reflected back into the glass, i.e. no transmitted beam. (Angle of reflection = angle of incidence.)

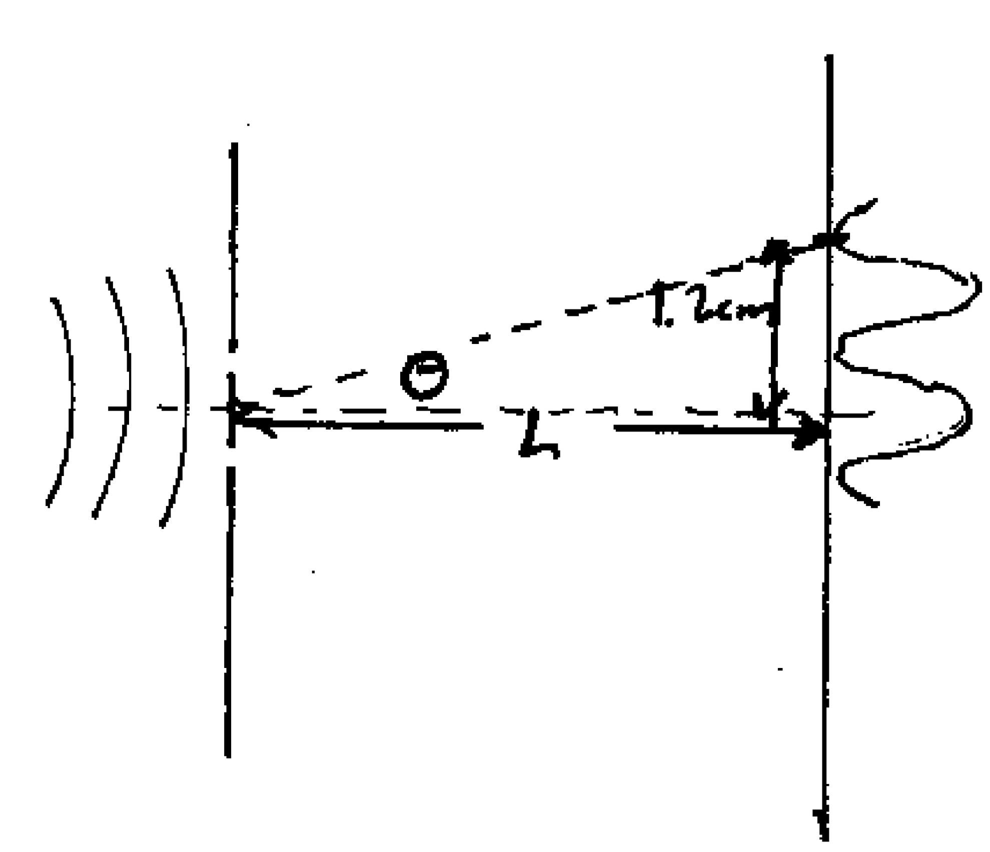
8. The second dark band in a double–slit interference pattern is 1.20 cm from the central maximum of the pattern. The separation of the two slits is a distance equal to 800. wavelengths of the monochromatic light which is incident on the two slits.

What is the distance between the plane of the slits and the viewing screen? (6)

Angle positions of dark fringes given by

$\sin\theta_{dark} = (m+\tfrac{1}{2})\lambda/d$, $m = 0, 1, 2, \ldots$

Here, $m = 1$ (for second fringe) and we are given $d/\lambda = 800$. Then:



$\sin\theta = (1+\tfrac{1}{2})\frac{1}{800} \rightarrow \theta = 0.1074°$

Since $\frac{1.2\,\text{cm}}{L} = \tan\theta$, then:

$L = \frac{1.2\,\text{cm}}{\tan\theta} = 640.\,\text{cm} = 6.40\,\text{m}$

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \,\tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \qquad \epsilon_0 = 8.85 \times 10^{-12} \,\tfrac{\mathrm{C^2}}{\mathrm{N \cdot m^2}} \qquad e = 1.602 \times 10^{-19}\,\mathrm{C}$$

$$m_{\mathrm{elec}} = 9.1094 \times 10^{-31}\,\mathrm{kg} \qquad m_{\mathrm{prot}} = 1.673 \times 10^{-27}\,\mathrm{kg} \qquad 1\,\mathrm{eV} = 1.609 \times 10^{-19}\,\mathrm{J}$$

$$\mathbf{F} = m\mathbf{a} \qquad g = 9.80\,\tfrac{\mathrm{m}}{\mathrm{s^2}} \qquad F = k\frac{|q_1 q_2|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

$$\mathbf{F} = q\mathbf{E} \qquad E_{\mathrm{pt\ ch}} = k\frac{|q|}{r^2} \qquad dq = \lambda dx \qquad dq = \sigma dA \qquad dq = \rho dV$$

$$E_{\mathrm{plane}} = \frac{\sigma}{2\epsilon_0} \qquad E_{\mathrm{cond\ surf}} = \frac{\sigma}{\epsilon_0} \qquad p = qd \qquad E_{\mathrm{dipole}} = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3}$$

$$E_{\mathrm{ring}} = \frac{qz}{4\pi\epsilon_0(z^2+R^2)^{3/2}} \qquad \vec{\tau} = \mathbf{p} \times \mathbf{E} \qquad U = -\mathbf{p} \cdot \mathbf{E}$$

$$\Phi = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\mathrm{encl}}}{\epsilon_0} \qquad \Delta U + \Delta K = 0 \qquad \Delta U = q\Delta V \qquad \Delta V = -\int_i^f \mathbf{E} \cdot d\mathbf{s}$$

$$V_{\mathrm{pt-ch}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \qquad E_x = -\frac{\partial V}{\partial x} \qquad E_{x,\mathrm{uniform}} = -\frac{\Delta V}{\Delta x} \qquad E_x = -\frac{\partial V}{\partial x}$$

$$q = CV \qquad C_{\mathrm{p.-pl.}} = \epsilon_0\frac{A}{d} \qquad C_{\mathrm{cyl}} = 2\pi\epsilon_0\frac{L}{\ln(b/a)} \qquad C_{\mathrm{sph}} = 4\pi\epsilon_0\frac{ab}{b-a}$$

$$C = \kappa C_{\mathrm{air}} \qquad C_{\mathrm{par}} = C_1 + C_2 + \ldots \qquad \frac{1}{C_{\mathrm{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots \qquad U = \frac{q^2}{2C} = \tfrac{1}{2}CV^2$$

$$u = \tfrac{1}{2}\epsilon_0 E^2 \qquad i = \frac{dq}{dt} \qquad J = i/A \qquad J = (ne)v_d \qquad V = iR \qquad P = iV = i^2R$$

$$R = \rho\frac{L}{A} \qquad R_{\mathrm{series}} = R_1 + R_2 + \ldots \qquad \frac{1}{R_{\mathrm{par}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

$$\tau = RC \qquad q(t) = C\mathcal{E}(1 - e^{-t/\tau}) \quad i(t) = \frac{\mathcal{E}}{R}e^{-t/\tau} \qquad q(t) = q_0 e^{-t/\tau} \quad i(t) = \left(\frac{q_0}{RC}\right)e^{-t/\tau}$$

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \qquad \mathbf{F} = I\mathbf{L} \times \mathbf{B} \qquad \frac{mv}{r} = qB \qquad \mu = NiA \qquad \tau = \mu B \sin\phi \qquad \vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\tfrac{\mathrm{T \cdot m}}{\mathrm{A}} \qquad d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i\,d\mathbf{s} \times \mathbf{r}}{r^3} \qquad B_{\mathrm{wire}} = \frac{\mu_0 i}{2\pi R} \qquad B_{\mathrm{arc}} = \frac{\mu_0 i\phi}{4\pi R} \qquad B_{\mathrm{loop}} = \frac{\mu_0 i}{2R}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i_{\mathrm{enc}} \qquad B_{\mathrm{sol}} = \mu_0 n i \qquad B_{\mathrm{tor}} = \frac{\mu_0 i N}{2\pi}\frac{1}{r} \qquad \mathcal{E} = -N\frac{d\Phi_B}{dt} \qquad \mathcal{E}_L = -L\frac{di}{dt}$$

$$L = \frac{N\Phi_B}{i} \qquad L_{\mathrm{sol}} = \mu_0 n^2 A l \qquad M_{21} = \frac{N_2\Phi_{21}}{i_1} \qquad M_{21} = M_{12} \qquad \mathcal{E}_2 = -M\frac{di_1}{dt}$$

$$\tau_L = \frac{L}{R} \qquad i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau_L}) \qquad i = i_0 e^{-t/\tau}$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad \mathcal{E} = \mathcal{E}_m \sin\omega_d t \qquad i = I\sin(\omega_d t - \phi) \qquad \tan\phi = \frac{X_L - X_C}{R}$$

$$X_L = \omega_d L \qquad X_C = \frac{1}{\omega_d C} \qquad Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad V_L = IX_L \qquad V_C = IX_C \qquad \mathcal{E}_m = IZ$$

$$P = I_{\mathrm{rms}}^2 R = \mathcal{E}_{\mathrm{rms}} I_{\mathrm{rms}} \cos\phi \qquad V_{\mathrm{rms}} = \frac{V}{\sqrt{2}} \qquad V_s = V_p\frac{N_s}{N_p}$$

$$I = \tfrac{1}{2}I_0 \qquad I = I_0\cos^2\theta \qquad n_1\sin\theta_1 = n_2\sin\theta_2 \qquad \sin\theta_c = \frac{n_2}{n_1}$$

$$\sin\theta_{\max} = m\frac{\lambda}{d} \qquad \sin\theta_{\min} = (m + \tfrac{1}{2})\frac{\lambda}{d} \qquad \text{for } m = 0, 1, 2, \ldots$$