Phys 2110-5 11/9/12

Note Title 11/9/2012

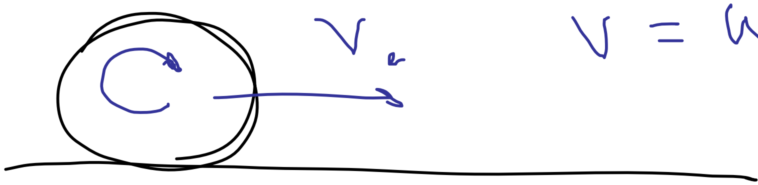
Rotations

$\tau = I\alpha$

$F = ma$

$K_{rot} = \frac{1}{2} I \omega^2$

$K = \frac{1}{2} m v^2$



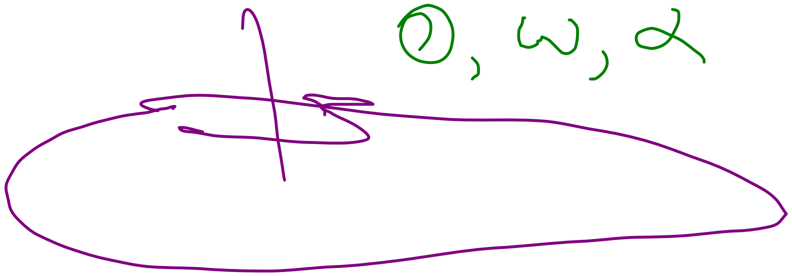
$v = \omega r$

$K_{tot} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

Chap 11 More rota~~ta~~tions

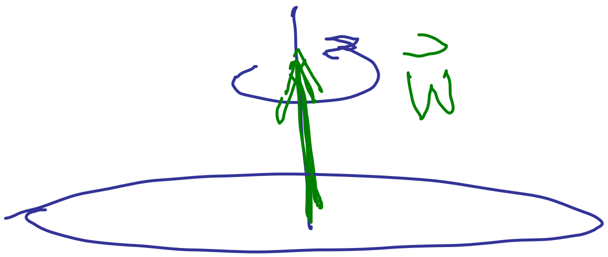
more sophistication

$\longrightarrow$ Angular momentum



θ, ω, α Scalars

<u>All</u> possible rotations



$\vec{\omega}$ is a vector. Points along axis of rotation.

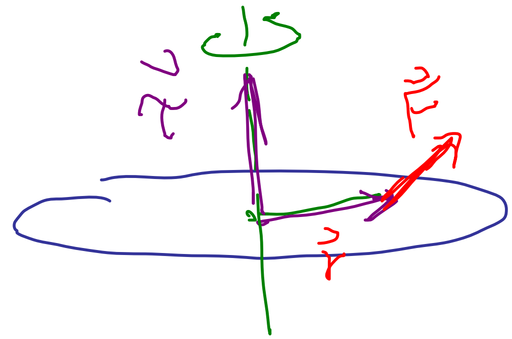
$\vec{\omega}$ can change in <u>mag</u> <u>and</u> direction.

$$\vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t} \text{ as } \Delta t \to 0$$

$\vec{\alpha}$ is a vector

$\Delta\vec{\omega}$ parallel to $\vec{\alpha}$.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$



Also torque is a vector

How do we get torque from

$\vec{r}$ and $\vec{F}$

Mathematical Interlude

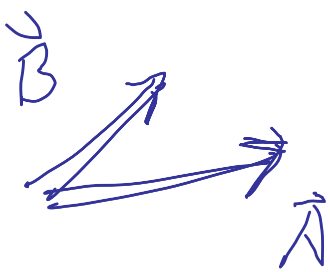
Cross product

$$\vec{A} \times \vec{B} = \vec{C}$$

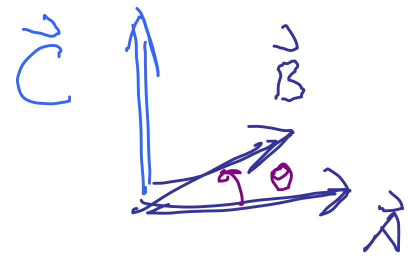
$\vec{C} \perp \vec{A}$

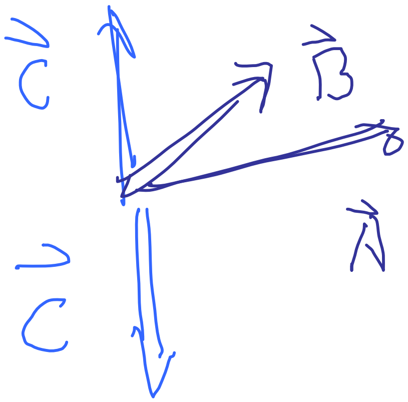
$\vec{C} \perp \vec{B}$

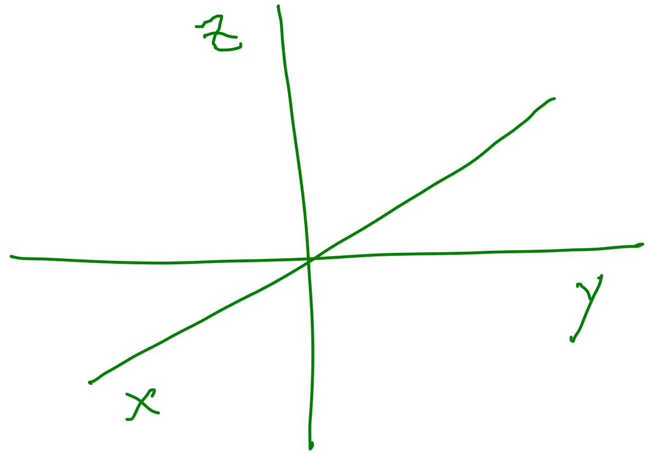
$C = AB|\sin\theta|$



$\vec{A} \times \vec{B} = \vec{C}$



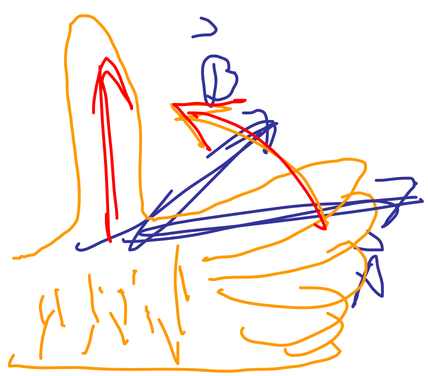




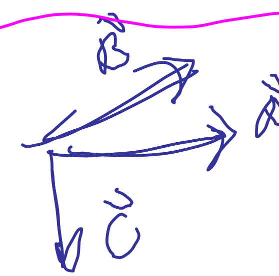
Right Hand.

$\vec{C}$'s dir is given by rt-hand rule

$\vec{A} \to \vec{B}$ — Thumb in dir of $\vec{C}$
Fingers



$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

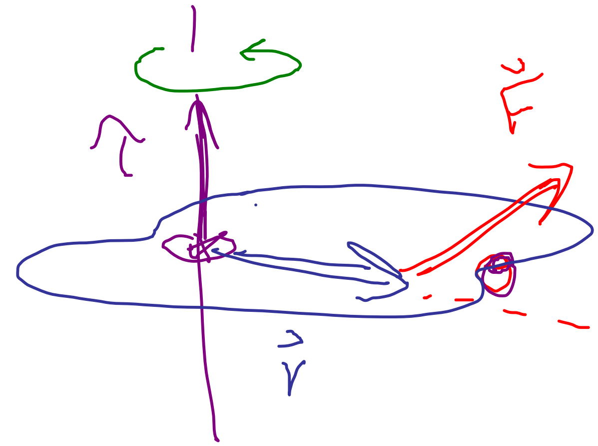


$$\vec{B}\times\vec{A} = \vec{C}$$

Math: $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$

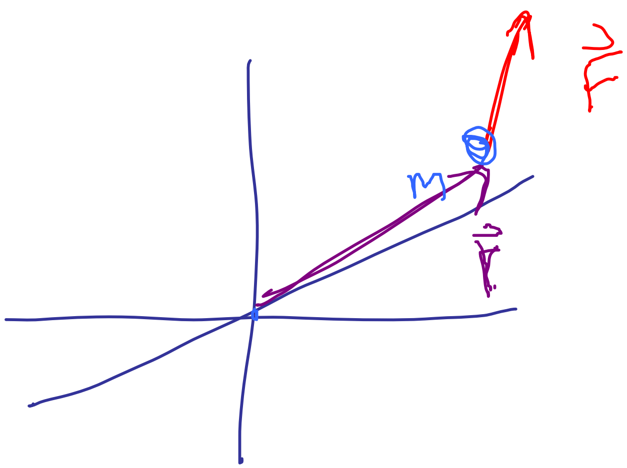
Torque:

$$\vec{\tau} = \vec{r}\times\vec{F}$$

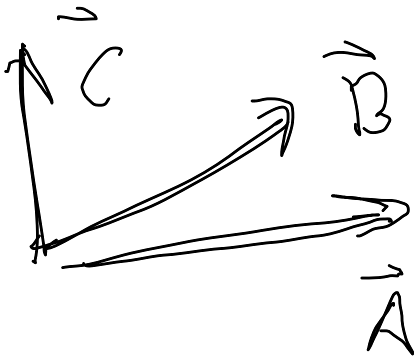


$$|\vec{\tau}| = rF\,|\sin\theta|$$

$$\tau = rF\sin\theta$$



$$\vec{\tau} = \vec{r}\times\vec{F}$$



$$\boxed{\vec{A} \times \vec{B} = \vec{C}}$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

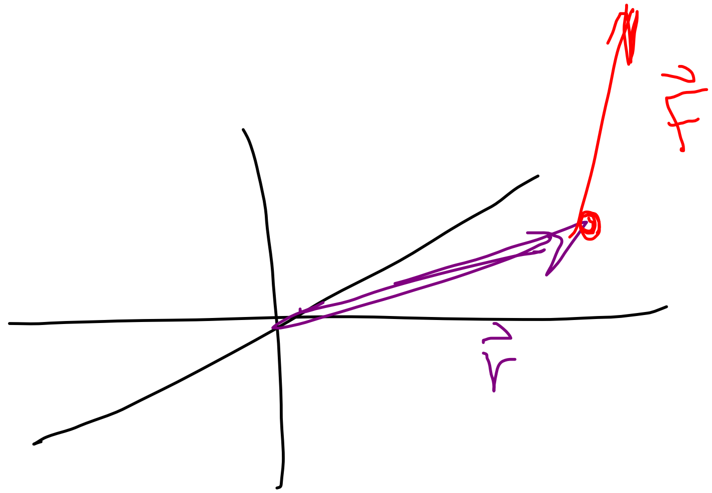
$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{C} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

Long!

More simple:

$$\vec{C} = \begin{array}{c} - \quad\quad + \\ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \end{array} = \hat{i}(A_y B_z - A_z B_y) + \hat{j}(\;\_\_\_\_\_\_\;) + \hat{i}$$



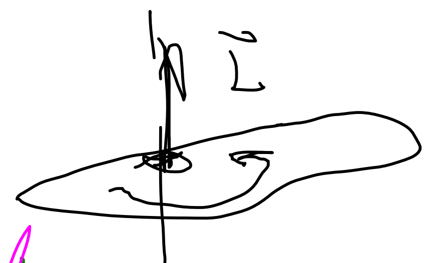
$$\vec{\tau} = \vec{r} \times \vec{F}$$

Go on:

We're done: Forces, energy (kinetic), momentum

$p_x = m v_x$

angular momentum



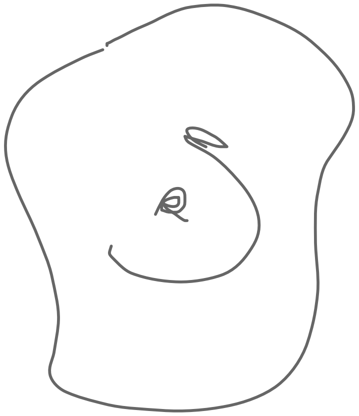
$L = I\omega$ (Vector)

(Units?) $[L] = kg\, m^2 \frac{\cancel{rad}}{sec}$

$= \frac{kg\, m^2}{s} = J \cdot s$ (New set.)

Planck's constant, h units of ang. mom.

J·s   erg·sec.



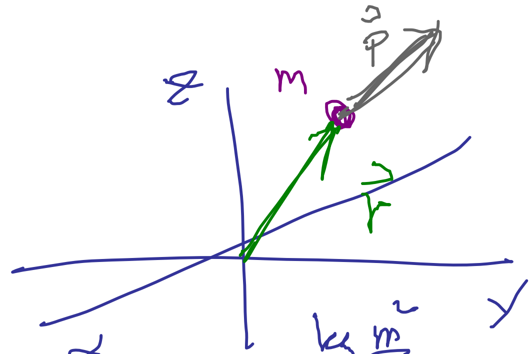
I, ω

$L = I\omega$

$\vec{L} = I\vec{\omega}$ I is a matrix

Right definition:
particle

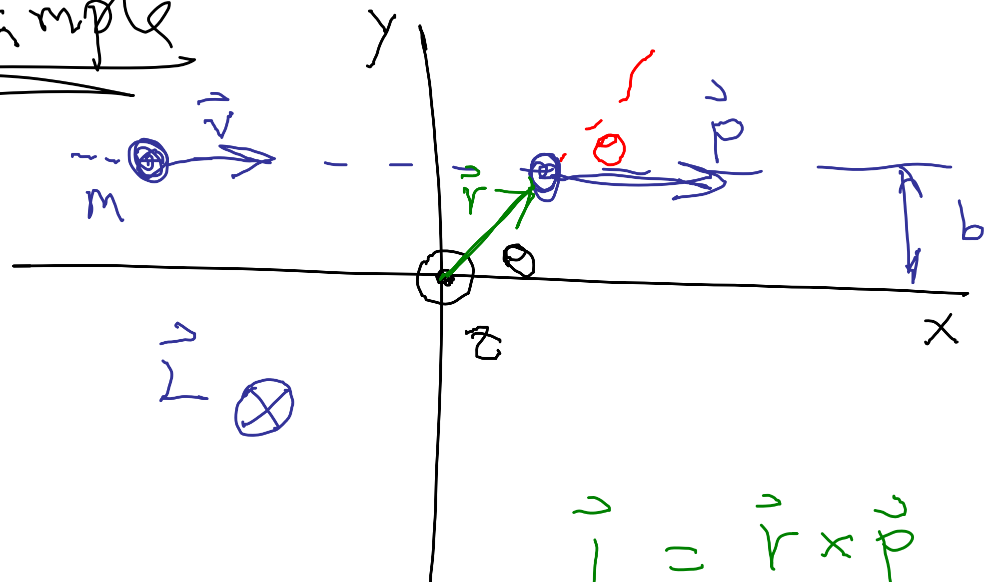
$\vec{L} = \vec{r} \times \vec{P}$



$\mathrm{kg}\,\frac{m^2}{s}$

$$\begin{pmatrix} \vdots \end{pmatrix} = \begin{pmatrix} \cdot & - & - \\ \cdot & - & - \\ \cdot & - & - \end{pmatrix}\begin{pmatrix} \vdots \end{pmatrix}$$

Example



Motion in xy plane.

Mass m, velocity $\vec{v}$ moves along line b away from line.

What is angular moment

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L = rp\sin\theta$$

$$\sin\theta = b/r \qquad p = mv$$

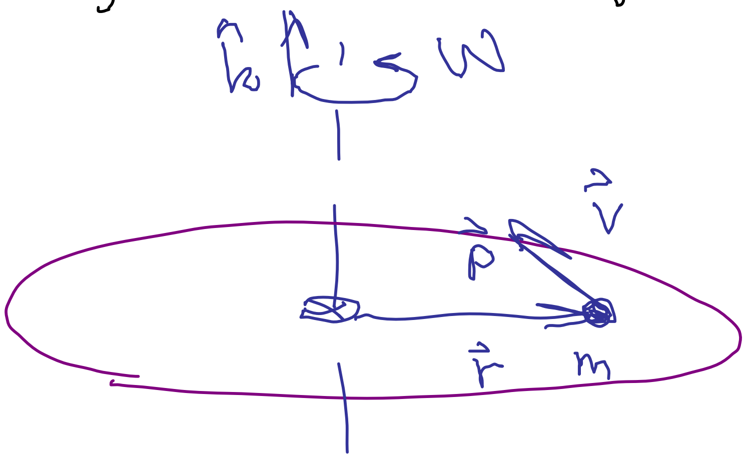
Direction given by rt-hand rule

$\vec{L}$ goes into board (page).

Ang mom is constant.

$$L = rp\,\frac{b}{r} = \cancel{r}\,mv\,\frac{b}{\cancel{r}} = mvb$$

Ang. momentum of planar rotating object



For the particle drawn $\vec r \times \vec v$ points up (along rot. axis).

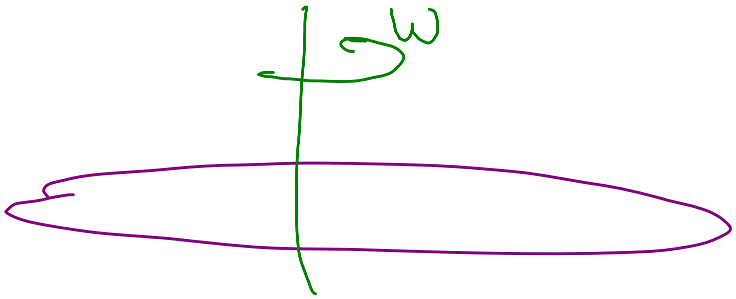
Mag is $rp = rmv$

$= rm(r\omega) = r^2 m\omega$

$$\vec L = r^2 m \omega \hat k$$

Add 'em all up: $\vec L_{Total} = \sum_i m_i r_i^2 \omega \hat k = I\omega \hat k$

$= \left(\sum_i m_i r_i^2\right) \omega \hat k$



$$\hat{L} = I\omega\hat{k}$$

$\vec{L}$ ... huh! what is it good for?

Theorem: $\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$ $\qquad$ $\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$

This applied to an entire system

$$\vec{F}_{net} = d\vec{p}/dt \qquad \vec{\tau}_{net} = \frac{d\vec{L}_{tot}}{dt}$$

$$\vec{F}_{net} = 0 \rightarrow \vec{p} = const \qquad \vec{\tau}_{net} = 0 \quad \vec{L}_{tot} = const.$$