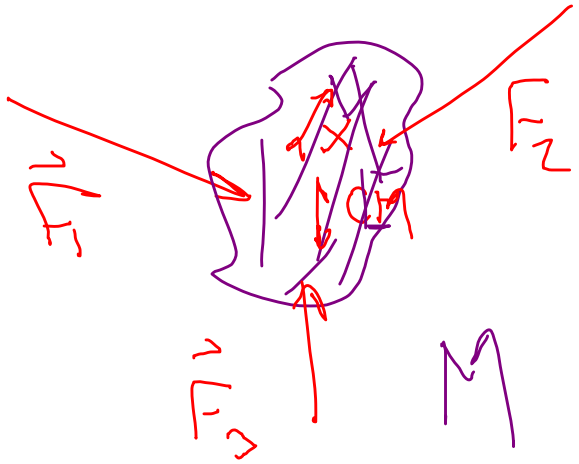
Phys 2110-4 3/2/12

Note Title 3/2/2012

Main Theorem

$$\vec{F}_{\substack{net \\ ext}} = M\vec{a}_{cm}$$



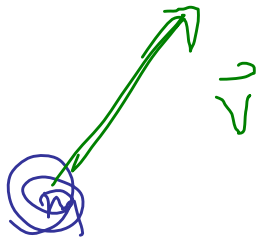
Isolated system: No net external force $\Longrightarrow \vec{a}_{cm} = 0$

Definition

Momentum of particle

$$\vec{p} = m\vec{v}$$

Vector



$$[\text{Units}] = \text{kg}\,\frac{\text{m}}{\text{s}} = \frac{\text{kg m}}{\text{s}}$$

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{M}$$

$$M = \sum_i m_i$$

$$\frac{d\vec{r}_{cm}}{dt} = \vec{v}_{cm} = \frac{1}{M}\sum m_i \frac{d\vec{r}_i}{dt}$$

$$= \frac{1}{M}\sum m_i \vec{v}_i = \frac{1}{M}\sum \vec{p}_i$$

$$M\vec{v}_{cm} = \sum_i \vec{p}_i = \vec{P}$$

Total momentum

$$\vec{p} = m\vec{v}$$

$$\vec{P} = M\vec{v}_{cm}$$

Next

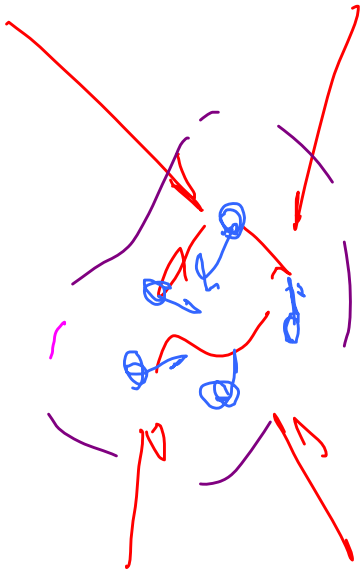
$$\frac{d\vec{P}}{dt} = \sum m_i \frac{d\vec{v}_i}{dt} = M\left(\frac{1}{M}\sum m_i a_i\right)$$

$$= M\vec{a}_{cm} = \vec{F}_{ext}^{net}$$

$$\vec{F}_{net\ ext} = \frac{d\vec{p}}{dt}$$

If $\vec{F}_{net\ ext} = 0$ $\quad \frac{d\vec{p}}{dt} = 0$

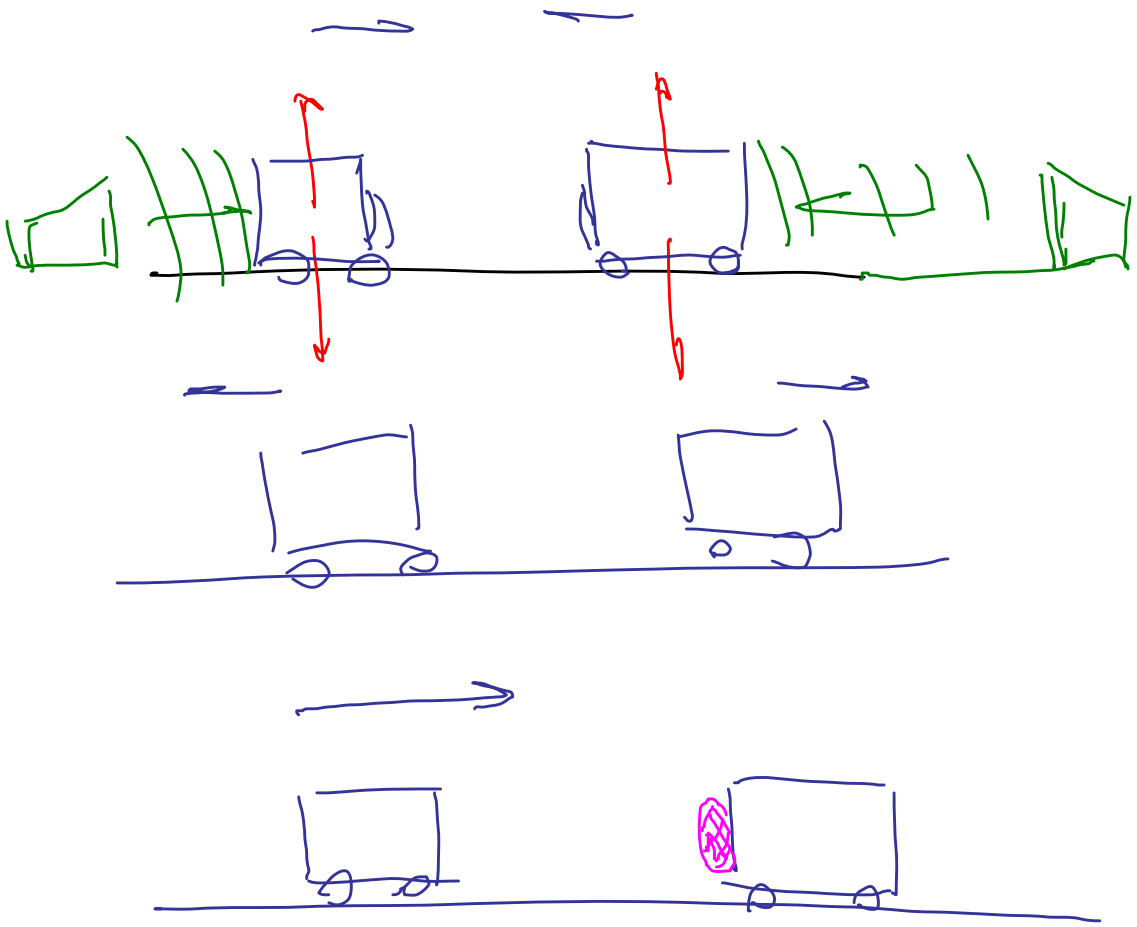
$\vec{p}$ = constant

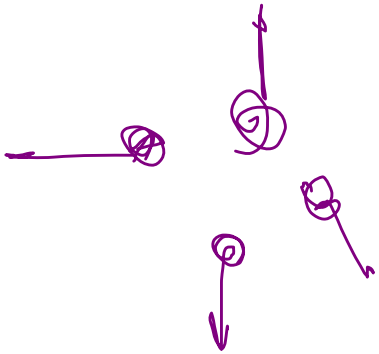


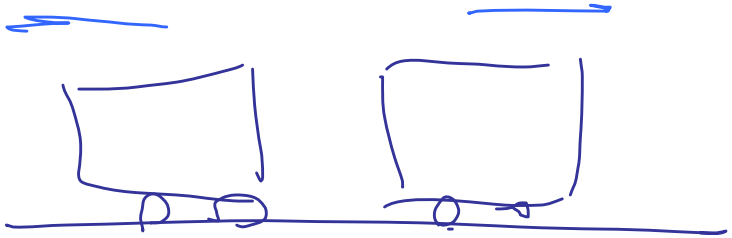
Systems

If isolated

$\vec{p}$ stays same.

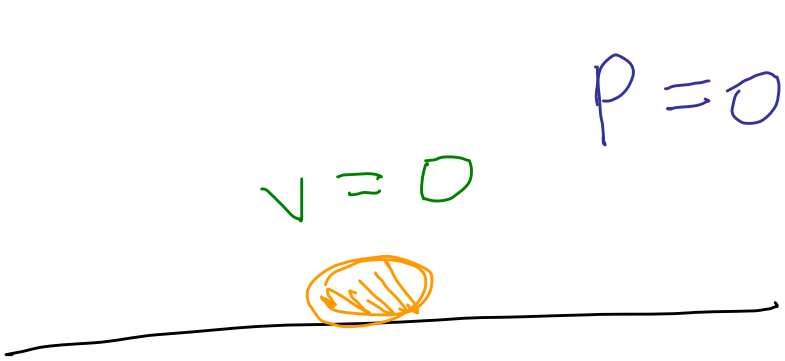


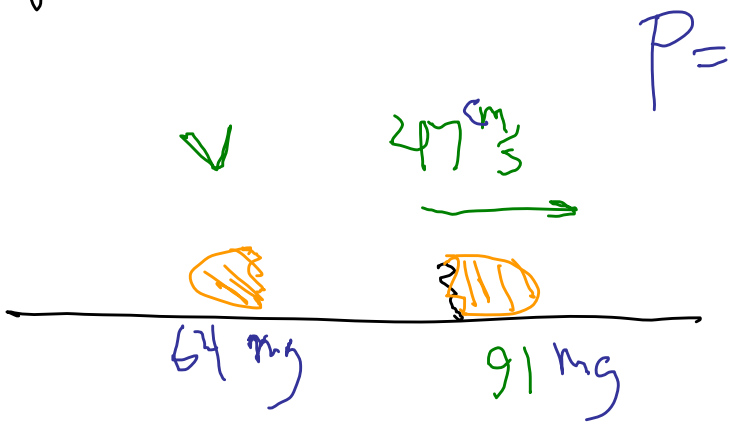




Isolated system: $\vec{P}$ is constant

9.17 A popcorn kernel at rest in hot pan burst two pieces of masses 91 mg, 64 mg. More massive piece moves horizontally at 47 $\frac{cm}{s}$. Describe motion of second piece.



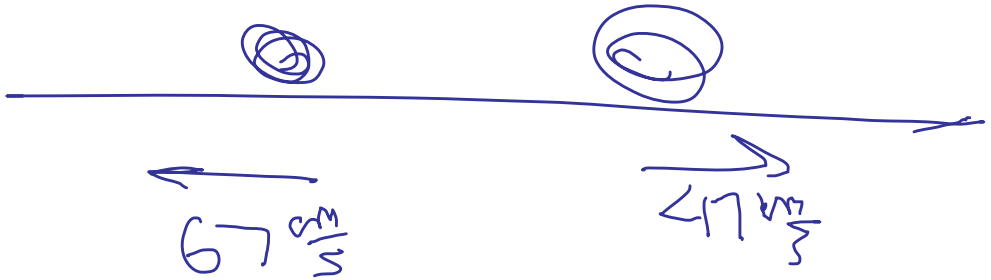


$$P_i = P_f = 0$$

$$0 = (64\ mg)V + (91\ mg)\left(47\ \frac{cm}{s}\right)$$

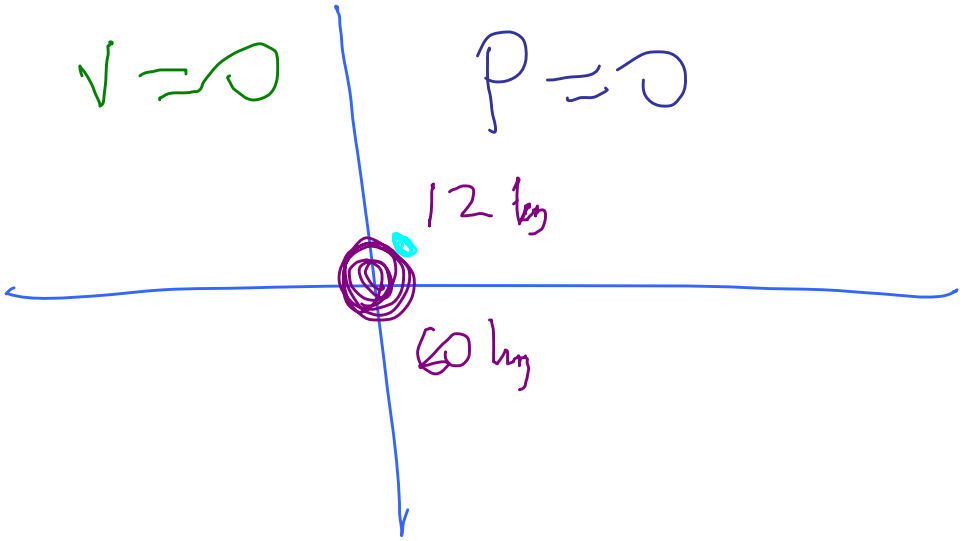
$$V = -67\ \frac{cm}{s}$$

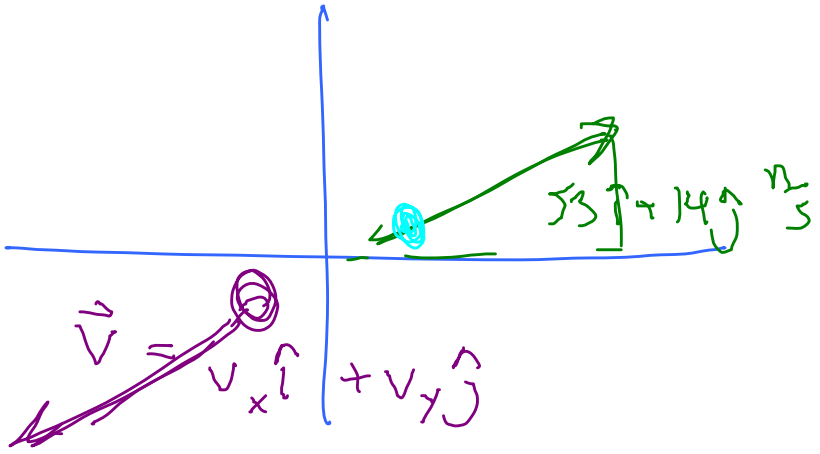
Velocity (component)



9.18 A 60-kg skater at rest on frictionless ice tosses 12 kg snowball with velocity $\vec{v} = (53.0\hat{i} + 14.0\hat{j})\ \frac{m}{s}$

Find skater's subsequent velocity.





Momentum is conserved $\vec{p}$ stays same

Both components conserved

$$p_x = 0 = (12\,kg)(53\,\tfrac{m}{s}) + (60\,kg)\,v_x$$

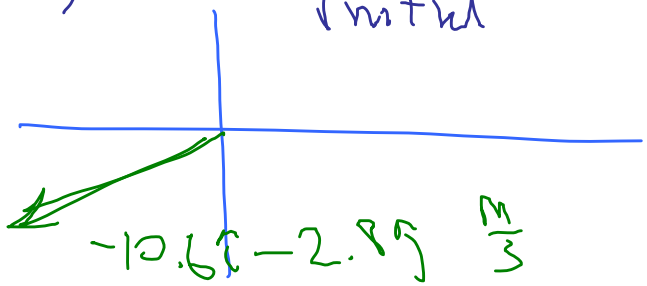
(initially)

$$v_x = -10.6\,\tfrac{m}{s}$$

$$p_y = 0 = (12\,kg)(14\,\tfrac{m}{s}) + (60\,kg)\,v_y$$

(initial)

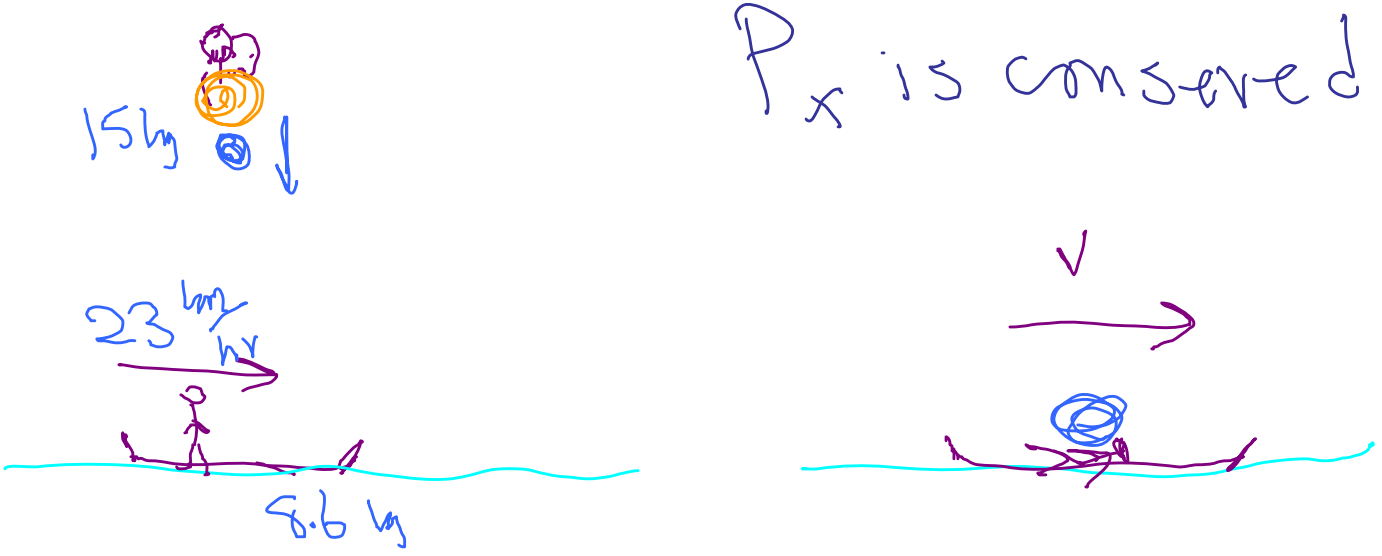
$$v_y = -2.8\,\tfrac{m}{s}$$



$$\vec{v} = \left(-10.6\,\hat{i} - 2.8\,\hat{j}\right)\tfrac{m}{s}$$

9.20 A toboggan of mass 8.6 kg is moving horizontally at 23 $\frac{km}{hr}$. As it passes under tree 15 kg snowball falls into it. Find subsequent speed.

P_x is conserved!



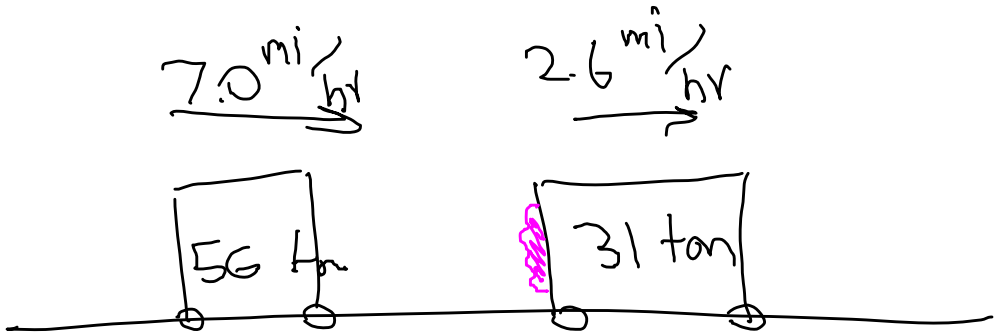
$$(8.6\,kg)\left(23\,\frac{km}{hr}\right) = (8.6\,kg + 15\,kg)\,v_f$$

$$v_f = 8.4\,\frac{km}{hr}$$

9.26 In railroad switchyard 56-ton freight car is sent at 7.0 mi/h toward 31-ton car moving in same dir. at 2.6 mi/hr.

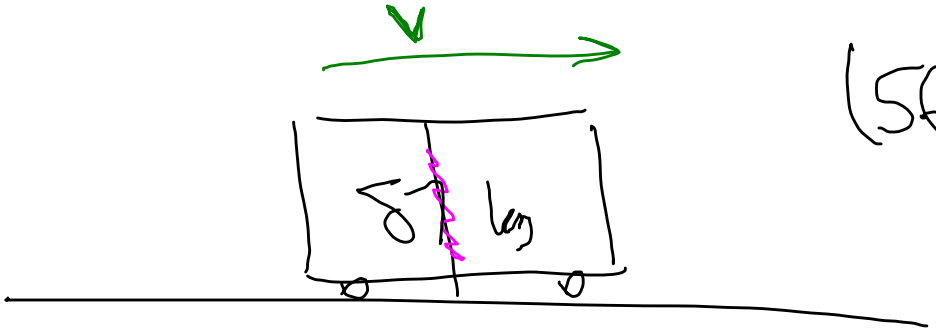
a) Speed of cars after they couple?

b) What fraction of initial KE was lost in collision.



P_x is conserved

$P_i = P_f$



$(56\ \text{ton})(7.0\ \frac{\text{mi}}{\text{hr}}) + (31\ \text{ton})(2.6\ \frac{\text{mi}}{\text{h}}) = (87\ \text{ton})\,V$

$V = 5.4\ \frac{\text{mi}}{\text{h}}$

Momentum is conserved.

Energy not conserved.

b) Frac of kinetic energy lost?

$K = \frac{1}{2} m v^2$ Calc $K_{\text{Total, initial}}$ $K_{\text{Total final}}$

$\frac{1}{2}\left(\quad \text{tan}\right)\left(\frac{mi}{hr}\right)^2$

$\text{frac}_{\text{lost}} = \frac{K_f - K_i}{K_i}$

$= -13\%$

inelastic

elastic