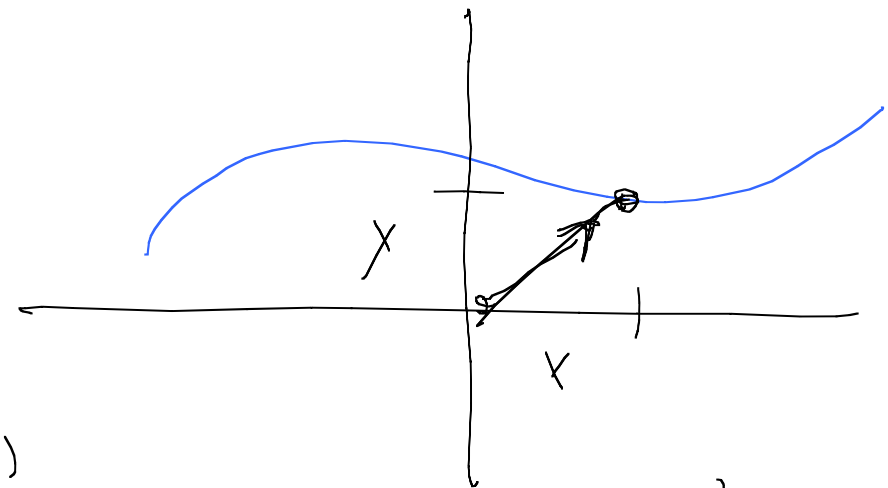
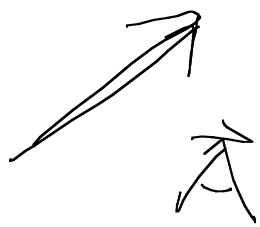
Phys 2110-3 9/13/10

2-D Motion



x, t

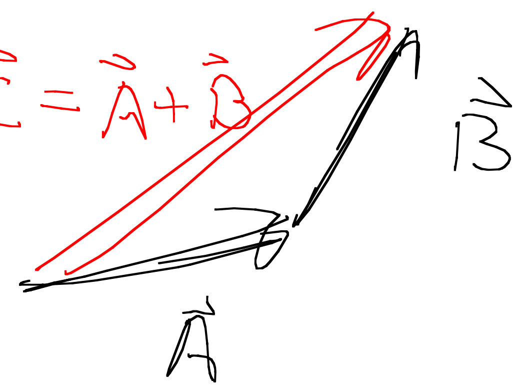
Vectors



Magnitude,
direction

Added ~~toge~~ together

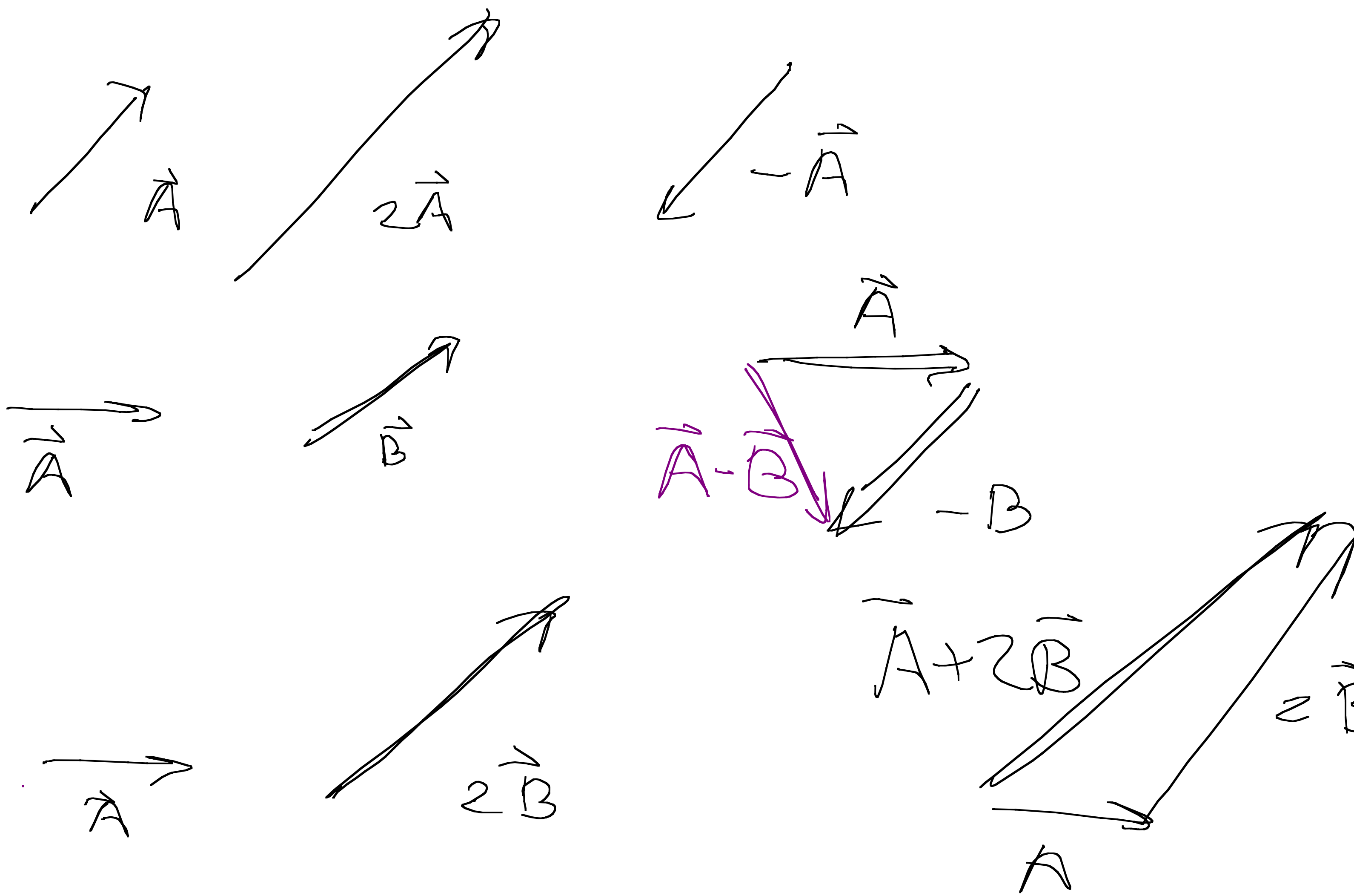
$\vec{C} = \vec{A} + \vec{B}$



Multiplied by number

Vectors

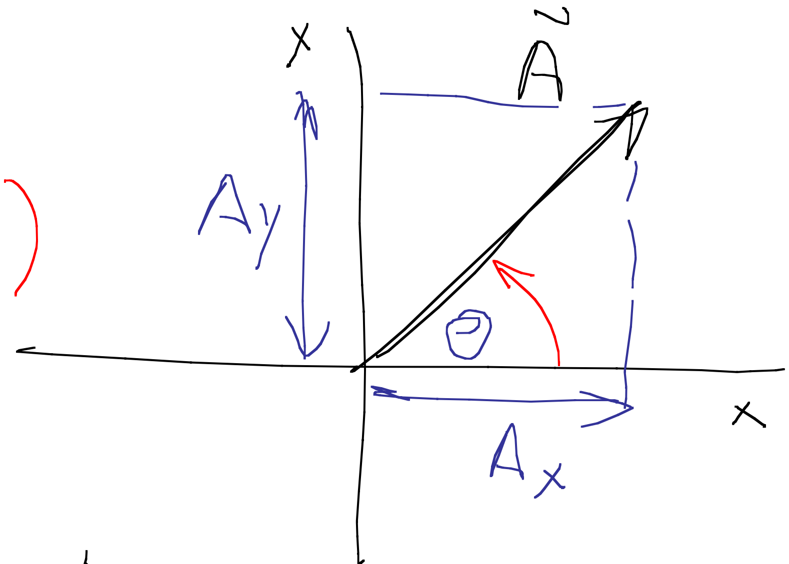
Scalar



Vs components

Magnitude of $\vec{A}$ (length)

$= A$



$A_x = A\cos\theta \qquad A_y = A\sin\theta$

$A = \sqrt{A_x^2 + A_y^2} \qquad \tan\theta = \frac{A_y}{A_x} \qquad \frac{\sin\theta}{\cos\theta}$

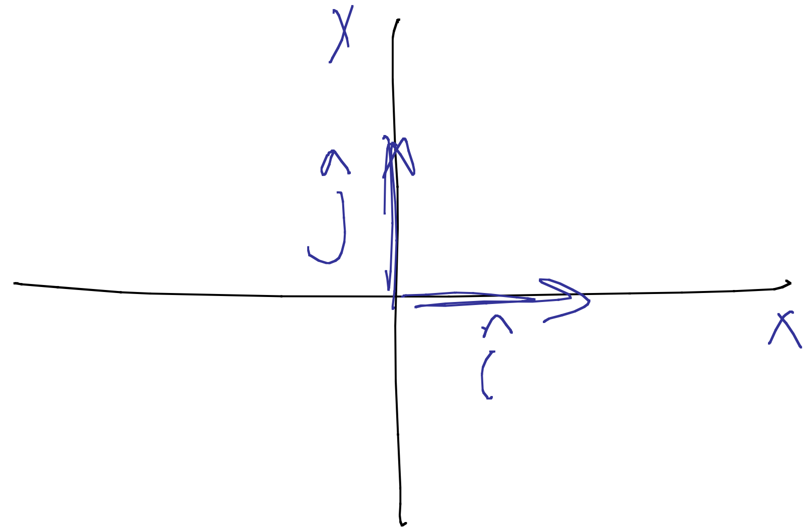
Be careful!! Get the quadrant right!

A_x, A_y both neg,

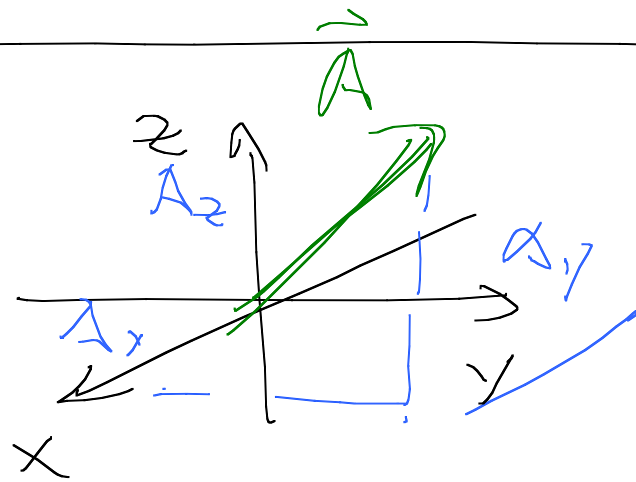
III $\quad 180° < \theta < 270°$

$\hat{i}, \hat{j}$ Unit vectors

$\vec{A} = A_x \hat{i} + A_y \hat{j}$



3-D

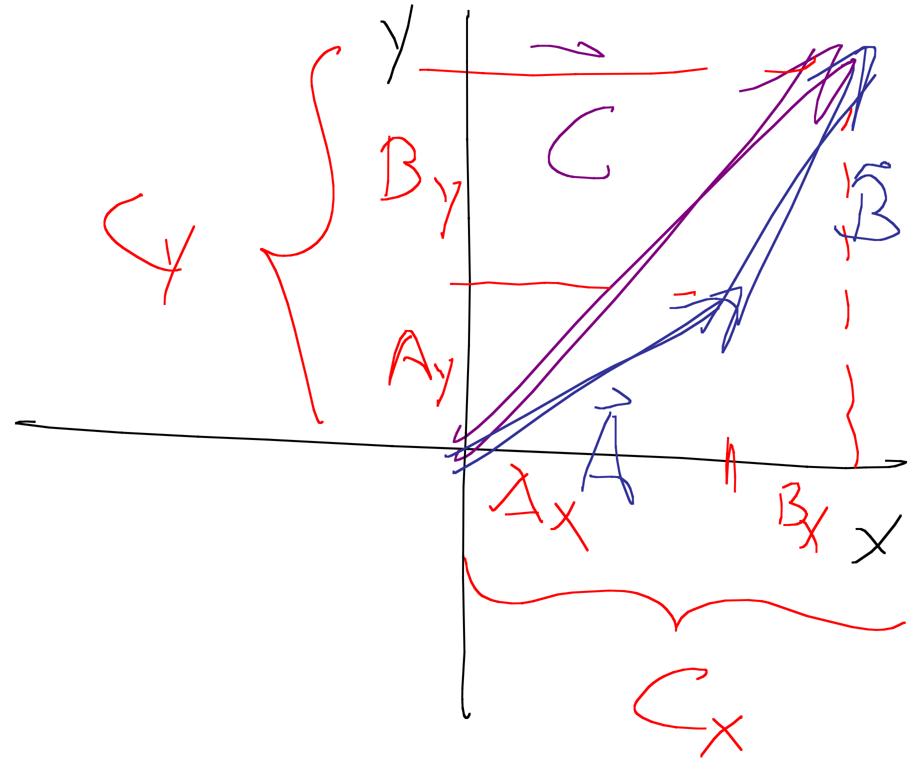


$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

Adding vectors: Use components

$\vec{A} \rightarrow A_x, A_y$

$\vec{B} \rightarrow B_x, B_y$

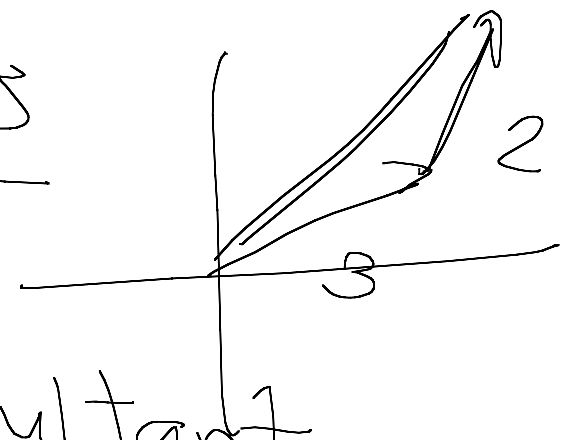


$$C_x = A_x + B_x$$

$$C_y = A_y + B_y$$

Resolve vectors into components

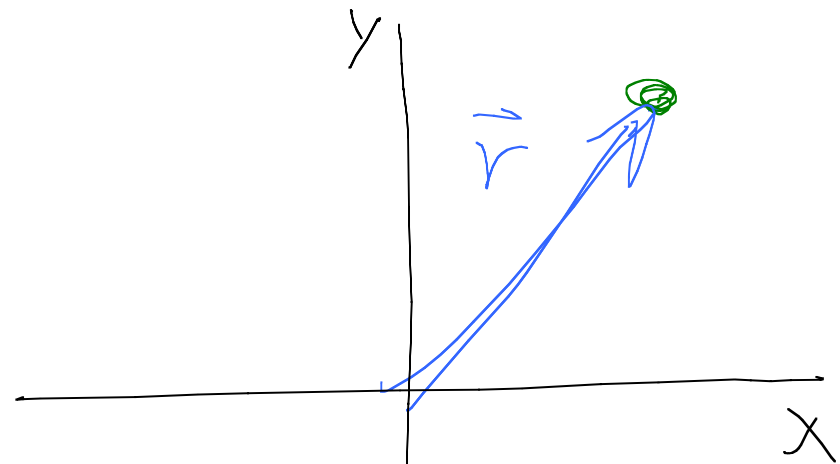
Add components separately



Then find dir & mag of resultant

$\vec{r}$ displacement
location (?)

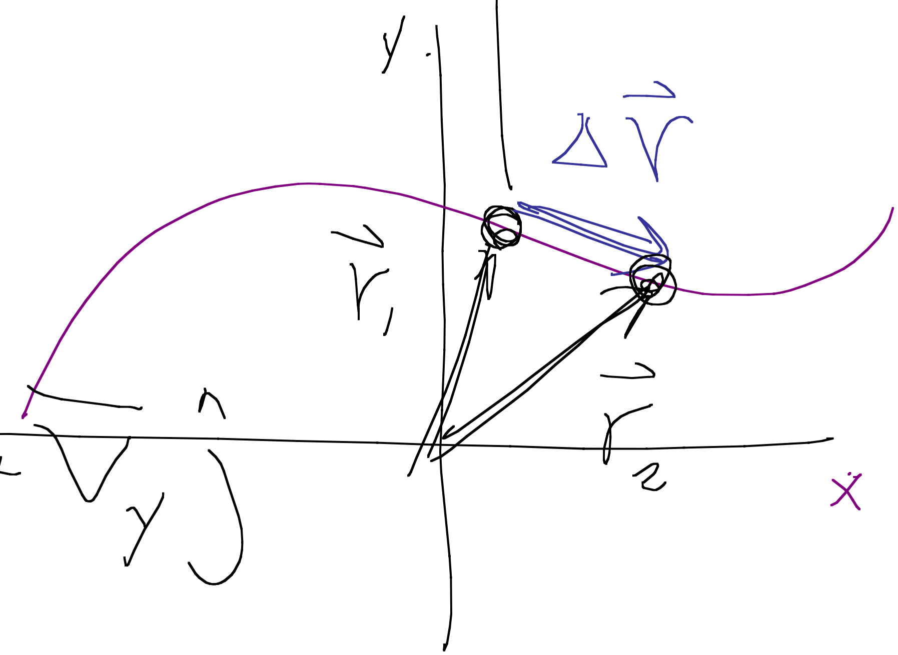
$$\vec{r} = x\hat{i} + y\hat{j}$$
$$= 3.0\,m\,\hat{i} + 4.0\,m\,\hat{j}$$



$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \bar{v}_x\hat{j} + \bar{v}_y\hat{j}$$

$$= \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta x}{\Delta t}\hat{j}$$



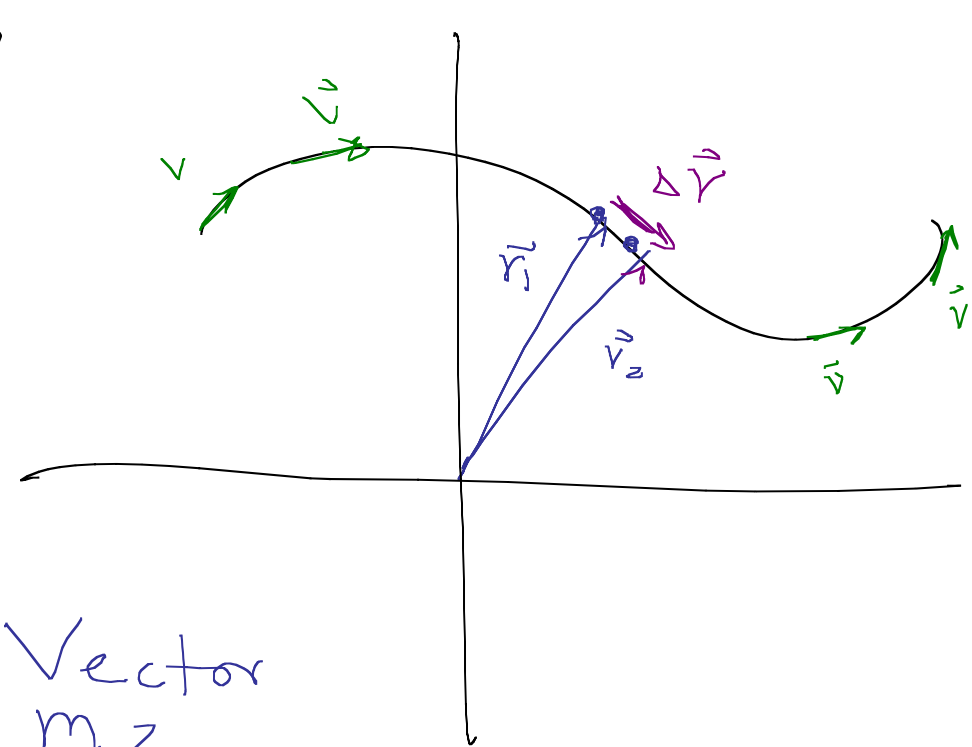
$$\Delta r = \vec{r}_2 - \vec{r}_1$$

Instantaneous velocity

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{i} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\hat{j}$$

$$= v_x \hat{i} + v_y \hat{j}$$



Vector

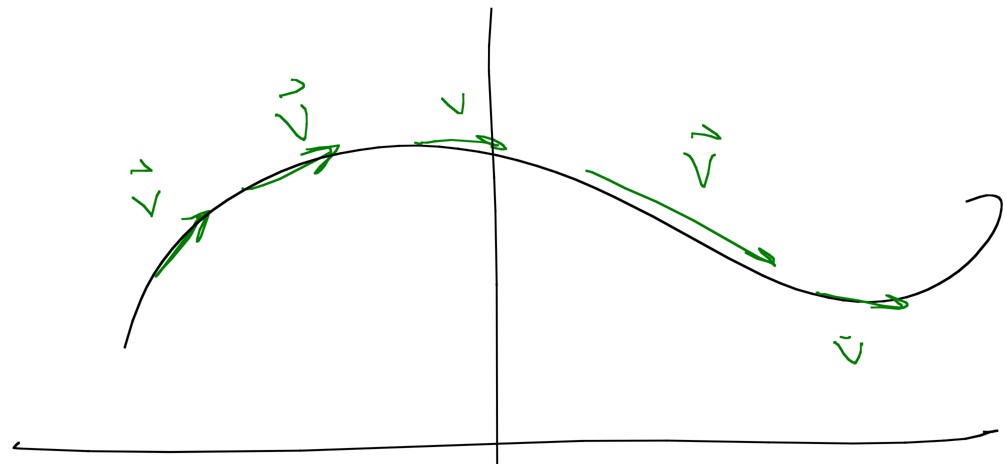
m/s

Instantaneous speed = Mag of $\vec{v} = v = |\vec{v}|$

$$= \sqrt{v_x^2 + v_y^2}$$

Velocity vector is always tangent to path

Velocity vector can change with time.



If $\vec{v}$ is constant

$$V_x = \text{const} = \frac{dx}{dt}$$

$$x = C_1 + C_2 t$$

$$V_y = \text{const} = \frac{dy}{dt}$$

$$y = C_3 + C_4$$

$$X = X_0 + V_x t$$

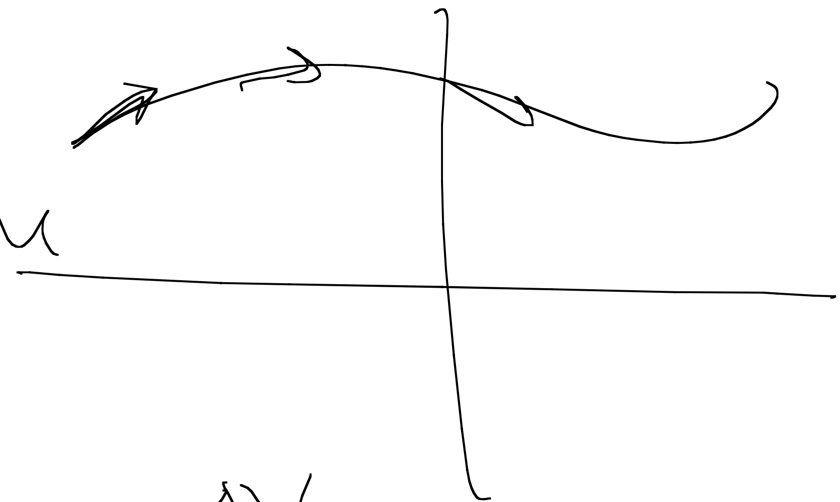
(Initial value: $V_x \to V_{0x}$)

$$y = y_0 + V_y t$$

$V_y - V_{0y}$

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

How does $\vec{v}$ change w/ time



Acceleration

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

Limit $\Delta t \to 0$

$$\vec{a} = \underset{\Delta t \to 0}{\frac{\Delta \vec{v}}{\Delta t}} = a_x \hat{i} + a_y \hat{j}$$

Vector

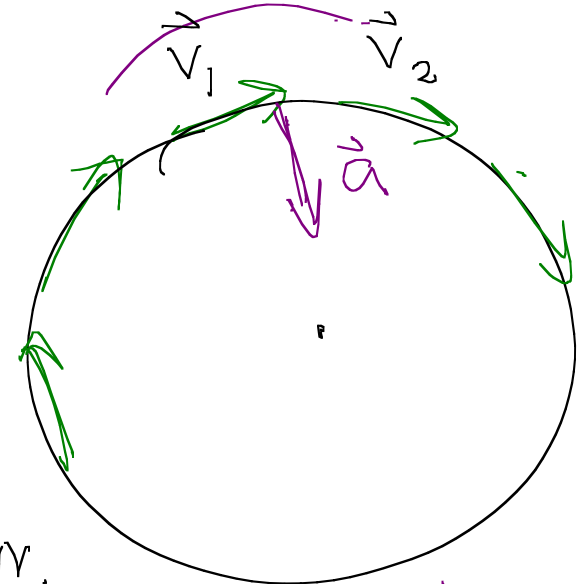
Units $= m/s^2$

$$a_x = \frac{\Delta v_x}{\Delta t} \quad \Delta t \to 0 \quad \text{etc.}$$

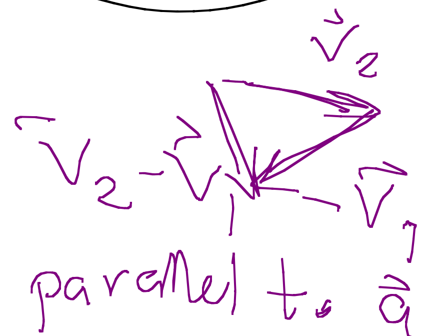
Uniform circular motion

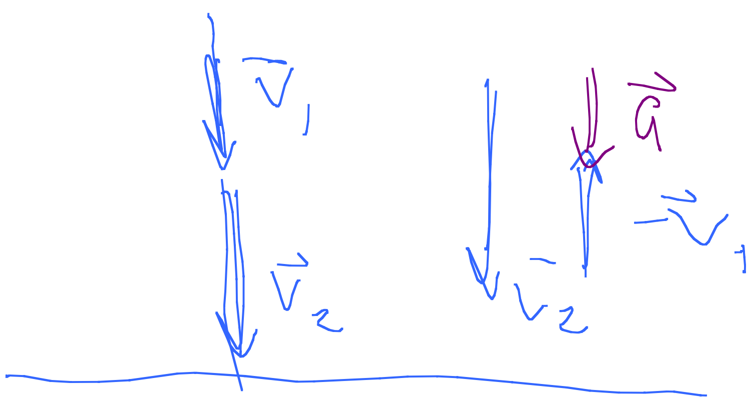
Is there an acceleration

<u>Yes</u>, $\vec{v}$ is changing in dir.

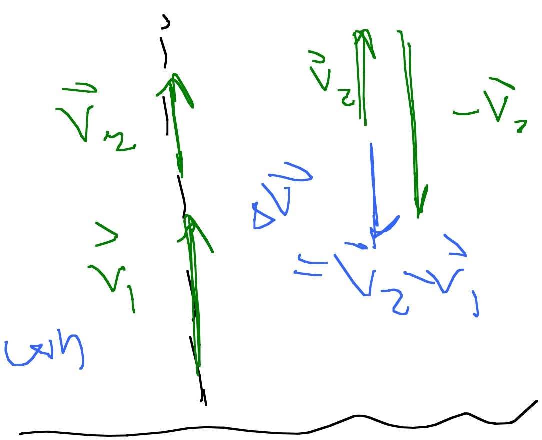


Acceleration vector points <u>inward</u>!





Toss object up,
accel vector points down



3.37 Position of object given by

$$\vec{r} = \underbrace{(3.2t + 1.8t^2)}_{x}\,\hat{i} + \underbrace{(1.7t - 2.4t^2)}_{y}\,\hat{j}$$

(in meters)

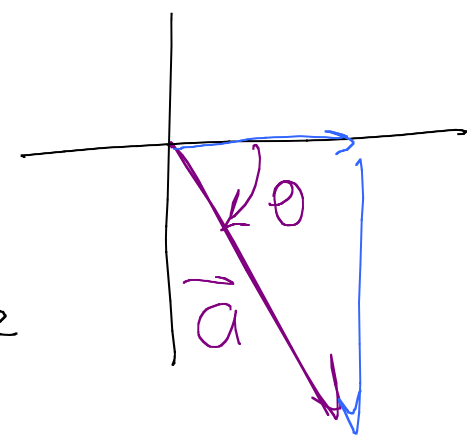
What are mag & dir of accel. vector?

$$\vec{v} = (3.2 + 3.6t)\,\hat{i} + (1.7 - 4.8t)\,\hat{j}$$

in $\frac{m}{s}$

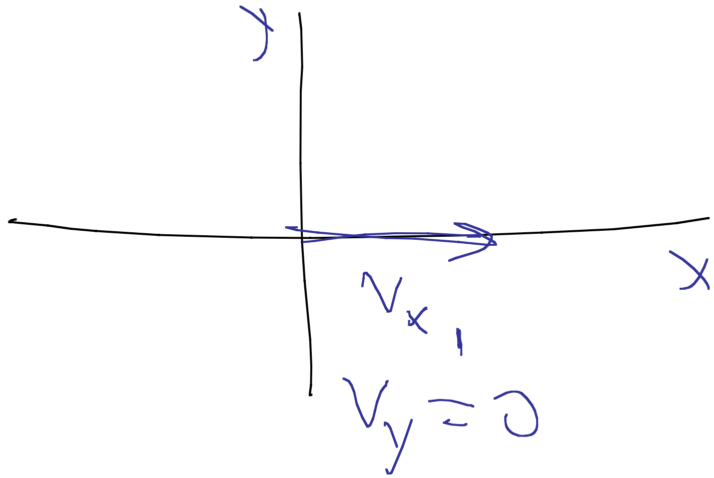
$$\vec{a} = 3.6\,\frac{m}{s^2}\,\hat{i} - 4.8\,\frac{m}{s^2}\,\hat{j}$$

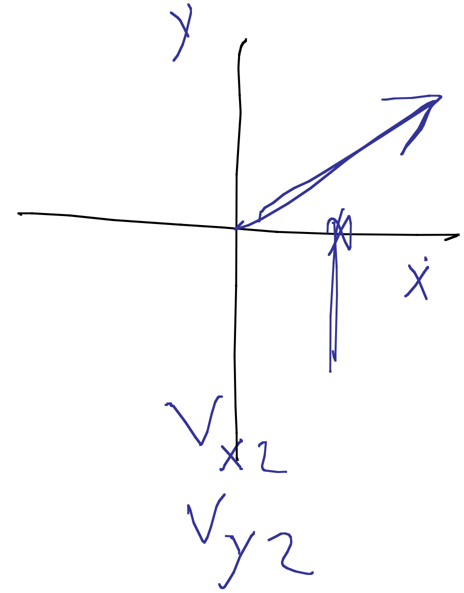
$$a = \sqrt{(3.6)^2 + (4.8)^2}\,\frac{m}{s^2} = 6.0\,\frac{m}{s^2}$$



$$a_x = \frac{\Delta V_x}{\Delta t}$$

$$a_y = \frac{\Delta V_y}{\Delta t}$$





Constant Acceleration

$$a_x = \text{const} = \frac{dv_x}{dt} \qquad a_y = \text{const} = \frac{dv_y}{dt}$$

$$v_x = a_x t + C$$

$$\boxed{V_x = V_{0x} + a_x t} \qquad \boxed{V_y = V_{0y} + a_y t}$$

$$\boxed{x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2}$$

$$\boxed{y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2}$$

$$\boxed{v_x^2 = v_{0x}^2 + 2a_x(x - x_0)}$$

$$\boxed{v_y^2 = v_{0y}^2 + 2a_y(y - y_0)}$$

$$\boxed{x = x_0 + \frac{1}{2}(v_{0x} + v_x)t}$$

$$\boxed{y = y_0 + \frac{1}{2}(v_{0y} + v_y)t}$$