

$$\tau = r F_{\perp}$$

$$\equiv r F \sin \theta$$

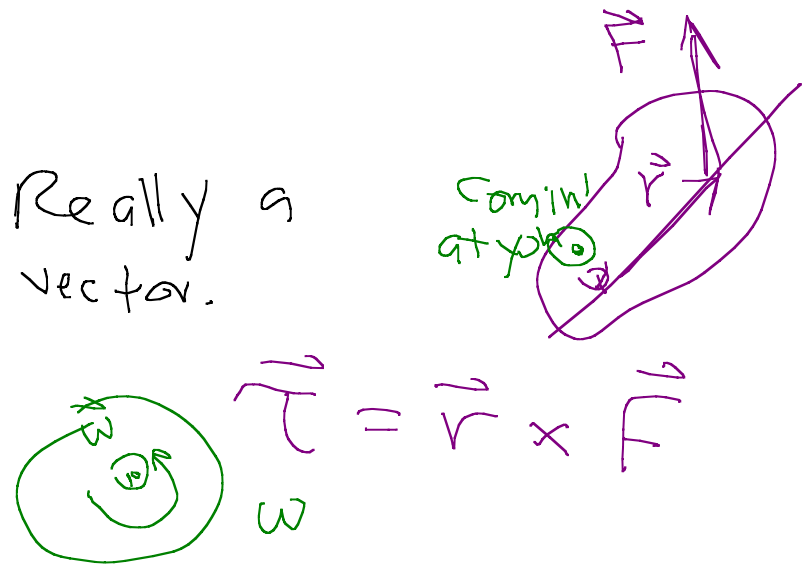
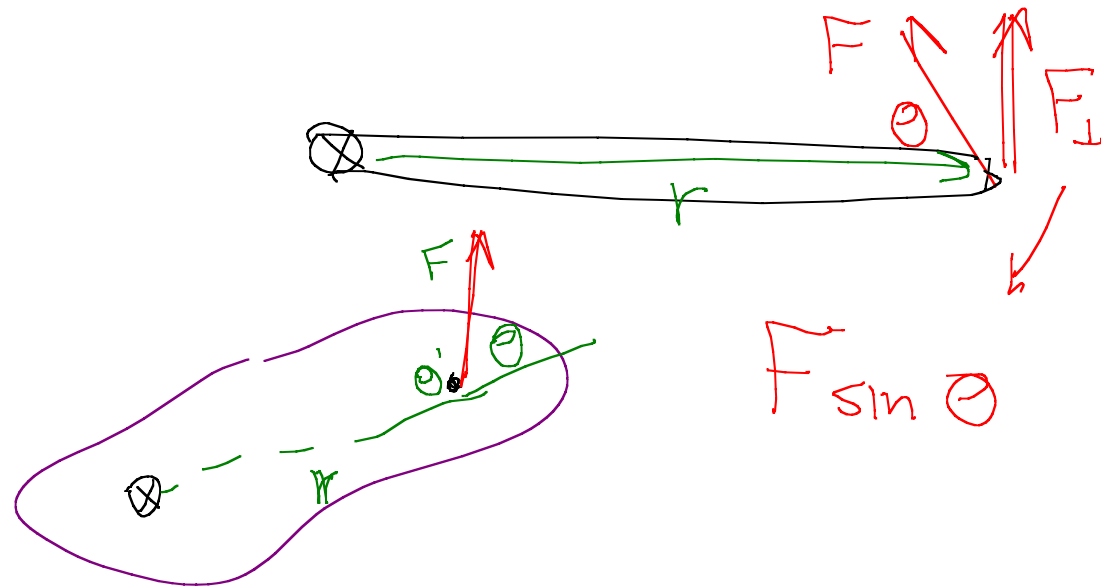
Torque

Eng: Moment of force

What is it?

Scalar (for now)

Really a vector.



$$\vec{\tau} = \vec{r} \times \vec{F}$$

Units ?

$$\tau = r F \sin \theta$$

$m \cdot N$ \downarrow

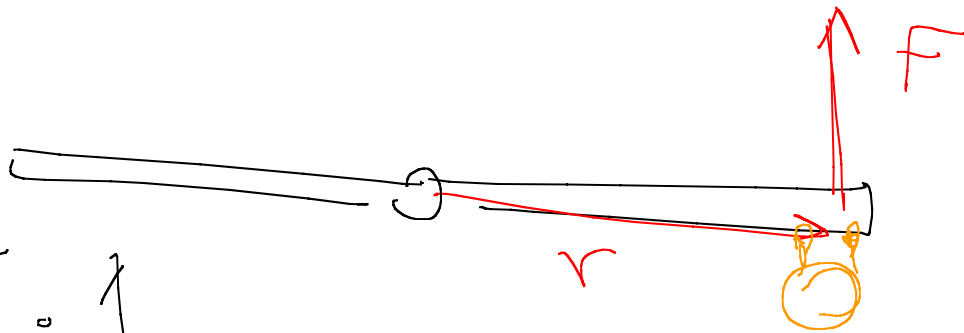
$$\underline{N \cdot m} = J$$

This is ok

Torque never combines energy

Please use $N \cdot m$

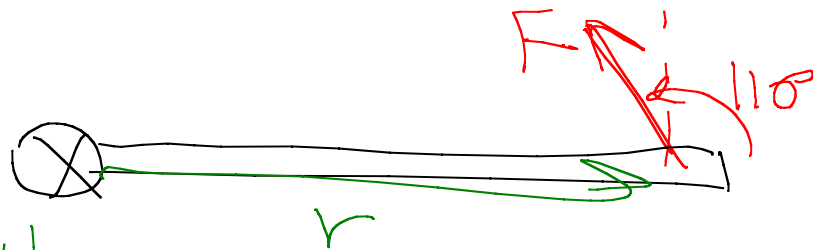
10.22 A torque of $110 \text{ N}\cdot\text{m}$ is
req'd to start revolving door.
If a child can push w/
max force 90 N , how far
from door's axis must she apply
force



$$\tau = rF \cdot 1$$

$$r = \frac{\tau}{F} = \frac{110 \text{ N}\cdot\text{m}}{90 \text{ N}} = 1.22 \text{ m}$$

10.23 Car tune-up manual calls for tightening spark plugs to $35.0 \text{ N}\cdot\text{m}$. To achieve this torque with what force you pull on end of 24-cm -long wrench if
b) Pull at 110° to wrench shaft?



$$\begin{aligned} \tau &= 35 \text{ N}\cdot\text{m} \\ r &= 0.24 \text{ m} \end{aligned}$$

$$\tau = r F \sin(110^\circ)$$

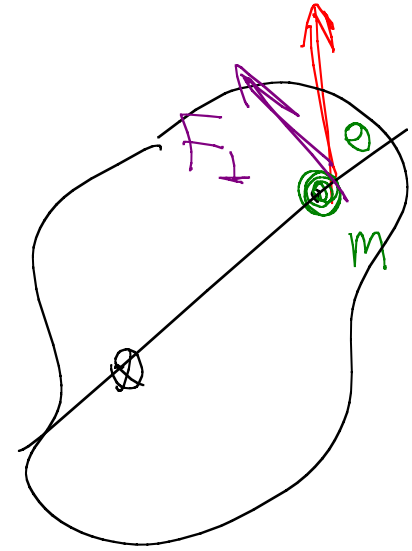
$$F = \frac{\tau}{r \sin(110^\circ)} = 155 \text{ N}$$

τ gives α

Heuristic derivation.

m (mass point) can
only move tangentially

Use N's 2nd law for tangential



$$F_{\perp} = m a_t$$

tangential

$$F \sin \theta = m (r \alpha)$$

$$r F \sin \theta = (m r^2) \alpha$$

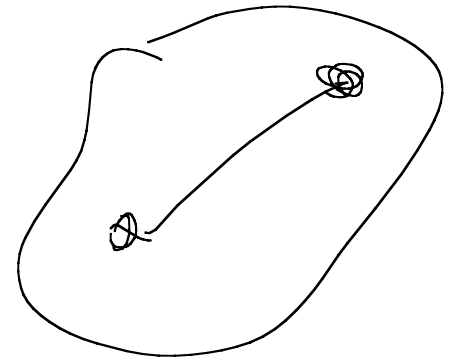
$$a_t = r \alpha$$

Mult both sides
by r

$$\tau = (m r^2) \alpha$$

For single particle. What about the whole object

Add up left & right sides



$$\begin{aligned} \tau_{\text{total}} &= \sum_i (m_i r_i^2) \alpha \\ &= \left[\sum_i m_i r_i^2 \right] \alpha \end{aligned}$$

$$\tau_{\text{total (ext)}} = \left[\sum_i m_i r_i^2 \right] \alpha$$

moment of inertia

$$\begin{array}{c} \tau \\ \downarrow \\ F = m a \end{array} \quad \begin{array}{c} I \\ \downarrow \\ F = m a \end{array} \quad \begin{array}{c} \alpha \\ \downarrow \\ F = m a \end{array}$$

$$I = \sum_i m_i r_i^2$$

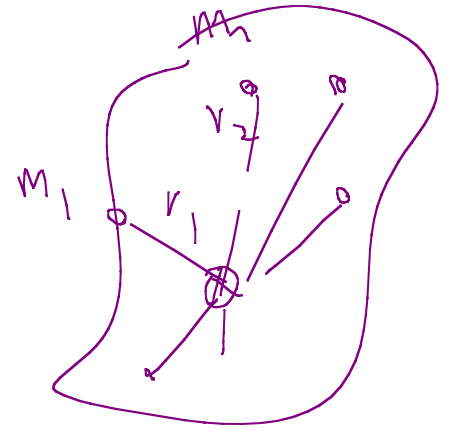
Moment of inertia

Scalar (Positive)

(Actually it's a
matrix!)

Units: kg m^2

Examples of I

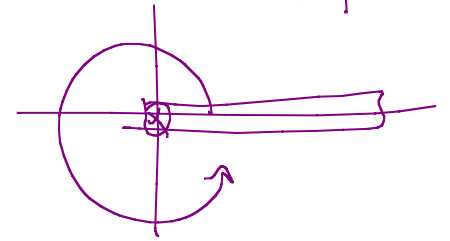
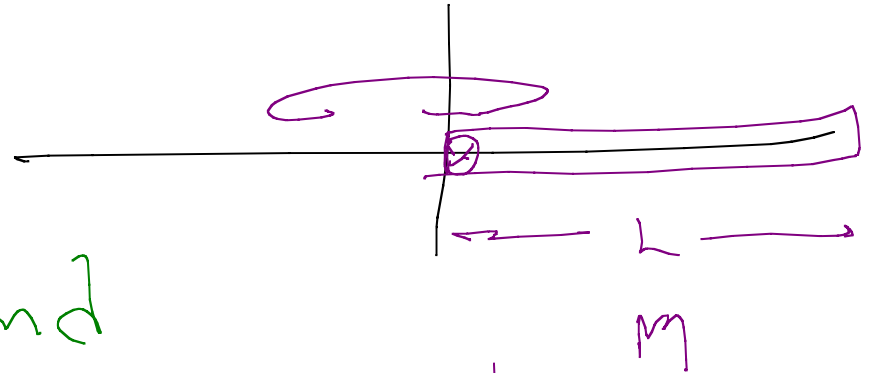


$$I = m_1 r_1^2 + m_2 r_2^2 + \dots$$

Example:

Uniform stick

rotates about end

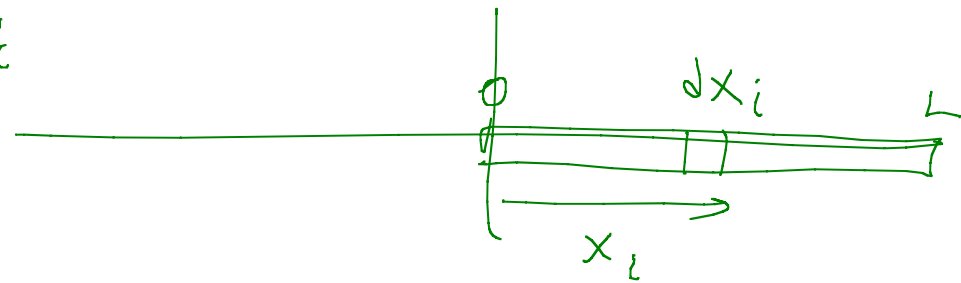


$$I = \sum_i dm_i \cdot x_i^2$$

$$= \frac{M}{L} \sum_i x_i^2 dx_i$$

$$= \frac{M}{L} \int_0^L x^2 dx$$

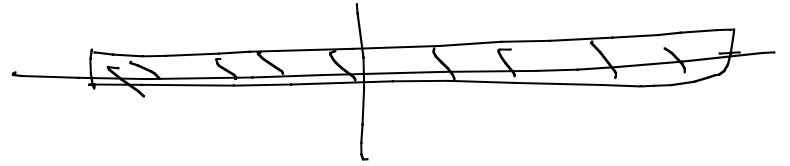
$$= \frac{M}{L} \left(\frac{1}{3} L^3 \right) = \frac{1}{3} M L^2$$



$$\text{Density} = \frac{M}{L}$$

$$dm_i = \frac{M}{L} \cdot dx_i$$

Also:



Do this as
two small sticks!

Next time