Phys 2920, Spring 2009
Problem Set #3

1. Generally matrices do not commute (for multiplication). The extent to which they do not is given by the **commutator**, which for matrices A and B is given by

$$[\mathsf{A}, \mathsf{B}] \equiv \mathsf{AB} - \mathsf{BA}$$

The following matrices are very useful in physics:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

a) Evaluate each of the three following commutators, and for each express the result in terms of the σ matrices themselves.

$$[\sigma_x, \sigma_y] \qquad [\sigma_y, \sigma_z] \qquad [\sigma_z, \sigma_x]$$

b) Are the σ matrices symmetric? Orthogonal? Hermitian? Unitary?

2. Show that the matrix

$$\mathsf{R} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

is an orthogonal matrix.

3. Find the determinant of the matrix

$$\mathsf{A} = \begin{pmatrix} 5 & 4 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 9 \\ 1 & -2 & -1 & 4 \end{pmatrix}$$

any way you can!

4. For the following matrices, figure out if each has an inverse (give a good mathematical reason) and *then* use a computer to find the inverse.

$$\text{(a)} \qquad \mathsf{A} = \begin{pmatrix} 1 & 3 & -4 \\ 1 & 5 & -1 \\ 3 & 13 & -6 \end{pmatrix} \qquad\qquad \text{(b)} \qquad \mathsf{B} = \begin{pmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{pmatrix}$$

5. Solve the set of equations

$$\begin{aligned} x + 2y - 4z &= -4 \\ 2x + 5y - 9z &= -10 \\ 3x - 2y + 3z &= 11 \end{aligned}$$

by first writing it in matrix/vector form and then using matrix inversion to get (x, y, z). You can get help from Maple for the last step.

6. Find the eigenvalues of the matrix

$$\begin{pmatrix} 2 & -3 \\ 2 & -5 \end{pmatrix}$$

Don't use a computer on this!!