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Dec. 8, 2009

Phys 2112, Fall 2009
Quiz #3

1. If a spherical body with the mass of the sun (2.00×10^{30} kg) has an escape speed equal to the speed of light c, find its radius. (Assume that Newtonian physics is correct!)

Use the formula for escape speed; solve for R:

$$v_{\rm esc} = \sqrt{\frac{2GM}{R}} \qquad \Longrightarrow \qquad v_{\rm esc}^2 = c^2 = \frac{2GM}{R} \qquad \Longrightarrow \qquad R = \frac{2GM}{c^2}$$

Plug in numbers:

$$R = \frac{2(6.67 \times 10^{-11})(2.00 \times 10^{30})}{(2.998 \times 10^8)^2}\,\mathrm{m} = 2.97 \times 10^3\,\mathrm{m} = 2.97\,\mathrm{km}$$

The sun must squoosh down to a radius of $3\,\mathrm{km}$ before light is trapped at its surface!

2. In the derivation of the shape of an orbit for the gravitational force, we started with a differential equation for r as a function of time: $r(t)$. But that function does not give the *shape* of the orbit.

What did we have to do the differential equation to get a function which *does* tell us the shape of the orbit? (Summarize as much of the math as you can in a paragraph.)

What we need to tell the shape of the orbit is the function $r(\phi)$, which will give a (polar) plot of the path of the orbiting object. To get this, we had to change the time derivatives $\frac{d}{dt}$ in the differential equations to angle derivatives $\frac{d}{d\phi}$, using the chain rule; thus we got a differential equation in terms of the variable ϕ. (Solving this differential equation was quite doable but required a couple tricks!)

The relation between the two derivatives involved a factor of the constant angular momentum ℓ.

3.a) Find the speed of a proton which has a kinetic energy of 2.0 GeV.

If $T = 2.0\,\mathrm{GeV}$ then the total energy is

$$E = m_{\rm prot}c^2 + T = 0.9383\,\mathrm{GeV} + 2.0\,\mathrm{GeV} = 2.938\,\mathrm{GeV}\ .$$

We can get the speed from this by

$$E = \frac{m_{\rm prot}c^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \Longrightarrow \qquad \sqrt{1-\frac{v^2}{c^2}} = \frac{m_{\rm prot}c^2}{E} = \frac{0.938}{2.938} = 0.319$$

From this it follows that

$$\frac{v^2}{c^2} = 0.898 \qquad \Longrightarrow \qquad \frac{v}{c} = 0.947 \qquad \Longrightarrow \qquad v = 2.84\times 10^8\,\tfrac{\rm m}{\rm s}$$

b) What answer would you get if you used Newtonian physics?

The kinetic energy is

$$T = 2.0\times 10^9\,\mathrm{eV}\left(\frac{1.602\times 10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}\right) = 3.2\times 10^{-10}\,\mathrm{J}$$

Using $T = \frac{1}{2}mv^2$ we get

$$v^2 = \frac{2T}{m} = \frac{2(3.2\times 10^{-10}\,\mathrm{J})}{(1.67\times 10^{-27}\,\mathrm{kg})} = 4.68\times 10^{17}\,\tfrac{\rm m^2}{\rm s^2} \qquad \Longrightarrow \qquad v = 6.2\times 10^8\,\tfrac{\rm m}{\rm s}$$

(Whoa! Faster than light!)

4. (Here we disregard the potential energy of a particle.) How does our old expression for the energy of a particle, $E = \frac{1}{2}mv^2$ relate to the new *relativistic* total energy E? (The result was found on the last problem set; tell what you remember of it.)

On the problem set, we showed that the *relativistic* kinetic energy can be written as

$$E - mc^2 \ = T \ = \tfrac{1}{2}mv^2 + \text{ lotsa terms with } v\text{'s and } c\text{'s}$$

where the extra terms are small when v is small compared to c. Thus the (correct) total energy is roughly equal to $mc^2 + \frac{1}{2}mv^2$. Basically Newtonian physics is ignoring all those extra terms!

Show work for all problems and include the right units!

$$m_{\rm prot} = 1.67\times 10^{-27}\,\mathrm{kg} \qquad m_{\rm prot}c^2 = 938.3\,\mathrm{MeV} \qquad 1\,\mathrm{eV} = 1.602\times 10^{-19}\,\mathrm{J}$$

$$F = G\frac{m_1 m_2}{r^2} \qquad G = 6.67\times 10^{-11}\,\tfrac{\rm N\cdot m^2}{\rm kg^2} \qquad a_c = \frac{v^2}{r} \qquad F_c = \frac{mv^2}{r} \qquad c = 2.998\times 10^8\,\tfrac{\rm m}{\rm s}$$

$$v_{\rm esc} = \sqrt{\frac{2GM}{R}} \qquad E = p^2c^2 + m^2c^4 \qquad E = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} \qquad T = E - mc^2$$