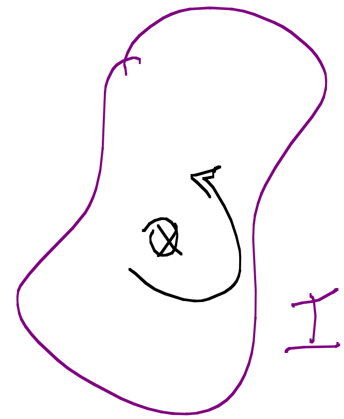
Phys 2110-3 11/08/10

Note Title 11/8/2010

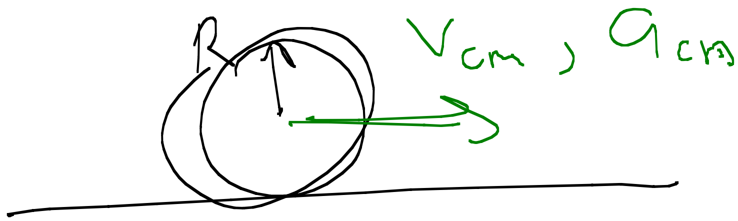
Rotational Dynamics

$$\tau = I\alpha$$

$$F = ma$$



Rolling Motion

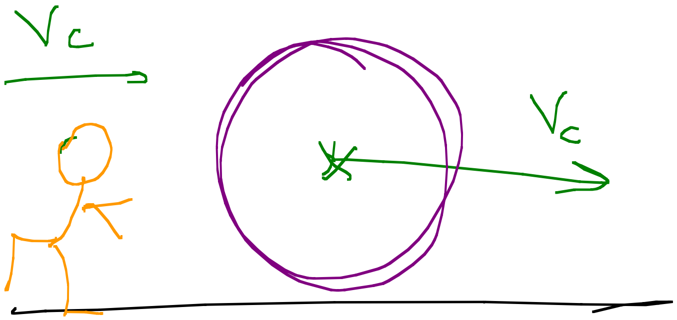


$$v_{cm} = R\omega$$

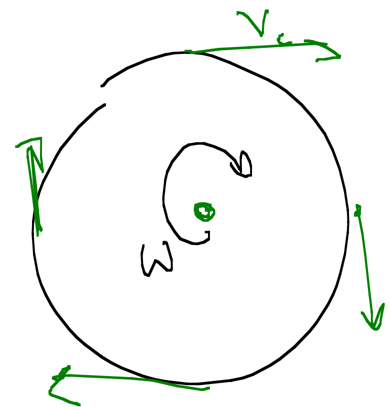
$$a_{cm} = R\alpha$$

Velocities of the points of the rolling object

what does man see?



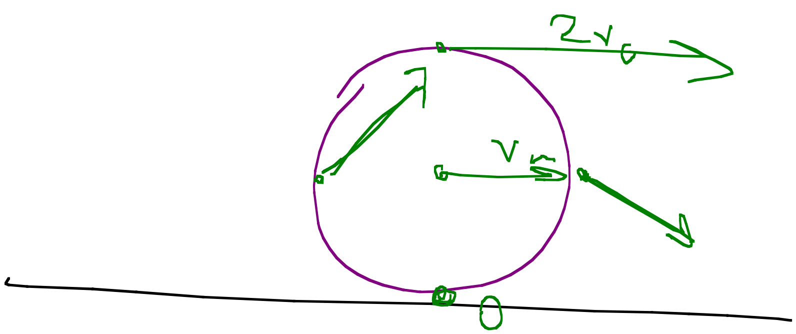
$v_c = R\omega$

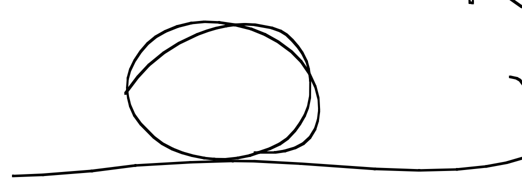


Pure rotation.

Back in the lab frame:

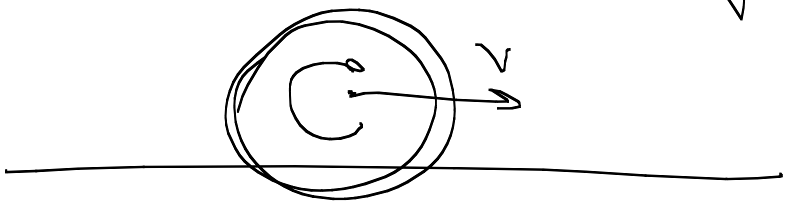
Add $\vec{v}_{cm}$ onto everything.





What is KE of rolling obj.

$v = R\omega$



Just sliding

$K_{trans} = \frac{1}{2} m v^2$

Just rotating

$K_{rot} = \frac{1}{2} I \omega^2$

$\omega = v/R$

It turns out:

$$\boxed{K_{tot} = K_{trans} + K_{rot}}$$

$$\boxed{= \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2}$$

10.41 What fraction of a solid disk's kinetic energy is rotational if it is rolling w/o slipping?

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Rolling disk

$I = \frac{1}{2}mR^2$

$\omega = v/R$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\frac{1}{2}mR^2\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$= \frac{3}{4}mv^2$$

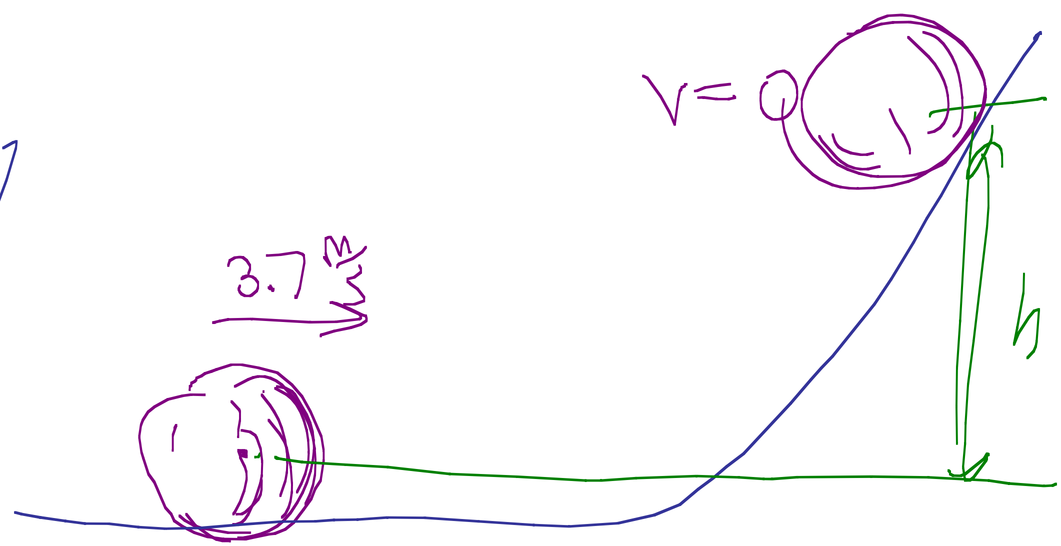
$$K_{rot} \underset{\text{2nd term}}{=} \frac{1}{4}mv^2$$

So $\text{Fraction} = \frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{4}mv^2}{\frac{3}{4}mv^2} = \frac{1}{3}$

10.62

A hollow ball is rolling along a horiz. surface at $3.7\ \frac{m}{s}$ when it encounters an upward incline. If it rolls w/o slipping up incline what max height will it reach?

Cons of Energy



on flat part

$$K_{Tot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{3}mv^2$$

$$= \frac{5}{6}mv^2$$

$$\text{final PE} = mgh$$

Math:

$$h = \frac{5}{6}\,v^2/g$$

$$\boxed{= 1.16\ m}$$

Example

what is accel of ball rolling down hill?

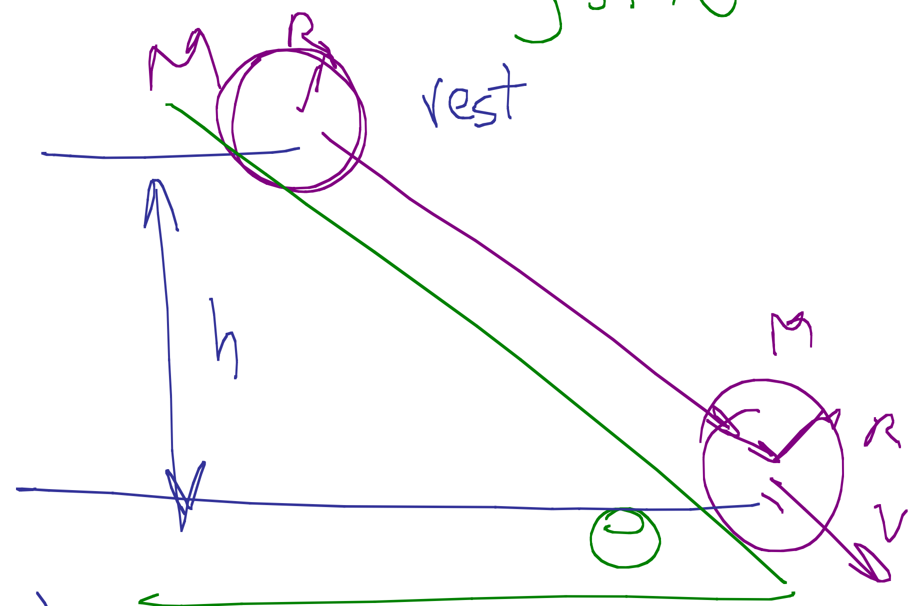
incline, θ

$9.8 \frac{m}{s^2}$

$g \sin\theta$

PE lost = KE gained

Solid ball, $I = \frac{2}{5} M R^2$



$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} m v^2 + \frac{1}{2} \left(\frac{2}{5} m R^2\right)\left(\frac{v}{R}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

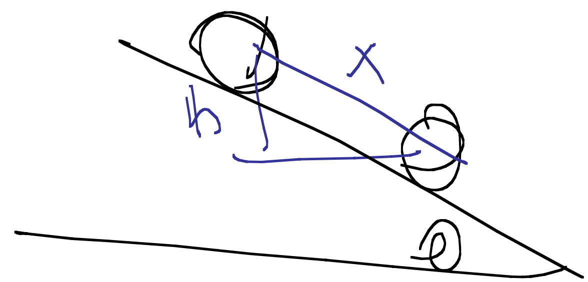
$$= \frac{7}{10}mv^2$$

$$gh = \frac{7}{10}v^2$$

$$g\, x \sin\theta = \frac{7}{10}v^2$$

$$v^2 = \frac{10}{7}(g\sin\theta)\, x$$

$$v^2 = 2\left[\frac{5}{7}g\sin\theta\right] x$$



$$x = \frac{h}{\sin\theta}$$

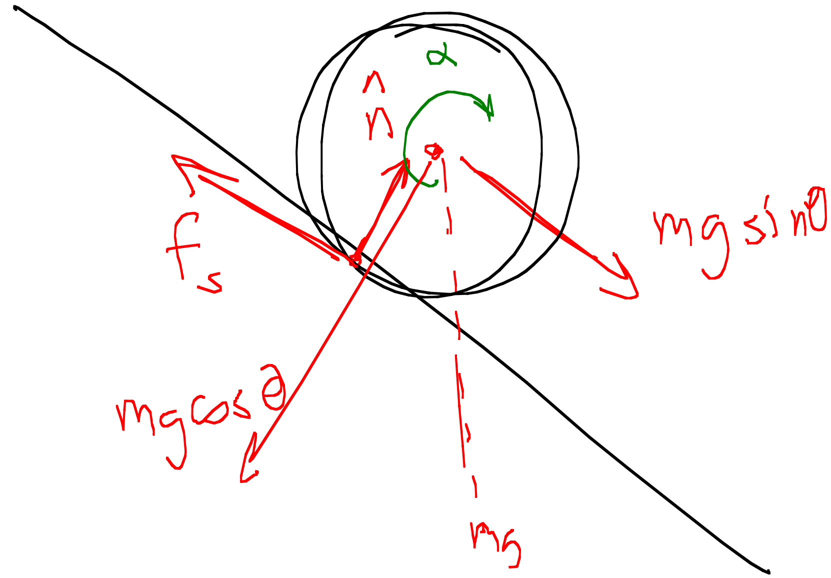
$$v^2 = v_0^2 + 2a_x x$$

$$v_0 = 0$$

$$a = \frac{5}{7}g\sin\theta$$

Another way:

Forces, torques



2nd Law forces:

$$F_{down} = mg\sin\theta - f_s = ma$$

$$\tau_{net} = I\alpha = \frac{2}{5}mR^2\left(\frac{a}{R}\right) = f_s R$$

Gravity gives no torque

$$F = mg\sin\theta - f_s = ma$$

$$f_s R = \frac{2}{5} maR$$

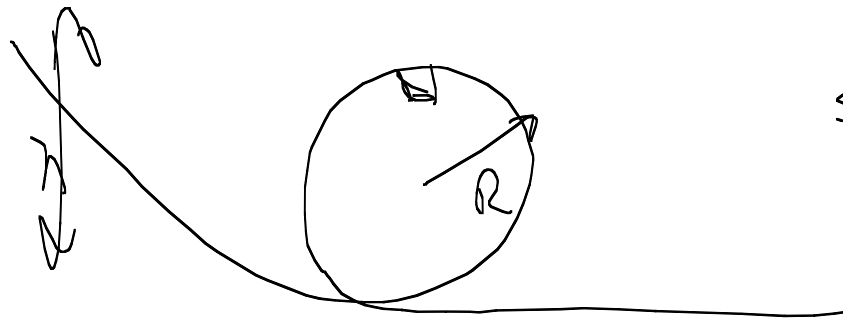
$$f_s = \frac{2}{5} ma$$

$$F = ma = mg\sin\theta - \frac{2}{5} ma$$

$$\frac{7}{5}\cancel{m}a = \cancel{m}g\sin\theta$$

$$\boxed{a = \frac{5}{7} g\sin\theta}$$

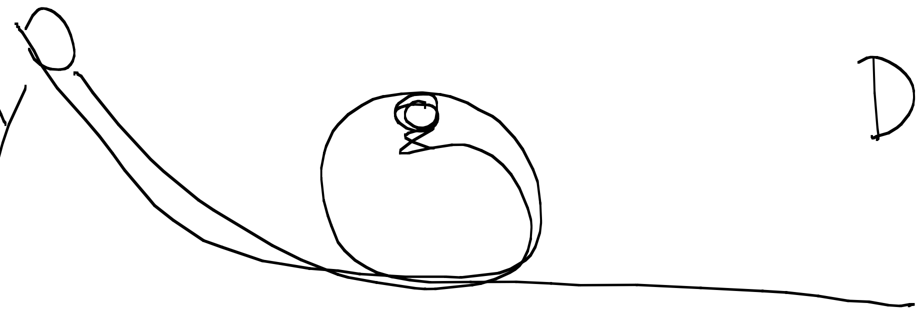
More probs



sliding

$h = \frac{5}{2}R$

Cons of energy



Different answer