Phys 3810, Spring 2009
Problem Set #6, Hint-o-licious Hints

1. *Griffiths*, **4.55** Both terms are eigenfunctions of L^2 with the *same* eigenvalue, but they are eigenfunctions of L_z with *different* eigenvalues. The coeffients squared give the probabilities for the results for measurement of L_z. Likewise, the terms are eigenfunctions of S^2 with the same eigenvalue and eigenfunctions of S_z with different eigenvalues.

For the first term the decomposition into states of *total* angular momentum is, using the C–G tables,

$$|1\ 0\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}|\tfrac{3}{2}\ +\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{3}}|\tfrac{1}{2}\ \tfrac{1}{2}\rangle$$

and do a similar decomposition of the second term, that is, the state

$$|1\ 1\rangle|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

You should arrive at a new expression for the given state in terms of states of total angular momentum,

$$R_{21}\left(\tfrac{2\sqrt{2}}{3}|\tfrac{3}{2}\ \tfrac{1}{2}\rangle + \tfrac{1}{3}|\tfrac{1}{2}\ \tfrac{1}{2}\rangle\right)$$

and this is a combination of eigenfunctions of J^2 and J_z with different eigenvalues for J^2 but the *same* eigenvalues for J_2 (namely $\hbar/2$).

2. *Griffiths*, **4.35** This one is a short easy (?) answer. Recall that spins s_1 and s_2 can "add" to give all spins from

$$s_1 + s_2 \qquad \text{down to} \qquad |s_1 - s_2|$$

3. *Griffiths*, **4.52** Follow the example of spin given out in class (G's problem 4.31). The eigenvectors of S_z are

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Using (4.136), find the action of the raising and lowering operators S_+ and S_- on all the eigenstates $|\frac{3}{2}\ m\rangle$ and then construct the matrices for these operators. Get S_x from $S_x = \frac{1}{2}(S_+ + S_-)$. You should get

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$

but *show* this!

You just need to get eigenvalues of S_x but it should be clear what they ought to be! For this you need to take the determinant of a 4×4 matrix which needs to be done by an expansion (not by zipping along all the diagonals as you can for 3×3).

4. *Griffiths*, **5.1** You should be able to get the algebraic relations between $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}$ and $\mathbf{R}$. Note that the x, y and z components of the vectors don't mix.

How do we change the ∇ operator to new coordinates? Originally in the Schödinger equation we have the operators

$$\frac{\partial}{\partial x_1}, \frac{\partial}{\partial y_1}, \ldots, \frac{\partial}{\partial x_2}, \quad \text{etc.}$$

To change the first one, we go from (x_1, x_2) to (x, X) by:

$$\frac{\partial}{\partial x_1} = \frac{\partial X}{\partial x_1}\frac{\partial}{\partial X} + \frac{\partial x}{\partial x_1}\frac{\partial}{\partial x} \qquad \frac{\partial}{\partial x_2} = \frac{\partial X}{\partial x_2}\frac{\partial}{\partial X} + \frac{\partial x}{\partial x_2}\frac{\partial}{\partial x}$$

and you can get $\frac{\partial X}{\partial x_1}$ etc. from the relations between the coordinates. Put everything together, use the definition of the reduced mass and eventually arrive at Griffiths' expression for the transformed ∇^2 operators. (Note that there are cross terms with ∇_R and ∇_r, but they cancel out.)

5. *Griffiths*, **5.2** (a) Show that the fractional difference between $m_{\rm e}$ and $\mu_{\rm H}$ is 5.4×10^{-4}. This is the same as the fractional change in the binding energy. (Show all of this!)

(b) It's same frastional correction to R; one finds that for the H atom

$$R_H = 1.096 \times 10^7 \ \mathrm{m}^{-1}$$

The fractional difference between μ_H and μ_D (reduced masses for the H and D atoms) is 2.7×10^{-4}. Take differentials to get the fractional change in the Balmer wavelength; it comes out to about 17.9 nm.

(c) The reduced mass for positronium is half the electron mass!

(d) The reduced mass for muonium is 185.9 times the electron mass. That's the factor by which you need to fix R from the value given in the book. With this new value of R, get Lyman-α. It comes out to about 6.54×10^{-10} m.

6. *Griffiths*, **5.3** The energy of the photon emitted in the transition (always between adjecent HO states) is $\hbar\omega$. The *frequency* of the radiation is

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$$

where μ is the reduced mass of the oscillator system. Show that if μ changes, the change in frequency is related to the change in μ by

$$d\nu = -\frac{1}{2}\nu\frac{d\mu}{\mu}$$

What is the frational difference in reduced mass between the two molecules?