

**Phys 2920, Spring 2009**  
**Problem Set #8**

1. Go back to Problem 5 on the last set and find the divergence of that vector field,  $\nabla \cdot \mathbf{a}$ . Then for the cylindrical volume of that problem, evaluate  $\int_V (\nabla \cdot \mathbf{a}) dV$ .

Did you get what you expected?

2. (VA 6.55) If  $S$  is any closed surface enclosing a volume  $V$  and  $\mathbf{A} = ax\mathbf{i} + by\mathbf{j} + cz\mathbf{k}$ , prove that

$$\int \int_S \mathbf{A} \cdot \mathbf{n} dS = (a + b + c)V$$

3. (VA 6.53) Verify the divergence theorem for  $\mathbf{A} = 2x^2y\mathbf{i} - y^2\mathbf{j} + 4xz^2\mathbf{k}$  taken over the region in the first octant bounded by  $y^2 + z^2 = 9$  and  $x = 2$ .

4. (VA 6.63) Verify Stokes' theorem for  $\mathbf{A} = (y - z + 2)\mathbf{i} + (yz + 4)\mathbf{j} - xz\mathbf{k}$ , where  $S$  is the surface of the cube  $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$  above the  $xy$  plane.

5. Check Stokes' theorem using the function  $\mathbf{v} = 2y\mathbf{i} + 3x\mathbf{j}$  where the path is the unit circle in the  $xy$  plane. (This is the same thing as checking "Green's theorem in a plane" for this case.

6. Evaluate:

a)  $\int_0^1 \cos x \delta(x - \frac{\pi}{4}) dx$

b)  $\int_0^4 (3x^2 - 2x - 1)(\delta(x - 2) + \delta(x - 5)) dx$

c)  $\int_V (5\mathbf{r}^2 - 2\mathbf{r} \cdot \mathbf{c} - 7) \delta^3(\mathbf{r} - 2\mathbf{k}) dV$  where  $\mathbf{c} = 3\mathbf{i} - 5\mathbf{k}$  and  $V$  is the sphere of radius 3 centered at the origin.

7. (CV 1.53 g)) Evaluate, in simple  $x + iy$  form,

$$\frac{(2+i)(3-2i)(1+2i)}{(1-i)^2}$$

8. (CV 1.54 b, j)) If  $z_1 = 1 - i, z_2 = -2 + 4i, z_3 = \sqrt{3} - 2i$ , find

$$(a) \quad \left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right| \qquad (b) \quad \text{Im} \{z_1 z_2 / z_3\}$$