Astr 1020

Problem Set #1, Solutions

1. Radio waves are EM waves; the frequency of these waves is

$$f = 1590 \text{ kHz} = 1590 \times 10^3 \text{ Hz} = 1.590 \times 10^6 \text{ Hz}$$

and their speed is the speed of light, $v = c = 3.00 \times 10^8 \, \frac{\text{m}}{\text{s}}$. Use $\lambda f = c$ to find the wavelength:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \, \frac{\text{m}}{\text{s}}}{1.590 \times 10^6 \text{ Hz}} = 187 \text{ m}$$

2. Here the wavelength of this type of EM radiation is $\lambda = 3.0 \text{ cm} = 3.0 \times 10^{-2}$ m. Then using $\lambda f = c$, we find its frequency:

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \, \frac{\text{m}}{\text{s}}}{3.0 \times 10^{-2} \text{ m}} = 1.00 \times 10^{10} \, \frac{1}{\text{s}} = 1.00 \times 10^{10} \text{ Hz}$$

3. Convert all distances to meters! The linear size of the Red Spot is

$$d = (24,000 \text{ km}) \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right) = 2.4 \times 10^7 \text{ m}$$

and its distance from us is

$$D = 4.5 \text{ AU} \left(\frac{1.50 \times 10^{11} \text{ m}}{1 \text{ AU}} \right) = 6.75 \times 10^{11} \text{ m}$$

Then the angular width of the Red Spot, *in radians* is

$$\theta = \frac{d}{D} = \frac{2.4 \times 10^7}{6.75 \times 10^{11}} = 3.56 \times 10^{-5} \text{ radians} .$$

Convert this to arcseconds:

$$\begin{aligned} \theta &= 3.56 \times 10^{-5} \text{ radians} \\ &= 3.56 \times 10^{-5} \text{ radians} \left(\frac{180 \text{ deg}}{\pi \text{ rad}} \right) \left(\frac{60 \text{ arcmin}}{1 \text{ deg}} \right) \left(\frac{60 \text{ arcsec}}{1 \text{ arcmin}} \right) = 7.3 \text{ arcsec} \end{aligned}$$

So earth–based telescopes *can* pick out the Red Spot but they won't be able to see much of its detail.

4. Again, convert all distances to meters. The distance of the planet from the star is

$$d = 5.20 \text{ AU} \left(\frac{1.50 \times 10^{11} \text{ m}}{1 \text{ AU}} \right) = 7.8 \times 10^{11} \text{ m}$$

and the distance of the star/planet system from the earth is

$$D = 25.0 \text{ ly} \left(\frac{9.46 \times 10^{15} \text{ m}}{1 \text{ ly}} \right) = 2.36 \times 10^{17} \text{ m}$$

so that the (maximum) angular separation of the planet from the star in radians is

$$\theta = \frac{d}{D} = \frac{7.8 \times 10^{11}}{2.36 \times 10^{17}} = 3.30 \times 10^{-6} \text{ radians}$$

Convert to arcseconds:

$$\begin{aligned} \theta &= 3.30 \times 10^{-6} \text{ radians} \\ &= 3.30 \times 10^{-6} \text{ radians} \left(\frac{180 \text{ deg}}{\pi \text{ rad}} \right) \left(\frac{3600 \text{ arcsec}}{\text{deg}} \right) = 0.68 \text{ arcsec} \end{aligned}$$

Observing planets orbiting other stars *may* someday be possible. (Presently, the intensity of the light from the central star prevents this.)

5. Convert the angular diameter of the Ring Nebula to radians:

$$74 \text{ sec} = (74 \text{ sec}) \left(\frac{1 \text{ deg}}{3600 \text{ sec}} \right) \left(\frac{\pi \text{ rad}}{180 \text{ deg}} \right) = 3.59 \times 10^{-4} \text{ rad}$$

We are given the distance to the Ring, $D = 1400$ ly. We use the small angle formula to solve for the (linear) diameter), d:

$$\theta = \frac{d}{D} \qquad \Longrightarrow \qquad d = \theta D$$

(In this formula, we can use any units for D and d but they must be the *same* units; θ must be in radians.) Plug in the value for D and get:

$$d = \theta D = (3.59 \times 10^{-4} \text{ rad})(1400 \text{ ly}) = 0.50 \text{ ly} \ ,$$

that is, it is about half a light-year across.

6. In centimeters, the mirror diameter is

$$D = (50 \text{ in}) \left(\frac{2.54 \text{ cm}}{1 \text{ in}} \right) = 127 \text{ cm}$$

Then our formula for the resolving power of an optical telescope gives:

$$\alpha = \frac{11.6}{127} = 9.1 \times 10^{-2} \text{ arcsec}$$

This is significantly smaller than the angular separation found in Problem 4, so one might think there is a chance of using such a telescope in space to look for planets around nearby stars. There is an additional problem in detecting a dim planet when it is close to a bright star, but the Hubble telescope will probably be making such observations in the near future (using IR light).