Phys 3810, Spring 2011
Problem Set #3, Hint-o-licious Hints

1. *Griffiths*, **2.26** Following the hint (and assuming that Plancheral's theorem is good for *anything* we want to stick into it), show that the Fourier transform of $\delta(x)$ is

$$F(k) = \frac{1}{\sqrt{2\pi}}$$

that is, a constant function. If we now merrily transform *back* to x space with the other transform formula, we get the desired (highly flaky) result.

2. *Griffiths*, **2.29** Actually, this one isn't so bad. You just need to go through the finite–well derivation in the book and make the appropriate changes for the asymmetric state.

The main results you should get to are the results of applying the boundary conditions:

$$-\kappa = \ell \cot(\ell a)$$

and, using the same definitions of z and z_0, the new condition for finding the energies (graphically, perhaps)

$$-\cot(z) = \sqrt{(z_0/z)^2 - 1}$$

which is *not* guaranteed to have a root since the right side is positive and the left side starts off being negative for small z.

3. *Griffiths*, **2.45** Follow the given hints and integrate to show that for any $x_!$ and x_2 we have

$$\int_{x_1}^{x_2} \left(\psi_2 \frac{\partial \psi_2}{dx^2} - \psi_1 \frac{\partial \psi_2}{dx^2} \right) dx = 0$$

Integration by parts of both terms leads to a cancellation and the result that

$$\psi_2 \frac{\partial \psi_2}{dx^2} - \psi_1 \frac{\partial \psi_2}{dx^2}$$

is constant (the *same* for any x. Since it is clearly zero as $a \to \infty$ it is zero everywhere.

But we're still not done! Considering $\frac{d}{dx}(\psi_1/\psi_2)$ will give the result that ψ_1 and ψ_2 are not independent.

4. *Griffiths*, **3.7** (a) is pretty easy; (b) is also easy; just try the simplest linear combinations of $f(x)$ and $g(x)$ you can think of. Big hint, they're common functions accessed on calulators with a key marked "hyp".

5. *Griffiths*, **3.12** Some steps: Start with

$$\langle x \rangle = \int_{-\infty}^{\infty} \Psi(x,t)^* x \Psi(x,t)\, dx$$

and substitute (3.55),

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{ipx/\hbar} \Phi(p,t)\, dp$$

Watch for complex conjugates and be sure that the dummy variable of p integration is *different* in the two integrals. When you do this you have a big integral over three variables: x, p and p'.

Do what Griffiths says, use

$$xe^{ipx/\hbar} = \frac{\hbar}{i}\frac{d}{dp}e^{ipx/\hbar}$$

and then do an integration by parts. Be careful with with variable names to know what the derivatives are acting on.

Use (and review) the result of problem 2.26,

$$\int_{-\infty}^{\infty} dx\, e^{iqx} = 2\pi\delta(q)$$

Use this for one of the integrations, but as usual be careful with the variables.

The next integration collapses the delta function and gives the desired result.

6. *Griffiths*, **3.23** As discussed in class, from considering the action of this H on the state $|1\rangle$ and $|2\rangle$, one can show that

$$H = \begin{pmatrix} E & E \\ E & -E \end{pmatrix}$$

With all of the $\sqrt{2}$'s flying around, the normalization is a little funky.