Astr 1010
Problem Set #3, Solutions

1. Saskatoon, SK[1] is at 52° N latitude, so the North Celestial Pole is 52° up from the Northern horizon.

Make a diagram showing the angle of the North Celestial Pole, and the Celestial Equator. Then we see that the Celestial Equator (at its highest point) is up from the Southern horizon by 38°. On the equinoxes, the sun is on the Celestial equator, so on Mar 21 and Sept 21, it is up from the Southern horizon by 38°.

On Jun 21 the sun is 23.5° above the Cel. Equator, so it is

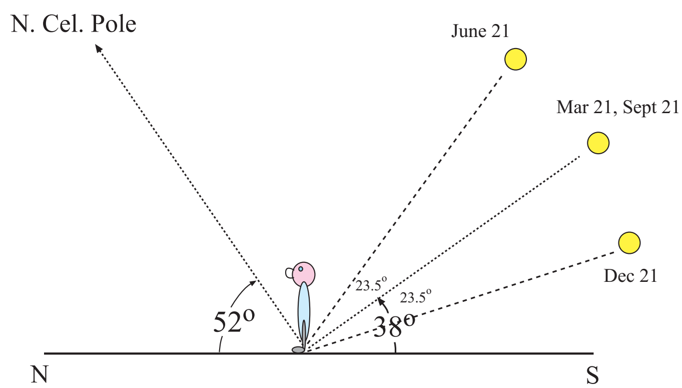
$$38^\circ + 23.5^\circ = 61.5^\circ$$

above the South horizon. On Dec 21 it is 23.5° below the Cel. Equator, so it is

$$38^\circ - 23.5^\circ = 14.5^\circ$$

above the South horizon. Pretty low in the sky, giving few hours of daylight!

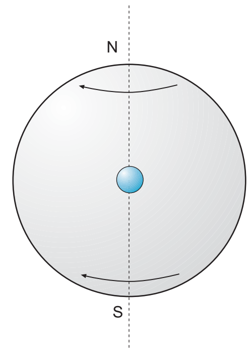
During the year the sun never passes directly overhead, and the sun rises and sets every day.



2. Taking (or imagining) a Celestial Sphere model, we turn it so that the stars come up in the East and go down in the West. We see that the stars revolve *counterclockwise* about the North Celestial Pole.

Looking at the South Celestial Pole, we see that the stars go *clockwise* about it, and that is what people in the Southern hemisphere see when they look at *their* celestial pole.

[1] "So far north they don't speak English anymore"– *Hee Haw*



3. We will use the small angle formula to relate the distances and the angle subtended by the object.

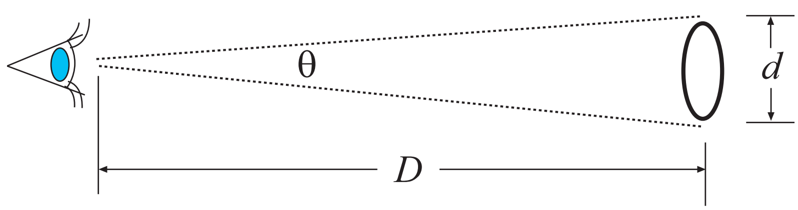
The diameter of the object —the crater— (called d in this class!) is $d = 5.0\,\mathrm{km}$. The distance to the crater is the same as the distance to the Moon, which is about $3.84 \times 10^5\,\mathrm{km}$. This gives us d and D so that we have the subtended angle in *radians*:

$$\theta = \frac{d}{D} = \frac{5.0\,\mathrm{km}}{3.84 \times 10^5\,\mathrm{km}} = 1.3 \times 10^{-5}\,\mathrm{radians}$$

Notice that we could leave both distances in kilometers since these units cancelled each other. Now we need to convert θ to arcseconds. We find:

$$\theta = (1.3 \times 10^{-5}\,\mathrm{radians}) \left(\frac{180\,\mathrm{deg}}{\pi\,\mathrm{rad}}\right) \left(\frac{3600\,\mathrm{sec}}{1\,\mathrm{deg}}\right) = 2.7\,\mathrm{sec}$$

which is a rather small angle. As we learn second semester, this is near the limit of angles which an earth–based telescope can distinguish.



4. Measuring a quarter, I find that it is about 2.5 cm in diameter. A football field has a length of 300 ft, and converting that to centimeters, I get:

$$D = (300\,\mathrm{ft}) \left(\frac{12\,\mathrm{in}}{1\,\mathrm{ft}}\right) \left(\frac{2.54\,\mathrm{cm}}{1\,\mathrm{in}}\right) = 9.144 \times 10^3\,\mathrm{cm}$$

so the angle subtended by the quarter (in radians!) is

$$\theta = \frac{d}{D} = \frac{2.5\,\mathrm{cm}}{9.144 \times 10^3\,\mathrm{cm}} = 2.73 \times 10^{-4}\,\mathrm{rad}$$

and converting this to arcminutes gives:

$$\theta = 2.73 \times 10^{-4}\,\mathrm{rad} = (2.73 \times 10^{-4}\,\mathrm{rad})\left(\frac{180\,\mathrm{deg}}{\pi\,\mathrm{rad}}\right)\left(\frac{60\,\mathrm{min}}{1\,\mathrm{deg}}\right) = 0.94\,\mathrm{min}\ .$$

So the angular width of the quarter as seen at this distance is slightly less than one (arc)minute.

5. Use the formula $d = vt$ (true for regular motion, as that of a radio signal). To find the time t, write it as $t = \frac{d}{v}$. Then when Mars is at a distance of $1.5 \times 10^{11}\,\mathrm{m}$, a radio signal travelling at a speed of $c = 2.998 \times 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}$ takes a time

$$t = \frac{d}{v} = \frac{1.5 \times 10^{11}\,\mathrm{m}}{2.998 \times 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}} = 500\,\mathrm{s} = 8.3\,\mathrm{min}$$

For a star at a distance of 100 ly, that is,

$$100\,\mathrm{ly} = (100\,\mathrm{ly})\left(\frac{9.46 \times 10^{15}\,\mathrm{m}}{1\,\mathrm{ly}}\right) = 9.46 \times 10^{17}\,\mathrm{m}$$

the ratio of this distance to our "typical" distance to Mars is

$$R = \frac{9.46 \times 10^{17}\,\mathrm{m}}{1.5 \times 10^{11}\,\mathrm{m}} = 6.3 \times 10^{6}$$

that is, 6.3 million (!).

If in a certain scale model of the universe we put Mars 3 ft away from the Earth, where would we put the stars in an "average constellation"? To find this number we make the corresponding ratios equal:

$$\frac{x}{3\,\mathrm{ft}} = \frac{d_{\mathrm{const}}}{d_{\mathrm{Mars}}}$$

and solve for x.

Suppose in a scale model of the universe we place Mars 3ft away from us. Then our star will be at a distance x for which the ratio is the same as the ratio of distances in the real universe:

$$R = \frac{x}{3\,\mathrm{ft}} = 6.3 \times 10^{6}$$

from which we find x:

$$x = (6.3 \times 10^{6})(3\,\mathrm{ft.}) = 1.9 \times 10^{7}\,\mathrm{ft}\left(\frac{1\,\mathrm{mi}}{5280\,\mathrm{ft}}\right) = 3600\,\mathrm{mi}\ .$$

So in our scale model the typical star would be 3600 miles away!!

While planets and the stars may be close together *as we see them on the Celestial Sphere* in reality they are nowhere near each other!