

Phys 2112, Spring 2011
Quiz #3

1. a) In the derivation of the general orbit for motion from Newton's law of gravity, we originally had factors of $\dot{\phi} \equiv \frac{d\phi}{dt}$ in the equations, but managed to eliminate them. How was that done?

We used the fact that a certain combination of $\dot{\phi}$ and r is a constant, specifically $\ell \equiv mr^2\dot{\phi}^2$. With this we could eliminate $\dot{\phi}$ in favor of using a number (ℓ) which is characteristic of the particular orbit.

b) In what way is the solution, expressed as $r(\phi)$, more useful (and interesting) than the solution we might have gotten, expressed as $r(t)$?

In this form we will get the *shape* of the orbit, i.e. a map of the places it moves to. But we lose information about how r (and ϕ) vary with time.

c) What are the possibilities for the shapes of the orbits for an object in motion around a much more massive object? Make a comment about the *position of the massive object* in relation to the orbital shape(s).

The orbit will have the shape of an ellipse if it is closed (with the circle as a special case) or if it is not closed it can be a parabola or a hyperbola. Thus, one of the famous conic sections.

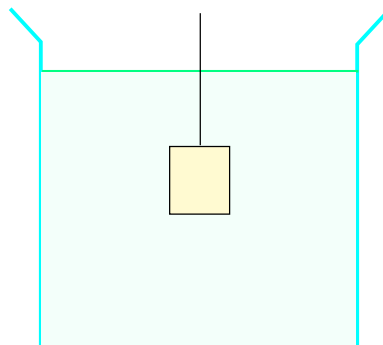
The massive object (which we placed at the origin of our coordinate system) is at one of the foci of the conic section.

2. a) A piece of solid copper has mass 0.600 kg. When it is suspended (motionless) from a string while under water, what is the tension in the string?

The density of copper is $8.94 \frac{\text{g}}{\text{cm}^3}$, that is, $8.94 \times 10^3 \frac{\text{kg}}{\text{m}^3}$.

The buoyant force on the copper is

$$F_b = (\rho_{\text{fl}} V)g = \left(\rho_{\text{fl}} \frac{M}{\rho_{\text{Cu}}} \right) g = \frac{\rho_{\text{fl}} M g}{\rho_{\text{Cu}}}$$



Total force on the metal piece is zero, so

$$T - Mg + \frac{\rho_{\text{fl}} M g}{\rho_{\text{Cu}}} = 0 \quad \Rightarrow \quad T = Mg \left(1 - \frac{\rho_{\text{fl}}}{\rho_{\text{Cu}}} \right)$$

Plug in the numbers:

$$T = (0.600 \text{ kg})(9.80 \frac{\text{m}}{\text{s}^2}) \left(1 - \frac{1.00}{8.94} \right) = 5.22 \text{ N}$$

Of course, without the water the tension would be $T + Mg = 5.88 \text{ N}$.

b) If the string is attached to a scale (which reads out its results in *kilograms*), what does the scale read?

The scale's maker assumes you don't have the masses sitting in water, so the *mass* reading of the scale is the mass given by $T = mg$. So in this case the scale reads

$$m' = \frac{T}{g} = \frac{5.22 \text{ N}}{9.80 \frac{\text{m}}{\text{s}^2}} = 0.532 \text{ kg}$$

Show work for all problems and include the right units!

$$F_{\text{grav}} = G \frac{m_1 m_2}{r^2} \quad G = 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \quad F_c = \frac{mv^2}{R} \quad g = 9.80 \frac{\text{m}}{\text{s}^2}$$

$$-G \frac{Mm}{r^2} = m(\ddot{r} - r\dot{\phi}^2) \quad r\ddot{\phi} + 2\dot{r}\dot{\phi} = 0$$

$$\rho = \frac{M}{V} \quad F_b = W_{\text{fl-disp}} = \rho_{\text{fl}} V g \quad \rho_{\text{water}} = 1000 \frac{\text{kg}}{\text{m}^3} \quad \rho_{\text{air}} = 1.29 \frac{\text{kg}}{\text{m}^3} \quad g = 9.80 \frac{\text{m}}{\text{s}^2}$$