Phys 3610, Fall 2008
Problem Set #6, Hint-o-licious Hints

1. *Taylor*, **8.6** Use the relations on p. 299 Substitute for $\mathbf{r}_1$ and $\mathbf{p}_1 = m_1\dot{\mathbf{r}}_1$ for the values they have in the CM frame, i.e.

$$\mathbf{r}_1 = \mathbf{R} + \frac{m_2}{M}\mathbf{r}\ .$$

Do some algebra and use

$$\mathbf{L} = \mathbf{r} \times \mu\dot{\mathbf{r}}$$

2. *Taylor*, **8.10** I get, with $m_1 = m_2 = m$,

$$\mathcal{L} = m\dot{\mathbf{R}}^2 + \frac{m}{4}\dot{\mathbf{r}}^2 - kR^2 - \frac{k}{2}(\alpha + \tfrac{1}{4})r^2$$

3. *Taylor*, **8.11** The solution has the general form

$$x(t) = A_x \cos\omega t + B_x \sin\omega t \qquad y(t) = A_y \cos\omega t + B_y \sin\omega t$$

which seems like it ought to be obviously an ellipse, but it *isn't* obvious.

Do as the man says, solve for $\cos\omega t$ and $\sin\omega t$ and use the basic trig identity.

The very end of the proof requires you to show that $ac > b^2$. For that, the following fact may help:

$$(A_x B_y - A_y B_x)^2 > 0$$

(Expand it out and see what you have.)

4. *Taylor*, **8.16** This is similar to the case of the hyperbola that I gave out in class except that here $1 - \epsilon^2$ is positive.

5. *Taylor*, **8.28** Fairly easy. Use the formula

$$r(\phi) = \frac{c}{1 + \epsilon\cos\phi}$$

6. *Taylor*, **9.8** Draw pictures and think about the directions of the cross products which give the centrifugal and Coriolis forces.

7. *Taylor*, **9.22** In the rotating frame the equation of motion for the charge takes the form (explain why!):

$$m\ddot{\mathbf{r}} = -k\frac{qQ}{r^2}\hat{\mathbf{r}} - q\mathbf{v} \times \mathbf{B} + 2m\dot{\mathbf{r}} \times \mathbf{\Omega} + m(\mathbf{\Omega} \times \mathbf{r}) \times \mathbf{\Omega}$$

But here $\mathbf{v}$ is still the velocity in the inertial frame. We need to express it in terms of $\dot{\mathbf{r}}$, the velocity in the rotating frame. We can use

$$\mathbf{v} = \dot{\mathbf{r}} + \mathbf{\Omega} \times \mathbf{r}$$

Substitute this and choose $\mathbf{\Omega}$ so that the terms with $\dot{\mathbf{r}}$ cancel. This wont make the double cross products go away, but follow Taylor's hint that for small B they are small.

When you're done with with this the orbit in the rotating frame must be an ellipse, parabola or a hyperbola... why? How would this motion look if you go back to the inertial (non-rotating) frame?