

Chap 9

Systems of Particles

CM,

$$\vec{F}_{\text{net ext}} = M \vec{a}_{\text{cm}}$$

Isolated system

$$\vec{F}_{\text{net ext}} = 0 = \frac{d\vec{p}}{dt}$$

$$\Rightarrow \vec{p} = \text{const}$$

Conservation of Momentum

$$\vec{p} = m \vec{v} \quad \text{vector}$$

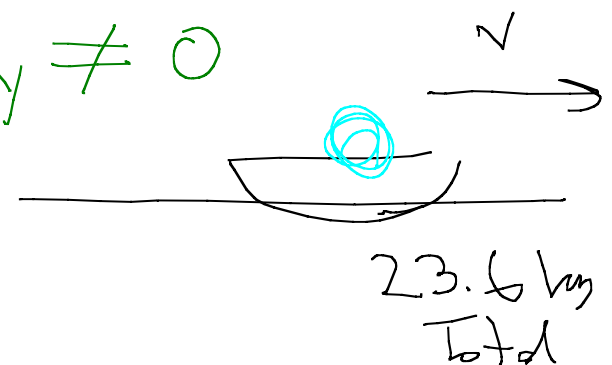
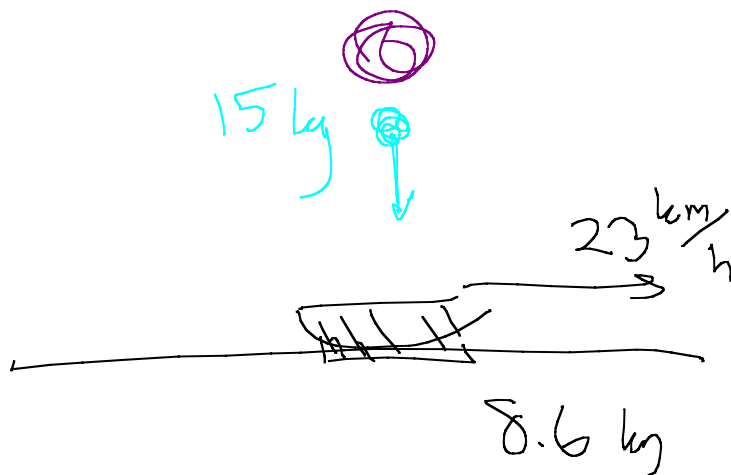
9.21

A runaway toboggan of mass 8.6 kg is moving horizontally at $23 \frac{\text{km}}{\text{h}}$. Passes under tree 15 kg of snow drops onto it. What is subsequent speed?

No horizontal forces on sled

y-momentum not cons'd.

$$F_{\text{ext}, y} \neq 0$$



No net $F_{ext, x}$ P_x is conserved

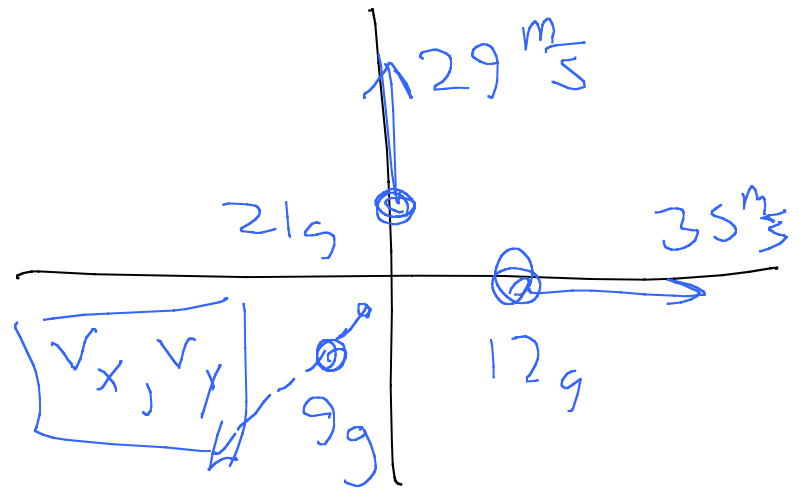
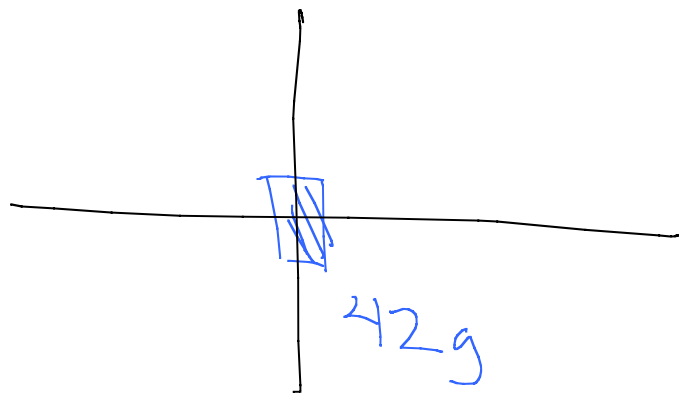
$$P_{x, \text{init}} = (8.6 \text{ kg}) (23 \frac{\text{km}}{\text{hr}})$$

$$= P_{x, \text{final}} = (23.6 \text{ kg}) V_x$$

$$V_x = 8.38 \frac{\text{km}}{\text{h}}$$

9.417 A 42 g firecracker is at rest at origin, explodes into 3 pieces.

First mass 12 g move along x-axis at $35 \frac{m}{s}$. Second mass 21 g moves along y axis at $29 \frac{m}{s}$. Find velocity of 3rd piece.



x-momentum

$$P_{x,i} = P_{x,f}$$

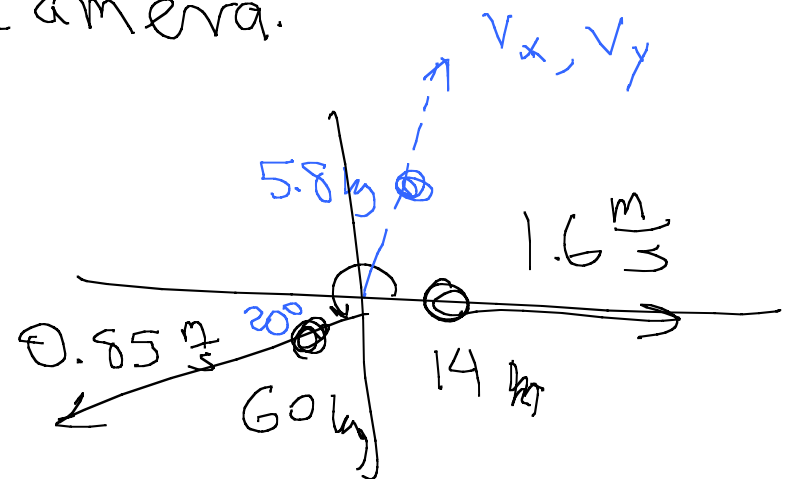
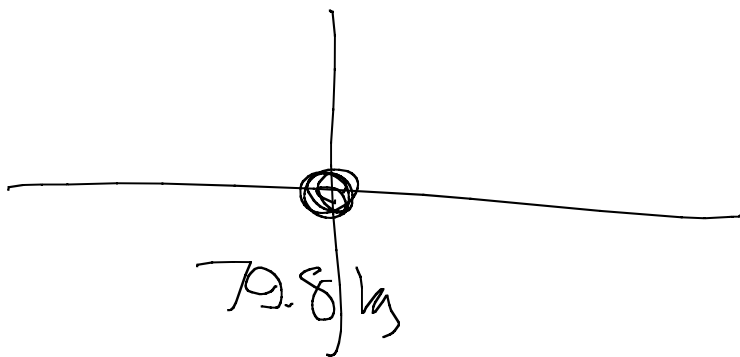
$$0 = (12g)(35 \frac{m}{s}) + (9g)v_x$$

$$v_x = -46.7 \frac{m}{s}$$

$$0 = (21g)(29 \frac{m}{s}) + (9g)v_y$$

$$v_y = -67.7 \frac{m}{s}$$

9.48 60 kg astronaut floating in space tosses 14 kg tank & 5.8 kg camera. Tank moves in x direction at $1.6 \frac{m}{s}$. Astronaut recoils at 20° from x axis at $2.85 \frac{m}{s}$. Find velocity of camera.



x-mom is conserved:

$$P_{x,i} = 0 = P_{x,f} = (14 \text{ kg})(1.6 \frac{\text{m}}{\text{s}})$$

etc.

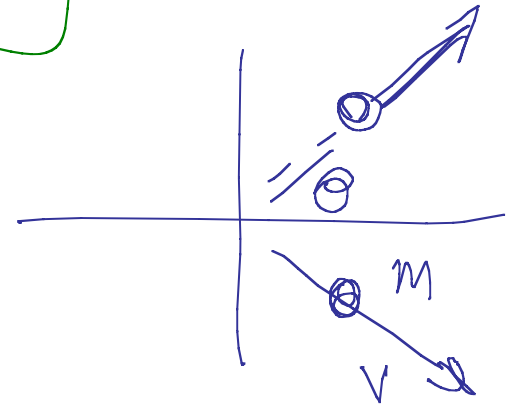
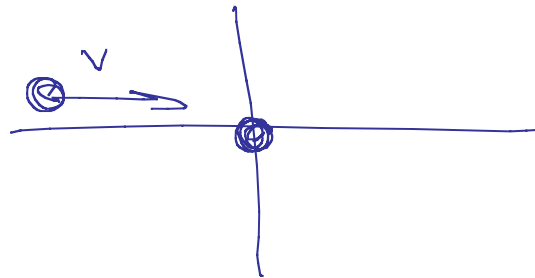
for y direction

$$+ (60 \text{ kg})(0.85 \frac{\text{m}}{\text{s}})(-\cos 20^\circ) + (5.8 \text{ kg}) v_x$$

$$v_x = 4.4 \frac{\text{m}}{\text{s}}$$

$$v_y = 3.01 \frac{\text{m}}{\text{s}}$$

Explosions



9.29 Two identical trucks have mass 5500 kg when empty. One truck carries 9500 kg load & moves $65 \frac{\text{km}}{\text{h}}$. Collides inelastically

w/ second truck init. at rest

$$\begin{aligned} &5500 \\ &+ 9500 \\ &= 15000 \end{aligned}$$

Pair moves off at $40 \frac{\text{km}}{\text{hr}}$. Find load in second truck



$$(1500) \left(65 \frac{\text{km}}{\text{h}} \right) = \left(1500 + 5500 + M \right) \left(40 \frac{\text{km}}{\text{h}} \right)$$

In each of these

mom conserved but not rec energy

$$K_i = \sum_i \frac{1}{2} m_i v_{i, \text{init}}^2 \neq K_f = \sum_i \frac{1}{2} m_i v_{i, \text{final}}^2$$

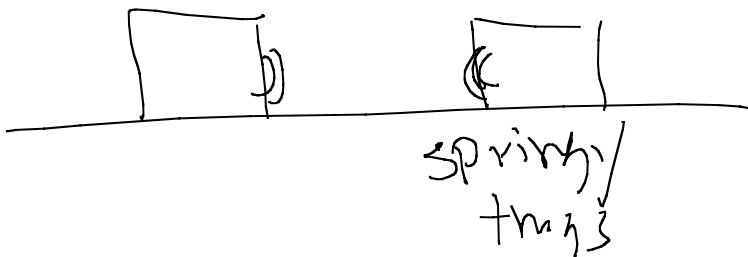
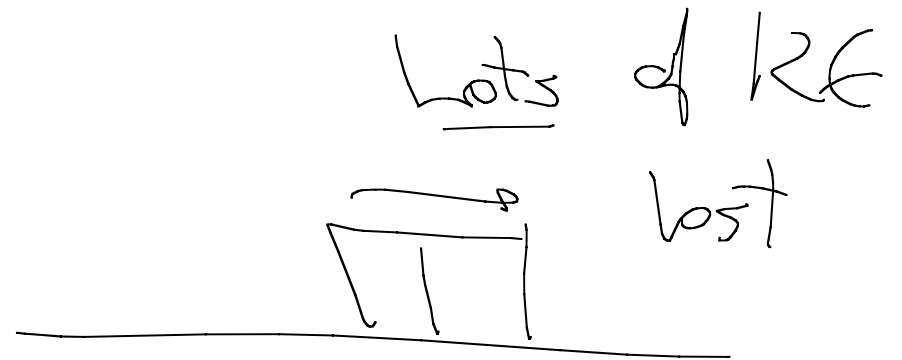
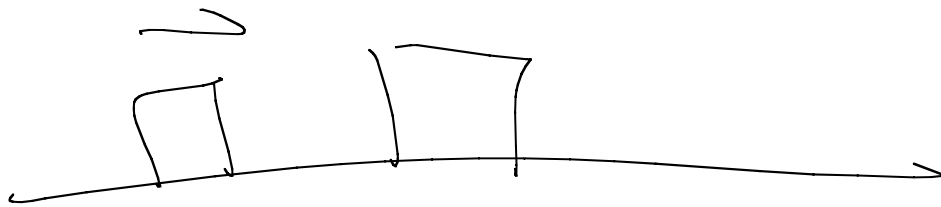
but... discuss energy

We'll show $K_i = \sum_i \frac{1}{2} m_i v_i^2 = \frac{1}{2} M v_{\text{cm}}^2 + \frac{1}{2} \sum_i m_i v_{i, \text{rel}}^2$

\vec{V}_{rel} = Velocity rel to cm

$$\vec{V} = \vec{V}_{cm} + \vec{V}_{rel}$$

Energy in collision



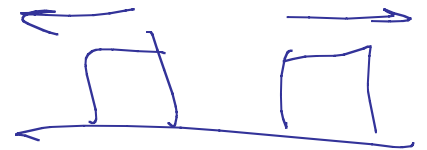
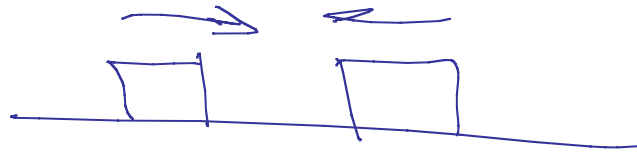
Very nearly all energy is conserved.

Energy conserved: Elastic collision.

Energy not conserved: Inelastic collision

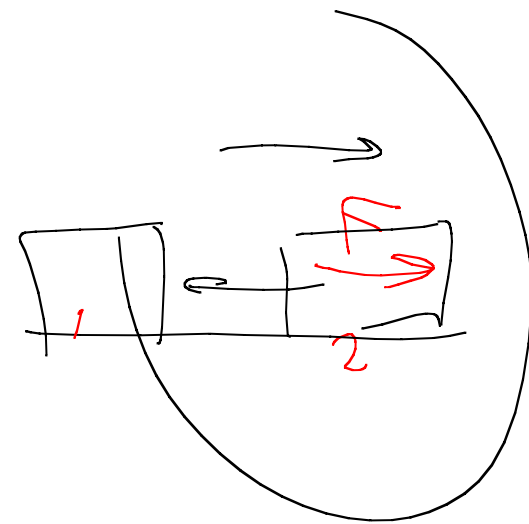
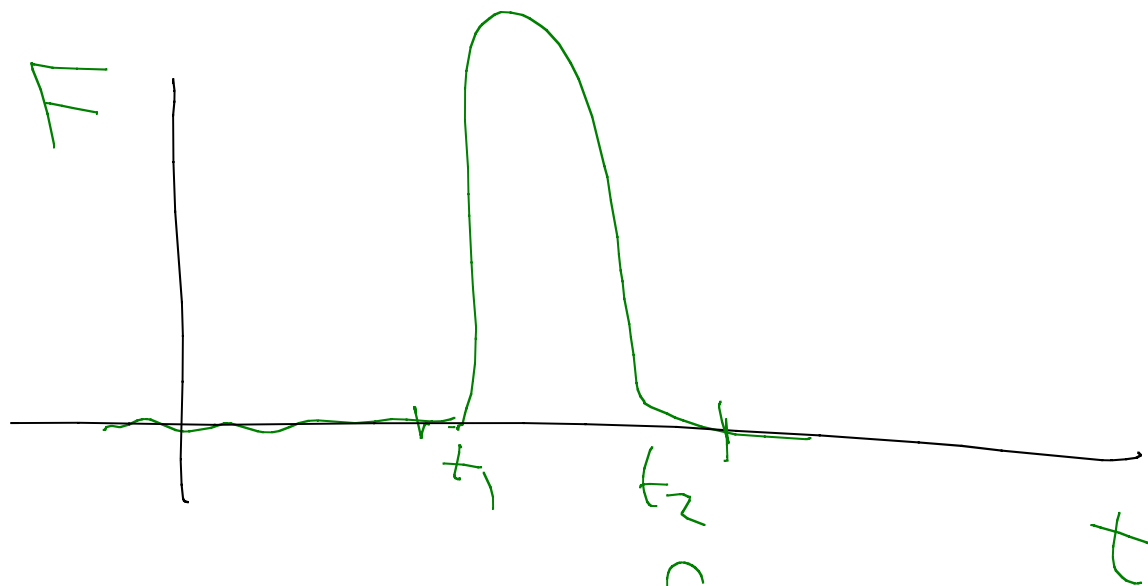
Totally inelastic : Stick.

Impulse



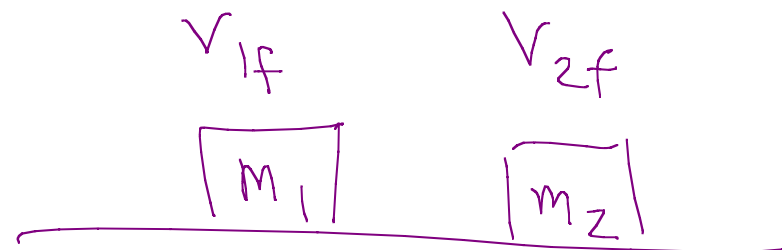
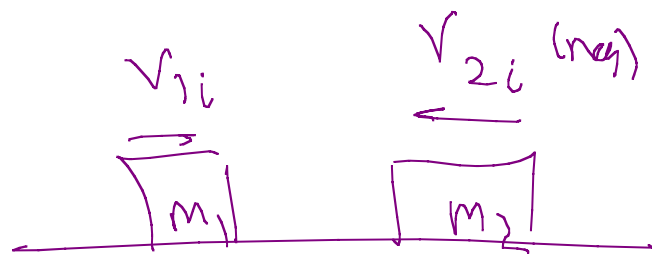
$\vec{J} = \Delta \vec{p}$ change in momentum

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt} = m\vec{a} \quad \Delta \vec{p} = \int_{t_1}^{t_2} \vec{F} dt$$



$$\Delta P_2 = \int F dt = \text{impulse.}$$

Elastic collision in 1 Dim



Elastic Collision

get conditions on final
velocities