Phys 3810, Spring 2012
Problem Set #3, Hint-o-licious Hints

1. *Griffiths*, **2.26** Following the hint (and assuming that Plancheral's theorem is good for *anything* we want to stick into it), show that the Fourier transform of $\delta(x)$ is

$$F(k) = \frac{1}{\sqrt{2\pi}}$$

that is, a constant function. If we now merrily transform *back* to x space with the other transform formula, we get the desired (highly flaky) result.

2. *Griffiths*, **2.29** Actually, this one isn't so bad. You just need to go through the finite–well derivation in the book and make the appropriate changes for the asymmetric state.

The main results you should get to are the results of applying the boundary conditions:

$$-\kappa = \ell \cot(\ell a)$$

and, using the same definitions of z and z_0, the new condition for finding the energies (graphically, perhaps)

$$-\cot(z) = \sqrt{(z_0/z)^2 - 1}$$

which is *not* guaranteed to have a root since the right side is positive and the left side starts off being negative for small z.

3. *Griffiths*, **2.34** For the case $E < V_0$ (insufficient energy to travel to positive x) show that the Schrödinger equation can be written as

$$\frac{d^2\psi}{dx^2}\psi = \kappa^2\psi \qquad \text{where} \qquad \kappa \equiv \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

The general solution for $x > 0$ is a linear combination of $e)+\kappa x$ and $e^{-\kappa x}$ but one of these terms is illegal for a wave function! For this case, matching at the boundary gives

$$\frac{B}{A} = \frac{(ik + \kappa)}{(ik - \kappa)}$$

(show this!) which has absolute value 1 (why?). That show that $R = 1$. What does this mean?

For the other case $E > V_0$ we now want to define

$$k' \equiv \frac{\sqrt{2m(E - V_0)}}{\hbar}$$

so that for $x > 0$ the Schrödinger equation is

$$\frac{d^2\psi}{dx^2}\psi = -k'^2\psi$$

Now with the boundary condition you can show

$$R \equiv \left| \frac{B}{A} \right| = \frac{k - k'}{k + k'}$$

But T requires more care. As you'll show in (c) T gets a slightly different expression, and you'll show

$$T = \frac{k'}{k} \left| \frac{F}{A} \right| = \frac{k'}{k} \left(\frac{2k}{k + k'} \right)$$

Do we get $T + R = 1$?

4. *Griffiths*, **2.45** Follow the given hints and integrate to show that for any ψ_1 and ψ_2 we have

$$\int_{x_1}^{x_2} \left(\psi_2 \frac{d\psi_1}{dx} - \psi_1 \frac{d\psi_2}{dx} \right) dx = 0$$

Integration by parts of both terms leads to a cancellation and the result that

$$\psi_2 \frac{d\psi_1}{dx} - \psi_1 \frac{d\psi_2}{dx}$$

is constant (the *same* for any x). Since it is clearly zero as $x \to \infty$ it is zero everywhere.

But we're still not done! Considering $\frac{d}{dx}(\psi_1/\psi_2)$ will give the result that ψ_1 and ψ_2 are not independent.