Phys 2110-3 10/20/10

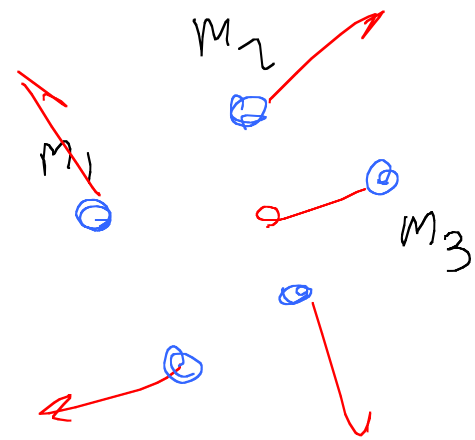
Chap 9

Center of mass

M = total mass

$$\vec{F}_{Total} = M \frac{d^2 \vec{r}_{cm}}{dt^2}$$

$$= M \vec{a}_{cm}$$



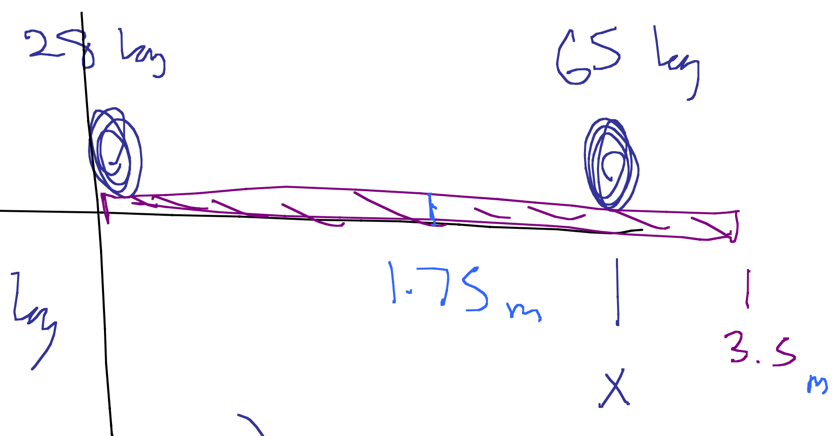
$(\vec{F}_{internal}, \vec{F}_{external})$

$$\vec{r}_{cm} = \frac{1}{M} \sum m_i \vec{r}_i$$

$$x_{cm} = \frac{1}{M} \sum m_i x_i$$

9.12 A 28 kg child sits at one end of 3.5 m-long see-saw. Where should her 65-kg father sit so that cm of system is at center of seesaw

$$x_{cm} = \frac{1}{M} \sum m_i x_i$$



$$M = 28\text{ kg} + 65\text{ kg} = 93\text{ kg}$$

$$x_{cm} = \frac{1}{93\text{ kg}}\left(28\text{ kg} \cdot 0 + (65\text{ kg})\,x\right) = 1.75\text{ m}$$

solve for x "75.4 cm from ctr"

$$\vec{r}_{cm} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$$\vec{V}_{cm} = \frac{1}{M} \sum_i m_i \vec{V}_i$$

$$\vec{a}_{cm} = \frac{1}{M} \sum_i m_i \vec{a}_i$$

Math class

Continuous distrib of mass

$$\underline{\underline{m_i}} \rightarrow \rho(x_i)\, d^3r$$

$$\overline{\overline{\rho(\vec{x})\, dV}}$$

$$M = \int_V \rho(\vec{x})\, dV$$

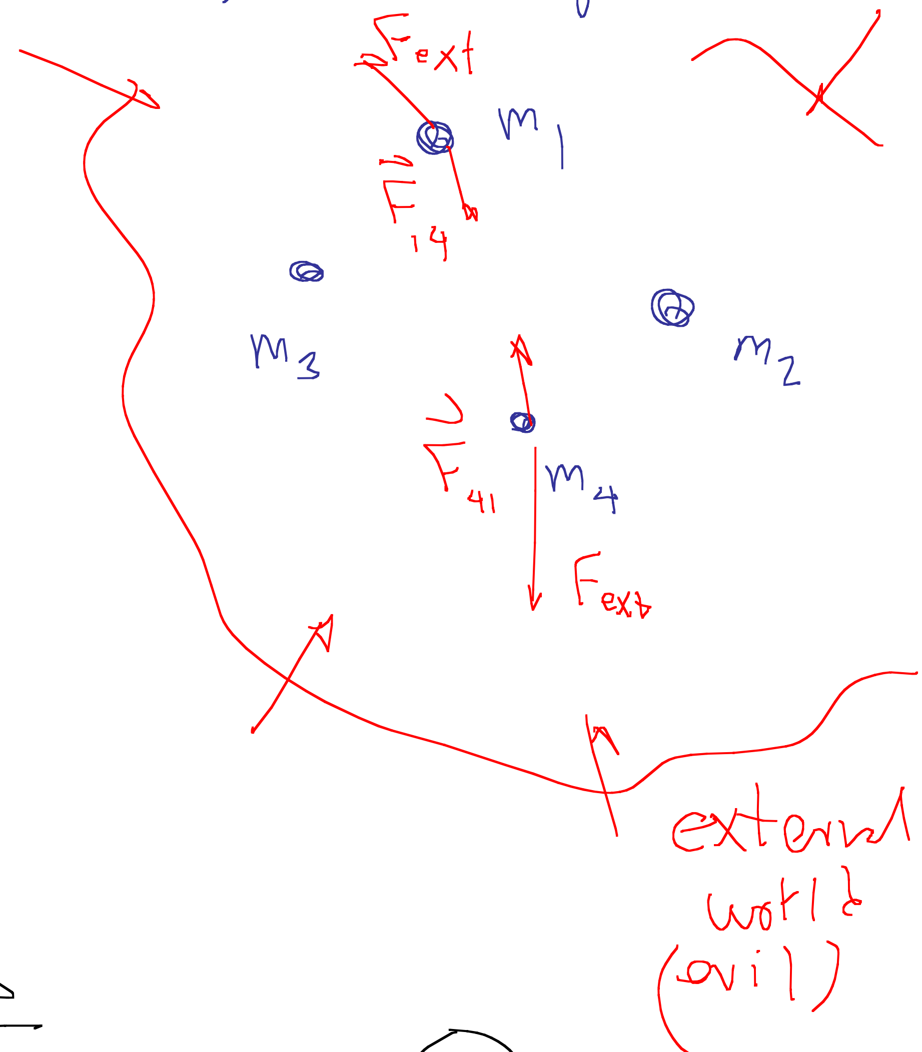
$$x_{cm} = \int_V x\, \rho(\vec{x})\, dV$$

etc.

$$\vec{F}_{tot} = \vec{F}_{ext\ tot} + \vec{F}_{internal\ total}$$

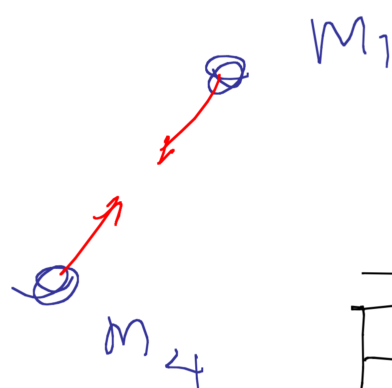
total is sum over <u>all</u> particles

Systems of masses



Newton's 3rd Law

when internal forces <u>are</u> all added up get zero

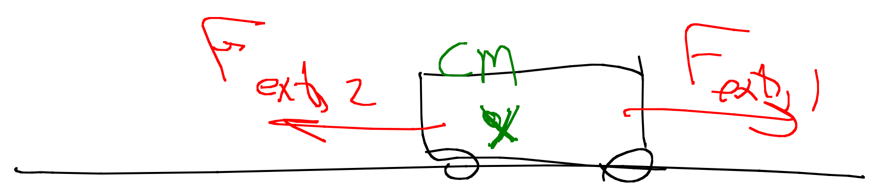


$$\vec{F}_{internal\ tot} = 0$$

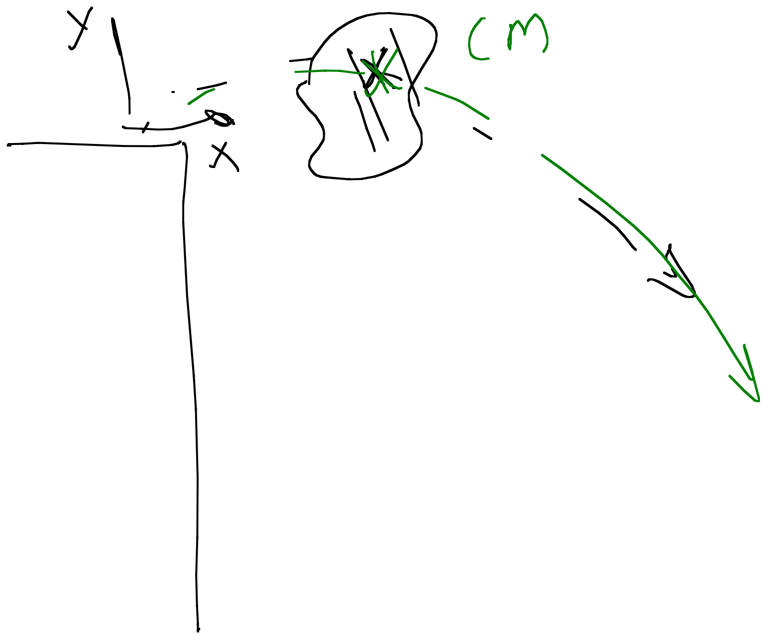
$$\vec{F}_{Total} = \vec{F}_{external,\,tot} = M \frac{d^2\vec{r}_{cm}}{dt^2} = M\vec{a}_{cm}$$

$$\vec{F}_{ext} = M\vec{a}_{cm}$$

Now we know what we've been talking about



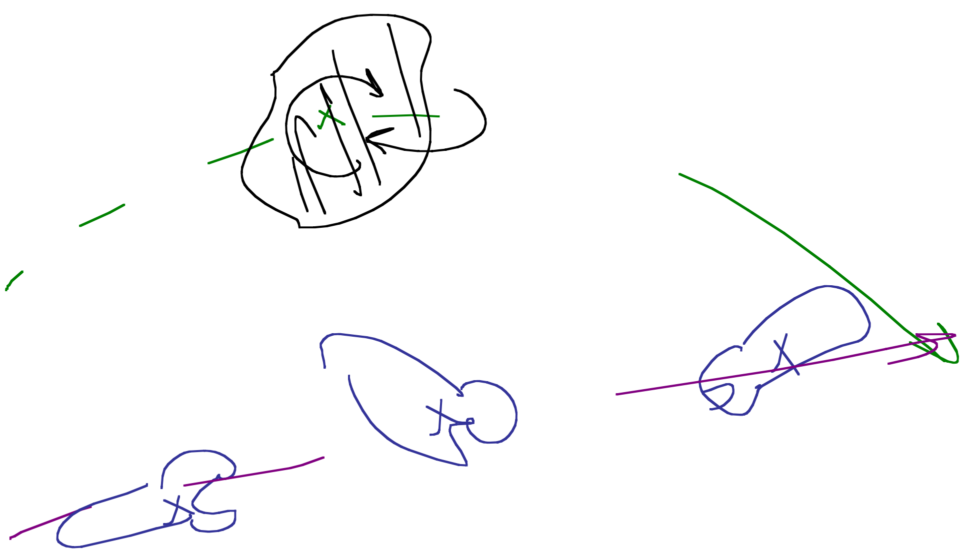
External forces
motion of cm of object

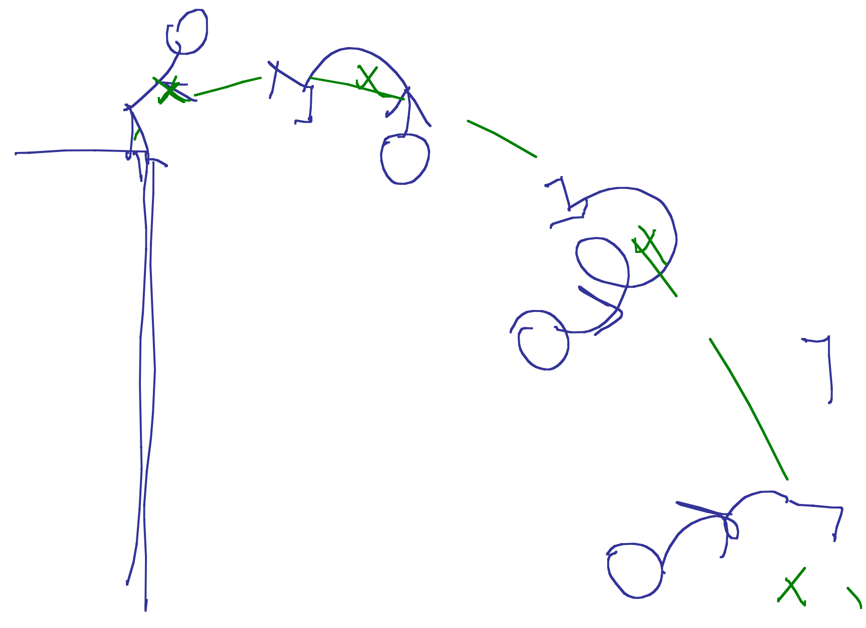


$$\vec{F}_{ext} = -Mg\hat{j}$$

$$M\vec{a}_{cm} = -Mg\hat{j}$$

$$\vec{a}_{cm} = -g\hat{j}$$





Definition:

$$\boxed{\vec{p} = m\vec{v}}$$

Momentum of particle of mass m, velocity $\vec{v}$

Vector

$p_x = mv_x$

$p_y = mv_y$

Units?

$m\vec{v} \Rightarrow kg\frac{m}{s} = \frac{kg\, m}{s}$

Leave it at that.

$$\vec{v}_{cm} = \frac{1}{M}\sum m_i \vec{v}_i = \frac{1}{M}\sum \vec{p}_i = N\cdot s \qquad \vec{v}_{cm} = \vec{P}/M$$

$\sum \vec{p}_i = \vec{P}$ total momentum

Take $\frac{d}{dt}\vec{P}$

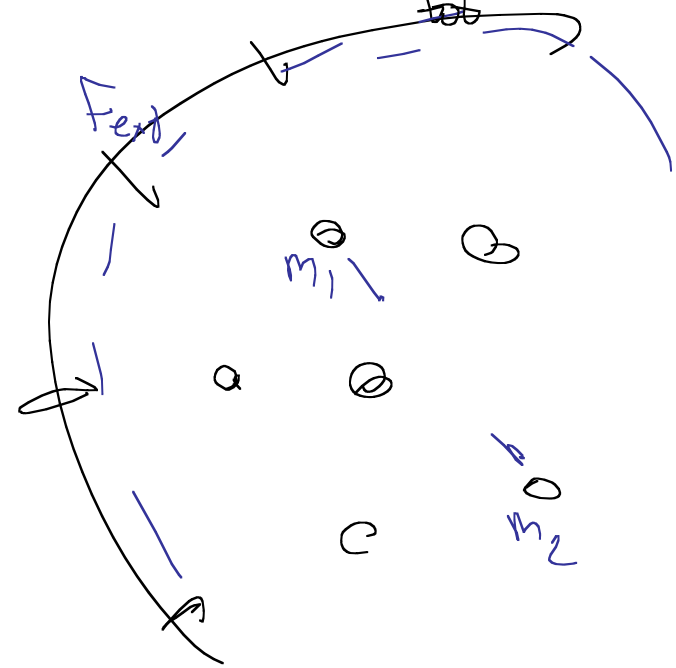
$$\frac{d\vec{v}_i}{dt} = \vec{a}_i$$

$$\frac{d\vec{P}}{dt} = \frac{d}{dt}\left(\frac{1}{M}\sum m_i v_i\right)$$

$$= \frac{1}{M}\sum m_i \vec{a}_i = \vec{F}_{tot} = \vec{F}_{\substack{ext\\tot}}$$

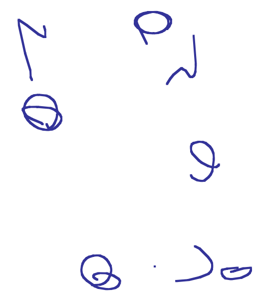
$$\boxed{\vec{F}_{\substack{ext\\tot}} = \frac{d\vec{P}}{dt}}$$

If $\vec{F}_{\substack{ext\\tot}} = 0$, isolated system

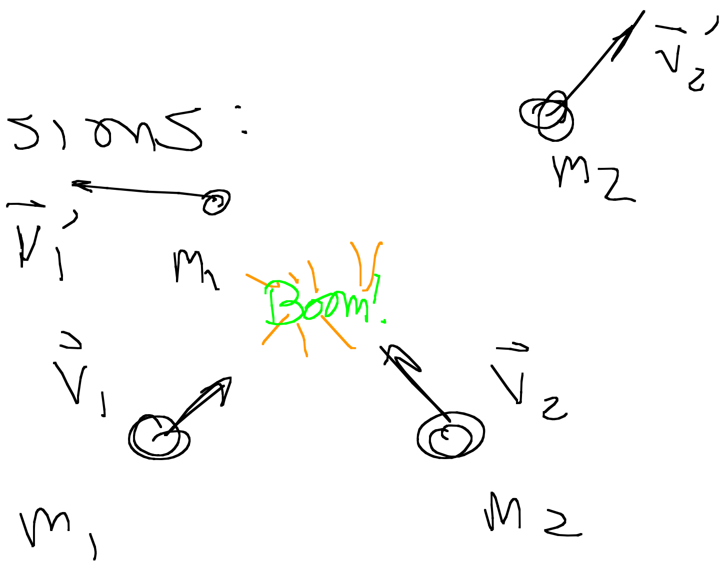


Special case: $\vec{F}_{ext,\,tot} = 0$ Isolated

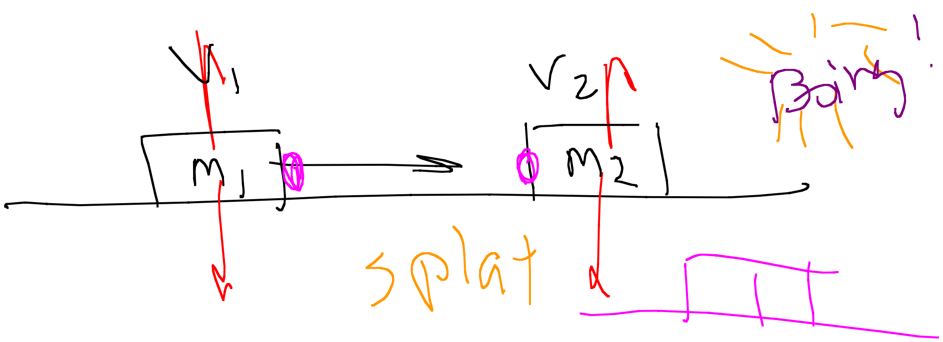
$$d\vec{P}/dt = 0 \qquad \vec{P} \text{ is } \underline{\text{constant}}.$$

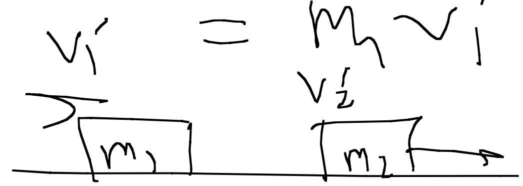


Collisions:



$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$



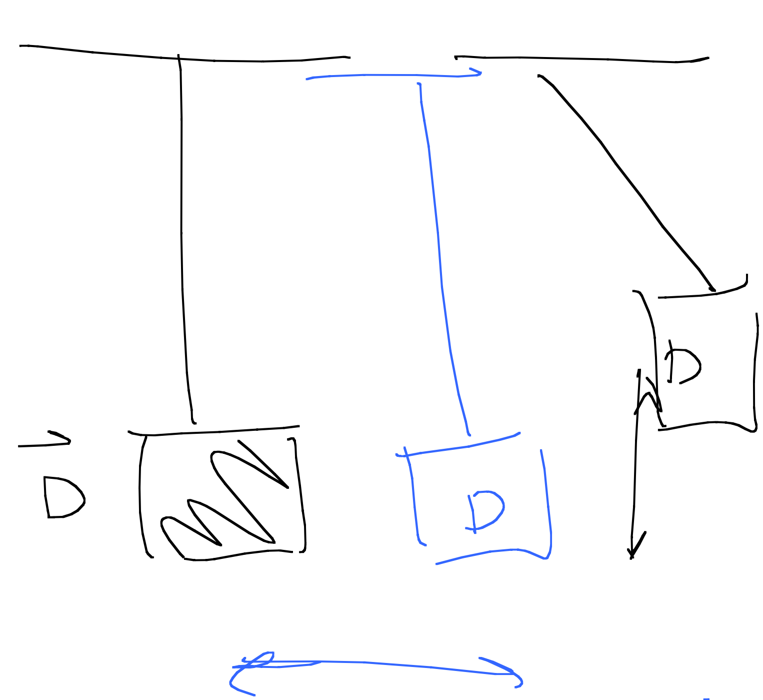


Ballistic Pendulum

For a short time can consider system a isolated

For isolated system

$$\vec{P} = \text{constant}$$



Mom. is conserv'd

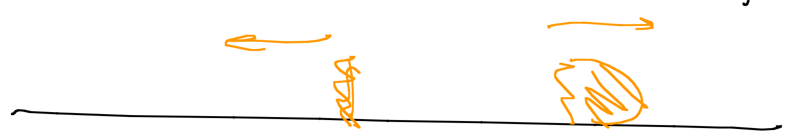
Work problems

9.18 A popcorn kernel in a hot pan bursts into two pieces 91 mg 64 mg. The more massive piece moves horiz at 47 $\frac{cm}{s}$. Describe motion of second piece.

Isolated system

$v = 0$

$v = ?$ $\quad v = 47\,\frac{cm}{s}$



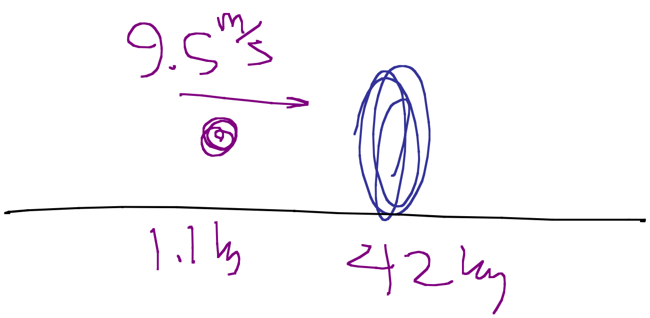
64 mg $\quad$ 91 mg

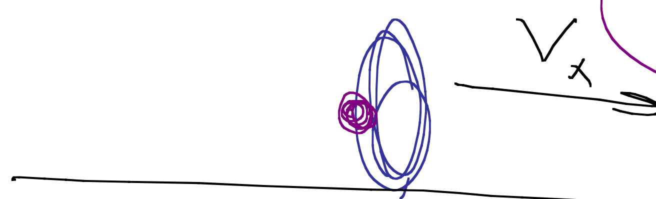
$\vec{P}$ is conserved

$$0 = (91\,mg)(47\,\frac{cm}{s}) + (64\,mg)V_x$$

Get: $V_x = -66.8\,\frac{cm}{s}$

9.22 42 kg child stands at rest on ice skates. She catches 1.1 kg ball moving at 9.5 m/s. What her speed after she catch ball?





$\vec{P}$ is conserved

V_x = 0.24 m/s