

Phys 3810, Spring 2012
Problem Set #5, Hint-o-licious Hints

1. *Griffiths, 4.2* Show that the energies are given by

$$E_{(n_x, n_y, n_z)} = \frac{\pi^2 \hbar^2}{2ma^2} (n_x^2 + n_y^2 + n_z^2)$$

The latter part of the problem is a math-puzzle sort of thing where you figure out how many ways one can get the same E with different sets of integers.

You can find the values and the degeneracies from E_1 up to E_{14} , but if you run out of patience, I'll tell you that

$$E_{14} = \frac{\pi^2 \hbar^2}{2ma^2} (27)$$

So what's different about this value from the first few?

2. *Griffiths, 4.38* The separated solution is

$$\psi(\mathbf{r}) = X(x)Y(y)Z(z)$$

Note that when you write out the Schrödinger equation in cartesian coordinates for this potential with the separated solution (substitute and then divide by $\psi = XYZ$) it becomes a sum of three terms each of which depends only on x , y or z . This means that you can write three separate Schrödinger equations, i.e.

$$-\frac{\hbar^2}{2m} \frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{2} m \omega^2 x^2 = E_x \quad , \text{etc.}$$

which we know how to solve. (Be as clear as you can about this part.) The total energy is just sum of energies from each separate Schrödinger equation:

$$E = E_x + E_y + E_z = \hbar \omega (n_x + n_y + n_z + \frac{3}{2})$$

The second part involves more thought. The energy of the oscillator state just depends on

$$n \equiv n_x + n_y + n_z \quad \text{for} \quad n_x = 0, 1, 2, \dots \quad \text{etc.}$$

so the question is how many ways can you get n from the three separate indices. That's a sort of puzzle-math problem. First, see if you can spot the pattern for the lowest n 's. (To give you one: $n = 3$ has 10 possible states.)

3. *Griffiths, 4.8* Test that the $u(r)$ radial solution

$$u_1(r) = A r j_1(kr)$$

really does solve Eq. (4.41).

For the infinite spherical well, the boundary condition $u(a) = 0$ (continuity of the wave function) leads to $ka = \tan(ka)$. (Show this, of course.) Use a graph to demonstrate that for the higher solutions, we have

$$ka \approx \frac{(2n+1)}{2}\pi \quad \text{for } n = 1, 2, 3, \dots$$

4. Griffiths, 4.10 Start off with c_0 arbitrary and then use (4.76) to get the succeeding coefficients for the polynomial $v(\rho)$. They will truncate after very few terms. The math isn't hard it's just to get familiar with the notation and see how the math works out. The correct value for c_0 would come from normalizing the radial function but that's not necessary here.

5. Griffiths, 4.45 Obviously you want to do the integral $\int_V |\psi_{100}|^2 dV$ for a sphere of radius b centered at the origin, b being the radius of the nucleus. Normally, I work these out by hand, but when my tables just gave a recursion formula for $\int x^2 e^{-ax} dx$, I just turned to Maple!