Phys 3810, Spring 2011
Problem Set #5, Hint-o-licious Hints

1. *Griffiths*, **4.10** Start off with c_0 arbitrary and then use (4.76) to get the succeeding coefficients for the polynomial $v(\rho)$. They will truncate after very few terms. The math isn't hard it's just to get familiar with the notation and see how the math works out. The correct value for c_0 would come from normalizing the radial function but that' snot necessary here.

2. *Griffiths*, **4.45** Obviously you want to do the integral $\int_V |\psi_{100}|^2 \, dV$ for a sphere of radius b centered at the origin, b being the radius of the nucleus. Normally, I work these out by hand, but when my tables just gave a recursion formula for $\int x^2 e^{-ax} \, dx$, I just turned to Maple!

3. *Griffiths*, **4.11** The normalization condition on $R(r)$ is

$$\int_0^\infty R(r)^2 r^2 \, dr = 1$$

Find the c_0 (that is, the overall constant) which makes this true and then write out $R(r)$. You can compare with Table 4.7.

4. *Griffiths*, **4.14** The trick here is that the probability to find the electron at a certain radius r (within dr) is *not* $R(r)^2 \, dr$. A hint comes from how $R(r)$ is normalized (as used in Prob 6, Griff 4.11); the integral of any probability distribution must give 1.

When you do get the right probability function, find its maximum to get the most probable value.

5. *Griffiths*, **4.16** Note that *only* place in the whole derivation where the electric charges showed up was in the potential

$$V(r) = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

Obviously if the central charge is replaced by $+Ze$, then the e^2 in the potential gets replaced by Ze^2, but then e^2 should be replaced *everywhere* by this. See how the result for the energy levels changes. (Is it proportional to Z? To Z^2?)

6. *Griffiths*, **4.19** Part (a) is a simple application of the basic commutation relations

$$[x, p_x] = xp_x - p_x x = i\hbar, \qquad [x, p_y] = 0, \qquad \text{etc.}$$

For (b), pull the commuting factors out in front to simplify things, and use the results in (a) get the answers. For (c), write out the string of operators and use, e.g.

$$L_z x = xL_z + i\hbar y \qquad L_z y = yL_z - i\hbar x$$

to switch the order of the operators and get cancellation.

7. *Griffiths*, **4.20** You need to use

$$\frac{d}{dt}\langle \mathbf{L} \rangle = \frac{i}{\hbar}\langle [H, \mathbf{L}] \rangle$$

where

$$H = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + V(\mathbf{r})$$

to get the result. Just choose any component of this equation, say z. Show that

$$\frac{1}{2m}[p_x^2, xp_y - yp_x] = \frac{-i\hbar}{m} p_x p_y \qquad \frac{1}{2m}[p_y^2, xp_y - yp_x] = \frac{+i\hbar}{m} p_x p_y$$

so these terms will cancel. Then show

$$[V(\mathbf{r}), yp_y - yp_x] = i\hbar \left(x\frac{\partial V}{\partial y} - y\frac{\partial V}{\partial x} \right) = i\hbar(\mathbf{r} \times \nabla V)_z$$

This will give

$$\frac{d}{dt}\langle L_z \rangle = \langle N_z \rangle$$