Phys 2110 - 4 2/1/13

Note Title 2/1/2013

2D motion

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{r} = x\hat{x} + y\hat{y}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

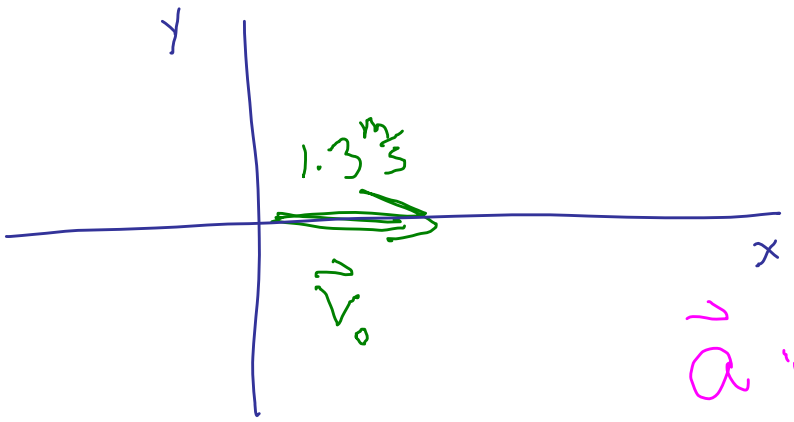
$$\vec{a} = a_y\hat{i} + a_y\hat{j}$$

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt}$$

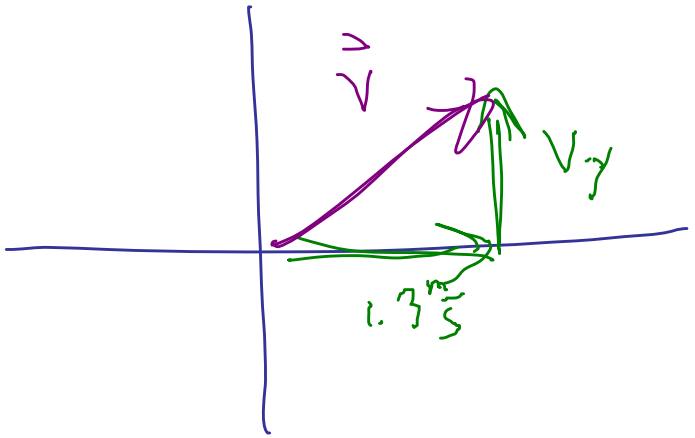
$$a_x = \frac{dv_x}{dt} \qquad a_y = \frac{dv_y}{dt}$$

3.25 An object is moving in the x-direction at $1.3\ \frac{m}{s}$ when it undergoes an accel. $\vec{a} = 0.52\hat{j}\ \frac{m}{s^2}$ constant

Find the velocity vector after 4.4 s.



$\vec{a}$ in y dir.

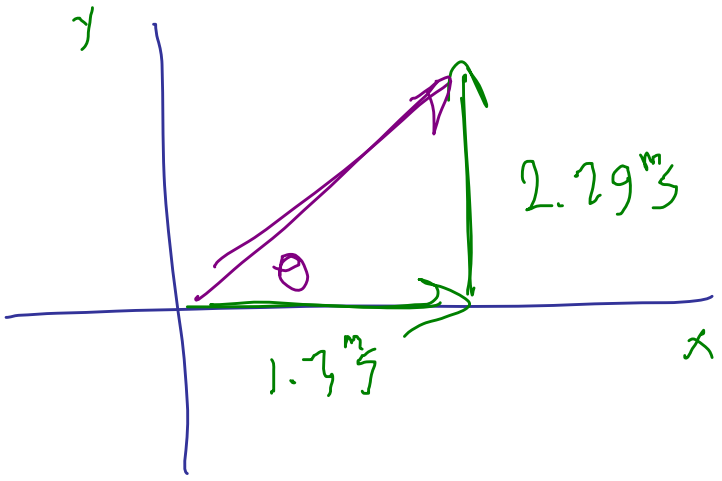


There is a change in v_y (not v_x)

$$a_y = \text{constant} = \frac{\Delta v_y}{\Delta t} = \frac{dv_y}{dt}$$

$$\Delta v_y = a_y \Delta t = \left(0.52 \frac{m}{s^2}\right)(4.4\,s)$$

$$= 2.29 \frac{m}{s}$$



$$\vec{v} = 1.3 \frac{m}{s}\,\hat{i} + 2.29 \frac{m}{s}\,\hat{j}$$

Magnitude:

$$|\vec{v}| = \sqrt{(1.3)^2 + (2.29)^2} = 2.63 \frac{m}{s}$$

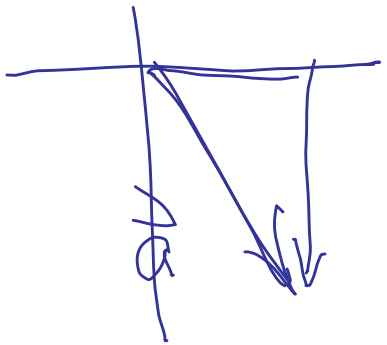
$$\tan\theta = \left(\frac{2.29}{1.3}\right) \qquad \theta = 60.4^\circ$$

Magnitude of $\vec{v}$ = speed

3.30 The position of an object as a function of time is

$$\vec{r} = (3.2t + 1.8t^2)\hat{i} + (1.7t - 2.4t^2)\hat{j}$$

($\vec{r}$ is in m when t is in s)



Find $\vec{a}$

$x = 3.2t + 1.8t^2$ $\quad y = 1.7t - 2.4t^2$

$v_x = 3.2 + 3.6t$ $\quad v_y = 1.7 - 4.8t$

$a_x = 3.6\ m/s^2$ $\quad a_y = -4.8\ m/s^2$

(Constant both components)

$|\vec{a}| = \sqrt{(3.6)^2 + (4.8)^2} = 6.0\ m/s^2$ etc.

Specialize to constant $\vec{a}$ a_x, a_y both constant

a_x

$$v_x = a_x t + C$$
$$= a_x t + v_{x0}$$

$$\boxed{v_x = v_{x0} + a_x t}$$

$$x = v_{x0} t + \frac{1}{2} a_x t^2 + C \quad x_0$$

$$\boxed{x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2}$$

a_y

$$v_y = a_y t + C$$

$$\boxed{v_y = v_{y0} + a_y t}$$

$$y = v_{y0} t + \frac{1}{2} a_y t^2 + C \quad y_0$$

$$\boxed{y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2}$$

Can derive

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$$

Also

$$x = x_0 + \frac{1}{2}(v_{x0} + v_x)t$$

etc.

Projectile Motion

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$= 0\hat{i} - 9.8\,\frac{m}{s^2}\hat{j}$$

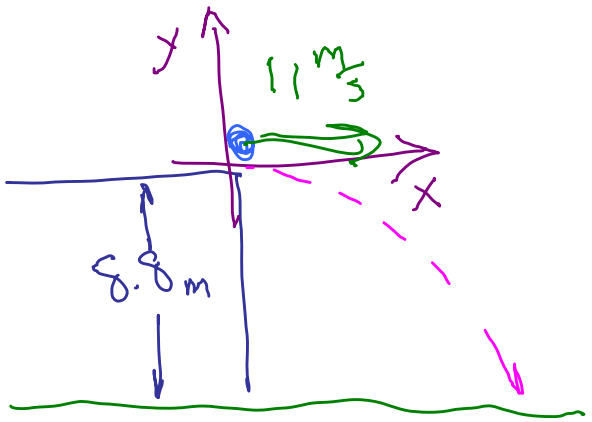
Things from bottom

$$= -g\hat{j}$$

3.33 A carpenter tosses a shingle horizontally off an 8.8 m-high roof at $11\ \frac{m}{s}$

a) How long does it take the shingle to reach the ground?

b) How far does it move horizontally?



$v_x = v_{x0} + a_x t$

$v_x = 11\ \frac{m}{s}$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t$

$= 0 + 11\ \frac{m}{s}\, t + 0$

$x = (11\ \frac{m}{s})\, t$

$v_y = v_{y0} + a_y t$

$= 0 - 9.8\ \frac{m}{s^2} t$

$v_y = -9.8\ \frac{m}{s^2}\, t$

$y = v_{y0} t + \frac{1}{2} a_y t^2$

$y = -\frac{1}{2}(9.8\ \frac{m}{s^2})\, t^2$

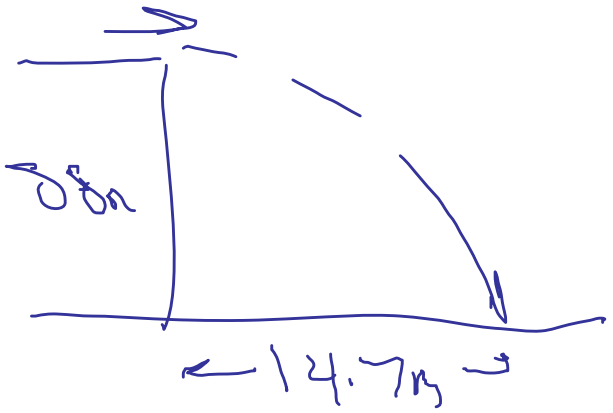
a) when does $y = -8.8\ m$?

$$-8.8\ m = -\tfrac{1}{2}\left(9.8\ \tfrac{m}{s^2}\right) t^2$$

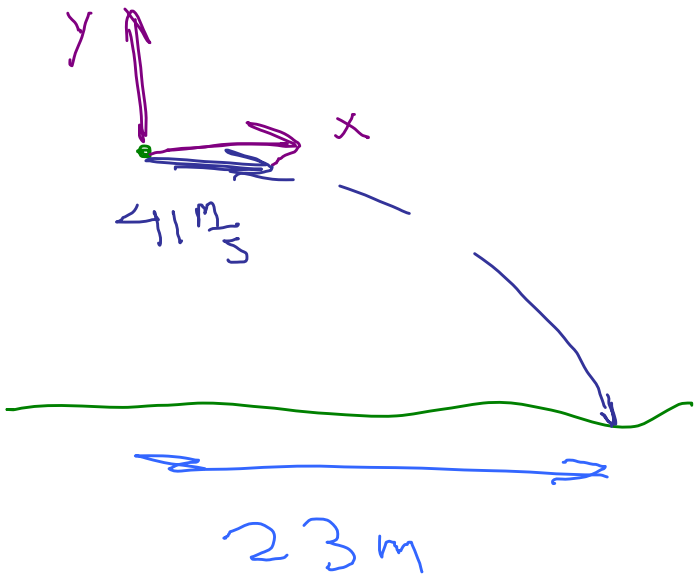
$$t = \boxed{1.34\ s}$$

b) what is x at this time?

$$x = \left(11\ \tfrac{m}{s}\right)(1.34\ s) = \boxed{14.7\ m}$$



3.34 An arrow fired horizontally at $41\ m/s$ travels 23 m horizontally. From what height was it fired?



Find the time at which it hit ground?

$$x = \left(41\ \tfrac{m}{s}\right)t + 0$$

At impact $\quad x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$$23_m = \left(41\ \tfrac{m}{s}\right)t$$

$$t = 0.56\,s$$

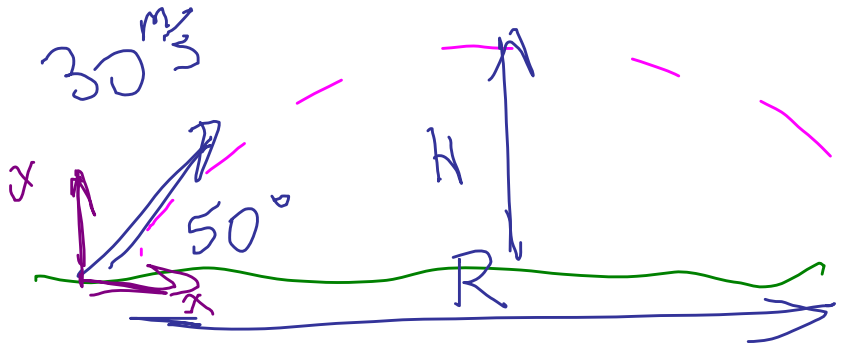
what was y when it hit?

$$y = 0 + 0 + \frac{1}{2}\left(-9.8 \frac{m}{s^2}\right) t^2$$

$$= -1.54\ m$$

Height fmd from was 1.54 m

"Projectile Problem"

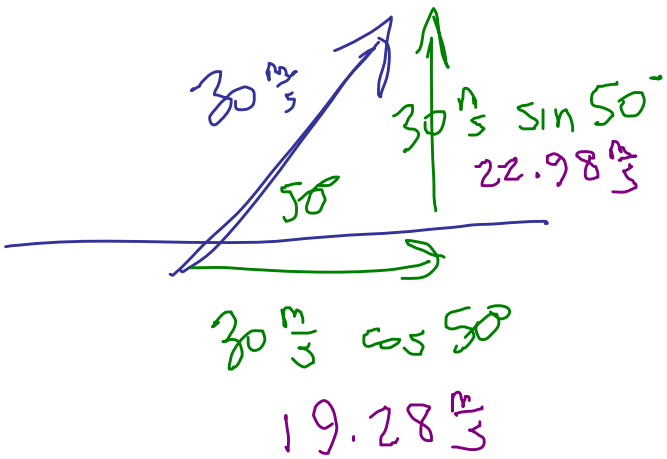


Projectile fired from ground level at 50° above horiz. at speed of 30 m/s

a) Fmd range of proj
b) Fmd time in air
c) Fmd max height,

$$V_x = V_{x0} + a_x t$$

$$V_x = 19.3 \frac{m}{s}$$



$$x = x_0 + V_{x0} t + \frac{1}{2} a_x t^2$$

$$x = (19.3 \frac{m}{s}) t$$

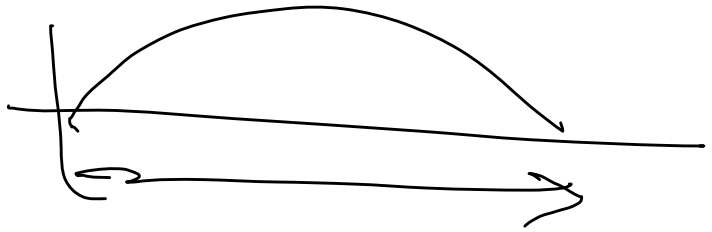
$$V_y = V_{y0} + a_y t$$

$$= 22.98 \frac{m}{s} + (-9.8 \frac{m}{s^2}) t$$

$$y = y_0 + V_{y0} t + \frac{1}{2} a_y t^2$$

$$= (22.98 \frac{m}{s}) t - \frac{1}{2} (9.8 \frac{m}{s^2}) t^2$$

Find range



Find the value of
x at time it hits ground.

Find t of impact, $y = 0$

$$0 = (22.98)t - \frac{1}{2}gt^2$$

Solve for t

$$= t\left(22.98 \frac{m}{s} - 4.9 \frac{m}{s^2} t\right)$$

$t = 0$

$t = 4.7\,s$

Find x at this time