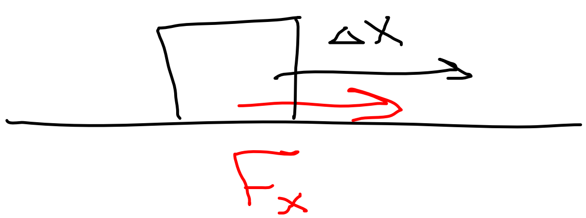
Phys 2110-5 10/3/12

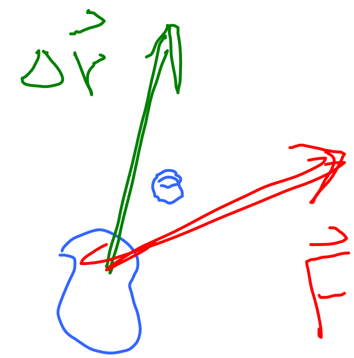
Note Title 10/3/2012

Some stuff from last time

→ Ch 6 Work, Energy



$$W = F_x \Delta x$$



Constant force, motion is str line

$$W = |\vec{F}| \cdot |\Delta\vec{r}| \cos\theta$$

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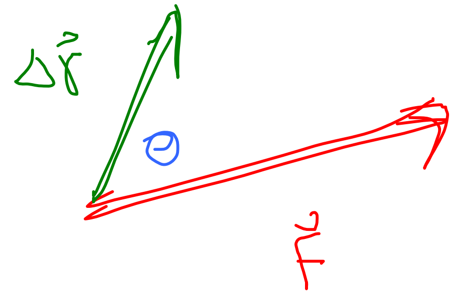
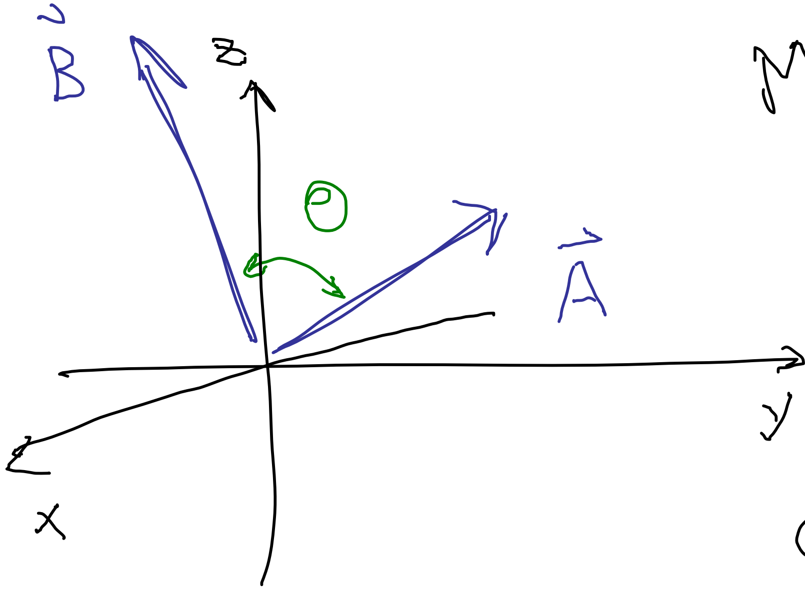
$$= \vec{F}\cdot\Delta\vec{r}$$





Multiply vectors

$$\vec{A}\cdot\vec{B} = AB\cos\theta \qquad (1)$$

Dot product gives scalar

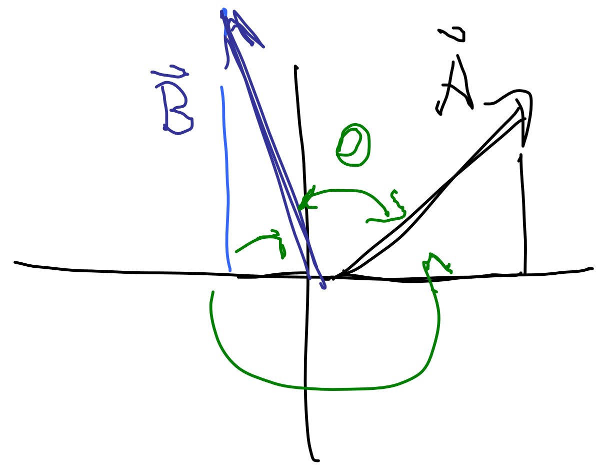
Scalar product.

Comp's of $\vec{A}$, $\vec{B}$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z \qquad (2)$$

Example: Find the angle between

$$\vec{A} = 3\hat{i} + 2\hat{j} \qquad \vec{B} = -\hat{i} + 6\hat{j}$$



$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y$$

$$= 9$$

$$|\vec{A}| = \sqrt{13} \qquad |\vec{B}| = \sqrt{37}$$

$$= A \qquad\qquad = B$$

$$\vec{A}\cdot\vec{B} = 9 = AB\cos\theta = \sqrt{13}\sqrt{37}\cos\theta$$

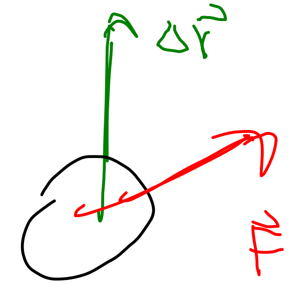
$$\cos\theta = 0.4104 \qquad \theta = 65.8^\circ$$

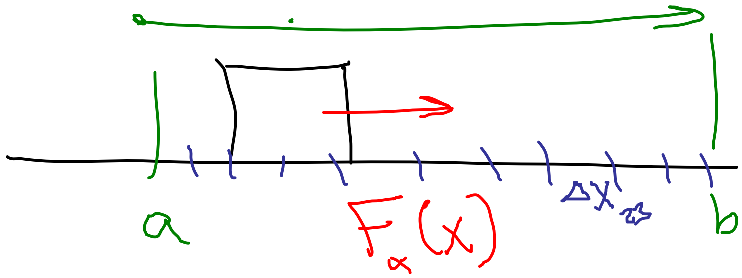
$$W = \vec{F} \cdot \Delta \vec{r}$$

Scalar

Pos or neg

Units: N·m = Joule





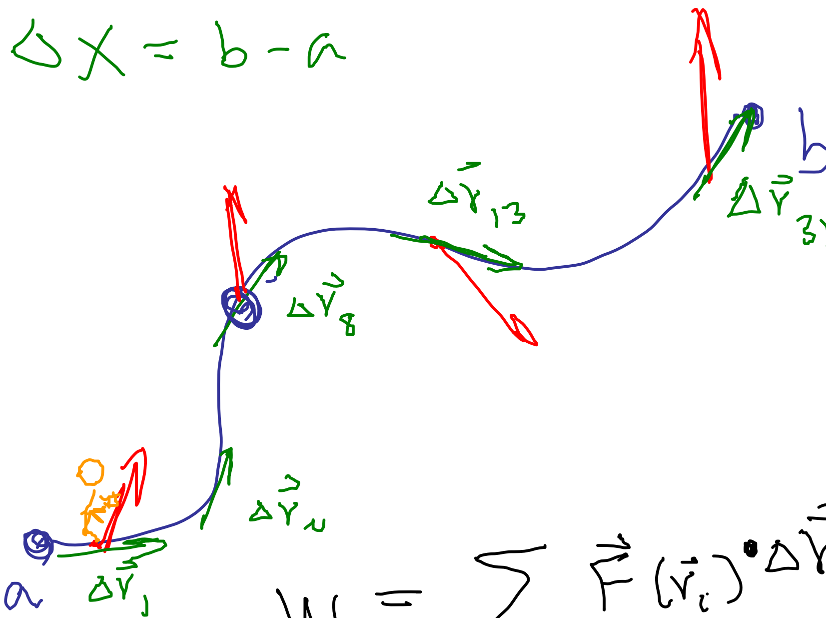
$$\Delta x = b - a$$

$$W = \sum F(x_i)\,\Delta x_i$$

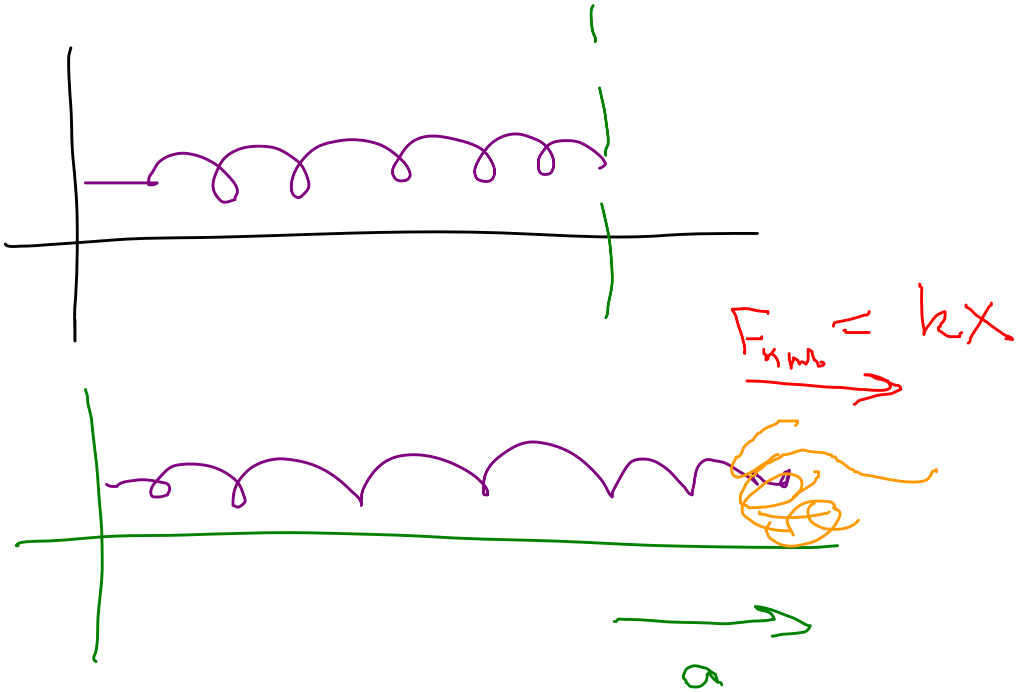
$$= \int_a^b F(x)\,dx$$

$$\Rightarrow W = \int_a^b \vec{F} \cdot d\vec{r}$$

Line Integral



$$W = \sum \vec{F}(\vec{r}_i) \cdot \Delta\vec{r}_i$$



$$W_{hand} = \int_0^a F_x \, dx$$

$$= \int_0^a kx \, dx$$

$$= \frac{1}{2} k a^2$$

J $\quad \frac{N}{m} m^2$

$N \cdot m$

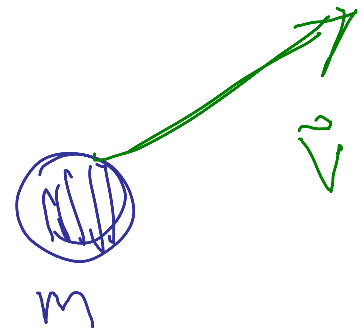
$a \to b$

$$W_{hand} = \int_a^b kx \, dx = \frac{1}{2} k \left(b^2 - a^2\right)$$

W is defined as we said.

What is it good for?

Kinetic Energy



Definition:

$$K = \frac{1}{2} m v^2$$

Kinetic energy scalar.

Units: $[K] = kg \frac{m^2}{s^2} = \frac{kg\, m^2}{s^2} = N \cdot m = J$

9.25 What's the KE of 2.4×10^5 kg airplane cruising at $900 \frac{km}{h}$?

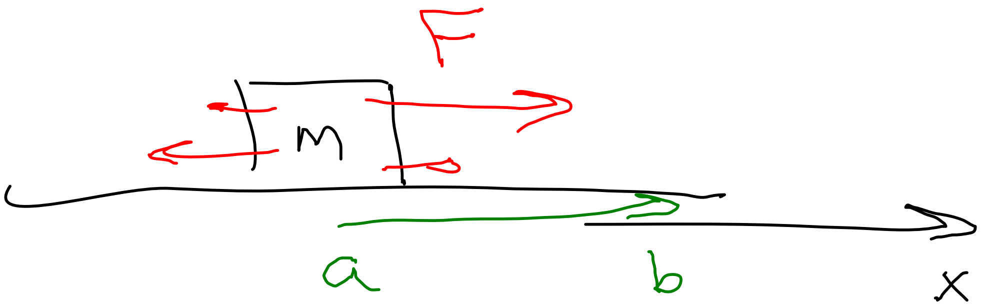
$$v = 900 \frac{km}{h} \left(\frac{1\,h}{3600\,s}\right)\left(\frac{10^3\,m}{1\,km}\right) = 250 \frac{m}{s}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(2.4 \times 10^5\,kg)(250 \tfrac{m}{s})^2$$

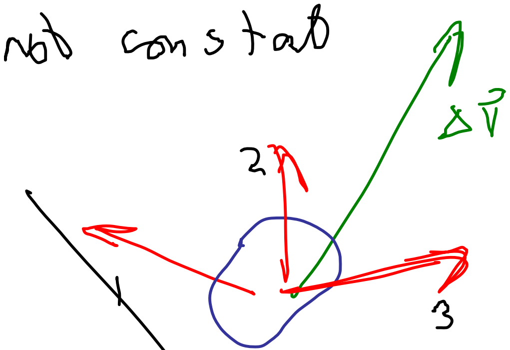
$$= 7.5 \times 10^9\,J = 7.5\,GJ$$

What is it good for?

Derivations   1-D motion



F not constant



$$\underline{W_{net}} = \int_a^b F_{net,x}\, dx = \int_a^b m a_x \, dx$$

function of x

$$W_{net} = W_1 + W_2 + W_3 = (\vec{F}_{net}) \cdot \Delta \vec{r}$$

$$a_x = \frac{dv_x}{dt} = \underbrace{\frac{dx}{dt}}_{=v} \cdot \frac{dv_x}{dx} = v_x \frac{dv_x}{dx}$$

substitute.

$$F_{net,x} = m a_x$$

$$W_{net} = \int_a^b m\, v_x \frac{dv_x}{dx}\, dx$$

$$v_x \frac{dv_x}{dx} = \frac{1}{2}\frac{d}{dx}\left[v^2\right] \checkmark$$

$$= \frac{1}{\cancel{2}}\, \cancel{2} v \frac{dv}{dx} = v\frac{dv}{dx}$$

$$W_{net} = m\int_a^b \frac{1}{2}\frac{d}{dx}\left[v^2\right] dx = \frac{m}{2} v^2 \Big|_a^b = K_b - K_a$$

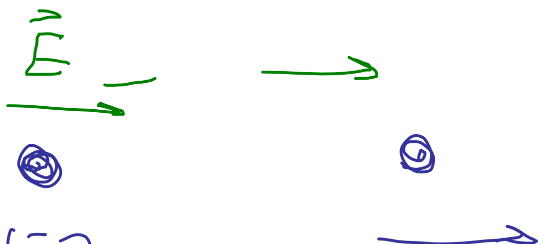
$$= \frac{m}{2}\left(v_b^2 - v_a^2\right) = \underbrace{\frac{1}{2} m v_b^2}_{K_b} - \underbrace{\frac{1}{2} m v_a^2}_{K_a} = \Delta K$$

Shown Work-energy theorem-

$$\boxed{W_{net} = \Delta K}$$

Work & KE

6.26 A cyclotron accelerates protons from rest to 21 $M\frac{m}{s}$. How much work does it do on each proton.



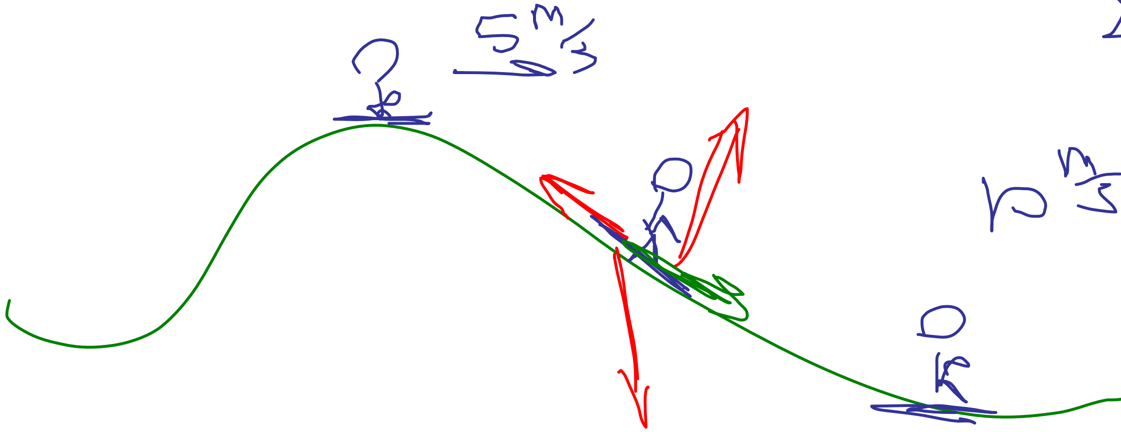
$v=0$

$v = 2.1 \times 10^7 \frac{m}{s}$

$1 eV = 1.602 \times 10^{-19} J$

$$\Delta K = \Delta \tfrac{1}{2} m v^2$$
$$= \tfrac{1}{2}\left(1.67 \times 10^{-27} kg\right)\left(2.1 \times 10^7 \tfrac{m}{s}\right)^2$$
$$= 3.68 \times 10^{-13} J$$
$$= W_{net} \quad = 2.3 \times 10^6 eV$$
$$= 2.3 MeV$$

6.28 A 60-kg skateboarder on hill at 5.0 $\frac{m}{s}$ & reaches 10 $\frac{m}{s}$ at bottom. Find total work done on he or she/it between top & bottom of hill



$$\Delta K = \frac{m}{2}\Delta v^2$$

$$= 30\,kg\left(100\frac{m^2}{s^2} - 25\frac{m^2}{s^2}\right)$$

$$= 2.25\,kJ$$

$$= W_{net}$$