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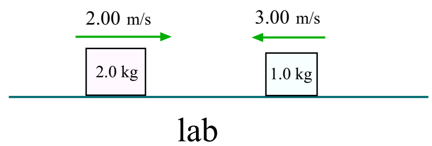
Apr. 5, 2011

Phys 2112, Spring 2011
Quiz #2

1. In a one-dimensional collision which takes place on a frictionless level track, a 2.0 kg mass moves to the right at a speed of $2.0\,\frac{\mathrm{m}}{\mathrm{s}}$ and a 1.0 kg mass moves to the left at $3.0\,\frac{\mathrm{m}}{\mathrm{s}}$.



a) Suppose we observe the collision in a frame which moves to the right at $0.333\,\frac{\mathrm{m}}{\mathrm{s}}$ (the frame of the center of mass), and in this frame the velocities simply reverse in the collision. *In this frame*, what are the final velocities of the masses?

With $V = +0.333\,\frac{\mathrm{m}}{\mathrm{s}}$, we have the velocities before the collision:

$$v_1' = 2.0\,\tfrac{\mathrm{m}}{\mathrm{s}} - 0.333\,\tfrac{\mathrm{m}}{\mathrm{s}} = 1.666\,\tfrac{\mathrm{m}}{\mathrm{s}} \qquad v_2' = -3.0\,\tfrac{\mathrm{m}}{\mathrm{s}} - 0.333\,\tfrac{\mathrm{m}}{\mathrm{s}} = -3.333\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

and after the collision they are

$$v_1' = -1.666\,\tfrac{\mathrm{m}}{\mathrm{s}} \qquad v_2' = +3.333\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

b) What are the final velocities of the masses in the original (lab) frame?

Back in the lab frame the final velocities are

$$v_1 = v_1' + V = -1.666\,\tfrac{\mathrm{m}}{\mathrm{s}} + 0.333\,\tfrac{\mathrm{m}}{\mathrm{s}} = -1.333\,\tfrac{\mathrm{m}}{\mathrm{s}} \qquad v_2 = v_2' + V = +3.333\,\tfrac{\mathrm{m}}{\mathrm{s}} + 0.333\,\tfrac{\mathrm{m}}{\mathrm{s}} = 3.666\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

c) *Show* that velocity of the center of mass *was* $+0.333\,\frac{\mathrm{m}}{\mathrm{s}}$ (as seen in the lab).

$$v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1+m_2} = \frac{(2.0\ \mathrm{kg})(+2.0\,\frac{\mathrm{m}}{\mathrm{s}}) + (1.0\ \mathrm{kg})(-3.0\,\frac{\mathrm{m}}{\mathrm{s}})}{(3.0\ \mathrm{kg})} = +0.333\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

2. Saturn's moon Titan has a mass of 1.345×10^{23} kg and a radius of 2576 km. Find the gravitational acceleration g on the surface of Titan and the escape speed for Titan

$$R = 2.576 \times 106\ \mathrm{m} \qquad M = 1.345 \times 10^{23}\ \mathrm{kg}$$

$$g = G\frac{M}{R^2} = \left(6.67 \times 10^{-11}\,\tfrac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)\frac{(1.345 \times 10^{23}\ \mathrm{kg})}{(2.576 \times 10^6\ \mathrm{m})^2} = 1.34\,\tfrac{\mathrm{m}}{\mathrm{s^2}}$$

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11})(1.345 \times 10^{23})}{(2.576 \times 10^6\ \mathrm{m})}}\,\tfrac{\mathrm{m}}{\mathrm{s}} = 2.64 \times 10^3\,\tfrac{\mathrm{m}}{\mathrm{s}}$$

3. The planet Uranus has a moon Umbriel which orbits at a distance of 2.663×10^5 km from the planet's center with a period of 4.14 days.

a) From these numbers, find the mass of Uranus.

With

$$R = 2.663 \times 10^8 \text{ m} \qquad \text{and} \qquad T = 4.14 \text{ day} \left(\frac{(24)(3600) \text{ s}}{1 \text{ day}}\right) = 3.58 \times 10^5 \text{ s}$$

Use

$$M = \frac{4\pi^2 R^3}{GT^2} = \frac{4\pi^2(2.663 \times 10^8)^3}{(6.67 \times 10^{-11})(3.58 \times 10^5)} \text{ kg} = 8.72 \times 10^{25} \text{ kg}$$

b) If Uranus itself has a radius of 2.527×10^4 km, what is the mean density of Uranus?

With $R = 2.527 \times 10^7$ m and $V = \frac{4}{3}\pi R^3$ for a sphere we have

$$\rho = \frac{M}{V} = \frac{(8.72 \times 10^{25} \text{ kg})}{\frac{4}{3}\pi(2.527 \times 10^7 \text{ m})^3} = 1.29 \times 10^3 \, \frac{\text{kg}}{\text{m}^3} = 1.29 \, \frac{\text{g}}{\text{cm}^3}$$

Show work for all problems and include the right units!

$$\mathbf{v}' = \mathbf{v} - \mathbf{V} \qquad F_{\text{grav}} = G\frac{m_1 m_2}{r^2} \qquad G = 6.67 \times 10^{-11} \, \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \qquad F_c = \frac{mv^2}{R}$$

$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM} \qquad g = G\frac{M}{R^2} \qquad v = \sqrt{\frac{2GM}{R}} \qquad \rho = \frac{M}{V}$$