Phys 3810, Spring 2009
Problem Set #4, Hint-o-licious Hints

1. *Griffiths*, **4.2** Show that the energies are given by

$$E_{(n_x.n_y,n_z)} = \frac{\pi^2\hbar^2}{2ma^2}(n_x^2 + n_y^2 + n_z^2)$$

The latter part of the problem is a math–puzzle sort of thing where you figure out how many ways one can get the same E with different sets of integers.

You can find the values and the degeneracies from E_1 up to E_{14}, but if you run out of patience, I'll tell you that

$$E_{14} = \frac{\pi^2\hbar^2}{2ma^2}(27)$$

So what's different about this value from the first few?

2. *Griffiths*, **4.38** The separated solution is

$$\psi(\mathbf{r}) = X(x)Y(y)Z(z)$$

Note that when you write out the Schrödinger equation in cartesian coordinates for this potential with the separated solution (substitute and then divide by $\psi = XYZ$) it becomes a sum of three terms each of which depends only on x y or z. This means that you can write three separate Schrödinger equations, i.e.

$$-\frac{\hbar^2}{2m}\frac{1}{X}\frac{d^2X}{dx^2} + \tfrac{1}{2}m\omega^2 = E_x \qquad \text{, etc.}$$

which we know how to solve. (Be as clear as you can about this part.) The total energy is just sum of energies from each separate Schrödinger equation:

$$E = E_x + E_y + E_z = \hbar\omega(n_x + n_y + n_z + \tfrac{3}{2})$$

The second part involves more thought. The energy of the oscillator state just depends on

$$n \equiv n_x + n_y + n_z \qquad \text{for} \qquad n_x = 0, 1, 2, \ldots \quad \text{etc.}$$

so the question is how many ways can you get n from the three separate indices. That's a sort of puzzle–math problem. First, see if you can spot the pattern for the lowest n's. (To give you one: $n = 3$ has 10 possible states.)

3. *Griffiths*, **4.8** Test that the $u(r)$ radial solution

$$u_1(r) = Arj_1(kr)$$

really does solve Eq. (4.41).

For the infinite spherical well, the boundary condition $u(a) = 0$ (continuity of the wave function) leads to $ka = \tan(ka)$. (Show this, of course.) Use a graph to demonstrate that for the higher solutions, we have

$$ka \approx \frac{(2n+1)}{2}\pi \qquad \text{for} \quad n = 1, 2, 3, \ldots$$

4. *Griffiths*, **4.10** Start off with c_0 arbitrary and then use (4.76) to get the succeeding coefficients for the polynomial $v(\rho)$. They will truncate after very few terms. The math isn't hard it's just to get familiar with the notation and see how the math works out. The correct value for c_0 would come from normalizing the radial function but that' snot necessary here.

5. *Griffiths*, **4.45** Obviously you want to do the integral $\int_V |\psi_{100}|^2 \, dV$ for a sphere of radius b centered at the origin, b being the radius of the nucleus. Normally, I work these out by hand, but when my tables just gave a recursion formula for $\int x^2 e^{-ax} \, dx$, I just turned to Maple!

6. *Griffiths*, **4.11** The normalization condition on $R(r)$ is

$$\int_0^\infty R(r)^2 r^2 \, dr = 1$$

Find the c_0 (that is, the overall constant) which makes this true and then write out $R(r)$. You can compare with Table 4.7.

7. *Griffiths*, **4.14** The trick here is that the probability to find the electron at a certain radius r (within dr) is *not* $R(r)^2 \, dr$. A hint comes from how $R(r)$ is normalized (as used in Prob 6, Griff 4.11); the integral of any probability distribution must give 1.

When you do get the right probability function, find its maximum to get the most probable value.

8. *Griffiths*, **4.16** Note that *only* place in the whole derivation where the electric charges showed up was in the potential

$$V(r) = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

Obviously if the central charge is replaced by $+Ze$, then the e^2 in the potential gets replaced by Ze^2, but then e^2 should be replaced *everywhere* by this. See how the result for the energy levels changes. (Is it proportional to Z? To Z^2?)