Name Units? Units?

Class Time: 9am 10am 11am

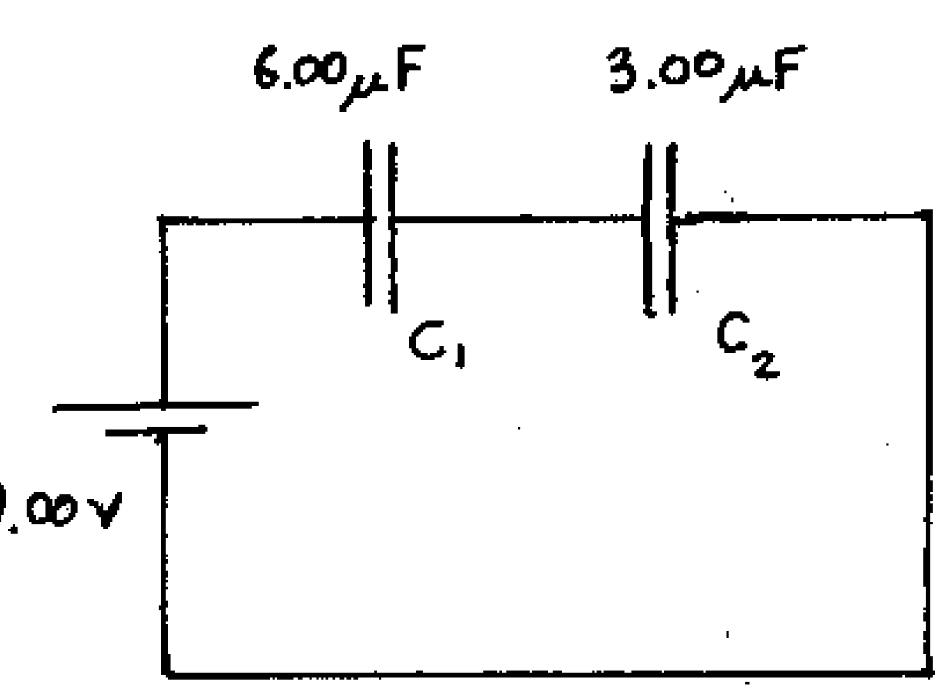
Mar 27, 2003

Phys 2120 — Spring 2003

Exam #2

1.		(7)
2.		(10)
3.		(8)
4.		(15)
5.		(15)
6.		(13)
7.		(13)
8.		(9)
9.		(10)
Total		(100)

You must show all your work and include the right units with your answers!



1. A 9.00 V potential difference is applied across two capacitors $C_1 = 6.00\,\mu\text{F}$ and $C_2 = 3.00\,\mu\text{F}$, connected in series.

Find the individual charges (Q_1 and Q_2) on the two capacitors. (7)

Equivalent capacitance of the pair of C's is:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{(6.00\mu F)} + \frac{1}{(3.00\mu F)} = 0.500\,\mu F^{-1}$$

$$\rightarrow \quad C_{eq} = 2.00\,\mu F$$

Then the charge on either end plate of the combination is

$$q = C_{eq} V = (2.00\times10^{-6} F)(9.00 V) = 1.80\times10^{-5} C$$

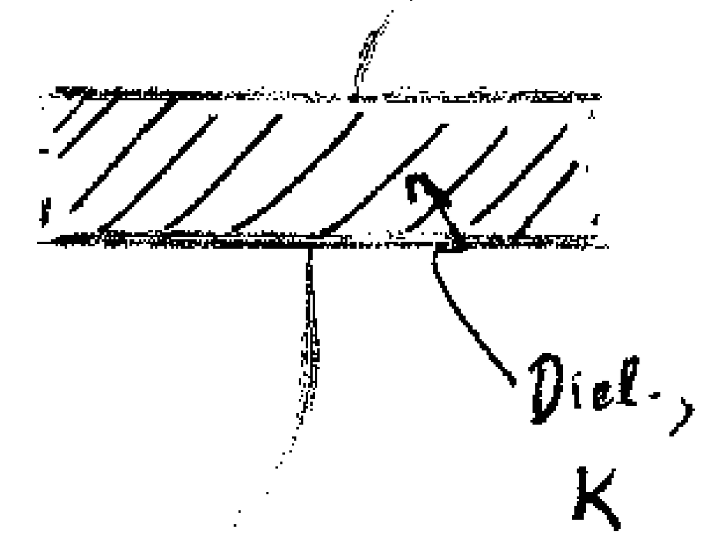
Then this is the charge on C_1 or C_2, i.e.

$$Q_1 = Q_2 = 1.80\times10^{-5} C$$

2. A parallel–plate capacitor has plates of area $0.100\,\mathrm{m}^2$ separated by 1.00 mm. The volume between the plates is filled with a dielectric.

When a 12.0 V potential difference is applied across the plates, the capacitor stores a charges of 15.0 nC.

a) What is the capacitance of the capacitor? (3)



The capacitance of the (filled) capacitor can be found from $q = CV$; thus,

$$C = \frac{q}{V} = \frac{(15.0\times10^{-9}\,C)}{(12.0\,V)} = 1.25\times10^{-9}\,F$$

b) What is the value of the dielectric constant for the material between the plates? (7)

If the capacitor were air-filled (empty!) then from the given dimensions its capacitance would be

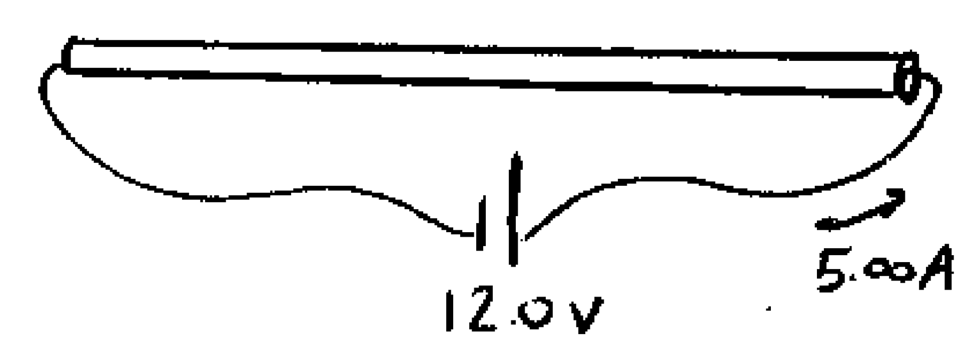
$$C_{air} = \epsilon_0 \frac{A}{d} = (8.85\times10^{-12})\frac{(0.100)}{(1.00\times10^{-3})}\,F = 8.85\times10^{-10}\,F$$

Using $C = \kappa\, C_{air}$, get:

$$\kappa = \frac{C}{C_{air}} = \frac{1.25\times10^{-9}F}{8.85\times10^{-10}F} = 1.41$$

3. A copper wire has circular cross-section with radius 1.00 mm and a length L; when a 12.0 V potential difference is applied across its ends, the current in the wire is 5.00 A. What is the length of the wire? (8)

[The value of the resistivity of copper is $1.69\times10^{-8}\,\Omega\cdot\mathrm{m}$.]



Resistance of the wire is

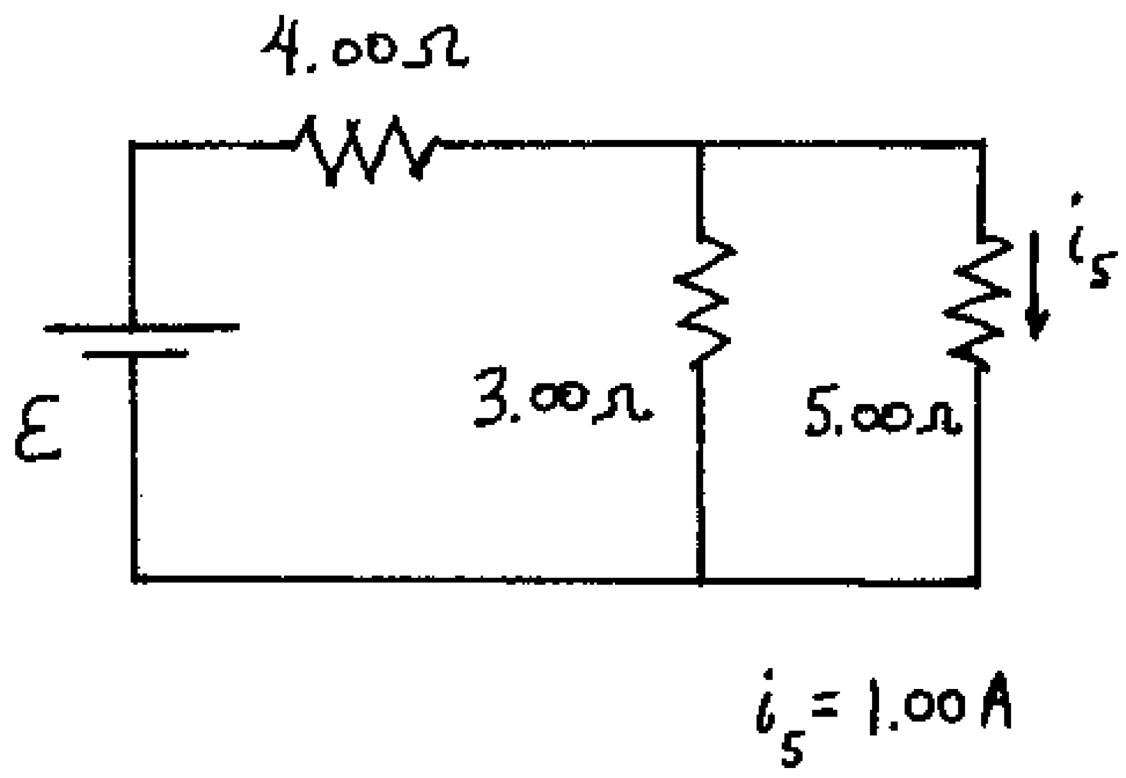
$$R = \frac{V}{i} = \frac{12.0\,V}{5.00\,A} = 2.40\,\Omega$$

From $R = \rho\frac{L}{A}$ w/ $A = \pi r^2$, get:

$$L = \frac{RA}{\rho} = \frac{R\pi r^2}{\rho} = \frac{(2.40\,\Omega)\,\pi\,(1.00\times10^{-3}\,m)^2}{(1.69\times10^{-8}\,\Omega\cdot m)}$$

$$= 446\,m$$

4. In the circuit shown here, the current in the 5.00 Ω resistor is 1.00 A.



a) What is the potential difference across the 5.00 Ω resistor? (2)

Ohm's Law:

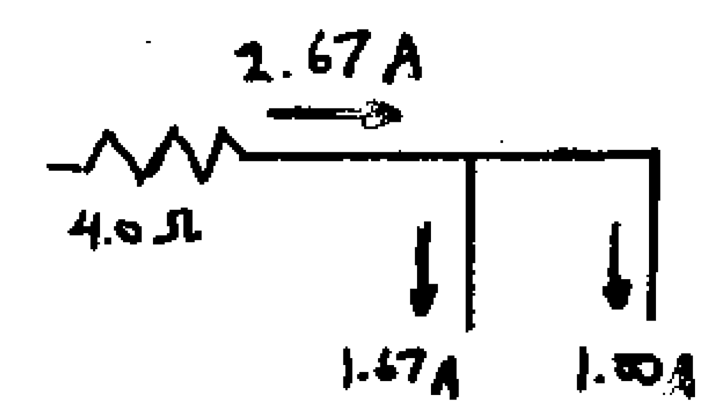
$$V = iR = (1.00A)(5.00\Omega) = 5.00\,V$$

b) What is the current in the 3.00 Ω resistor? (3)

From (a), potential across the 3.00Ω resistor is 5.00V so from Ohm's Law,

$$i = \frac{V}{R} = \frac{(5.00V)}{(3.00\Omega)} = 1.67\ A$$

c) What is the power dissipated in the 4.00 Ω resistor? (5)



From the Kirchhoff junction rule we know that the current in the 4.00Ω resistor must be 1.67 A + 1.00 A = 2.67 A

so the power dissipated in that resistor is

$$P = i^2 R = (2.67\,A)^2 (4.00\Omega) = 28.4\ W$$

d) What is the emf $\mathcal{E}$ of the battery? (5)

The Kirchhoff loop rule taken around the "small" loop gives

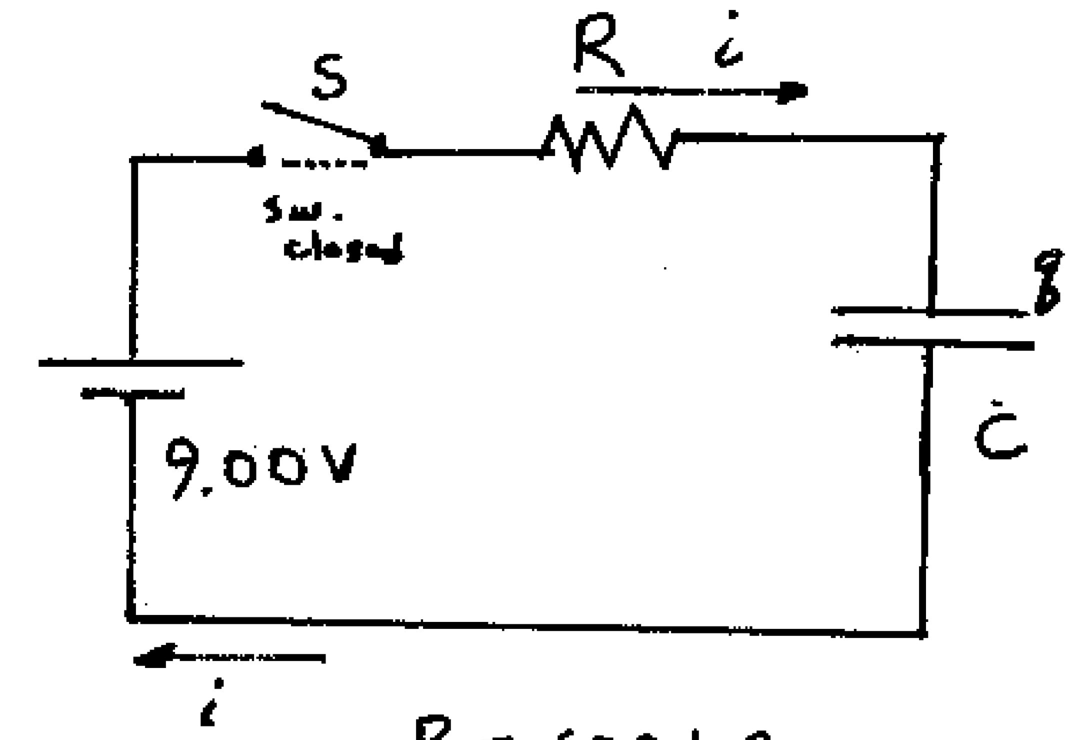
$$\mathcal{E} - (2.67\,A)(4.00\Omega) - (1.67\,A)(3.00\Omega) = 0$$

Solve for $\mathcal{E}$. Get:

$$\mathcal{E} = 15.7\ V$$

5. Consider the RC circuit shown here, where a 9.00 V battery is in series with a 600 kΩ resistor and capacitor; at $t = 0$ the switch S is closed, so that the capacitor begins to charge.

At $t = 0.300$ s the potential difference across the capacitor is 6.00 V.



$R = 600\ \text{k}\Omega$

From dir of current we know they are voltage drops across R and C.

a) What is the potential difference across the resistor at this time? (2)

Use Kirchhoff loop rule, get:

$$+9.00\text{ V} - V_R - 6.00\text{ V} = 0$$

$$V_R = 3.00\text{ V}$$

b) What is the current in the resistor at this time? (3)

From Ohm's Law & answer to (a),

$$i = \frac{V}{R} = \frac{3.00\text{ V}}{600\times10^{3}\,\Omega} = 5.00\times10^{-6}\text{ A} = 5.00\,\mu\text{A}$$

c) What is the time constant τ for the circuit? (5)

Current as a function of time is $i(t) = \frac{\mathcal{E}}{R}e^{-t/\tau}$. Using data at $t = 0.30$ s, get:

$$(5.00\times10^{-6}\text{A}) = \frac{(9.00\text{ V})}{(600\times10^{3}\,\Omega)}\exp[-(0.30\text{s}/\tau)]$$

Solve for τ:

$$-(0.30\text{s}/\tau) = -1.099 \longrightarrow \tau = 0.273\text{ s}$$

d) What is the value of the capacitance C? (3)

From $\tau = RC$, get

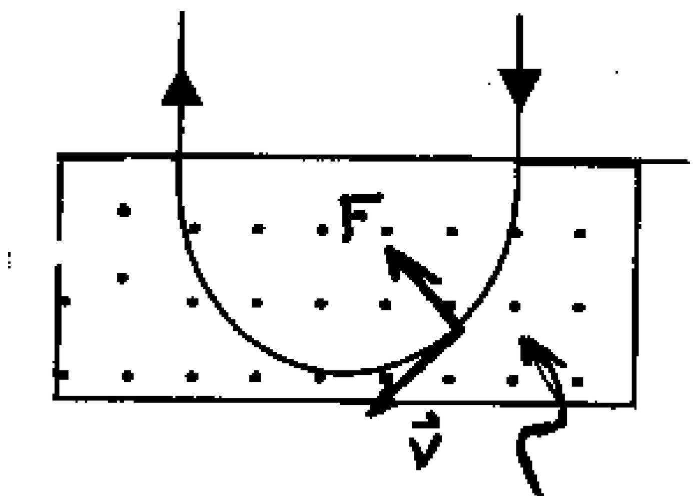
$$C = \frac{\tau}{R} = \frac{(0.273\text{s})}{(600\times10^{3}\,\Omega)} = 4.55\times10^{-7}\text{C} = 0.455\,\mu\text{C}$$

e) What is the value of the potential difference across the capacitor at $t = 1$ day ? (2)

No calculation necessary! After 1 day (enormously long compared to the time constant) the current is essentially zero and $V_R = 0$. Then the potential diff across C is the same as the battery voltage,

$$V_C = 9.00\text{ V}$$

6. In the figure at the right, a particle moves into a region of uniform magnetic field **B**, goes through half a circle and then exits the region.



a) The particle is either a proton or an electron; which one is it? (3) $\vec{F}$ must point inward. Can get $\vec{F} = q\vec{v}\times\vec{B}$ this way only if q is positive so the particle is a proton.

b) What is the magnitude of **B** if the particle spends 1.30 ns within the region? (Hint: From this information, can you calculate r/v for the particle's motion? (10)

Particle travels ½-circle in 1.30 ns hence

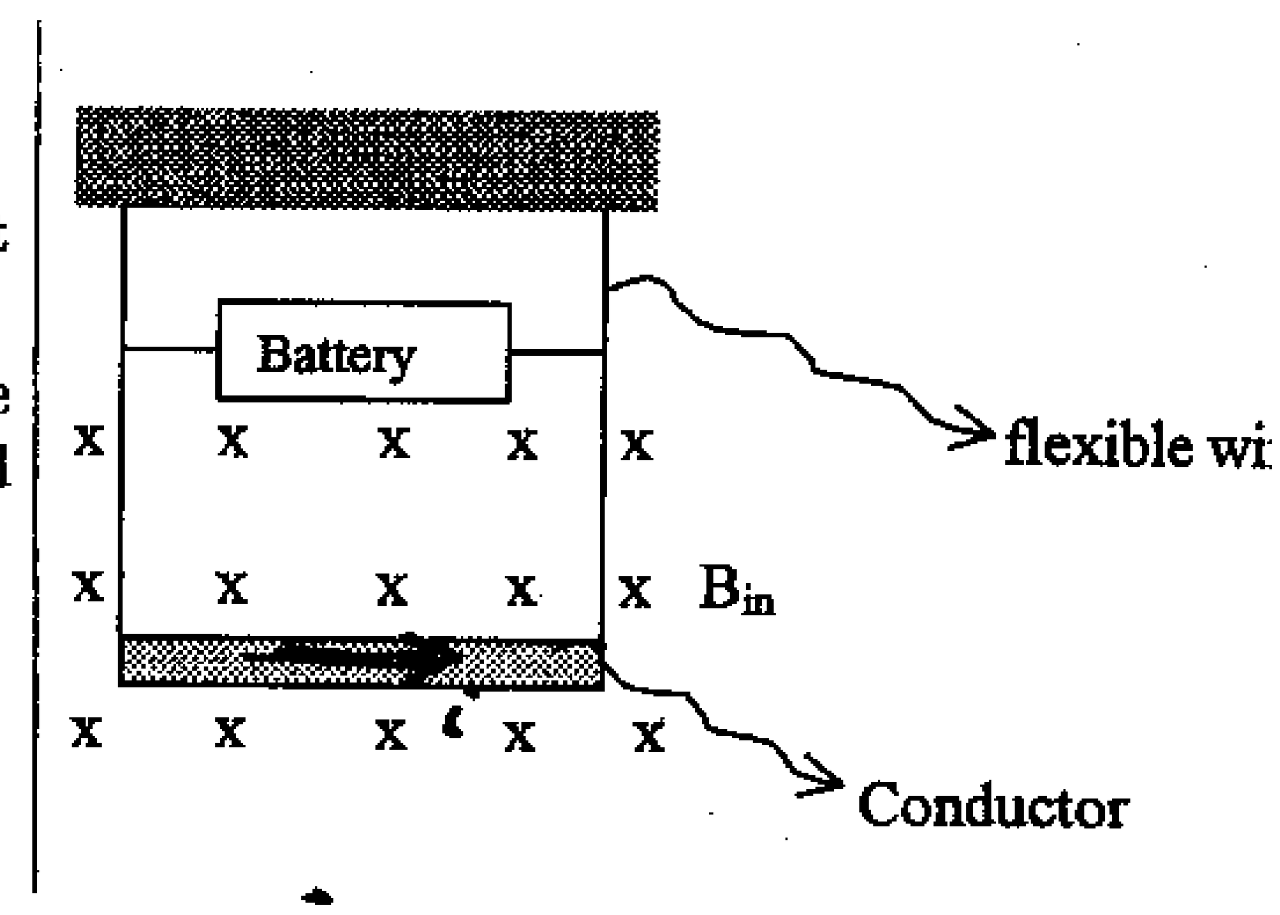
$$v = \frac{\pi r}{t} \rightarrow \frac{r}{v} = \frac{t}{\pi} = \frac{(1.30\times10^{-9}\,s)}{\pi} = 4.14\times10^{-10}\,s$$

Use relation for circular orbit in B field, get:

$$\frac{mv^2}{r} = qvB \rightarrow B = \frac{mv}{qr} = \frac{m}{q}\left(\frac{v}{r}\right) = \frac{(1.673\times10^{-27}\,kg)}{(1.602\times10^{-19}\,C)}\left(4.14\times10^{-10}\,s\right)^{-1}$$

$$= 25.2\ T$$

7. A conductor suspended by two flexible wires as shown at the right has a mass per unit length of 4.00×10^{-2} kg/m.



a) What current must exist in the conductor in order for the tension in the supporting wires to be zero if the magnetic field over the region is 3.60 T into the page? (10)

Magnitude of downward grav force on bar is

$F_{grav} = mg = L\lambda g$ where λ = mass density.

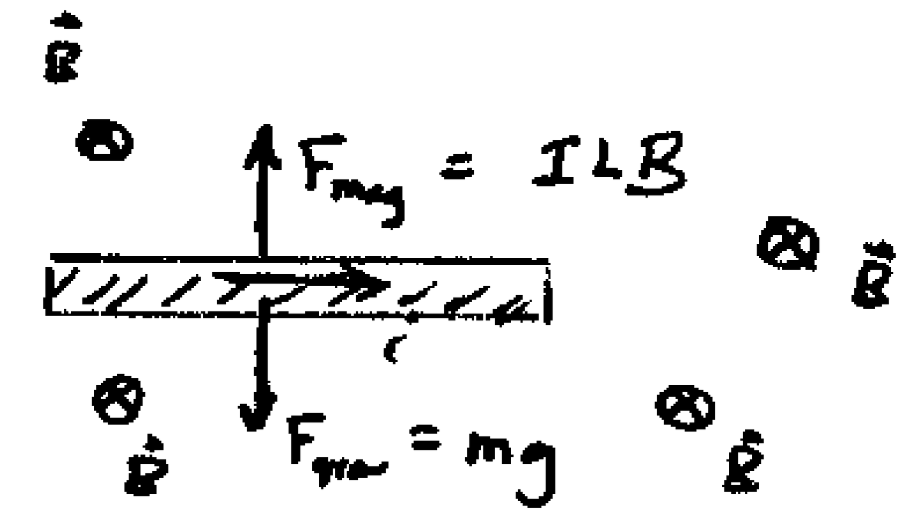
Magnitude of upward mag. force on bar is

$F_{mag} = ILB$ (since current ⊥ to $\vec{B}$ field).

Tension is zero when these forces balance, so

$L\lambda g = ILB$

$$\rightarrow I = \frac{\lambda g}{B} = \frac{(4.00\times10^{-2}\,kg/m)(9.80\,m/s^2)}{(3.60\,T)} = 0.109\ A$$

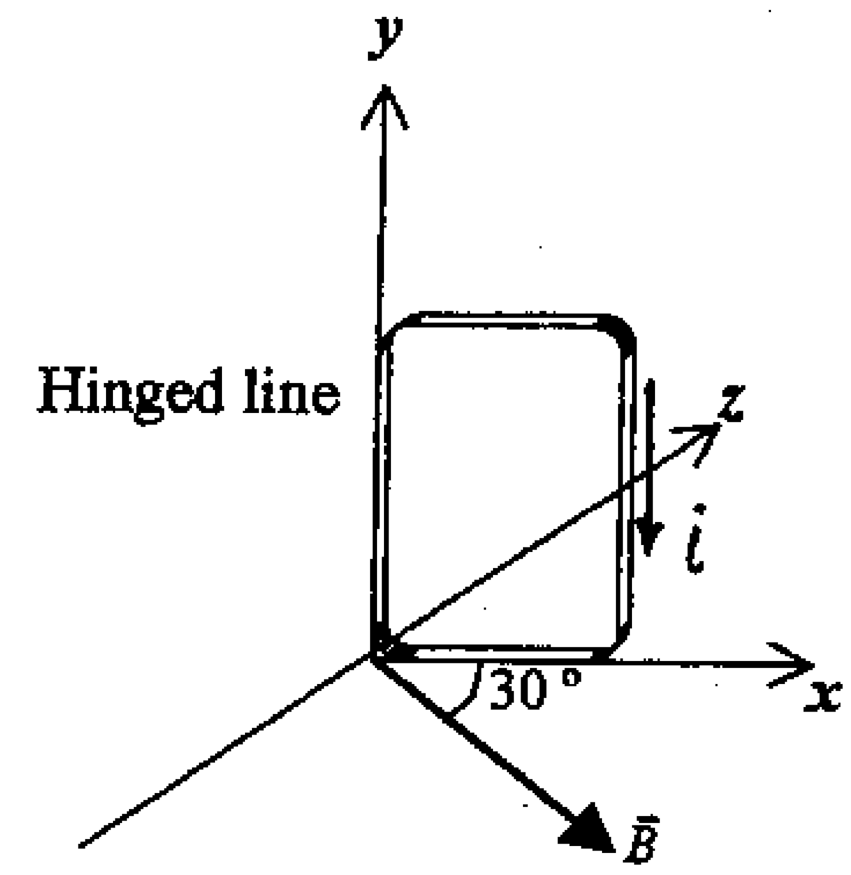


b) What is the required direction of the current? (Draw the direction of the current in the figure.) (3)

In order for the magnetic force to point up current must go as shown (left to right).

8. The figure shows a rectangular 20–turn coil of wire, of dimensions 10.0 cm by 5.00 cm. It carries a current of 0.10 A in the direction shown and is hinged along one long side. It is mounted in the xy plane, at 30° to the direction of a uniform magnetic field of magnitude 0.500 T.

Find the magnitude of the torque acting on the coil about the hinge line. (9)



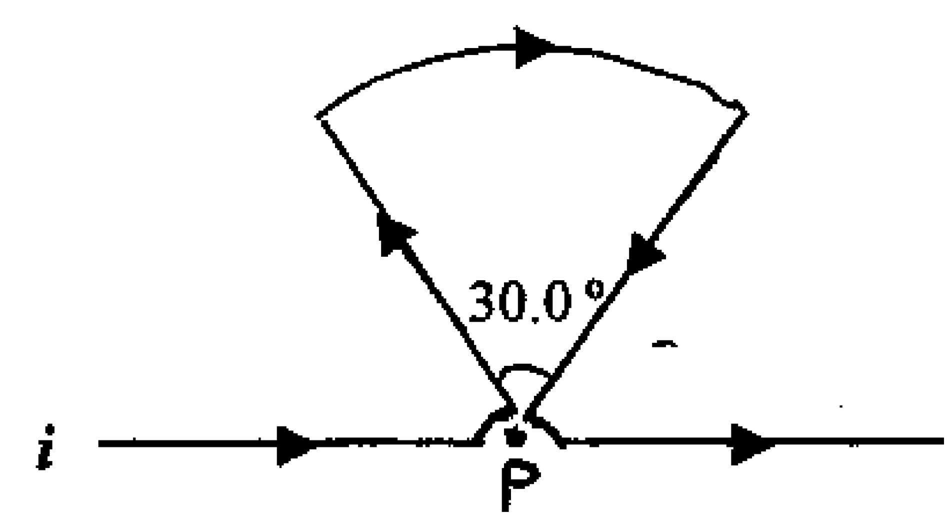
Formula for magnitude of torque on current-carrying coil in uniform $\vec{B}$ field is $\tau = NiAB\sin\phi$ where ϕ is angle between B field and normal to plane. Here, $\phi = 90° + 30° = 120°$ (or 60° for purposes of $\sin\phi$). Then:

$$\tau = NiAB\sin\phi = (20)(0.10\,A)(0.100\,m)(5.00\times10^{-2}\,m)(0.500\,T)(\sin 120°)$$

$$= 4.33\times10^{-3}\ N\cdot m$$

9. A current path shaped as shown in the figure produces a magnetic field at P, the center of the arc. If the arc subtends an angle of 30° and the radius of the pie-shaped part of the path is 0.48 m, what are the magnitude and direction of the field produced at P if the current is 3.00 A?

(Ignore the contributions to the field due to the current in the small arcs near P.) (10)



The straight parts of the don't contribute anything to the magnetic field at P since in applying the Biot-Savart law $d\vec{s}$ is parallel to $\vec{r}$ so $d\vec{s}\times\vec{r}$ is zero. Only the arc contributes. By the right-hand rule for currents the field from the arc points into the page and its magnitude is

$$B = B_{arc} = \frac{\mu_0 i \phi}{4\pi R} = \frac{(4\pi\times10^{-7})(3.00\,A)(\pi/6)}{4\pi(0.48\,m)}$$

Use $30° = \pi/6$

$$= 3.3\times10^{-7}\,T$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}} \qquad \epsilon_0 = 8.85 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}} \qquad e = 1.602 \times 10^{-19}\,\mathrm{C}$$

$$m_{\text{elec}} = 9.1094 \times 10^{-31}\,\mathrm{kg} \qquad m_{\text{prot}} = 1.673 \times 10^{-27}\,\mathrm{kg} \qquad 1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$$

$$\mathbf{F} = m\mathbf{a} \qquad g = 9.80\,\frac{\mathrm{m}}{\mathrm{s^2}} \qquad F = k\frac{|q_1q_2|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2}$$

$$\mathbf{F} = q\mathbf{E} \qquad E_{\text{pt ch}} = k\frac{|q|}{r^2} \qquad dq = \lambda dx \qquad dq = \sigma dA \qquad dq = \rho dV$$

$$E_{\text{plane}} = \frac{\sigma}{2\epsilon_0} \qquad E_{\text{cond surf}} = \frac{\sigma}{\epsilon_0} \qquad p = qd \qquad E_{\text{dipole}} = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3}$$

$$E_{\text{ring}} = \frac{qz}{4\pi\epsilon_0(z^2+R^2)^{3/2}} \qquad \vec{\tau} = \mathbf{p} \times \mathbf{E} \qquad U = -\mathbf{p}\cdot\mathbf{E}$$

$$\Phi = \oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{encl}}}{\epsilon_0} \qquad \Delta U + \Delta K = 0 \qquad \Delta U = q\Delta V \qquad \Delta V = -\int_i^f \mathbf{E}\cdot d\mathbf{s}$$

$$V_{\text{pt-ch}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r} \qquad E_x = -\frac{\partial V}{\partial x} \qquad E_{x,\text{uniform}} = -\frac{\Delta V}{\Delta x}$$

$$q = CV \qquad C_{\text{p.-pl.}} = \epsilon_0\frac{A}{d} \qquad C_{\text{cyl}} = 2\pi\epsilon_0\frac{L}{\ln(b/a)} \qquad C_{\text{sph}} = 4\pi\epsilon_0\frac{ab}{b-a}$$

$$C = \kappa C_{\text{air}} \qquad C_{\text{par}} = C_1 + C_2 + \dots \qquad \frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots \qquad U = \frac{q^2}{2C} = \tfrac{1}{2}CV^2$$

$$u = \tfrac{1}{2}\epsilon_0 E^2 \qquad i = \frac{dq}{dt} \qquad J = i/A \qquad J = (ne)v_d \qquad V = iR \qquad P = iV = i^2R$$

$$R = \rho\frac{L}{A} \qquad R_{\text{series}} = R_1 + R_2 + \dots \qquad \frac{1}{R_{\text{par}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

$$\tau = RC \qquad q(t) = C\mathcal{E}(1 - e^{-t/\tau}) \quad i(t) = \frac{\mathcal{E}}{R}e^{-t/\tau} \qquad q(t) = q_0 e^{-t/\tau} \quad i(t) = \left(\frac{q_0}{RC}\right)e^{-t/\tau}$$

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \qquad \mathbf{F} = I\mathbf{L} \times \mathbf{B} \qquad \frac{mv}{r} = qB \qquad \mu = NiA \qquad \tau = \mu B \sin\phi \qquad \vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\frac{\mathrm{T\cdot m}}{\mathrm{A}} \qquad d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i\,d\mathbf{s} \times \mathbf{r}}{r^3} \qquad B_{\text{wire}} = \frac{\mu_0 i}{2\pi R} \qquad B_{\text{arc}} = \frac{\mu_0 i\phi}{4\pi R} \qquad B_{\text{loop}} = \frac{\mu_0 i}{2R}$$