Phys 4620, Spring 2006
Exam #3

1. In the problem set we found the (classical) rate of energy loss of the electron in a hydrogen atom. It was too damn big!

In the Phys 2020 magnetic fields lab, we accelerate electrons through a potential difference of about 150 V. Then they are made to go in a circular path by a magnetic field which is perpendicular to the plane of their motion. The radius of the path is about 3.0 cm.

a) What is the speed of the electrons as they move on the circular path? Do we need to worry about relativity in studying their motion? If not, why not?

The speed of the electrons is the same as what they have after being accelerated through the potential; since their kinetic energy is then

$$T = |q\Delta V| = e(150\ \mathrm{V}) = 150\ \mathrm{eV} = 2.4 \times 10^{-17}\ \mathrm{J}$$

then assuming Newtonian mechanics for which $T = \frac{1}{2}mv^2$, the speed of the electrons is

$$v^2 = \frac{2T}{m} = \frac{2(2.4 \times 10^{-17}\ \mathrm{J})}{(9.11 \times 10^{-31}\ \mathrm{kg})} = 5.26 \times 10^{13}\ \tfrac{\mathrm{m}^2}{\mathrm{s}^2} \qquad \Longrightarrow \qquad v = 7.26 \times 10^{6}\ \tfrac{\mathrm{m}}{\mathrm{s}}$$

This speed is significantly less than c (though not $< 1\%$) so if we just want a couple decimal places of accuracy we can ignore relativity.

b) What is the magnitude of the acceleration of the electrons?

Recalling our happy days in Phys 2110 we calculate

$$a_c = \frac{v^2}{r} = \frac{(7.26 \times 10^{6}\ \frac{\mathrm{m}}{\mathrm{s}})^2}{(0.030\ \mathrm{m})} = 1.76 \times 10^{15}\ \tfrac{\mathrm{m}}{\mathrm{s}^2}$$

c) Make the low–velocity approximation made in the text and find the total power radiated by the electrons.

We'll approximate $v \ll c$; then we can use the Larmor formula,

$$\begin{aligned} P &\approx \frac{\mu_0 q^2 a^2}{6\pi c} \\ &= \frac{(4\pi \times 10^{-7})(1.6 \times 10^{-19})^2 (1.76\ \frac{\mathrm{m}}{\mathrm{s}^2})^2}{6\pi(3.00 \times 10^{8}\ \frac{\mathrm{m}}{\mathrm{s}})} \frac{\mathrm{J}}{\mathrm{s}} = 1.76 \times 10^{-23}\ \tfrac{\mathrm{J}}{\mathrm{s}} \end{aligned}$$

d) At this rate of energy loss, how long would it take the electrons to lose 10% of their kinetic energy? Is this number bigger than a 2020 lab period?

10% of the initial KE is 2.4×10^{-18} J. With $\Delta E = Pt$, the time to lose this much energy is

$$t = \frac{\Delta E}{P} = \frac{(2.4 \times 10^{-18}\ \mathrm{J})}{(1.76 \times 10^{-23}\ \mathrm{J/s})} 1.36 \times 10^{4}\ \mathrm{s} = 38\ \mathrm{hr}$$

This is much longer than a lab period! The electrons in the 2020 lab don't lose much energy due to radiation.

2. In the lab reference frame, particle A moves in the $+x$ direction with speed $\frac{1}{2}c$ and particle B moves in the $+x$ direction with speed $\frac{3}{4}c$.

What is the speed of particle B in the reference frame of particle A?

In the frame of A, the lab moves at $v_x = -c/2$. And B moves at $+\frac{3}{4}c$ with respect to the lab. The do a relativistic addition of these velocities and get:

$$v_{BA} = \frac{-\frac{c}{2} + \frac{3}{4}c}{1 + \frac{(-1/2)(3/4)c^2}{c^2}} = \frac{c/4}{1 - 3/8} = \frac{8}{5}\frac{c}{4} = \frac{2}{5}c$$

3. What is the (relativistic) kinetic energy of a proton which has a speed of $0.9c$? (Use $m_p c^2 = 938.27$ MeV.)

Energy of the proton is

$$E = \frac{m_{\mathrm{p}} c^2}{\sqrt{1 - u^2/c^2}} = \frac{m_{\mathrm{p}} c^2}{\sqrt{1 - \left(\frac{0.9c}{c}\right)^2}} = 2.29 m_{\mathrm{p}} c^2$$

This gives

$$E = 2.15 \times 10^{3}\ \mathrm{MeV} = m_{\mathrm{p}} c^2 + T$$

so then

$$T = 2153\ \mathrm{MeV} - 938.27\ \mathrm{MeV} = 1214\ \mathrm{MeV}$$

4. A proton with a kinetic energy of 1.0 GeV is incident on a stationary proton (in the lab frame).

a) Find the momentum of the incident proton in the lab frame. (Use $m_p c^2 = 938.27$ MeV.)

In the lab frame, the energy of the moving proton is

$$E_p = m_p c^2 + 1.0\ \mathrm{GeV} = 1938\ \mathrm{MeV}$$

then with $E_p^2 = p^2c^2 + m^2c^4$ we get

$$p^2c^2 = (1938\ \mathrm{MeV}^2 - (938\ \mathrm{MeV})^2 = 2.87 \times 10^{6}\ \mathrm{MeV}^2$$

so

$$pc = 1696\ \mathrm{MeV} \qquad \text{or} \qquad p = 1696\ \mathrm{MeV}/c$$

Leave the units this way!

b) Using the invariance of the square of the total momentum 4–vector, find the energy of (either) proton in the center-of-momentum frame.

In the lab frame the total energy is

$$E_p + m_p c^2 = 2877 \ \mathrm{MeV}$$

The total 3-momentum is $1696 \ \mathrm{MeV}/c$. So the total momentum 4-vector is

$$p^{\mu}_{\mathrm{Tot}} = (2877 \ \mathrm{MeV}/c, \ 1696 \ \mathrm{MeV}/c)$$

In the CM frame each proton has energy E'_p and the total 3-momentum is zero. So we have

$$p'^{\mu}_{\mathrm{Tot}} = (2E'_p, 0)$$

Invariance of the scalar product $p^{\mu}p_{\mu}$ gives

$$-(2877 \ \mathrm{MeV}/c)^2 + (1696 \ \mathrm{MeV}/c)^2 = -4{E'_p}^2$$

Get

$$-4{E'_p}^2 = -5.40 \times 10^6 \ \mathrm{MeV}^2 \qquad \Longrightarrow \qquad E'_p = 1161 \ \mathrm{MeV}$$

c) Find the speed of (either) proton in the center-of-mementum frame.

Use

$$E'_p = \frac{mc^2}{\sqrt{1 - u'^2/c^2}} = \frac{938.3 \ \mathrm{MeV}}{\sqrt{1 - u'^2/c^2}} = 1161 \ \mathrm{MeV}$$

Get

$$\sqrt{1 - u'^2/c^2} = 0.808 \qquad \Longrightarrow \qquad u'^2/c^2 = 0.348$$

or

$$u'/c = 0.590 \qquad \Longrightarrow \qquad u' = (0.590)c = 1.77 \times 10^8 \ \tfrac{\mathrm{m}}{\mathrm{s}}$$

d) If this collision produces a particle X, i.e.

$$\mathrm{p} + \mathrm{p} = \mathrm{p} + \mathrm{p} + \mathrm{X}$$

what is the largest value possible for the mass of X? (Consider conservation of energy–momentum in CM frame...)

The total CM energy is

$$2E'_p = 2322 \ \mathrm{MeV}$$

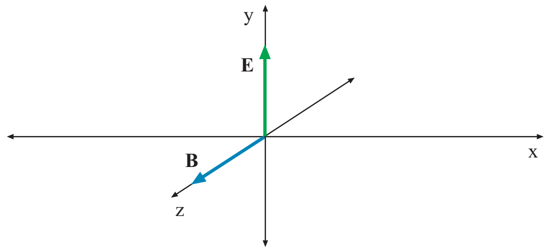
If 2 p's and an X are formed at rest then by conservation of energy

$$2322 \ \mathrm{MeV} = 2m_p^2 c^2 + m_X c^2$$

which gives

$$m_X c^2 = 447 \ \mathrm{MeV}$$

5. In the "lab" frame there is a uniform E field of magnitude $1.0 \times 10^7 \,\frac{\text{N}}{\text{C}}$ in the $+y$ direction and a B field of magnitude 0.100 T in the $+z$ direction.



a) What are the values (magnitudes and directions) of the E and B fields in a reference frame which moves in the $+x$ direction with a speed of $5.0 \times 10^7 \,\frac{\text{m}}{\text{s}}$?

We have

$$E_y = 1.00 \times 10^7 \,\tfrac{\text{N}}{\text{C}} \qquad B_z = 0.100 \text{ T}$$

with all others zero.

If v_x (vel of the frame $\overline{\mathcal{S}}$) is

$$v_x/c = \frac{5.00 \times 10^7}{3.00 \times 10^8} = 0.166$$

then, with $\gamma = 1.014$, in frame $\overline{\mathcal{S}}$ we have

$$\bar{E}_y = \gamma(E_y - vB_z) = (1.014)(1.0 \times 10^7 - (5.0 \times 10^7)(0.1))\,\tfrac{\text{N}}{\text{C}} = 5.07 \times 10^6 \,\tfrac{\text{N}}{\text{C}}$$

$$\bar{B}_z = \gamma(B_z - \frac{v}{c^2}E_y) = (1.014)(0.100 - \frac{(0.166)}{(3 \times 10^8)}(1.0 \times 10^7)) \text{ T} = 0.096 \text{ T}$$

The other field components are still zero.

b) Is there a reference frame in which there is *no* electric field? Specify this reference frame and find the value of the magnetic field in that frame.

For a frame in which E_y is zero we need

$$E_y - vB_z = 0$$

or

$$v = \frac{E_y}{B_z} = \frac{(1.0 \times 10^7 \,\frac{\text{N}}{\text{C}})}{(0.1 \text{ T})} = 1.0 \times 10^8 \,\tfrac{\text{m}}{\text{s}}$$

which is $< c$ and possible.

In that frame we would have (with $\gamma = 1.06$),

$$\bar{B}_z = \gamma(B_z - \frac{v}{c^2}E_y) = (1.06)(0.1 - \frac{(1.0 \times 10^8)}{(3 \times 10^8)^2}(1 \times 10^7)) \text{ T} = 0.0942 \text{ T}$$

c) Is there a reference frame in which there is no magnetic field? If so, specify this frame and find the value of the E field in that frame.

For a frame in which B_z is zero, we need $B_z = \frac{v}{c^2}E_y$, or

$$v = c^2\frac{B_z}{E_y} = (3 \times 10^8)^2 \frac{(0.1)}{(1 \times 10^7)} \,\tfrac{\text{m}}{\text{s}} = 9 \times 10^8 \,\tfrac{\text{m}}{\text{s}}$$

which is impossible!

6. What does it mean to say that a mathematical object is a 4-tensor? Specifically, what property does the object $t^{\mu\nu}$ have to have?

If $t^{\mu\nu}$ is a 4-tensor then the corresponding quantity in the reference frame $\overline{\mathcal{S}}$ is given by

$$\bar{t}^{\mu\nu} = \Lambda^{\mu}_{\sigma}\Lambda^{\nu}_{\lambda}t^{\sigma\lambda}$$

where $t^{\sigma\lambda}$ is the (set of) values of t in frame $\mathcal{S}$ and Λ^{μ}_{σ} is the Lorentz transformation matrix relating frames $\mathcal{S}$ and $\overline{\mathcal{S}}$.

7. If we let $\mu = 3$ in the relativistic "inhomogeneous" Maxwell equation, we have

$$\frac{\partial F^{3\nu}}{\partial x^{\nu}} = \mu_0 J^3$$

Show how this is the same as one of the Maxwell equations written in our old vector notion.

Recall that

$$x^{\nu} = (ct, x, y, z) \qquad \text{and} \qquad J^3 = J_z$$

and using the given entries for $F^{3\nu}$ (3rd row) we get:

$$\frac{\partial F^{3\nu}}{\partial x^{\nu}} = \frac{\partial(-E_z/c)}{\partial(ct)} + \frac{\partial(B_y)}{\partial x} + \frac{\partial(-B_x)}{\partial y} + 0 = \mu_0 J_z$$

or

$$\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) - \frac{1}{c^2}\frac{\partial E_z}{\partial t} = \mu_0 J_z$$

This is the z^{th} component of the vector equation

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \ ,$$

the Ampere--Maxwell equation.

8. If we let $\mu = 0$ and $\nu = 1$ in the definition of the $\mu\nu$ element of the EM field tensor we get

$$F^{01} = \frac{\partial A^1}{\partial x_0} - \frac{\partial A^0}{\partial x_1}$$

Show that this agrees with the relations we previously had between the fields and the potentials.

Use

$$A^1 = A_x \qquad A^0 = V/c \qquad x_1 = x^1 = x \qquad x_0 = -x^0 = -ct \qquad F^{01} = E_x/c$$

then get

$$\frac{E_x}{c} = \frac{\partial A_x}{\partial(-ct)} - \frac{\partial(V/c)}{\partial x}$$

or

$$E_x = -\frac{\partial V}{\partial x} - \frac{\partial A_x}{\partial t}$$

This is the x component of

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

Useful Equations

$$\int_{\mathbf{a}}^{\mathbf{b}} (\nabla T) \cdot d\mathbf{l} = T(\mathbf{b}) - T(\mathbf{a}) \qquad \int_{\mathcal{V}} (\nabla \cdot \mathbf{v}) d\tau = \oint_{\mathcal{S}} \mathbf{v} \cdot d\mathbf{a} \qquad \int_{\mathcal{S}} (\nabla \times \mathbf{v}) \cdot d\mathbf{a} = \oint_{\mathcal{P}} \mathbf{v} \cdot d\mathbf{l}$$

Spherical:

$$d\mathbf{l} = dl_r\, \hat{\mathbf{r}} + dl_\theta\, \hat{\boldsymbol{\theta}} + dl_\phi\, \hat{\boldsymbol{\phi}} \qquad d\tau = r^2\, \sin\theta\, dr\, d\theta\, d\phi$$

Gradient:

$$\nabla T = \frac{\partial T}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}} \tag{1}$$

Divergence:

$$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} \tag{2}$$

Curl:

$$\nabla\times\mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}} \tag{3}$$

Laplacian:

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2} \tag{4}$$

Cylindrical:

$$d\mathbf{l} = dl_s\, \hat{\mathbf{s}} + dl_\phi\, \hat{\boldsymbol{\phi}} + dl_z\, \hat{\mathbf{z}} \qquad d\tau = s\, ds\, d\phi\, dz$$

Gradient:

$$\nabla T = \frac{\partial T}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}} \tag{5}$$

Divergence:

$$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z} \tag{6}$$

Curl:

$$\nabla \times \mathbf{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}} \tag{7}$$

Laplacian:

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial\phi^2} + \frac{\partial^2 T}{\partial z^2} \tag{8}$$

More Math

Gradients:

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$$

Divergences:

$$\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$

Product Rules:

(1) $\nabla\cdot(\nabla T)$ (Divergence of curl)

$$\nabla\cdot(\nabla T) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \nabla^2 T$$

(2) $\nabla\times(\nabla T)$ (Curl of gradient)

$$\nabla\times(\nabla T) = 0$$

(3) $\nabla(\nabla\cdot\mathbf{v})$ (Gradient of divergence)

Nothing interesting about this; does not occur often.

(4) $\nabla\cdot(\nabla\times\mathbf{v})$

$$\nabla\cdot(\nabla\times\mathbf{v}) = 0$$

(5) $\nabla\times(\nabla\times\mathbf{v})$ (Curl of a curl)

$$\nabla\times(\nabla\times\mathbf{v}) = \nabla(\nabla\cdot\mathbf{v}) - \nabla^2\mathbf{v}$$

Physics:

$$\nabla\cdot\mathbf{E} = \frac{1}{\epsilon_0}\rho \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

$c = 2.998\times10^8\ \frac{\mathrm{m}}{\mathrm{s}}$ $\qquad \epsilon_0 = 8.854\times10^{-12}\ \mathrm{C^2/N{\cdot}m^2}$ $\qquad \mu_0 = 4\pi\times10^{-7}\ \mathrm{N/A^2}$ $\qquad e = 1.602\times10^{-19}\ \mathrm{C}$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad \nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t} \qquad \mathcal{E} = -\frac{d\Phi}{dt}$$

$$\mathbf{E} = -\nabla V - \frac{\partial\mathbf{A}}{\partial t} \qquad \mathbf{A}' = \mathbf{A} + \nabla\lambda \qquad V' = v - \frac{\partial\lambda}{\partial t}$$

$$\text{Coulomb}: \quad \nabla\cdot\mathbf{A} = 0 \qquad \text{Lorentz}: \quad \nabla\cdot\mathbf{A} = -\mu_0\epsilon_0\frac{\partial V}{\partial t}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r}', t_r)}{\mathscr{r}}\,d\tau' \qquad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}', t_r)}{\mathscr{r}}\,d\tau'$$

$$V(\mathbf{r},t)=\frac{1}{4\pi\epsilon_0}\frac{qc}{\mathscr{r}c-\boldsymbol{\mathscr{r}}\cdot\mathbf{v}} \qquad \mathbf{A}(\mathbf{r},t)=\frac{\mu_0}{4\pi}\frac{qc\mathbf{v}}{(\mathscr{r}c-\boldsymbol{\mathscr{r}}\cdot\mathbf{v})}=\frac{\mathbf{v}}{c^2}V(\mathbf{r},t)$$

$$\mathbf{E}(\mathbf{r},t)=\frac{q}{4\pi\epsilon_0}\frac{\mathscr{r}}{(\boldsymbol{\mathscr{r}}\cdot\mathbf{u})^3}[(c^2-v^2)\mathbf{u}+\boldsymbol{\mathscr{r}}\times(\mathbf{u}\times\mathbf{a})] \qquad \mathbf{B}(\mathbf{r},t)=\frac{1}{c}\boldsymbol{\mathscr{r}}\times\mathbf{E}(\mathbf{r},t)$$

$$\mathbf{E}(\mathbf{r},t)=\frac{q}{4\pi\epsilon_0}\frac{1-v^2/c^2}{\left(1-v^2\sin^2\theta/c^2\right)^{3/2}}\frac{\hat{\mathbf{R}}}{R^2} \qquad \mathbf{B}=\frac{1}{c}(\hat{\boldsymbol{\mathscr{r}}}\times\mathbf{E})=\frac{1}{c^2}(\mathbf{v}\times\mathbf{E})$$

$$P=\frac{\mu_0 q^2 a^2}{6\pi c}$$

$$\gamma=\frac{1}{\sqrt{1-v^2/c^2}} \qquad \Delta\bar{t}=\sqrt{1-v^2/c^2}\,\Delta t \qquad \Delta\bar{x}=\frac{1}{\sqrt{1-v^2/c^2}}\Delta x$$

$$v_{AC}=\frac{v_{AB}+v_{BC}}{1+(v_{AB}v_{BC}/c^2)} \qquad \bar{t}=\gamma\left(t-\frac{v}{c^2}x\right) \qquad \bar{x}=\gamma(x-vt) \qquad \bar{y}=y \qquad \bar{z}=z$$

$$\bar{x}^\mu=\sum_{\nu=0}^{3}(\Lambda^\mu_\nu)x^\nu \qquad \Lambda^\mu_\nu=\begin{pmatrix}\gamma & -\gamma\beta & 0 & 0\\ -\gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$\eta^\mu=\gamma(c,v_x,v_y,v_z) \qquad \mathbf{p}=\frac{m\mathbf{u}}{\sqrt{1-u^2/c^2}} \qquad p^\mu=(E/c,p_x,p_y,p_z) \qquad E=\gamma mc^2$$

$$p^\mu p_\mu=-m^2c^2 \qquad E^2=p^2c^2+m^2c^4$$

$$K^\mu=\frac{dp^\mu}{d\tau} \qquad J^\mu=(c\rho,J_x,J_y,J_z) \qquad A^\mu=(V/c,A^x,A^y,A^z)$$

$$\begin{aligned}\bar{E}_x &= E_x \qquad \bar{E}_y=\gamma(E_y-vB_z) \qquad \bar{E}_z=\gamma(E_z+vB_y)\\ \bar{B}_x &= B_x \qquad \bar{B}_y=\gamma(B_y+\frac{v}{c^2}E_z) \qquad \bar{B}_z=\gamma(B_z-\frac{v}{c^2}E_y)\end{aligned}$$

$$F^{\mu\nu}=\left\{\begin{matrix}0 & E_x/c & E_y/c & E_z/c\\ -E_x/c & 0 & B_z & -B_y\\ -E_y/c & -B_z & 0 & B_x\\ -E_z/c & B_y & -B_x & 0\end{matrix}\right\} \qquad F^{\mu\nu}=\frac{\partial A^\nu}{\partial x_\mu}-\frac{\partial A^\mu}{\partial x_\nu}$$

Invariants: $\mathbf{E}\cdot\mathbf{B}\,,\qquad (E^2-c^2B^2)$

$$\frac{\partial J^\mu}{\partial x^\mu}=0 \qquad \frac{\partial F^{\mu\nu}}{\partial x^v}=\mu_0 J^\mu \qquad \frac{\partial G^{\mu\nu}}{\partial x^\nu}=0 \qquad K^\mu=q\eta_\nu F^{\mu\nu}$$