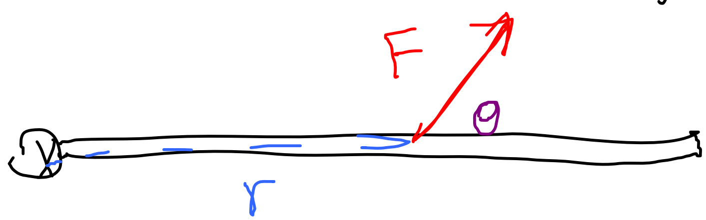
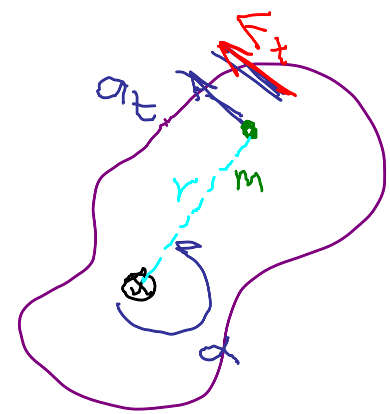
Rotational Dynamics



$$\tau = rF\sin\theta$$

You get the signs right

Relation between τ and α



$$F_t = ma_t = mr\alpha$$

$$\boxed{rF_t = mr^2\alpha}$$

$$rF_t = mr^2\alpha$$

Add up for all points in objec

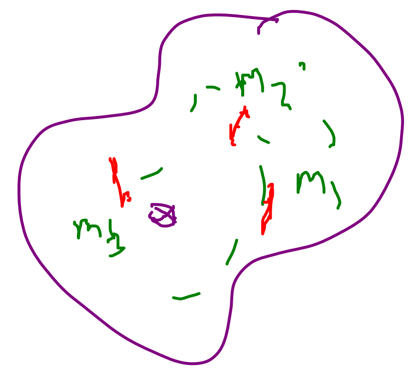
$$\tau_{\substack{net\\ext}} = \sum_i m_i r_i^2 \alpha$$

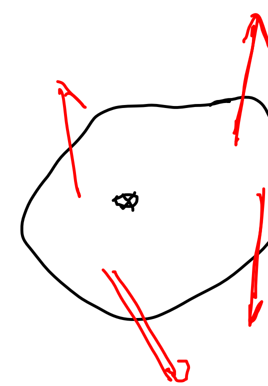
$$\tau_{net} = \underbrace{\left[\sum_i m_i r_i^2\right]}\alpha$$

$$\tau = I\alpha$$

I = moment of inertia

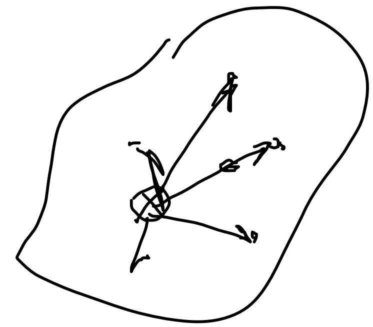
= rotational inertia





$\sum \tau$

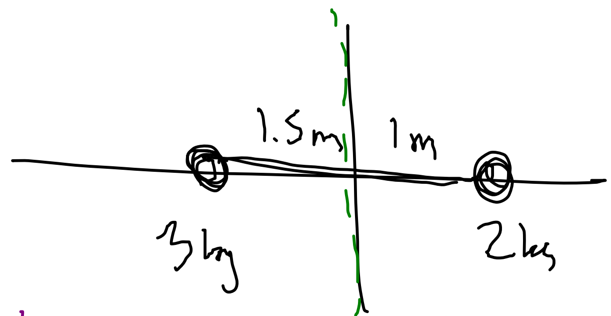
$$I = \sum_i m_i r_i^2$$



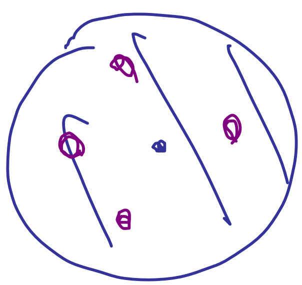
Scalar (Matrix)

Units?
Units?

$kg \cdot m^2$

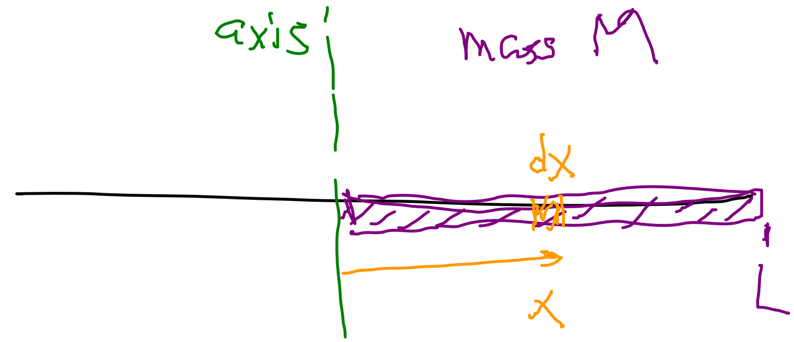


$$\sum mr^2$$



Summation is done w/ calculus.

Find moment of inertia of stick rotated about end.



$$I = \sum \underbrace{\left(\frac{M}{L}\right) dx}_{\text{mass of}} \cdot x^2 \qquad x: 0 \to L$$

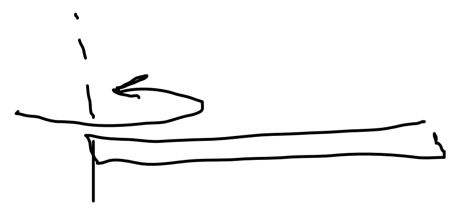
$x: 0 \to L$

Uniform mass density

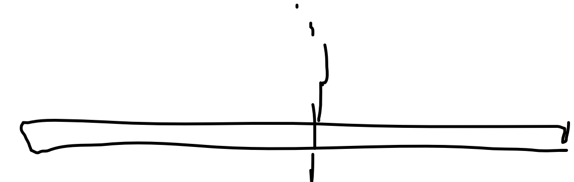
$\lambda = M/L$

$$= \int_0^L \frac{M}{L} x^2 \, dx$$

$$= \frac{M}{L} \left. \frac{x^3}{3} \right|_0^L = \frac{1}{3} M L^2$$



Stick about center rotated

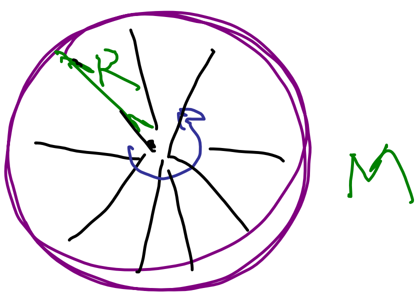


$$\frac{1}{3}\frac{M}{2}\left(\frac{L}{2}\right)^2 \qquad \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2$$

$$ML^2/24 + ML^2/24$$

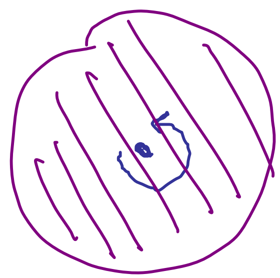
$$= \frac{1}{12}ML^2$$

Other shapes:



Hoop about center

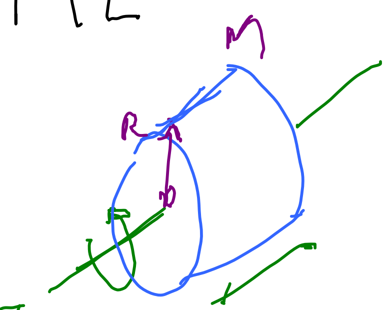
$$I = MR^2$$



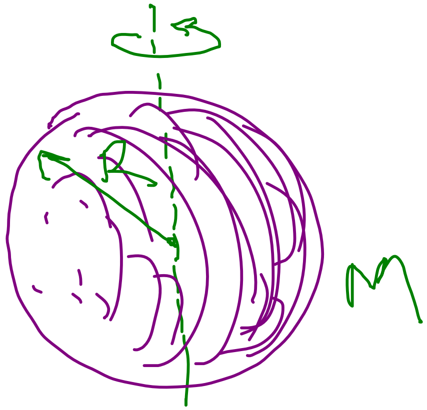
Solid Disk (uniform)

$$I = \frac{1}{2}MR^2$$

p.163

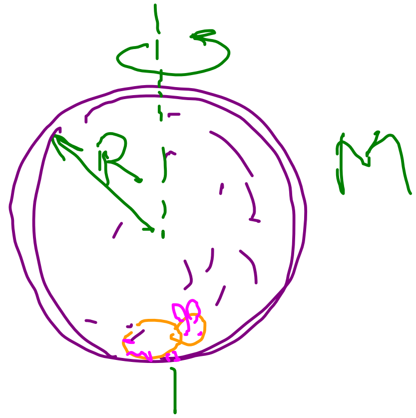


$$I = \frac{1}{2}MR^3$$



Solid Sphere
(Uniform)

$$I = \frac{2}{5} M R^2$$



Hollow Sphere - hamster

$$I = \frac{2}{3} M R^2$$

Why did we do this?

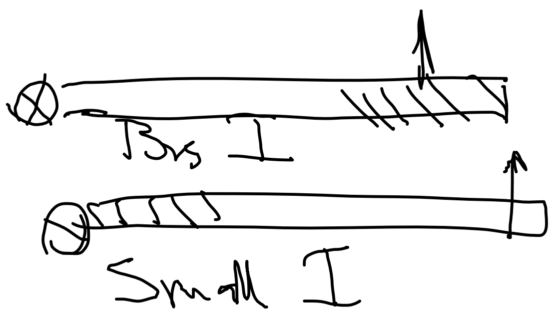
$$\tau = I\alpha$$

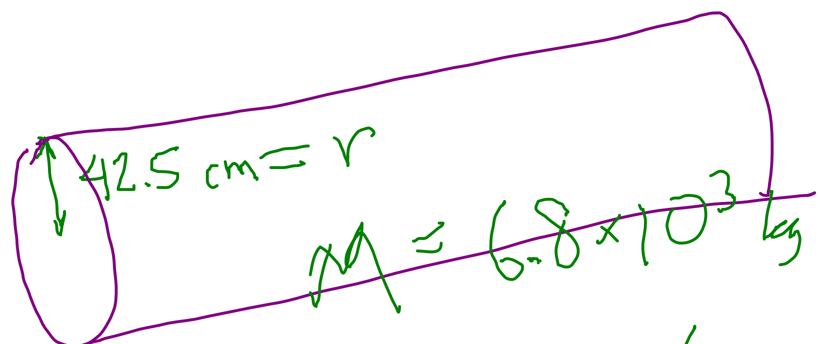
$F = ma$

$$\boxed{\tau_{net} = I\alpha}$$

Newton's 2nd Law for rotations.

10.26 The shaft connecting power plant's turbine and electric gen. is solid cylinder of mass 6.8 Mg and diameter 85 cm. Find its rot'l inertia

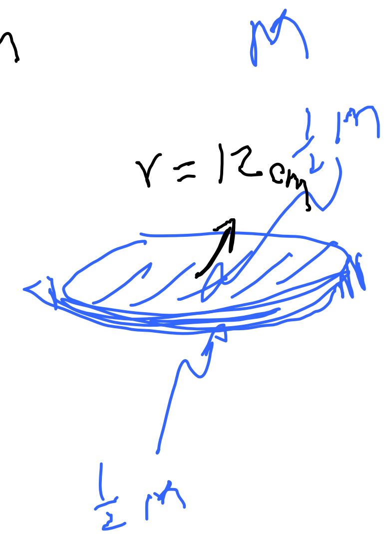




$$I_{\text{disk}} = \frac{1}{2}MR^2 = \frac{1}{2}(6.8\times10^3\,\text{kg})(0.425\,\text{m})^2 \stackrel{?}{=} 614\ \text{kg m}^2$$

10.32 A 108 g Frisbee is 24 cm in diameter. $\frac{1}{2}$ mass is in rim, $\frac{1}{2}$ mass is in spread in disk

a) What is its rot'l inertia



$$I = \left(\frac{M}{2}\right)R^2 + \frac{1}{2}\left(\frac{M}{2}\right)R^2$$

Rim Disk

MR^2

$$= \frac{3}{4}MR^2 \qquad I = 1.17\times10^{-3}\ \text{kg m}^2$$

b) With a quarter-turn flick of wrist student sets Frisbee in motion, rotating at 550 rpm. Find magn. of torque exerted.

$\tau = \underline{I}\alpha$ what is α

$\Longrightarrow \omega_0 = 0 \qquad \omega = 550 \frac{\text{rev}}{\text{min}} \left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)$

Give $\theta - \theta_0 : \pi/2$ in radians !

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ $\qquad \alpha = 1056 \, \text{rad}/s^2$

$\omega_0 \leftarrow 0 \qquad \theta - \theta_0 \leftarrow \pi/2$

$$\tau = I\alpha = \left(1.17\times10^{-3}\ \mathrm{kg\,m^2}\right)\left(1056\ \frac{\cancel{\mathrm{rad}}}{\mathrm{s}^2}\right)$$

$$= \mathrm{N\cdot m} = \frac{\mathrm{kg\,m^2}}{\mathrm{s}^2}$$

$$= 1.24\ \mathrm{N\cdot m}$$

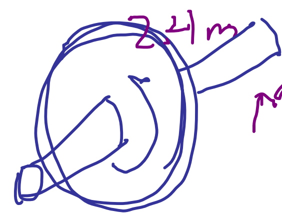
10.33 At the MIT Magnet Laboratory energy stored in huge solid fly wheels of mass 7.7×10^4 kg $r = 2.4$ m

Rides on shaft 41 cm in diameter

34 kN Frictional force acts tang'ly to shaft

How long does it take flywheel to stop from its speed of 360 rpm.

Assume shaft has neg'ble I



$M = 7.7 \times 10^4$ kg

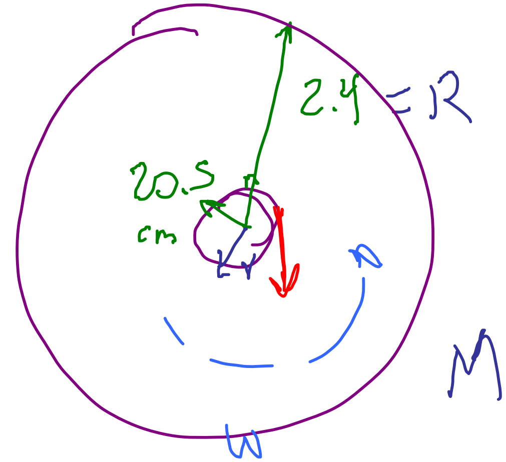
$$I = \frac{1}{2}(M)R^2$$

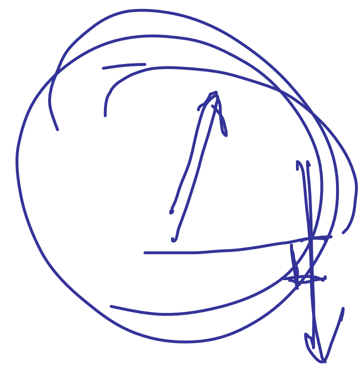
$$\tau = -rF(1)$$

$$= -(0.205\,m)(34\times10^3\,N)$$

$$= -6.97\,Nm$$

$$\alpha = \frac{\tau}{I} = \sim\!\sim$$





$$\omega = \omega_0 + \alpha t$$

0 $\quad$ $360 \text{ rpm} \left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60 \text{ s}}\right) = \_\_\_\_$

Solve for t. Get:

$\Rightarrow t = 20.0 \text{ min}$!