Phys 2920, Spring 2011
Problem Set #4

1. Find the eigenvalues and eigenvectors of the second Pauli matrix σ_y from the last problem set.

2. Find eigenvalues and eigenvectors for the matrices

$$L_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad\qquad L_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

3. Use Maple or something suitable to find the eigenvalues and eigenvectors of

$$\mathsf{A} = \begin{pmatrix} 2.5 & 1.6 & -4.8 \\ 4.2 & -3.1 & 2.1 \\ -1.5 & -2.6 & 5.5 \end{pmatrix}$$

Take any one of the eigenvectors output by the program and by hand check that it is a unit vector.

4. Let the matrix A be given by

$$\mathsf{A} = \begin{pmatrix} 5 & -7 \\ 2 & 3 \end{pmatrix}$$

Consider a scheme where the new basis vectors are

$$\mathbf{e}'_1 = \mathbf{e}_1 + 4\mathbf{e}_2 \qquad\qquad \mathbf{e}'_2 = 3\mathbf{e}_1 + 10\mathbf{e}_2$$

Find the representation of A in the new basis.

5. Find the similarity transformation which diagonalizes the matrix

$$\mathsf{A} = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$$

This problem is a bit different in that it has a repeated eigenvalue; but you can still find three independent eigenvectors with which to construct S.

6. Find the eigenvalues and eigenvectors of the matrix

$$\mathsf{B} = \begin{pmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{pmatrix}$$

Can B be diagonalized? (Can we construct a transformation matrix S to make B diagonal? We *can* if it has three independent eigenvectors.)

7. Recall the definition for the exponential of a *matrix*:

$$e^{\mathsf{A}} = \sum_{n=0}^{\infty} \frac{\mathsf{A}^n}{n!}$$

With this definition, find a simple expression for

$$e^{\mathsf{A}x} \qquad \text{for} \qquad \mathsf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(This is the matrix σ_x from a previous problem set.) (Here, x is a number which multiplies the matrix A, and $\mathsf{A}^0 = \mathbf{1}$.) You will need to spot a simple pattern in the powers of $\mathsf{A}x$ and you will need to review the Taylor series for basic functions.