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Nov. 9, 2010

Phys 2112, Fall 2010
Quiz #2

1. An *elastic* 1-D collision takes place in a physics lab, wherein a 2.00−kg mass moving at $6.00\,\frac{\mathrm{m}}{\mathrm{s}}$ strikes a 8.00 kg mass at rest.

a) Show that the center of mass moves at velocity $+1.20\,\frac{\mathrm{m}}{\mathrm{s}}$! *(Grrrr. . .)*

Using $\mathbf{P} = M\mathbf{v}_{\mathrm{cm}}$, we get the velcotiy of the center of mass by dividing the total momentum by the total mass; it will give the same thing both *before* and *after* the collision. We get:

$$\mathbf{v}_{\mathrm{cm}} = \frac{(2.00\ \mathrm{kg})(6.00\,\frac{\mathrm{m}}{\mathrm{s}}\,\hat{\mathbf{i}})}{(10.0\ \mathrm{kg})} = 1.20\,\frac{\mathrm{m}}{\mathrm{s}}\,\hat{\mathbf{i}}$$

b) In a reference frame that moves to the right at $1.20\,\frac{\mathrm{m}}{\mathrm{s}}$, what are the initial velocities of the masses?

In the center-of-mass reference frames we get all the velocities by *subtracting* $1.20\,\frac{\mathrm{m}}{\mathrm{s}}\,\hat{\mathbf{i}}$ from all the velocities in the lab frame. Then the $2.0\ \mathrm{kg}$ block initially moves at $\boxed{4.80\,\frac{\mathrm{m}}{\mathrm{s}}}$ to the right and the $8.0\ \mathrm{kg}$ mass is initially moving at $\boxed{1.20\,\frac{\mathrm{m}}{\mathrm{s}}}$ to the left.

c) In this reference frame, the velocities just *reverse* in the collision. (This will conserve both momentum and energy.) Knowing this, find the final velocities back in the lab frame.

The velocities reverse themselves, so after the collision, the $2.0\ \mathrm{kg}$ block moves at $-4.80\,\frac{\mathrm{m}}{\mathrm{s}}$ and the $8.0\ \mathrm{kg}$ block moves at $+1.20\,\frac{\mathrm{m}}{\mathrm{s}}$.

To get the final velocities back in the lab frame, add $1.20\,\frac{\mathrm{m}}{\mathrm{s}}$ onto all velocites, so the final velocities are:

$$\begin{aligned} 2.0\ \mathrm{kg}\ \text{mass:} &\quad -4.80\,\tfrac{\mathrm{m}}{\mathrm{s}} + 1.20\,\tfrac{\mathrm{m}}{\mathrm{s}} = -3.60\,\tfrac{\mathrm{m}}{\mathrm{s}} \\ 8.0\ \mathrm{kg}\ \text{mass:} &\quad +1.20\,\tfrac{\mathrm{m}}{\mathrm{s}} + 1.20\,\tfrac{\mathrm{m}}{\mathrm{s}} = +2.40\,\tfrac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

2. Io is a (highly volcanic) moon of Jupiter with a mass of 8.93×10^{22} kg and a radius of 1.821×10^3 km.

Find the acceleration of gravity on the surface of Io.

The formula for g gives:

$$g = G\frac{M}{R^2} = (6.67 \times 10^{-11}\ \tfrac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}})\frac{(8.93 \times 10^{22}\ \mathrm{kg})}{(1.82 \times 10^6\ \mathrm{m})^2} = 1.80\ \tfrac{\mathrm{m}}{\mathrm{s^2}}$$

3. During the Apollo manned missions, the Command Module orbited the Moon at a distance of 110 km above the surface; as with most Earth orbit this is small compared to Moon's radius.

The radius of the Moon is 1737 km and its mass is 7.35×10^{22} kg.

Find the period of orbit of the Lunar Command Module.

The ship was actually at a distance of

$$1737\ \mathrm{km} + 110\ \mathrm{km} = 1847\ \mathrm{km}$$

from the Moon's center. Anyways, the formula relating period and radius for a circular orbit gives

$$T^2 = \frac{4\pi^2 r^3}{GM} = \frac{4\pi^2(1.847 \times 10^6\ \mathrm{m})^3}{(6.67 \times 10^{-11}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}})(7.35 \times 10^{22}\ \mathrm{kg})} = 5.074 \times 10^7\ \mathrm{s}^2$$

so that

$$T = 7.12 \times 10^3\ \mathrm{s} = 1.98\ \mathrm{h}$$

Show work for all problems and include the right units!

$$\mathbf{r}_{\mathrm{cm}} = \frac{\sum_i m_i \mathbf{r}_i}{M} \qquad \mathbf{P} = M\mathbf{v}_{\mathrm{cm}} \qquad \mathbf{v}_{\mathrm{AC}} = \mathbf{v}_{\mathrm{AB}} + \mathbf{v}_{\mathrm{BC}}$$

$$F = G\frac{m_1 m_2}{r^2} \qquad G = 6.67 \times 10^{-11}\ \tfrac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}} \qquad a_c = \frac{v^2}{r} \qquad U(r) = -G\frac{m_1 m_2}{r} \qquad g = G\frac{M}{R^2}$$

$$F_c = \frac{mv^2}{r} \qquad \frac{4\pi^2 r^3}{T^2} = GM \qquad v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}} \qquad M_{\mathrm{earth}} = 5.97\times10^{24}\ \mathrm{kg} \qquad R_{\mathrm{earth}} = 6.37\times10^6\ \mathrm{m}$$