Phys 3820, Fall 2010
Problem Set #4, Hint-o-licious Hints

1. *Griffiths*, **9.1** You just need to evaluate matrix elements of the form

$$H'_{ij} = H'_{2lm,100} = \langle \psi_{2lm} | H' | \psi_{100} \rangle = eE \int \psi_{2lm}(\mathbf{r}) z \psi(\mathbf{r})\, d^3\mathbf{r}$$

but since $z = r\cos\theta$ and $\cos\theta$ is proportional to Y_1^0, that angular integral here is

$$\int Y_l^{m*}(\theta,\phi) Y_1^0(\theta,\phi)\, d\Omega$$

which from orthogonality is zero unless $l = 1$ and $m = 0$ so there's really only one matrix element to evaluate.

For the diagonal matrix elements, write down the angular integral and you can argue that from the antisymmetry in θ about $\theta = \frac{\pi}{2}$ the result is zero.

2. *Griffiths*, **9.5** With the general initial conditions

$$c_a(0) = a \qquad \text{and} \qquad c_b(0) = b$$

after one pass I get for the first–order answer

$$c_a = \frac{-ib}{\hbar} \int_0^t H'_{ab}(t') e^{-i\omega_0 t'}\, dt' + a \qquad \text{and} \qquad c_b = \frac{-ia}{\hbar} \int_0^t H'_{ba}(t') e^{i\omega_0 t'}\, dt' + b$$

Pass these through the TDPT equations again and integrate (using the initial conditions) again to get the very messy second–order answer. I get

$$c_a = a - \frac{ib}{\hbar} \int_0^t H'_{ab}(t') e^{-i\omega_0 t'} - \frac{a}{\hbar^2} \int_0^t H'_{ab}(t') e^{-i\omega_0 t'} \int_0^{t'} H'_{ba}(t'') e^{i\omega_0 t''}\, dt''\, dt'$$

Note the multiple integrations in the last term over different dummy t variables.

3. *Griffiths*, **9.8** Use (9.47), (9.56) and (9.52) to find the ratio

$$\frac{R_{b\to a}}{A}$$

You'll note that the messy dipole matrix element cancels out as well as nearly everything else, leaving only

$$\frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}$$

From this, at a particular T find the value of ω where the ratio R/A is one, i.e. the rates from stimulated and spontaneous emission are equal.

4. *Griffiths*, **9.10** A common, basic problem. Deduce that

$$t_{\frac{1}{2}} = (\ln 2)\tau$$

5. *Griffiths*, **9.11** You are out to calculate

$$\tau = \frac{1}{A} = \frac{3\pi\epsilon_0 \hbar c^3}{\omega_0^3 |\mathbf{p}|^2} \qquad \text{where} \qquad \mathbf{p} = q\langle\psi_b|\mathbf{r}|\psi_a\rangle$$

where ψ_1 is the ground state of the H atom and ψ_b is any one of the 4 states with $n = 2$.

First off, you can easily show that

$$\langle\psi_{200}|x|\psi_{100}\rangle = 0 \qquad \text{etc. for } y \text{ and } z$$

so from this, the lifetime of the 200 state is infinite!

Next, consider the 210 state. You can show that the matrix elements

$$\langle\psi_{210}|x|\psi_{100}\rangle \qquad \text{and} \qquad \langle\psi_{210}|y|\psi_{100}\rangle$$

are zero, and that the z matrix element is

$$\langle\psi_{210}|z|\psi_{100}\rangle = -\frac{256}{243\sqrt{2}} ea \approx -(0.74493)ea$$

and with only one component to include in $|\mathbf{p}|^2$, this gives a lifetime of the 210 state of 1.6 ns.

So now consider the $21\,\pm 1$ states. One can show that for these, the z matrix element is zero:

$$\langle\psi_{21\,\pm1}|z|\psi_{100}\rangle = 0$$

so we need only the x and y elements. The sum of the *absolute squares* of these give $|\mathbf{p}|^2$. I found

$$p_x = \langle\psi_{21\,\pm1}|x|\psi_{100}\rangle = \pm\frac{128}{243} ea \quad \text{and} \quad p_y = \langle\psi_{21\,\pm1}|y|\psi_{100}\rangle = -i\frac{128}{243} ea$$

which when put into $|\mathbf{p}|^2$ give the same value as in the 210 case and hence the same lifetime... which we would really expect. So the lifetime of any of the 2p states is 1.6 ns.