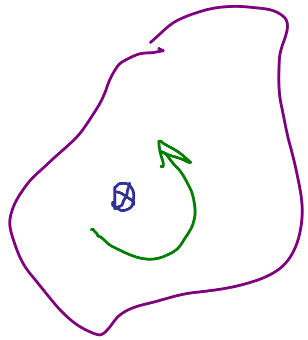
Phys 2110-4 3/27/13

Note Title 3/27/2013

Rotations



$\theta(t)$, $\omega(t)$, $\alpha(t)$

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt}$$

Constant accel $\alpha = \text{const}$

$$\omega = \omega_0 + \alpha t$$

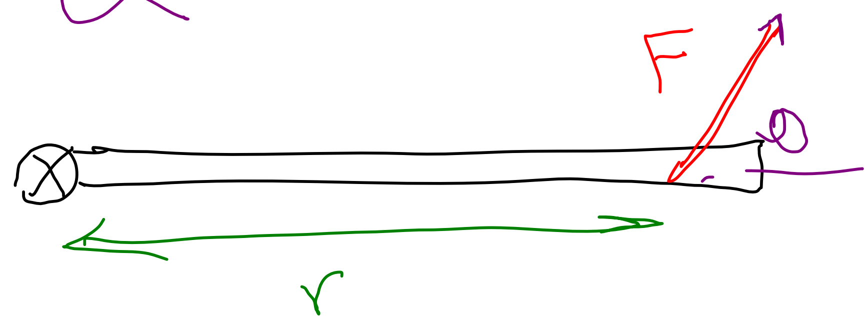
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

usually,

$\theta_0 = 0$

α comes from forces

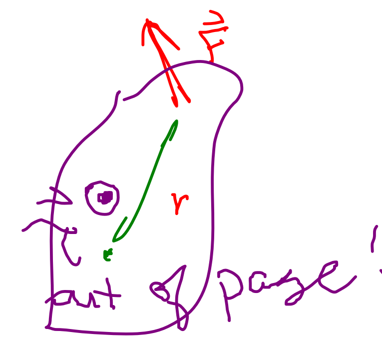


$$\tau = rF\sin\theta$$

Torque: <u>Moment of the force</u>

τ is + or −
(ccw) (cw)

Scalar for now
Vector



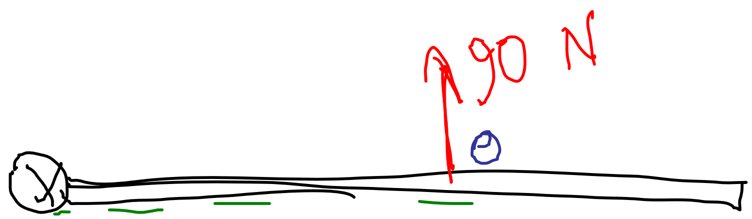
Units

$$[\tau] = m\,N \cdot 1 = N \cdot m = J \;?$$

$$= \underline{N \cdot m}$$

$E = ft \cdot lb$ $\qquad \tau = lb \cdot ft$

10.21 A 110 N·m torque is needed to start a revolving door rotating. If a child pushes w/ max force of 90 N, how far from axis must she apply force



$\theta = 90°$

$\sin 90° = 1$

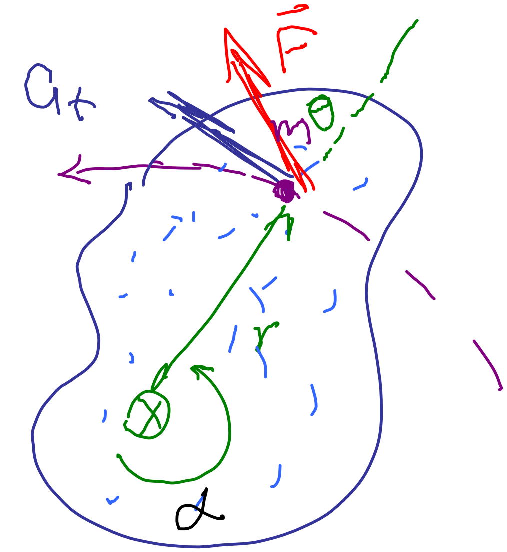
$\tau = rF \cdot 1$

$r = \tau / F = \frac{110\ N \cdot m}{90\ N} = 1.2\,m$

τ gives angular accel.

Really have to discuss all the mass points.

Heuristic derivation.



Tangential part of force $F\sin\theta = F_\perp$

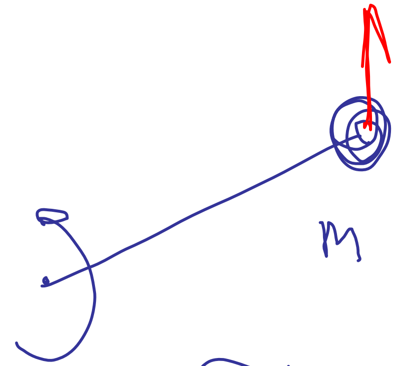
$$m a_t = F_\perp = F\sin\theta$$

$$a_t = r\alpha$$

$$m r\alpha = F\sin\theta$$

$$m r^2\alpha = F r\sin\theta = \tau$$

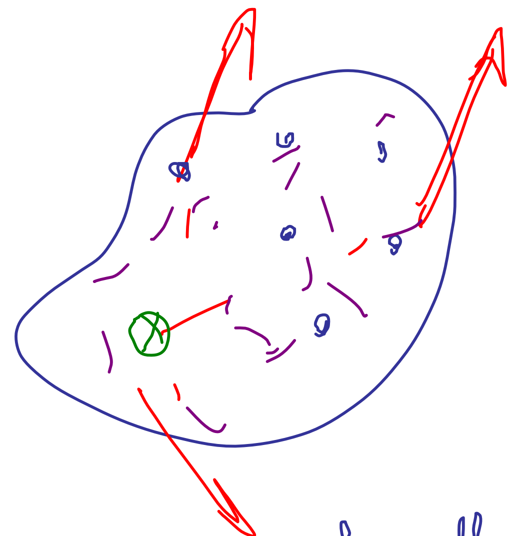
$$\tau = mr^2\alpha$$



$$\tau = mr^2\alpha$$

$$\tau_{\substack{net\\(ext)}} = \sum_{\text{mass point},\, i} m_i r_i^2 \alpha_i$$

$$= \alpha\left[\sum_{\text{mass points}} m_i r_i^2\right]$$



α same for all points

$$\tau_{net} = \left[\sum_{\text{mass points}} m_i r_i^2\right]\alpha$$

$$\boxed{\tau_w = I\alpha} \qquad F = ma$$

Discuss I

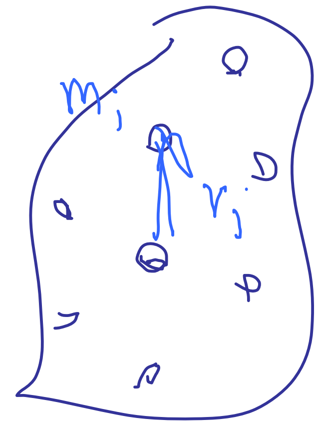
$$I = \sum m_i r_i^2$$

= Moment of inertia

= rotational inertia

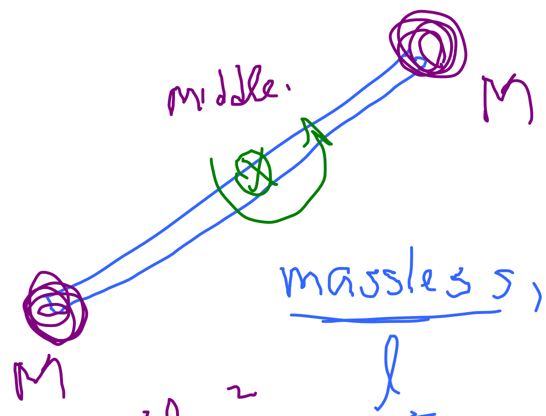
$\Longrightarrow$ Scalar.

Actually Matrix



Units: $\underline{\underline{kg \cdot m^2}}$

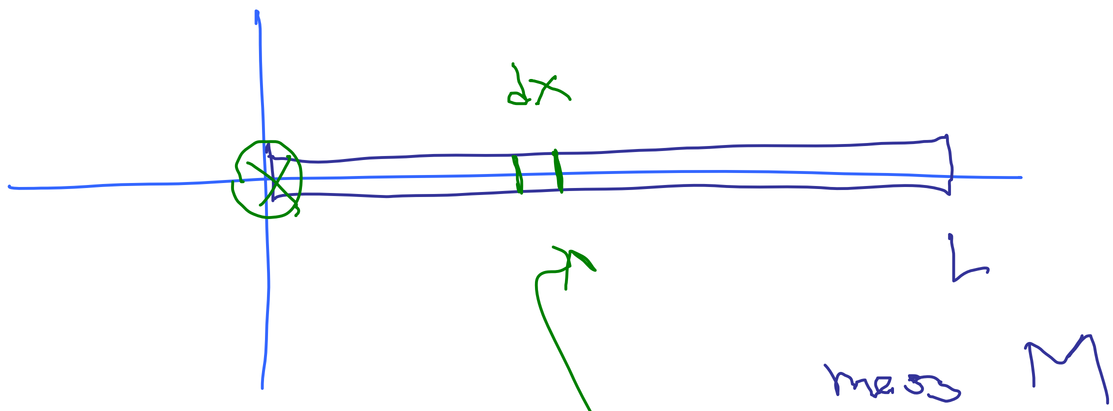
Simple



$$I = M\left(\frac{l}{2}\right)^2 + M\left(\frac{l}{2}\right)^2 = \frac{Ml^2}{2}$$

Formulae for solid objects.

Simple example of continuous object



$\Sigma m r^2$

Uniform stick.

Rotating around one end

mass of piece $\longrightarrow$ $dx \frac{M}{L} = \left(\frac{dx}{L}\right) M$

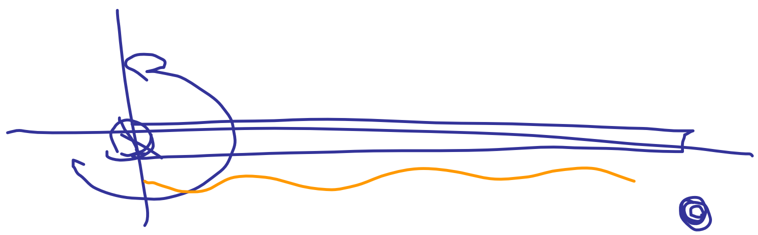
Density = $\frac{M}{L}$

Add up mr^2 all the little pieces:

$$\int_0^L x^2 \cdot \left(dx \frac{M}{L}\right) = \frac{M}{L} \int_0^L x^2 \, dx$$

$$I = \frac{M}{L}\int_0^L x^2\,dx = \frac{M}{L}\frac{x^3}{3}\Big|_0^L$$

$$= \frac{M}{L}\frac{L^3}{3} = \frac{1}{3}ML^2$$

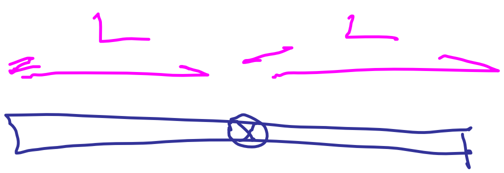


$$I = \frac{1}{3}ML^2$$

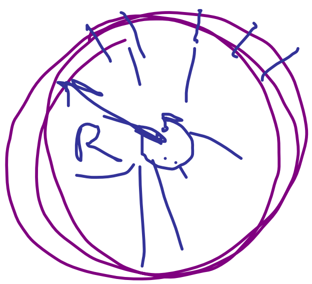
$kg\ m^2$

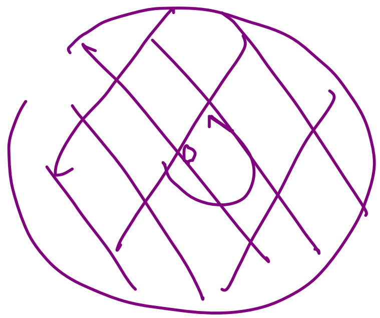
Stick about center

L, M $\quad = 2ML^2\,\frac{1}{3}\,\frac{1}{8}$



$$I = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 + \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{12}ML^2$$





Hoop
(Ignore spokes)

All mass points
are all at R

$$I = \sum m r^2$$

$$= MR^2$$