

Phys 3810, Spring 2013
Problem Set #2, Hint-o-licious Hints

1. Griffiths, 2.37 Find A from normalization and find the c_n 's from (2.37). Then $\Psi(x, t)$ is given by (2.36).

A clever trick on part (a) is to write $\Psi(x, 0)$ in terms of the stationary states (which are proportional to $\sin(n\pi x/a)$) before squaring it. You can look up in a book how $\sin^3 w$ is related to $\sin(3w)$. Then when you square and integrate you can use orthonormality of the stationary states.

Get

$$A = \frac{4}{\sqrt{5a}}$$

In finding $\langle x \rangle$ you'll need to evaluate the integral

$$\int_0^a x \sin(\pi x/a) \sin(3\pi x/a) dx$$

which you might not find in a table of integrals. You can use a trig identity which makes a product of trig functions into a sum and then it becomes a couple of integrals of x times a single trig function, and that *can* be found.

2. Griffiths, 2.10 Use (2.66) and you can use the result for $\psi_1(x)$ from 2.47. You will find:

$$\psi_2(x) = \frac{1}{\sqrt{2}} a_+ \psi_1(x) = \frac{1}{\sqrt{2}} \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} \left(\frac{2m\omega}{\hbar} x^2 - 1 \right) e^{-\frac{m\omega}{2\hbar} x^2}$$

3. Griffiths, 2.11 This one can be a little tedious; it will be OK if you just do the $\psi_0(x)$ state *or* the $\psi_1(x)$ state. For ψ_0 , the nonzero answers are

$$\langle x^2 \rangle = \frac{\hbar}{2m\omega} \quad \langle p^2 \rangle = \frac{\hbar m\omega}{2}$$

and for ψ_1 the nonzero answers are

$$\langle x^2 \rangle = \frac{3\hbar}{2m\omega} \quad \langle p^2 \rangle = \frac{3\hbar m\omega}{2}$$

For part (c), recall how T and V are related to p^2 and x^2 , respectively. Then use the results from part (a).

4. Griffiths, 2.15 The classical turning point for the ground state is $a = \sqrt{\frac{\hbar}{m\omega}}$. The probability we want is

$$P = \int_{|x|>a} |\psi_0(x)|^2 dx$$

I get $P = 0.157299$ (probability to be where, classically, it shouldn't be). But whatever probability you get, make sure it's less than 1.

5. Griffiths, 2.19 Use the definition of J from Problem 1.14 and be careful with the complex conjugates. You get an answer which makes sense, as it is proportional to the classical velocity.

6. Griffiths, 2.21 (a) Show that $A = \sqrt{a}$. (b) Find $\phi(k)$ with its definition; it may help to use

$$e^{-ikx} = \cos kx - i \sin kx$$

and the fact that \cos is an even function and \sin is odd. You should get

$$\phi(k) = \sqrt{\frac{2}{\pi}} \frac{a^{3/2}}{(a^2 + k^2)}$$

(c) Use this $\phi(k)$ to write out $\Psi(x, t)$ as an integral. (d) examine the behavior of $\psi(k)$ and $\Psi(x, 0)$ for large and small a .