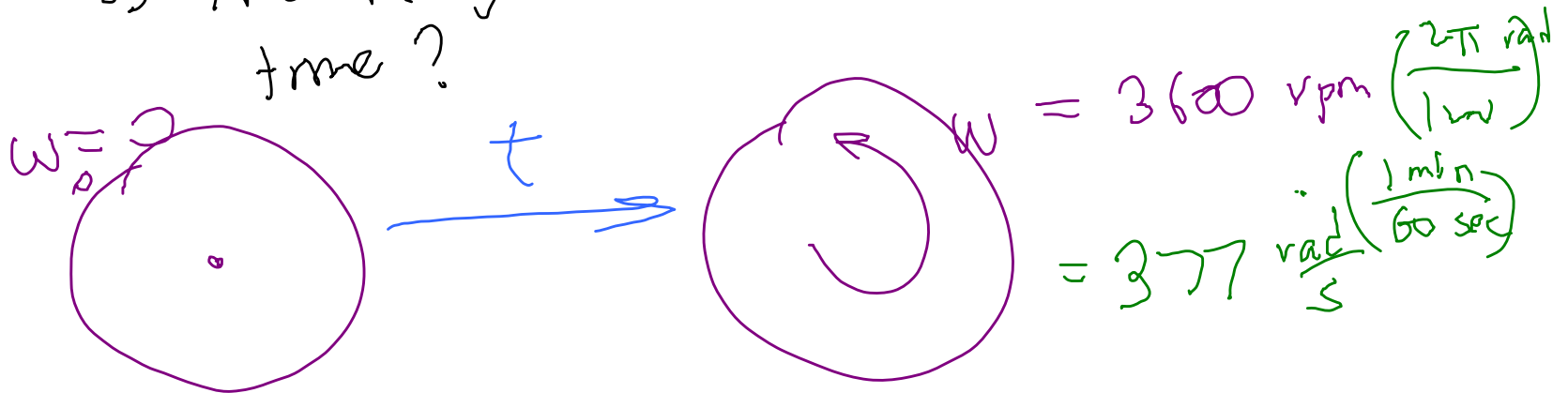


# Kinematics, $\theta$ , $\omega$ , $\alpha$

- 10.18 Turbine accel's from rest at  $0.52 \frac{\text{rad}}{\text{s}^2}$
- How long it take to reach 3600 rpm?
  - How many rev's does it make during this time?



$$\omega = \omega_0 + \alpha t$$

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{377 \frac{\text{rad}}{\text{s}} - 0}{0.52 \frac{\text{rad}}{\text{s}^2}} = \therefore 12 \text{ min}$$

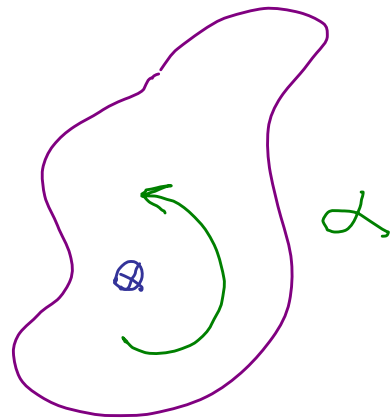
b) Find  $\theta$  ( $\theta_0 = 0$ )

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$= 0 + \frac{1}{2} (0.52 \frac{\text{rad}}{\text{s}^2}) (720 \text{ s})^2$$

$$= 1.37 \times 10^5 \text{ rad} = 2.17 \times 10^4 \text{ rev}$$

Why is there angular acceleration?

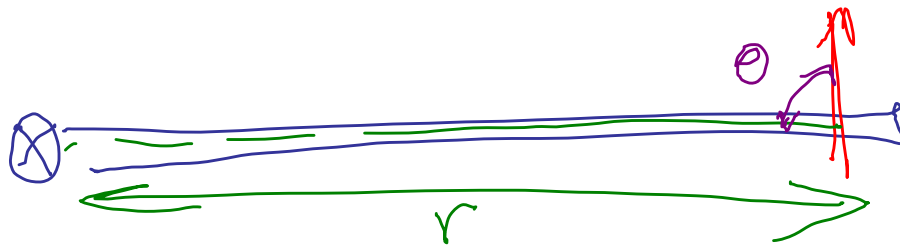
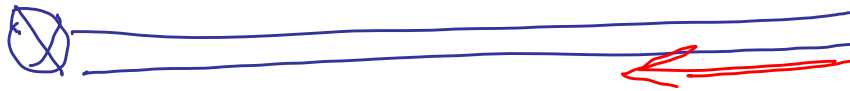


Forces

Yes, but ...



→ Push far from axis



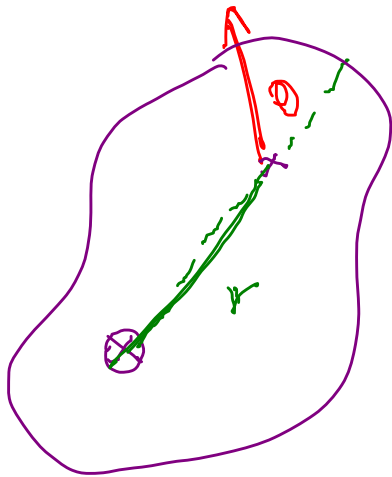
Get  $\alpha$ ,  
Push far from axis  $r$   
Want  $\sin\theta$  to be big.

To open doors secret to success:

$$\tau = Fr \sin \theta$$

p. 158

Torque (exerted by  $F$  on rotating)



$$|\tau| = |r F \sin \theta|$$

$\Rightarrow$  Yo put in proper sign  
for what torque does.

What is torque  
Scalar, for scalar  
(+/-).



$$\tau = r F \sin \theta$$

Units?

Units?

$$[\tau] = \text{m} \cdot \text{N} \cdot 1$$

$$= \text{N} \cdot \text{m} = \text{J}$$

Torque is not energy. Say  $\text{N} \cdot \text{m}$

English Work  $\text{ft} \cdot \text{lb}$  Torque  $\text{lb} \cdot \text{ft}$

Torque depends on where force applied & direction.

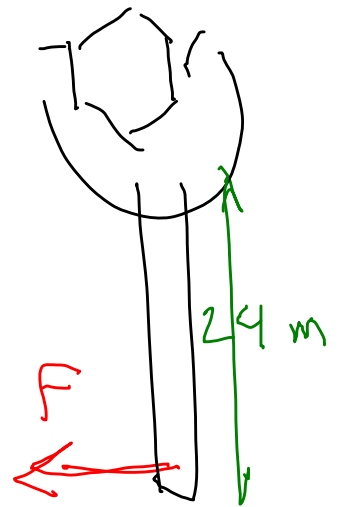
10. 22 Tighten spark-plugs to torque of 35 Nm.

Pull on end of 24-cm long wrench at

a) Rt angle to shaft

b) At  $110^\circ$  to shaft.

$$\begin{aligned} \text{a) } \tau &= 35 \text{ N}\cdot\text{m} = Fr \sin 90^\circ \\ &= F(0.24 \text{ m}) = 146 \text{ N} \end{aligned}$$



$$\tau = r F \sin 110^\circ = 35 \text{ N}\cdot\text{m}$$

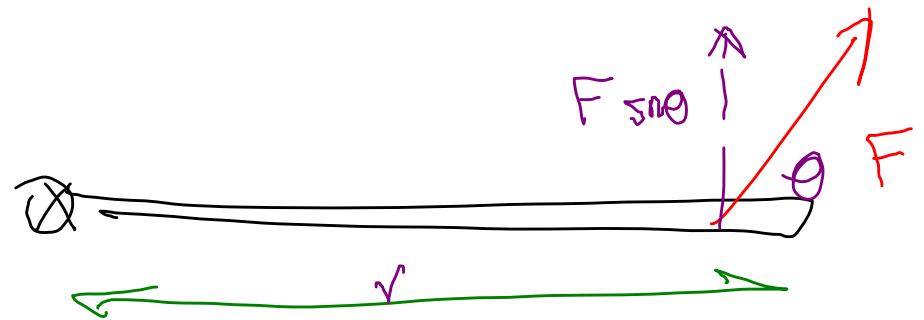
$$r = 0.24 \text{ m}$$

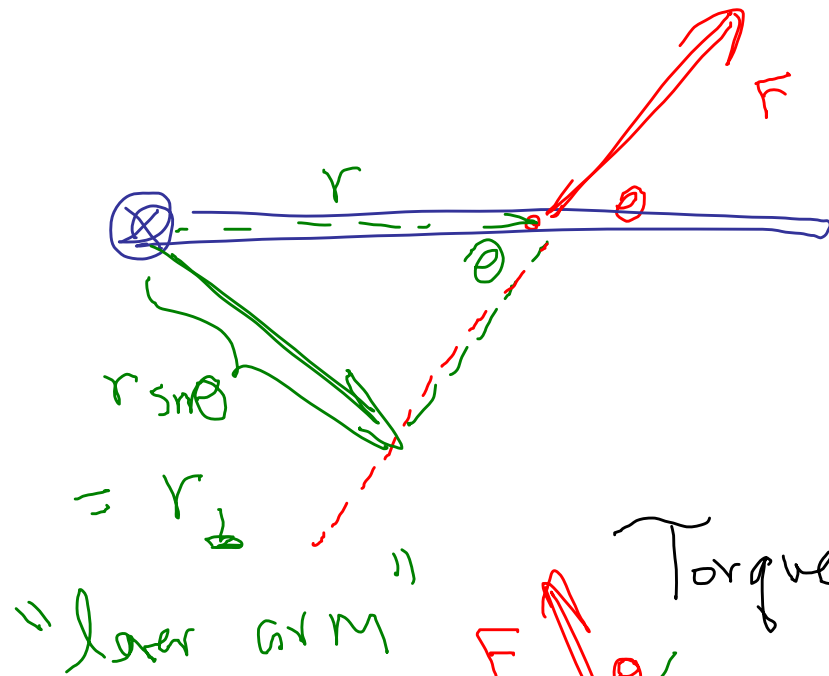
$$\Rightarrow 155 \text{ N}$$

For torque only perp part of  
force matters

$$\tau = r \frac{F \sin \theta}{F_{\perp}}$$

$$\tau = r F_{\perp}$$

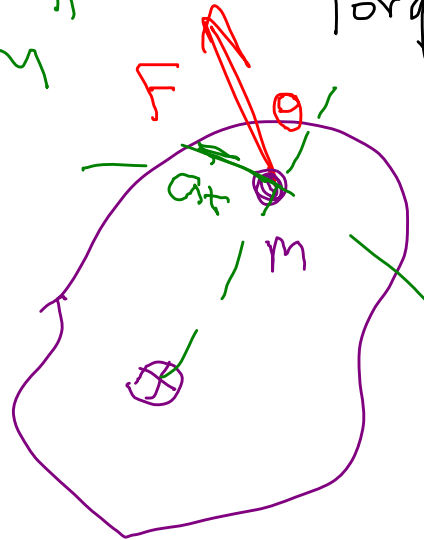




$$\tau = F (r \sin \theta)$$

$$\tau = F r_{\perp}$$

Torque - huh! What is it good for?



Force  $F$  will give  $m$  a linear accel. (also angular accel)

$$\underline{\underline{F_{\perp} = m a_t}}$$



$$\vec{F} = m \vec{a}_t$$

$$F \sin \theta = m r \alpha$$

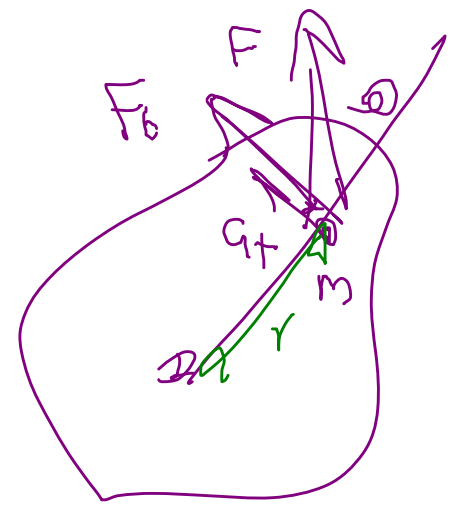
Mult both sides by  $r$

$$r F \sin \theta = m r^2 \alpha$$

$$\cancel{F} = (\cancel{m} r^2) \cancel{\alpha}$$

$F \quad m \quad a$

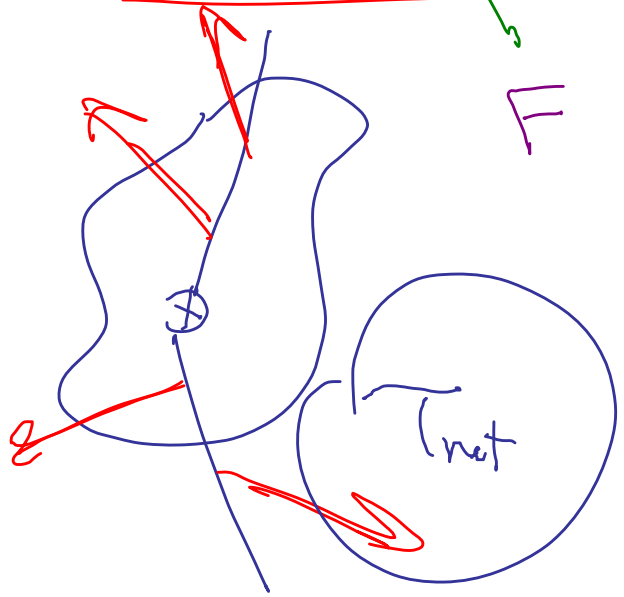
Correct relation comes from summing over all mass points



$$a_t = r \alpha$$

$$F = m a$$

$$\tau_{\text{net (ext)}} = \left( \sum m_i r_i^2 \right) \alpha$$



$$F = m a$$

call this  $I$ ,  
moment of inertia  
rotational inertia

What  $I$ ? Discuss

same for  
all mass  
points

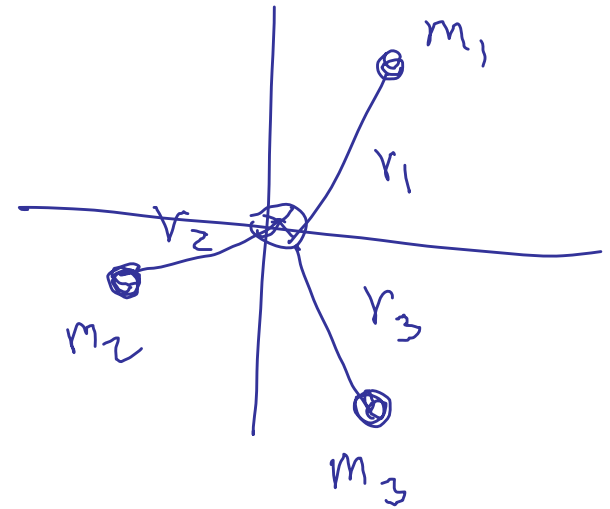
$$I = \sum_i m_i r_i^2$$

Scalar  
matrix  
Units?

$$mr^2$$

$\text{kg m}^2$

Next time examples!



$$m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$