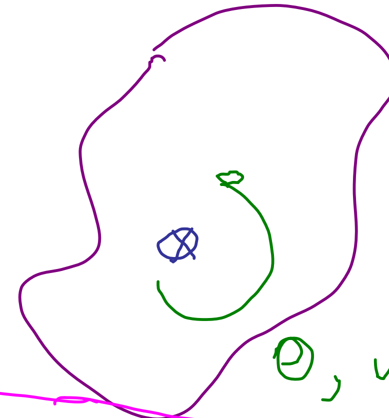
Phys 2110-5 10/31/12

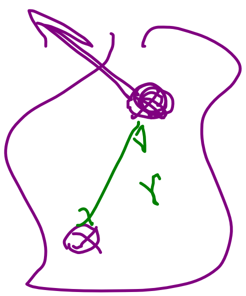
Note Title 10/31/2012

Rotations

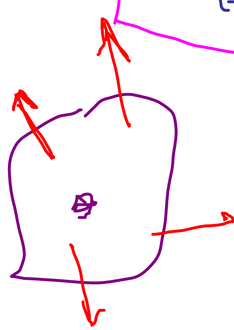
α, ang accel $\frac{rad}{s^2}$, $1/s^2$



θ, ω, α



$$\boxed{\tau_{net\,(ext)} = \underbrace{\left(\sum_i m_i r_i^2\right)}_{=I} \alpha}$$



α, ω, θ same for all points

$$I = \sum_i m_i r_i^2$$

Moment of inertia

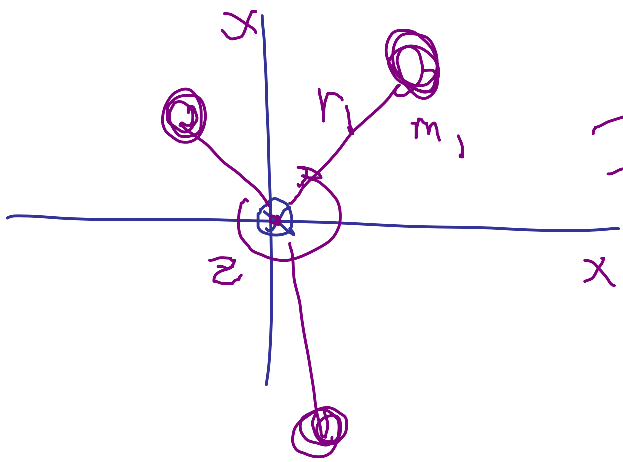
Rotational inertia

Scalar

matrix!

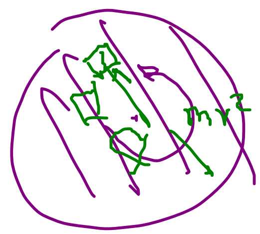
Units : kg m^2

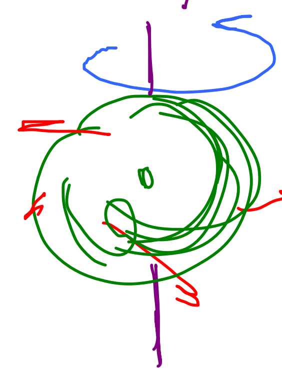
Depends which axis



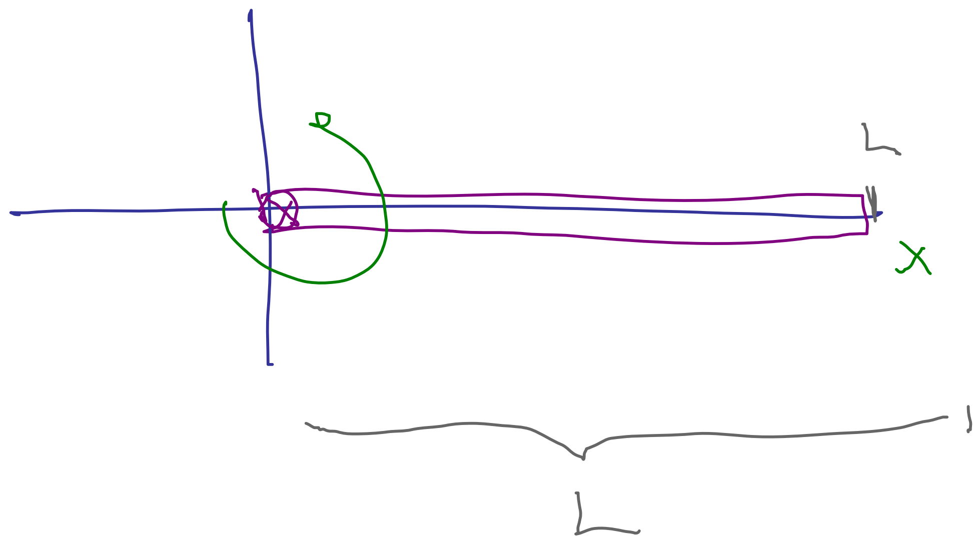
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^3$$

e.g. Solid cylinder





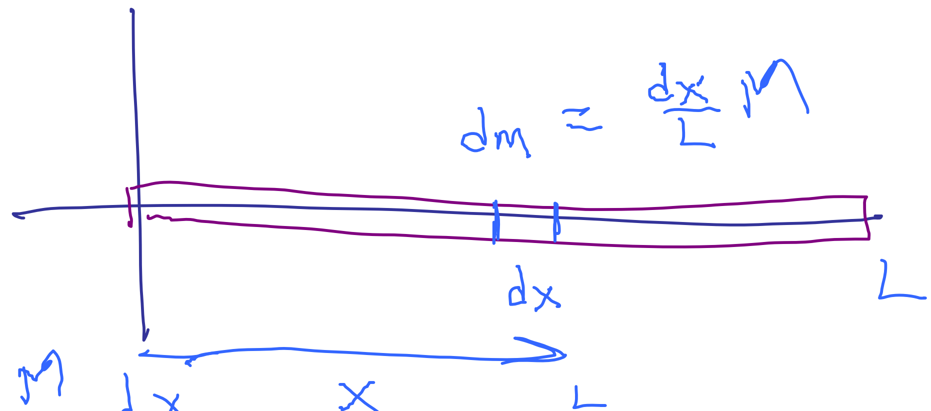
Example:



Rotate stick around axis, find I

Mass M, length

$$dm = \frac{dx}{L} M$$



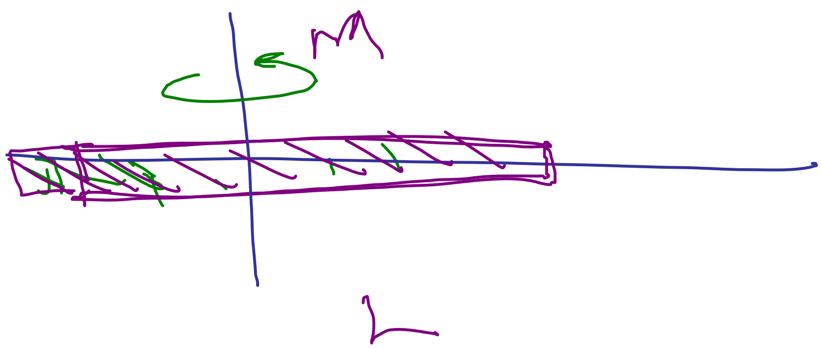
$$I = \sum dm\, x^2 = \sum_i x_i^2 \frac{M}{L} dx_i$$

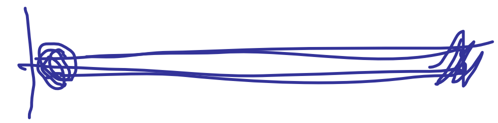
$$= \int_0^L \frac{M}{L} x^2\, dx$$

$$I_{\text{stick, end}} = \frac{M}{L}\int_0^L x^2\,dx = \frac{M}{L}\left.\frac{x^3}{3}\right|_0^L$$

$$= \frac{1}{3}\frac{M}{L}L^3 = \boxed{\frac{1}{3}ML^2}$$

Make sense





$= \quad M \quad ML^2$

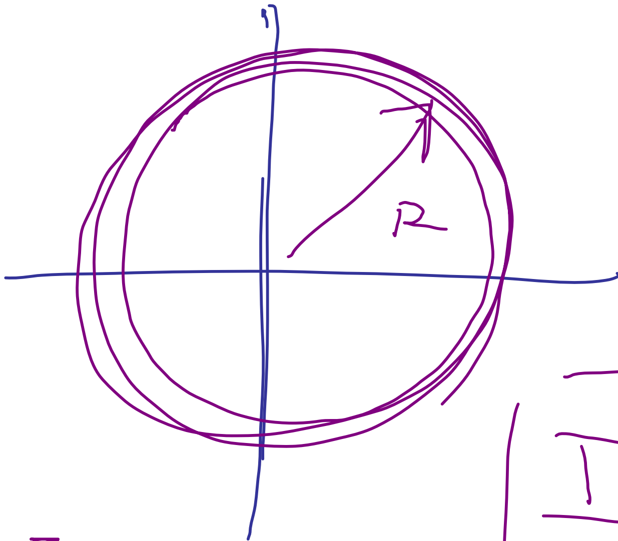
You could derive this

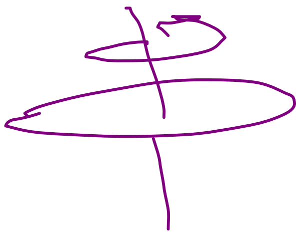
$$\boxed{I_{\text{stick, mid}} = \frac{1}{12}ML^2}$$

Units

kg m^2

p.163





Hoop

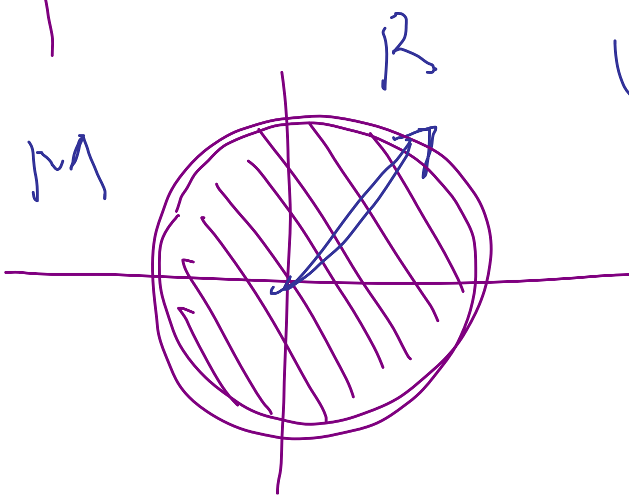
Mass M

$$\boxed{I_{hoop} = MR^2}$$

$$\sum m R^2$$

$$R^2 \sum m$$

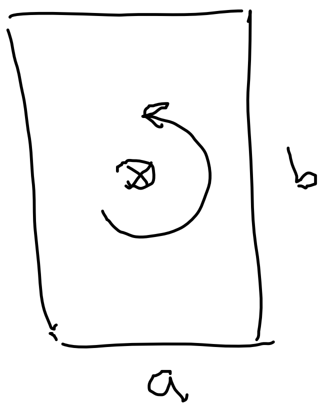
$$= MR^2$$



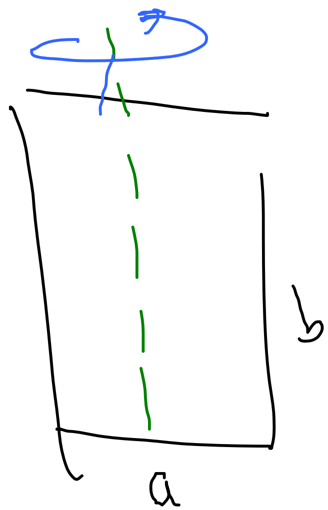
Uniform disk

$$I < MR^2$$

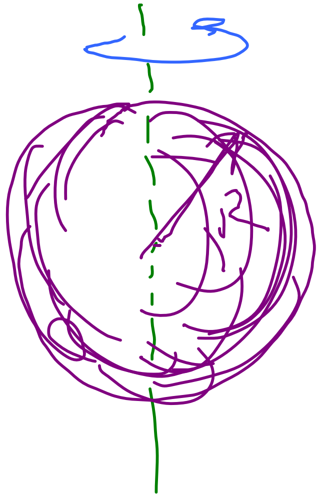
$$\boxed{I_{disk} = \frac{1}{2} MR^2}$$



$$I = \frac{1}{12} M (a^2 + b^2)$$



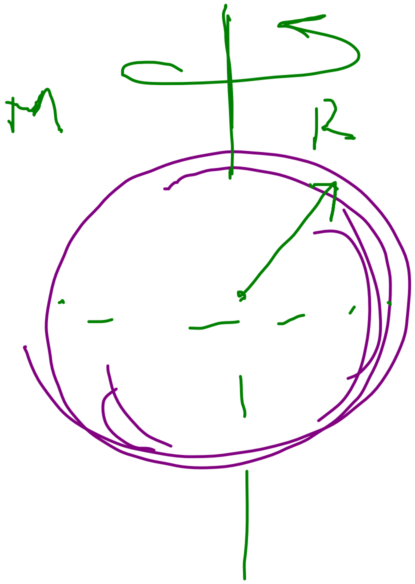
$$I = \frac{1}{12} M a^2$$



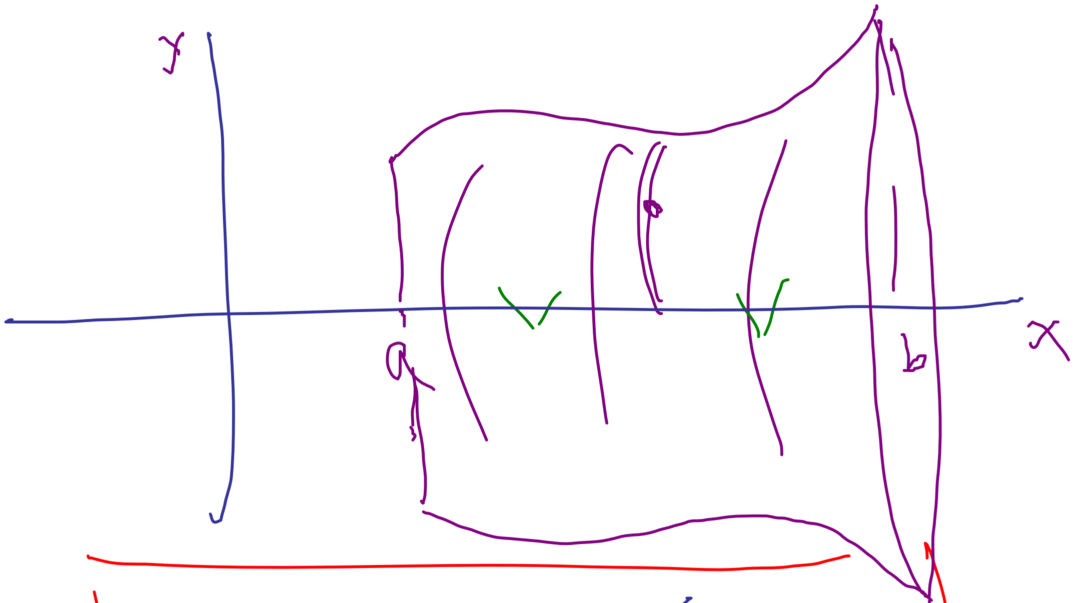
Solid sphere, R

Mass M

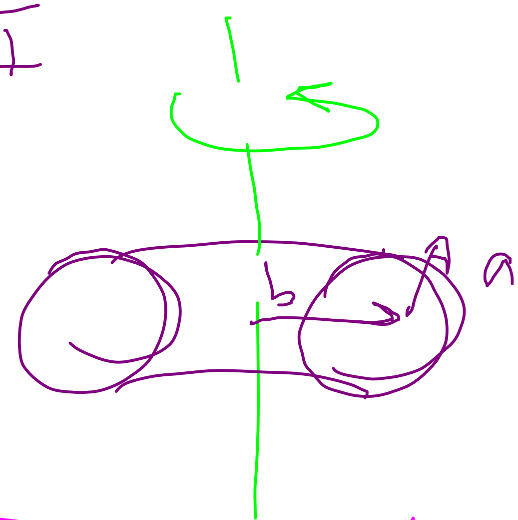
$$\boxed{I_{\substack{\text{sphere}\\\text{sol}}} = \frac{2}{5} M R^2}$$



$$\boxed{I_{\substack{\text{sphere}\\\text{hollow}}} = \frac{2}{3} M R^2}$$



find I



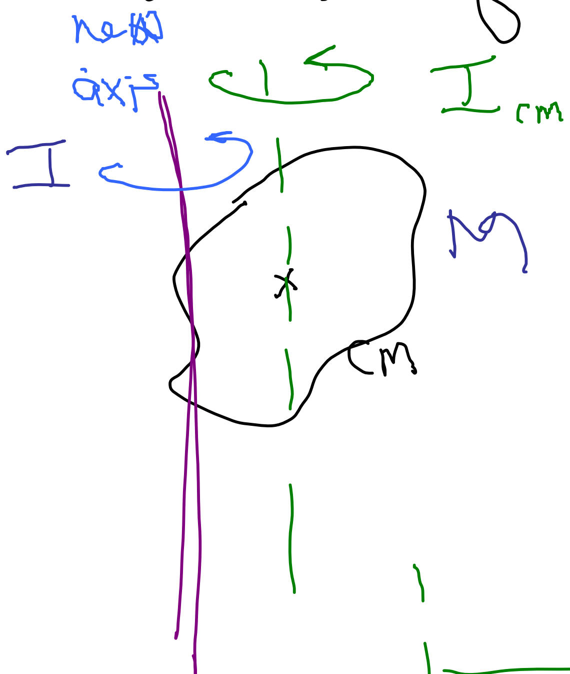
$$\boxed{\tau_{net} = I\alpha}$$

$$\boxed{F = ma}$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$x = x_0 + v_{x0} t + \tfrac{1}{2} a_x t^2$$

Moments of Inertia



New axis is parallel to old one distance d away. Obj has has M

$$I = I_{cm} + Md^2$$

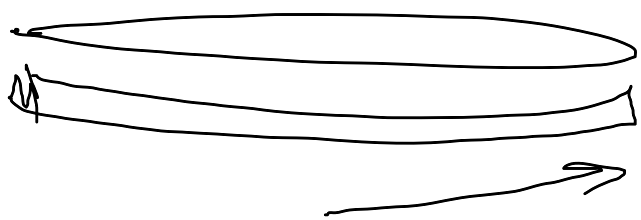
$$I \neq MR^2$$

$$= \frac{1}{2}Mr^2 + MR^2$$

10.32 A 108 g Frisbee is 24 cm in diameter & half mass spread in a uniform disk, half at edge.

$m = 0.054$ kg

a) What's rot'l inertia



12 cm

0.054 kg disk, $I = \frac{1}{2} m r^2$

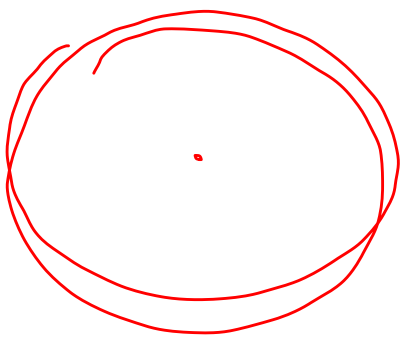
0.054 g hoop $I = m r^2$

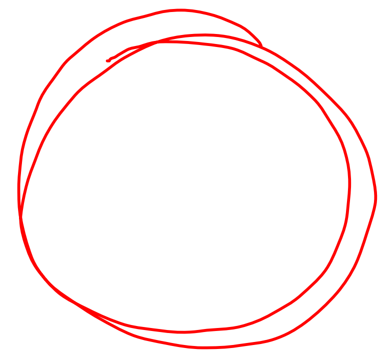
Add

$$I = 1.17 \times 10^{-3} \text{ kg m}^2$$

b) $\frac{1}{4}$ turn flick of wrist student gets Frisbee in rotating at 550 rpm

What's mag of torque (constant) applied?





$\omega_0 = 0$

$\omega = 550 \text{ rpm} \left(\frac{2\pi}{1 \text{ rad}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$

$= 57.6 \text{ rad/s}$

$\frac{\pi}{2}$ rad

$\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow \alpha = 1056 \text{ rad/s}^2$

$$\tau = I\alpha = (1.17 \times 10^{-3}\ kg\,m^2)(1056\ \frac{rad}{s^2})$$

$$= 1.24\ N \cdot m$$

$$\frac{kg\ m^2\ \cancel{rad}}{s^2}$$

$$\frac{kg\ m^2}{s^2} = \frac{kg\ m}{s^2} \cdot m$$

$$= J$$

10.33