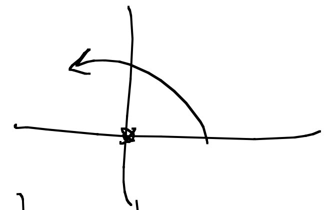
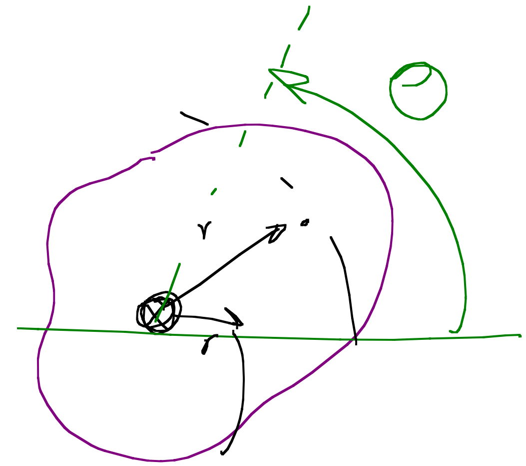
Chap 10

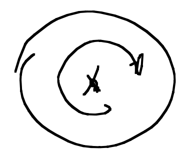
Θ is in radians.



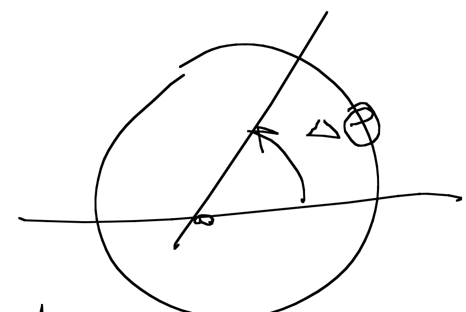


Meas'd cc-wise. Subject to change

2π radians $= 1$ rev
$= 360$ deg



Study how Θ changes w/ time



Define avg angular velocity

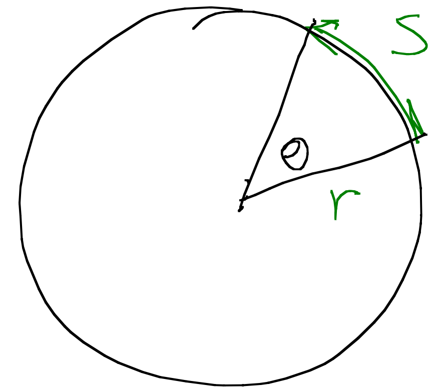
$$\frac{\Delta\Theta}{\Delta t} = \bar{\omega}$$

$\rightarrow$ Scalar

Units: $\frac{rad}{sec}$ or $\frac{1}{sec}$, s^{-1}

Instantaneous angular velocity:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$



Some geometry

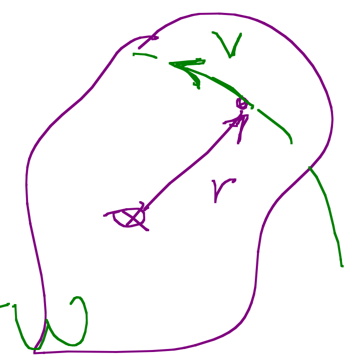
$s = \theta r$

m [rad] m

θ is in radians! optional unit.

s = arclength

All points have tangential velocity no radial velocity

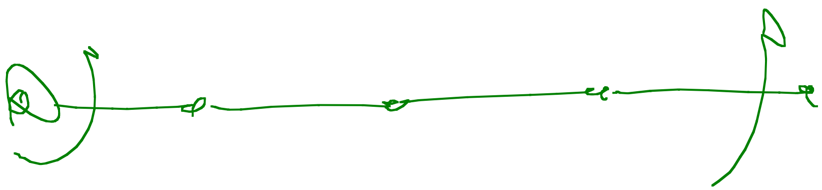


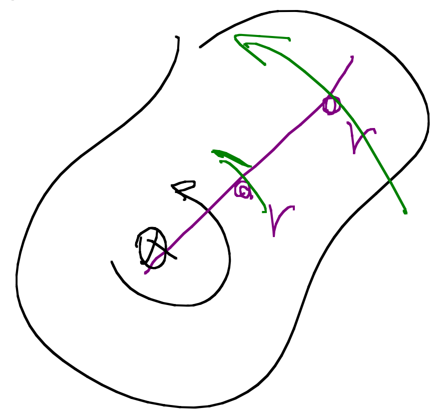
$$v_t = \frac{ds}{dt} = \frac{d}{dt}(\theta r) = r\frac{d\theta}{dt} = r\omega$$

$$\boxed{v = v_t = r\omega}$$

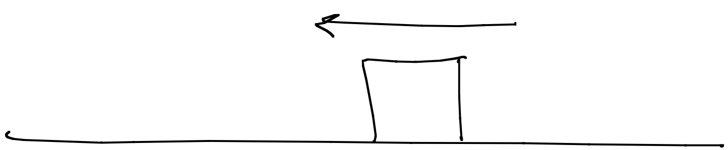
Relation between rotational & "linear" quantities

ω applies to entire object

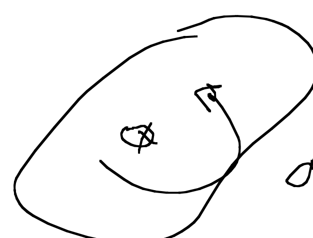




Constant ω is not interesting



vel. changed by force



angular vel is changed torque

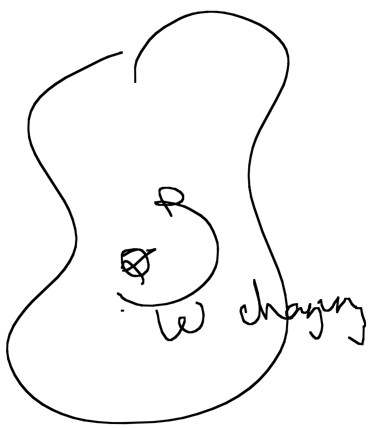
consider ω changing w/ time.
How radidly is ω changing?

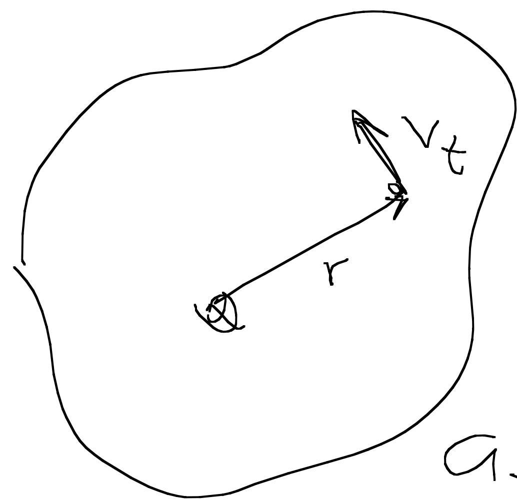
$$\frac{\Delta\omega}{\Delta t}$$ angular acceleration, α

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous α:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$





$$\longrightarrow \frac{dv_t}{dt} = \frac{dv}{dt}$$

$$v = \omega r$$

$$a_t = \frac{dv_t}{dt} = \frac{d}{dt}(\omega r)$$

$$a_t = \frac{d}{dt}(\omega r) = r\frac{d\omega}{dt} = r\alpha$$

$\frac{m}{s^2}$ $\qquad$ $m\,\frac{\cancel{rad}}{sec^2}$

Don't forget: $a_c = v^2/r$

$$a_c = \frac{(\omega r)^2}{r} = \omega^2 r$$

$$\boxed{a_t = r\alpha \qquad a_c = \omega^2 r}$$

Ang. accel's are caused by torques

Const torque $\longrightarrow$ constant ang accel.

Assume $\alpha = \text{constant} = \frac{d\omega}{dt}$

$$\Longrightarrow \quad \omega = \omega_0 + \alpha t$$

initial value of ang. velocity

$$v = v_0 + at$$

$$\omega = \frac{d\theta}{dt}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

init. value of θ

init. val of ω

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

Often set $\theta_0 = 0$

Can show:

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta = \theta_0 + \frac{1}{2}(\omega + \omega_0)t$$

10.16 Change to radians per second.

a) 720 rpm

$$720 \frac{\cancel{\text{rev}}}{\cancel{\text{min}}} \cdot \frac{1 \cancel{\text{min}}}{60 \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \cancel{\text{rev}}}$$

$$= 75.4 \frac{\text{rad}}{\text{s}}$$

b) $50°/h = \frac{50 \text{ deg}}{h} \cdot \left(\frac{\pi \text{ rad}}{180 \text{ deg}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)$

$= 2.42 \times 10^{-4} \frac{\text{rad}}{\text{sec}} = \frac{1}{\text{sec}} = s^{-1}$

10.19 During start up of a power plant turbine accelerates from rest at $0.52 \text{ rad}/s^2$.

a) How long it take to reach op speed 3600 rpm.

b) How many rev's it make?

$$\left(3600 \text{ rpm}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$

$$= 377 \text{ rad/s}$$

$$\alpha = \frac{\Delta\omega}{\Delta t} \qquad \Delta t = \frac{\Delta\omega}{\alpha} = \frac{377 \text{ rad/s} - 0}{0.52 \text{ rad/s}^2}$$

$$= 725 \text{ s}$$

b) During this time how many rev's

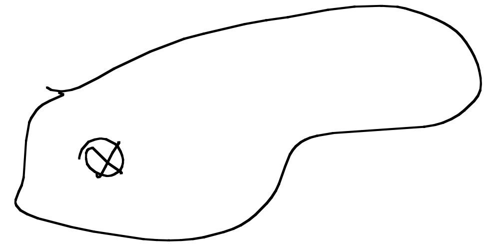
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(0.52 \tfrac{\text{rad}}{\text{s}^2}\right)\left(725 \text{ s}\right)^2$$

$$= 1.37 \times 10^5 \text{ rad} = 2.17 \times 10^4 \text{ rev}$$

Divide by 2π

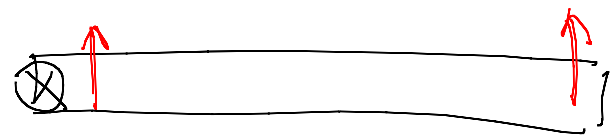
What gives α, changes in rot'l motion.

Practical discussion

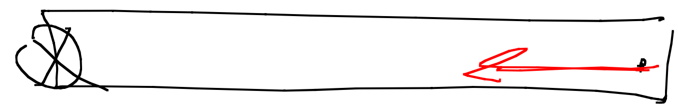


Force

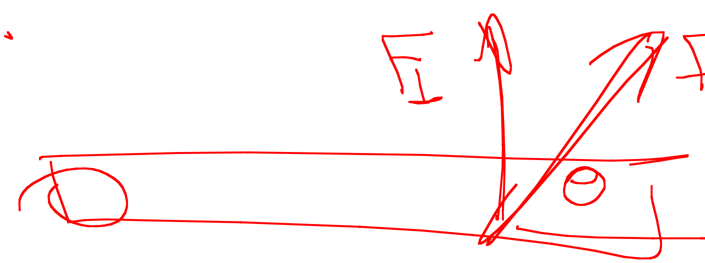
1) Apply force far from axis



2) Apply force perp. to line joining point & axis.



Only $F\sin\theta$ matters



$$(r)(F\sin\theta) = \text{Torque} = \tau$$