Phys 2920, Spring 2020
Problem Set #8

1. Go back to Problem 5 on the last set and find the divergence of that vector field, $\nabla \cdot \mathbf{a}$. Then for the cylindrical volume of that problem, evaluate $\int_V (\nabla \cdot \mathbf{a})\, dV$.

Did you get what you expected?

2. *(VA 6.55)* If S is any closed surface enclosing a volume V and $\mathbf{A} = ax\,\mathbf{i} + by\,\mathbf{j} + cz\,\mathbf{k}$, prove that

$$\int\int_S \mathbf{A} \cdot \mathbf{n}\, dS = (a + b + c)V$$

3. *(VA 6.53)* Verify the divergence theorem for $\mathbf{A} = 2x^2y\,\mathbf{i} + -y^2\,\mathbf{j} + 4xz^2\,\mathbf{k}$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $x = 2$.

4. *(VA 6.63)* Verify Stokes' theorem for $\mathbf{A} = (y - z + 2)\,\mathbf{i} + (yz + 4)\,\mathbf{j} - xz\,\mathbf{k}$, where S is the surface of the cube $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 2$, $z = 2$ above the xy plane.

5. Check Stokes' theorem using the function $\mathbf{v} = 2y\,\mathbf{i} + 3x\mathbf{j}$ where the path is the unit circle in the xy plane. (This is the same thing as checking "Green's theorem in a plane" for this case.

6. Evaluate:

a) $\int_0^1 \cos x\, \delta(x - \frac{\pi}{4})\, dx$

b) $\int_0^4 (3x^2 - 2x - 1)(\delta(x - 2) + \delta(x - 5))\, dx$

c) $\int_V (5\mathbf{r}^2 - 2\mathbf{r} \cdot \mathbf{c} - 7)\, \delta^3(\mathbf{r} - 2\mathbf{k})\, dV$ where $\mathbf{c} = 3\mathbf{i} - 5\mathbf{k}$ and V is the sphere of radius 3 centered at the origin.

7. *(CV 1.53 g))* Evaluate, in simple $x + iy$ form,

$$\frac{(2+i)(3-2i)(1+2i)}{(1-i)^2}$$

8. *(CV 1.54 b, j))* If $z_1 = 1 - i$, $z_2 = -2 + 4i$, $z_3 = \sqrt{3} - 2i$, find

(a) $\left|\dfrac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$ (b) $\mathrm{Im}\,\{z_1 z_2 / z_3\}$