2024-03-07

Number 6

An email message can travel through one of three server routes. The percentage of errors in each of the servers and the percentage of messages that travel through each route are shown in the following table. Assume that the servers are independent.

```
Percentage of Messages
                                                                                        Percentage of Errors
Server 1
                                          40
                                                                                                 1
                                                                                                 2
Server 2
                                          25
Server 3
                                          35
                                                                                                 1.5
```

- a. What is the probability of receiving an email containing an error?
- b. What is the probability that a message will arrive without error?

```
c. If a message arrives without error, what is the probability that it was sent through server 1?
## NUMBER 6
# (a)
# Define the percentages
percent_messages <- c(40, 25, 35)
percent_errors <- c(1, 2, 1.5)
# Calculate the weighted average of error percentages
prob_error <- sum(percent_messages * percent_errors) / sum(percent_messages) / 100</pre>
prob_error
```

[1] 0.01425

```
#(b)
# Calculate the complement of the error probability
prob_no_error <- 1 - prob_error</pre>
prob_no_error
```

```
## [1] 0.98575
#(C)
# Calculate the probability of sending a message through server 1 given no error
prob_server1_no_error <- (percent_messages[1] / 100) * prob_no_error</pre>
prob_server1_no_error
```

[1] 0.3943

Number 9

A software company surveyed managers to determine the probability that they would buy a new graphics package that includes three-dimensional graphics. About 20% of office managers were certain that they would not buy the package, 70% claimed that they would buy, and the others were undecided. Of those who said that they would not buy the package, only 10% said that they were interested in upgrading their computer hardware. Of those interested in buying the graphics package, 40% were also interested in upgrading their computer hardware. Of the undecided, 20% were interested in upgrading their computer hardware.

Let A denote the intention of not buying, B the intention of buying, C the undecided, and G the intention of upgrading the computer hardware.

- a. Calculate the probability that a manager chosen at random will not upgrade the computer hardware (P(complement G)).
- b. Explain what is meant by the posterior probability of B given G, P(B|G).
- c. Construct a tree diagram and use it to calculate the following probabilities: P(G), P(B|G), P(B|complement G), P(C|G), P(complement C|complement G).

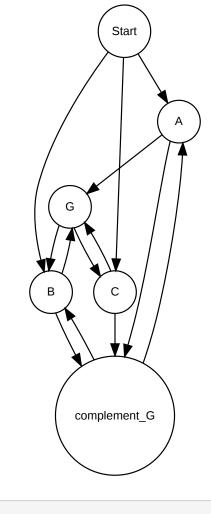
```
## NUMBER 9
# Define probabilities
P_A <- 0.20 # Probability of not buying
P_B <- 0.70 # Probability of buying
P_C <- 1 - P_A - P_B # Probability of undecided
P_G_given_A <- 0.10 # Probability of upgrading given not buying
P_G_given_B <- 0.40 # Probability of upgrading given buying
P_G_given_C <- 0.20 # Probability of upgrading given undecided
# Calculate complement probabilities
P_complement_G <- 1 - (P_A * P_G_given_A + P_B * P_G_given_B + P_C * P_G_given_C)
# (a) Calculate the probability that a manager chosen at random will not upgrade the computer hardware (P(complem
ent G))
print(P_complement_G)
```

[1] 0.68

```
# Define posterior probabilities
P_B_given_G <- (P_B * P_G_given_B) / P_complement_G</pre>
# (b) Explain what is meant by the posterior probability of B given G, P(B|G)
# Posterior probability of B given G represents the probability of a manager intending to buy the graphics packag
e given that they are interested in upgrading their computer hardware.
library(DiagrammeR)
```

Warning: package 'DiagrammeR' was built under R version 4.3.3

```
grViz("
digraph tree {
 node [shape = circle, fontsize = 10, fontname = Helvetica]
 edge [fontsize = 10, fontname = Helvetica]
 Start -> A [tooltip = '0.20']
 Start -> B [tooltip = '0.70']
 Start -> C [tooltip = '0.10']
 A -> G [tooltip = '0.10']
 A -> complement_G [tooltip = '0.90']
 B -> G [tooltip = '0.40']
 B -> complement_G [tooltip = '0.60']
 C -> G [tooltip = '0.20']
 C -> complement_G [tooltip = '0.80']
 G -> B [tooltip = '0.50']
 G -> C [tooltip = '0.50']
 complement_G -> A [tooltip = '0.90']
 complement_G -> B [tooltip = '0.10']
```



```
# Probability of G
P_G <- 0.2 * 0.10 + 0.7 * 0.40 + 0.1 * 0.20
# Probability of B given G
P_B_given_G <- 0.7 * 0.40 / P_G
# Probability of B given complement G
P_B_given\_complement_G <- 0.1 * 0.10 / (1 - P_G)
# Probability of C given G
P_C_given_G <- 0.1 * 0.50 / P_G
# Probability of complement C given complement G
P_complement_C_given_complement_G <- 0.9 * 0.90 / (1 - P_G)
# Print the probabilities
cat("P(G) = ", P_G, "\n")
## P(G) = 0.32
cat("P(B|G) = ", P_B_given_G, "\n")
```

P(B|G) = 0.875

```
cat("P(B|complement G) =", P_B_given_complement_G, "\n")
```

P(B|complement G) = 0.01470588

```
cat("P(C|G) = ", P_C_given_G, "\n")
## P(C|G) = 0.15625
```

cat("P(complement C|complement G) =", P_complement_C_given_complement_G, "\n")

P(complement C|complement G) = 1.191176

Number 13

A malicious spyware can infect a computer system though the Internet or through email. The spyware comes through the Internet 70% of the time and 30% of the time, it gets in through email. If it enters via the Internet the anti-virus detector will detect it with probability 0.6, and via email, it is detected with probability 0.8. (a) What is the probability that this spyware infects the system? (b) If the spyware is detected, what is the probability that it came through the Internet?

```
## NUMBER 13
# Probability of infection
P_infection <- 0.70 * 1 + 0.30 * 1
# Probability of detection
P_detection <- 0.6 * 0.70 + 0.8 * 0.30
# Probability of infection given detection
P_infection_given_detection <- (0.6 * 0.70) / P_detection
# Print results
print(P_infection)
```

[1] 1

print(P_infection_given_detection)

[1] 0.6363636