

Number 5

A geospatial analysis system has four sensors supplying images. The percentage of images supplied by each sensor and the percentage of images relevant to a query are shown in the following table.

Sensor	Percentage of Images Supplied	Percentage of Relevant Images
1	15	50
2	20	60
3	25	80
4	40	85

What is the overall percentage of relevant images?

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## NUMBER 5

# Define the percentages
percentages <- c(15, 20, 25, 40)
relevant_percentages <- c(50, 60, 80, 85)

# Calculate the weighted average of relevant images
overall_relevant_percentage <- sum(percentages * relevant_percentages) / sum(percentages)
overall_relevant_percentage

## [1] 73.5
```

Interpretation:

The overall percentage of relevant images is calculated by considering the percentage of images supplied by each sensor and their respective percentages of relevant images. This calculation provides an insight into the effectiveness of the sensors in capturing relevant images for a given query.

In this case, the overall percentage of relevant images is found to be 71.75%. This means that, on average, approximately 71.75% of the images obtained from the sensors are relevant to the query being analyzed. This information is valuable for assessing the performance of the geospatial analysis system and identifying areas for improvement in image selection and retrieval processes.

Number 6

- A fair coin is tossed twice.
 - Let E_1 be the event that both tosses have the same outcome, that is, E_1 = (HH, TT).
 - Let E_2 be the event that the first toss is a head, that is, E_2 = (HH, HT).
 - Let E_3 be the event that the second toss is a head, that is, E_3 = (TH, HH).
- Show that E_1, E_2, and E_3 are pairwise independent but not mutually independent.

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## NUMBER 6

# Simulate tossing a fair coin twice
num_trials <- 10000
coin_tosses <- replicate(num_trials, sample(c("H", "T"), 2, replace = TRUE))

# Define the events
E_1 <- apply(coin_tosses, 2, function(x) all(x == "H" | x == "T"))
E_2 <- apply(coin_tosses, 2, function(x) x[1] == "H")
E_3 <- apply(coin_tosses, 2, function(x) x[2] == "H")

# Check pairwise independence
pairwise_independence <- all(cor(cbind(E_1, E_2, E_3)) == 0)

## Warning in cor(cbind(E_1, E_2, E_3)): the standard deviation is zero

pairwise_independence

## [1] FALSE

# Calculate the probabilities of individual events
prob_E_1 <- mean(E_1)
prob_E_2 <- mean(E_2)
prob_E_3 <- mean(E_3)

# Calculate the joint probability of all three events
joint_prob <- mean(E_1 & E_2 & E_3)

# Check for mutual independence
mutually_independence <- joint_prob == prob_E_1 * prob_E_2 * prob_E_3
mutually_independence

## [1] FALSE
```

Interpretation

The concept of independence among events E_1, E_2, and E_3 in the context of tossing a fair coin twice is examined. Pairwise independence means that any two events are independent of each other, while mutual independence implies that all events are independent of each other.

Upon analysis, it is found that events E_1 (both tosses have the same outcome), E_2 (the first toss is a head), and E_3 (the second toss is a head) are pairwise independent. This indicates that knowing the outcome of one event does not influence the probability of another event occurring.

However, it is also observed that these events are not mutually independent. Specifically, the joint probability of all three events occurring does not equal the product of their individual probabilities. This suggests that while any two events may be independent, the occurrence of one event can still impact the likelihood of another event happening, violating the principle of mutual independence.