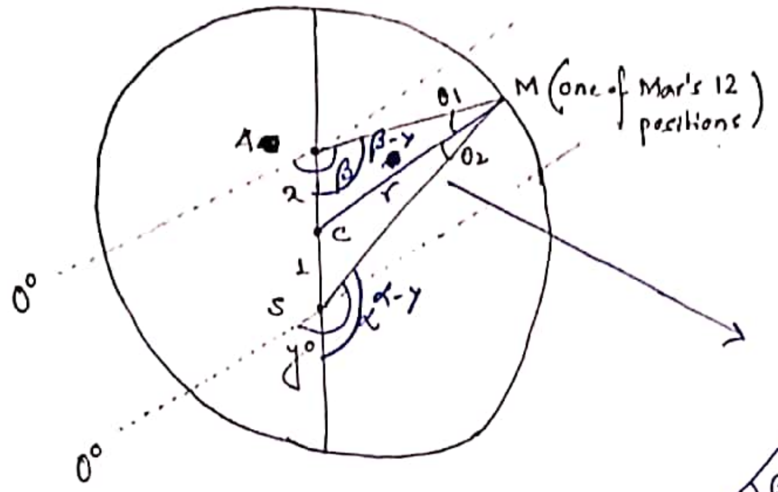
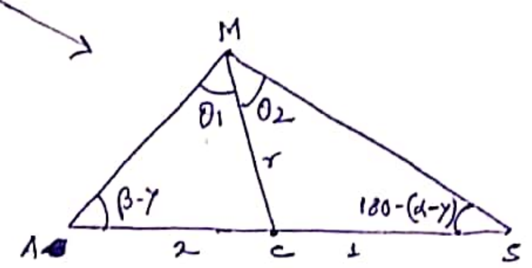
DATA ANALYTICS

Assignment-2: Part-1

Kishalay Das
(15938)

*** Derivation of Radius w.r.t x, y, α, β:**





A → Average Sun

S → Sun

r → radius

given data {
α → ~~Latitude~~ Longitude/angle relative to actual Sun for mars

β → ~~Latitude~~ Longitude/angle relative to Avg Sun for Mars
}

$$\theta_1+\theta_2 = 180 - [\beta - y + 180 - \alpha + y]$$

$$= \alpha - \beta.$$

① From ΔMAC

$$\frac{x}{\sin\theta_1} = \frac{r}{\sin(\beta-y)} \Rightarrow r\sin\theta_1 = x\sin(\beta-y)$$

$$= z_1 \text{ (say)} \quad \text{— ①}$$

$$\left.\begin{array}{l} \text{or } \sin\theta_1 = z_1/r \\ \cos\theta_1 = \sqrt{1-\frac{z_1^2}{r^2}} \end{array}\right\} \text{— ②}$$

② From ΔMCS

$$\frac{1}{\sin\theta_2} = \frac{r}{\sin(180-(\alpha-y))}$$

$$\text{or } \frac{1}{\sin\theta_2} = \frac{r}{\sin(\alpha-y)} \quad [\text{As } \sin(180-\theta) = \sin\theta]$$

$$\therefore r\sin\theta_2 = \sin(\alpha-y) = z_2 \text{ (say)} \quad \text{— ③}$$

$$\left.\begin{array}{l} \text{or } \sin\theta_2 = \frac{z_2}{r} \\ \therefore \cos\theta_2 = \sqrt{1-\frac{z_2^2}{r^2}} \end{array}\right\} \text{— ④}$$

③ And finally,

$$\cos(\theta_1+\theta_2) = \cos(\alpha-\beta) = z_3 \text{ (say)} \quad \text{— ⑤}$$

$$= \cos(\beta-\alpha) \quad [\text{As } \cos(-\theta) = \cos\theta]$$

$$Z_3 = \cos(\beta-\alpha) = \cos(\theta_1+\theta_2)$$
$$= \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$$

$$Z_3 = \sqrt{\left(1-\frac{Z_1^2}{r^2}\right)\left(1-\frac{Z_2^2}{r^2}\right)} - \frac{Z_1}{r}\cdot\frac{Z_2}{r} \quad \left\{\text{from } ①,②,③,④\right\}$$

$$\therefore \left(Z_3 + \frac{Z_1Z_2}{r^2}\right) = \sqrt{\frac{(r^2-Z_1^2)(r^2-Z_2^2)}{r^4}}$$

$$\therefore \left(Z_3 + \frac{Z_1Z_2}{r^2}\right)^2 = \frac{(r^2-Z_1^2)(r^2-Z_2^2)}{r^4}$$

$$\therefore \frac{(r^2Z_3+Z_1Z_2)^2}{\cancel{r^4}} = \frac{(r^2-Z_1^2)(r^2-Z_2^2)}{\cancel{r^4}}$$

$$\therefore r^4Z_3^2 + 2r^2Z_3Z_1Z_2 + \cancel{Z_1^2Z_2^2} = r^4 - Z_1^2r^2 - Z_2^2r^2 + \cancel{Z_1^2Z_2^2}$$

$$\therefore r^4(1-Z_3^2) = 2r^2Z_1Z_2Z_3 + Z_1^2r^2 + Z_2^2r^2$$

$$\cancel{r^4}\,r^2(1-Z_3^2) = \cancel{r^2}(Z_1^2+Z_2^2+2Z_1Z_2Z_3)$$

$$\text{or } r^2 = \frac{Z_1^2+Z_2^2+2Z_1Z_2Z_3}{1-Z_3^2}$$

$$\therefore \boxed{\text{Radius }(r) = \sqrt{\frac{Z_1^2+Z_2^2+2Z_1Z_2Z_3}{1-Z_3^2}}}$$

Where
$Z_1 = x\sin(\beta-\gamma)$
$Z_2 = \sin(\alpha-\gamma)$
$Z_3 = \cos(\beta-\alpha)$

→ I use this formula to findout r. Then I calculated Loss fn as →

$$J = \log[\text{Arithmatic mean (radius)}] - \log[\text{Geometric Mean (radius)}]$$

We optimized J by scipy.optimize.minimize method.

Optimized X: 1.2 Optimized Y: 148.874°