

Module 1 of E0 259, Data Analytics, August 2018

Kepler's data set on Mars

Lectures

[Lecture 1](#) (Ramesh Hariharan)

[Lectures 2 and 3](#) (Ramesh Hariharan)

Data set

[Mars Opposition data](#) (.csv file) :

This file contains data on longitude/latitude of Mars under "opposition" with the Sun, in the ecliptic coordinate system.

a) Columns A/B/C are day/month/year.

b) Columns D,E,F,G denote ZodiacIndex, Degree, Minute, Second, respectively, of Mars's (heliocentric) longitude in the ecliptic coordinate system. ZodiacIndex (s) refers to the zodiac (Aries 0, Taurus 1, ... Pisces 11).

Longitude = $s \times 30 + \text{Degree} + \text{Minute}/60 + \text{Second}/3600$ (degrees)

c) Columns H,I refer to degree, minute of the geocentric latitudinal position of Mars in the ecliptic coordinate system.

d) Columns J,K,L,M refer to Mars's mean longitude, with reference to Kepler's approximated equant.

[Triangulation data](#) (.csv file) :

This file contains data on the longitude of Earth (centred at the Sun) and the longitude of Mars (centred at Earth).

a) Column A indicates the index of the pair. Columns B/C/D are day/month/year. Paired observations are 687 days apart.

b) Columns E,F denote the heliocentric longitude of Earth.

c) Columns G,H denote the geocentric longitude of Mars.

Assignment 1

Due: 11:55 pm on Monday 20 August 2018 (as per moodle server time). You will submit your code via the course's moodle platform. There will be one single deadline for all. It will be a sharp cut-off. No late submissions will be entertained. No email submissions will be entertained, and such emails will be automatically deleted.

This is a long assignment, and will test your long-forgotten trigonometry skills. Start early.

Discussion is encouraged. But write your own code. Please follow the ethics policy.

Remember to upload, accept the submission statement, and click the 'submit' button.

1. In this first part of the assignment, you will use the Mars opposition data. Ignore the latitudes. You will assume that the projection of Mars orbit on the ecliptic plane is a circle and that this centre, the average Sun, and the Sun are collinear. Refer to slide 21 of Lecture 1.

(i) For each fixed value of x (offset of the average Sun) and y (angle of the line with respect to the reference Aries), write down an expression for the projection of Mars position on the ecliptic plane and the distance of this projection to the centre, in terms of a paired observation.

(ii) Then using the loss function $\log(\text{arithmetic mean}) - \log(\text{geometric mean})$, find the best fit value of x and y .

(iii) Can we identify the radius of the projected circle (in astronomical units (AU)) from this? 1 AU = average Earth-Sun distance. What do you expect to see when we use the sample variance as the loss function instead of the loss function in (ii)?

In the rest of the assignment, you will find the angle of Mars's orbital plane with respect to the ecliptic and the parameters of its elliptic orbit. We do not take the approach of slide 21 of Lecture 1. You are welcome to discuss the merits/demerits of this approach.

2. Use the paired observations in the triangulation data for this part of the assignment. Assume that the Sun is at the origin, and the Earth is orbiting around the Sun at radius 1 AU.

- (i) Triangulate and solve for the five different projections of Mars's location on the ecliptic plane.
- (ii) Find the best fit circle centred at the Sun. (This is a regression to find the radius in AU). Take the loss function to be the squared Euclidean distance to the nearest point on the circle.

3. Use the Mars opposition data for this part of the assignment. Again assume that the Sun is at the origin, and the Earth is orbiting around the Sun at a radius of 1 AU.

- (i) Using opposition and the geocentric latitudes of Mars, find the corresponding heliocentric latitudes of Mars. Together with the heliocentric longitudes, we now have 12 locations of Mars (heliocentric latitudes and longitudes) on the celestial sphere, with the Sun as the centre. *You will need to use the answer from 2(ii) to compute the heliocentric latitude.*
- (ii) Take the celestial sphere's radius to be 1. Propose an iterative algorithm, based on the nonlinear regression techniques discussed in the lectures, to find the best fit for Mars's orbital plane. This plane should contain the Sun; so you are really finding the best great circle on the celestial sphere, which is also the best 'straight line' on the celestial sphere. Take the loss function to be the squared Euclidean distance between Mars's position on the celestial sphere and its projection on the plane.
- (iii) Implement your proposed iterative algorithm and report the identified inclination of this plane in degrees/minutes.

4. Go back to the paired observations in the triangulation data.

- (i) Find Mars's five different (3-d) locations on Mars's orbital plane. Use the orbital plane you identified in 3(iii) and the projections you identified in 2(i).
- (ii) Now find the best fit circle on Mars's orbital plane with the Sun as centre. Report the sum of losses.
- (iii) Find the best fit ellipse on Mars's orbital plane with the Sun at one of the foci. Report the sum of the losses. Draw the circle of 3(ii) and the ellipse of 3(iii) on the same plot. Mark the location of the Sun and the 12 positions of Mars.