Probability

1. 15 students — Randomly choose student to answer
Ask 8 questions — P(no student answers more than 1 question)

# of combinations where 8 different students picked: $\binom{15}{8} = 6{,}435$
total outcomes: $\binom{15+8-1}{8-1} = \binom{22}{7} = 346{,}104$
$P(\text{event}) = \frac{6{,}435}{346{,}104} = \boxed{0.019 = 1.9\%}$

2. Integer 00000 - 99999 randomly generated
Even integers that start w/ 2 odd digits where all digits are unique
total outcomes: 100,000
event outcomes: $\binom{5}{2}$ for first two positions (5 odd, 2 positions) $\binom{8}{4}$ for remaining positions (8 integer options, 2 spots)

$$P(\text{event}) = \frac{\binom{5}{2}\binom{8}{4}}{100{,}000} = \frac{700}{100{,}000} = \boxed{\frac{7}{1000} = 0.7\%}$$

3. Roll 3 six sided fair dice
Event A: at least two dice show 4 or above
Event B: all three dice show the same value
Yes they are independent: both can happen regardless of other
Ex: A not B - 4, 5, 6 B not A - 3, 3, 3 Both - 5, 5, 5 Neither - 1, 2, 3

4. Flush (all five cards same suit)

Number of hands possible: $\binom{52}{5}$ (52 cards choose 5) $= 2598960$

Number of possible flushes: $\binom{13}{5}(4) = 5148$

($\binom{13}{5}$: 13 cards per suit choose 5 cards; 4 suits)

$P(\text{flush}) = \frac{5148}{2598960} = .00198 = 0.198\%$

$\frac{0.198}{100} = \frac{1}{x}$

$x = \frac{100}{.198} \approx 506$ games (round up)

5. Win 70% of the time when superstar plays

Win 50% of the time if superstar doesn't play

75% chance of playing next 5 games

Won 4 out of 5 games

P(played 5 games)

80% win rate

$P(\text{win} \mid \text{plays}) = 0.7 \qquad P(\text{plays}) = .75$

$P(\text{win} \mid \text{doesn't play}) = 0.5$

$P(\text{win 4/5} \mid \text{plays}) = \binom{5}{4}(0.7)^4(0.3)^1 = 0.36015$

$P(\text{win 4/5} \mid \text{doesn't play}) = \binom{5}{4}(0.5)^4(0.5)^1 = 0.15625$

$P(\text{win 4/5}) = (0.36015)(.75) + (0.15625)(0.25) = 0.309175$

$P(\text{plays} \mid \text{win 4/5}) = (0.36055)(0.75)/(0.309175) = \boxed{0.8737 = 87.37\%}$