Coding Assignment 5

Counting Problems

1. unusual — how many unique subsets of 5 letters
unique subsets: 7 letters 5 positions: $\frac{7!}{(7-5)!} = \frac{7!}{2!} = \boxed{2,520}$
unique strings: accounting for duplicates: $\frac{7!}{(7-5)!\,5!} = \frac{7!}{2!\,5!} = \boxed{21}$

2. Standard Deck of Playing Cards: 52
Two unique pairs and a 5th random card
13 unique cards 4 of each kind
$\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}$
$(78)(6)(6)(11)(4) = \boxed{123,552}$

3. 16 songs in 1 hour 7 couples 1 couple will allow at most 1 song
Only care about # of songs each couple receives

C_1	C_2	C_3	C_4	C_5	C_6	C_7	couples
0-16	0-16	0-16	0-16	6-16	0-16	0-1	possible # of songs

order not important no replacement
P_1: C_7 gets 0 songs: 16 songs 6 couples $-\binom{16+6-1}{6-1} = \binom{21}{5}$
P_2: C_7 gets 1 song: 15 songs 6 couples $-\binom{15+6-1}{6-1} = \binom{20}{5}$
$P_1 + P_2 = \binom{21}{5} + \binom{20}{5} = 33649 + 26334 = \boxed{59983}$

4. BST w/ 12 nodes each w/ distinct value 1-12
Root: 3 Right: 9
BST w/ 2 nodes: 2
BST w/ 3 nodes: 5
BST w/ 4 nodes: 14

Catalan number: $\frac{(2n)!}{(n+1)!\,n!}$

$C(12) = 208,012$

w/ $R = 3$ $R_R = 9$: $\frac{C(12)}{?} =$

5. 10 patients 4 or 3 nurses guaranteed to serve at least 1 person

Two main outcomes:

4 nurses

or

3 nurses

4 nurses

at least 4 served

at most 10 served

4: $\binom{4+4-1}{4-1} = \binom{7}{3} = 35$

5: $\binom{5+4-1}{4-1} = \binom{8}{3} = 56$

6: $\binom{6+4-1}{4-1} = \binom{9}{3} = 84$

7: $\binom{7+4-1}{4-1} = \binom{10}{3} = 120$

8: $\binom{8+4-1}{4-1} = \binom{11}{3} = 165$

9: $\binom{9+4-1}{4-1} = \binom{12}{3} = 220$

10: $\binom{10+4-1}{4-1} = \binom{13}{3} = 286$

$\text{Total}_4 = 966$

3 nurses

at least 3 served

at most 10 served

3: $\binom{3+3-1}{3-1} = \binom{5}{2} = 10$

4: $\binom{6}{2} = 15$

5: $\binom{7}{2} = 21$

6: $\binom{8}{2} = 28$

7: $\binom{9}{2} = 36$

8: $\binom{10}{2} = 45$

9: $\binom{11}{2} = 55$

10: $\binom{12}{2} = 66$

$\text{Total}_3 = 276$

$T_4 + T_3 = 966 + 276 = \boxed{1242}$