Homework No. 9 (Due on 04/17/14)

Problem 1. If X has cumulative distribution function $F_X(x)$, what is the distribution function of $Y = \max\{0, X\}$. Next suppose that X be a random variable which is uniformly distributed on the interval $(-1, 1)$. Compute the cumulative distribution function of $Y = \max\{0, X\}$.

Problem 2. Let X and Y be independent having an exponential distribution with parameters λ and μ respectively. Find the probability density function of $Z = X + Y$.

Problem 3. Suppose that the two dimensional random variable (X, Y) is uniformly distributed over the triangular region $R = \{(x, y) \mid 0 \le x \le y \le 1\}$. Find $\rho(X, Y)$.

Problem 4. A coin has an a priori probability X of coming up heads, where X is a random variable with probability density function

$$f_X(x) = \begin{cases} x\mathrm{e}^x, & \text{for } x \in [0, 1] \\ 0, & \text{otherwise.} \end{cases}$$

Essentially this means that $\mathbb{P}(\text{Heads} \mid X = x) = x$.

(a) Find $\mathbb{P}(\text{Heads})$.

(b) If A denotes the event that the last flip came up heads, find the conditional probability density function of X given A, i.e., $f_{X|A}(x \mid A)$.

(c) Given A, find the conditional probability of heads at the next flip.

Problem 5. Let X be a geometric random variable with parameter P, where P is itself random and uniformly distributed on the interval $[0, \frac{n-1}{n}]$. Let $Z = \mathbb{E}[X \mid P]$. Find $\mathbb{E}[Z]$.

Problem 6. Consider two random variables X and Y. Assume for simplicity that they both have zero mean.

(a) Show that X and $\mathbb{E}[X \mid Y]$ are positively correlated.

(b) Show that the correlation coefficient of Y and $\mathbb{E}[X \mid Y]$ has the same sign as the correlation coefficient of X and Y.

In this problem, do not assume that X and Y have a PDF; use only the properties of conditional expectation.

Problem 7. Random variables X and Y are distributed according to the joint probability density function

$$f_{X,Y}(x, y) = \begin{cases} ax & \text{if } 1 \le x \le y \le 3 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Evaluate the constant a.

(b) Determine the marginal probability density function $f_Y(y)$.

(c) Determine the expected value of $\frac{1}{X}$, given that $Y = \frac{3}{2}$.

(d) Determine the probability density function $f_Z(z)$, if $Z = Y - X$.