Homework No. 8 (Due on 04/10/14)

Problem 1. If X has the normal distribution with mean μ and variance σ^2, find $\mathbb{E}\big[X^3\big]$ (as a function of μ and σ^2), without computing any integrals.

Problem 2. Suppose that the weight of a person selected at random from some population is normally distributed with parameters μ and σ^2. Suppose also that $\mathbb{P}(X \leq 160) = \frac{1}{2}$ and $\mathbb{P}(X \leq 140) = \frac{1}{4}$. Find μ and σ. Also, find $\mathbb{P}(X \geq 200)$. (Use the *standard normal table* for this problem).

Problem 3. Let X and Y be independent exponential random variables with a common parameter λ.

(a) Find the moment generating function associated with $aX + Y$, where a is a constant.

(b) Use the result of part (a) to find the probability density function of $aX + Y$, for the case where a is positive and different than 1.

(c) Use the result of part (a) to find the probability density function of $X - Y$.

Problem 4. Each egg laid by the hen falls onto the concrete floor of the henhouse and cracks with probability p. If the number of eggs laid today by the hen has the Poisson distribution, with parameter λ, use moment generating functions to find the probability distribution of the number of uncracked eggs.

Problem 5. Suppose that X has moment generating function

$$M_X(s) = \frac{6 - 3s}{2(1-s)(3-s)}\,.$$

Find the probability density function of the associated random variable X.

Problem 6. The moment generating function associated with the random variable X is

$$M(s) = a\mathrm{e}^s + b\mathrm{e}^{4(\mathrm{e}^s - 1)}\,, \qquad \mathbb{E}[X] = 3\,.$$

Find:

(a) The scalars a and b.

(b) $\mathbb{P}(X = 1)$, $\mathbb{E}[X^2]$, and $\mathbb{E}[2^X]$.

Problem 7. The random variables X_1, X_2 and X_3 are independent and identically distributed, having the exponential distribution with parameter 1, i.e., a probability density function e^{-x}, $x > 0$. Find the probability density function of

$$Z = \tfrac{1}{2}X_1 + \tfrac{1}{3}X_2 + X_3\,.$$