

# EE 351K - HW3 Solutions

## Problem 1

$$P(\cdot S) = 2/5$$

$$P(-S) = 3/5$$

$$P(-R | \cdot S) = \frac{1}{6}$$

$$P(\cdot R | \cdot S) = \frac{1}{5}$$

• S : Dot sent

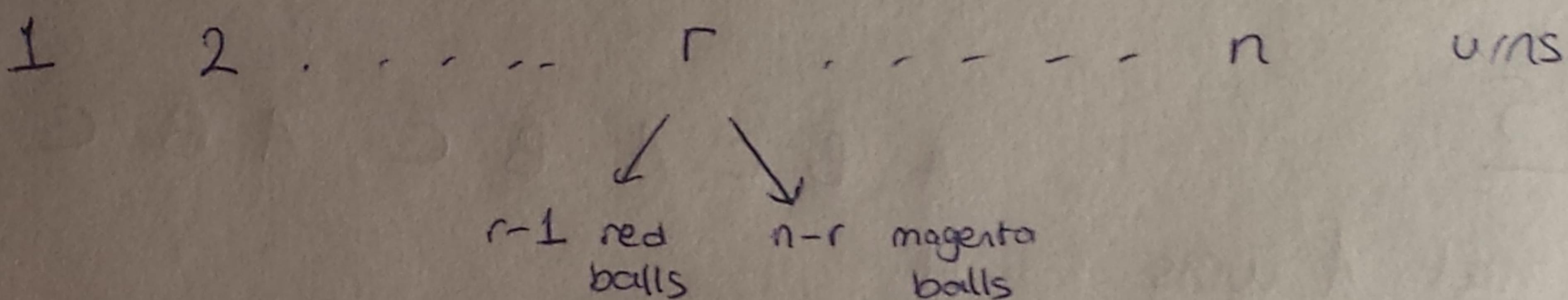
- S : Dash sent

• R : Dot received

- R : Dash received

$$P(\cdot S | \cdot R) = \frac{P(\cdot S, \cdot R)}{P(\cdot R)} = \frac{(2/5)(5/6)}{(2/5)(5/6) + (3/5)(1/5)} = \frac{25}{34}$$

## Problem 2



$$a) P(\text{Second ball is magenta}) = \sum_{r=1}^n P(\text{Second ball is magenta} | \text{urn } r) \underbrace{P(\text{urn } r)}_{1/n}$$

$$= \frac{1}{n} \sum_{r=1}^n \underbrace{\frac{r-1}{n-1} \cdot \frac{n-r}{n-2}}_{\text{1st red}} + \underbrace{\frac{n-r}{n-1} \cdot \frac{n-r-1}{n-2}}_{\text{2nd magenta}}$$

$$= \frac{1}{n} \cdot \frac{1}{n-1} \cdot \frac{1}{n-2} \sum_{r=1}^n (n-r)(r-1+n-r-1) = \frac{1}{n} \cdot \frac{1}{n-1} \sum_{r=1}^n n-r$$

$$= \frac{1}{n} \cdot \frac{1}{n-1} \sum_{u=0}^{n-1} u$$

$$= \frac{1}{n} \cdot \frac{1}{n-1} \cdot \frac{(n-1)n}{2} = \frac{1}{2}$$

$$b) P(\text{Second ball is magenta} | \text{First ball is magenta})$$

$$= \frac{P(\text{both balls are magenta})}{P(\text{First ball is magenta})} = \frac{\cancel{\frac{1}{n}} \sum_{r=1}^n \frac{n-r}{n-1} \cdot \frac{n-r-1}{n-2}}{\cancel{\frac{1}{n}} \sum_{r=1}^n \frac{n-r}{n-1}}$$

$$= \frac{1}{n-2} \frac{\sum_{u=-1}^{n-2} u}{\sum_{u=0}^{n-1} u} = \frac{1}{n-2} \cdot \frac{\left[ \frac{(n-2)(n-1)}{2} - 1 \right]}{\frac{(n-1)n}{2}} = \frac{2}{(n-2)(n-1)} \left[ \frac{n^2 - 3n}{2} \right] = \frac{n-3}{(n-2)(n-1)}$$

Problem 3 25 items  $\rightarrow$  5 defective

$$P(X = i) = \frac{\binom{5}{i} \binom{20}{4-i}}{\binom{25}{4}} \quad i = 0, 1, 2, 3, 4$$

Problem 4

a)  $P(\text{defective}) = \frac{1}{3} \cdot \frac{1}{8} + \frac{1}{3} \cdot \frac{3}{8} + \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{4}$

b)  $P(\text{II} | \text{Defective}) = \frac{P(\text{II}, \text{Defective})}{P(\text{Defective})} = \frac{\frac{1}{3} \cdot \frac{3}{8}}{\frac{1}{4}} = \frac{1}{2}$

Problem 5

A B C A B C A B C

Prob. that A wins : H

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$$P(A \text{ wins}) = p + (1-p)^3 p + (1-p)^6 p + \dots = \frac{p}{1 - (1-p)^3}$$

$$P(B \text{ wins}) = (1-p) \cdot P(A \text{ wins})$$

Problem 6

$$P(k \text{ balls in } r_1) = \binom{n}{k} \left(\frac{r_1}{r}\right)^k \left(\frac{r-r_1}{r}\right)^{n-k}$$

$$P(\text{a ball is in one of the first } r_1 \text{ boxes}) = \frac{r_1}{r}$$

~~Prob. a ball is not in one of the first  $r_1$  boxes~~

$$P(\text{a ball is not in one of the first } n \text{ boxes}) = 1 - \frac{r_1}{r}$$