HW 9.1

Clearly

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ P(X \le y) & y \ge 0 \end{cases}$$

So if $X \sim$ Uniform $([-1, 1])$

then

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \frac{1}{2} + \frac{y}{2} & 0 \le y \le 1 \end{cases}$$

HW 7.2

Clearly $\quad W^2 = \sqrt{X^2 + Y^2}$

and $\quad X = W \cos Q$

$\quad Y = -W \sin Q$

The Jacobian is $\det \begin{bmatrix} \cos Q & -W \sin Q \\ -\sin Q & -W \cos Q \end{bmatrix} = -W$

Therefore

(a) $$f_{W,Q}(w, q) = \frac{w}{2\pi \sigma^2} e^{-\frac{w^2}{2\sigma^2}} \qquad \begin{matrix} w \ge 0 \\ -\pi \le q < \pi \end{matrix}$$

(b) $f_W(\omega) = \int_{-\pi}^{\pi} f_{W,Q}(\omega,q)\, dq = \frac{\omega}{\sigma^2} e^{-\frac{\omega^2}{2\sigma^2}}, \quad \omega \geq 0$

$$f_Q(q) = \int_0^{\infty} f_{W,Q}(\omega,q)\, d\omega = \frac{1}{2\pi} \qquad -\pi \leq q \leq \pi$$

(c) Yes, independent since

$$F_W \cdot F_Q = F_{W,Q}$$

~~Problem~~ HW 7.6

Since X, Y are nonnegative the probability of any set of values over which $X+Y < X$ is zero

Therefore with $Z = X+Y$ we have

$$f_{X,Z}(x,z) = 0 \quad \text{if} \quad z < x$$

If $z \geq x$ then (see figure)

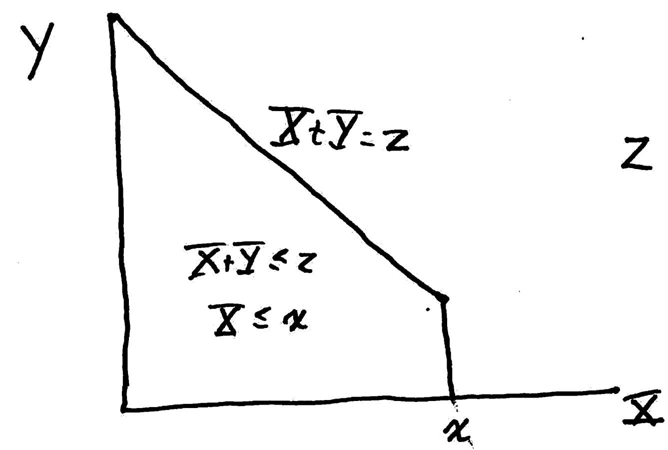
$$P(X \leq x, Z \leq z) = \int_0^x \left(\int_0^{z-x'} \lambda e^{-x'} \lambda e^{-\lambda y'} dy' \right) dx'$$

$$= \lambda - \lambda e^{-\lambda x} - \lambda x e^{-\lambda z} \qquad 0 \leq x \leq z$$

Therefore

$$f_{X,Z}(x,z) = \begin{cases} 0 & \text{if} \quad x > z \\ \lambda^2 e^{-\lambda z} & \text{if} \quad 0 \leq x \leq z \end{cases}$$

figure



$Z \geq x$

For the density of Z we have

$$f_Z(z) = \int_0^z f_X(x') f_Y(z-x')\, dx'$$

$$= \int_0^z \lambda e^{-\lambda x'} \lambda e^{-\lambda(z-x')}\, dx'$$

$$= \lambda^2 z e^{-\lambda z}, \quad z \geq 0$$

Therefore

$$f_{X|Z}(x \mid Z=z) = \frac{1}{z}, \quad 0 \leq x \leq z.$$