(b) $$f_W(\omega) = \int_{-\pi}^{\pi} f_{W,Q}(\omega, q)\, dq = \frac{\omega}{\sigma^2} e^{-\frac{\omega^2}{2\sigma^2}}, \quad \omega \geq 0$$

$$f_Q(q) = \int_0^{\infty} f_{W,Q}(\omega, q)\, d\omega = \frac{1}{2\pi} \qquad -\pi \leq q \leq \pi$$

(c) Yes, independent since

$$F_W \cdot F_Q = F_{W,Q}$$

~~Problem~~ HW 7.6

Since X, Y are nonnegative the probability of any set of values over which $X + Y < X$ is zero

Therefore with $Z = X + Y$ we have

$$f_{X,Z}(x, z) = 0 \quad \text{if} \quad z < x$$

If $z \geq x$ then (see figure)

$$P(X \leq x, Z \leq z) = \int_0^x \left(\int_0^{z-x'} \lambda e^{-x'} \lambda e^{-\lambda y'} dy' \right) dx'$$

$$= \underline{\underline{1 - e^{-\lambda x} - \lambda x e^{-\lambda z}}} \qquad 0 \leq x \leq z$$

Therefore

$$f_{X,Z}(x, z) = \begin{cases} 0 & \text{if} \quad x > z \\ \lambda^2 e^{-\lambda z} & \text{if} \quad 0 \leq x \leq z \end{cases}$$