Solutions #2

1) $P(2K + 1Ace) = \dfrac{\binom{4}{2}\binom{4}{1}\binom{44}{10}}{\binom{52}{13}}$

$P(1A \mid 2K) = \dfrac{P(1A + 2K)}{P(2K)} = \dfrac{\binom{4}{2}\binom{4}{1}\binom{44}{10}}{\binom{52}{13}}$

2) A: missing blue
B: " red
C: " green

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$= 3\left(\frac{2}{3}\right)^k - 3\left(\frac{1}{3}\right)^k + 0$

$= \left(\frac{1}{3}\right)^{k-1}(2^k - 1)$

3) (a) $P = \dfrac{\binom{35}{6}}{\binom{75}{6}}$ (b) $P = \dfrac{\binom{25}{2}\binom{15}{3}\binom{35}{1}}{\binom{75}{6}}$

4) A = defective
B = declared defective
C = good

$P(A \cap B) = 0.006$

$P(B) = P(B|A)P(A) + P(B|C)P(C)$

$= 1 \times 0.006 + 0.02(1 - 0.006)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)} = 0.23$

5)

$$P = P(\text{4 heads, 4 tosses})\, P(\text{die} = 4) + P(\text{4 heads in 5 tosses})\, P(\text{die} = 5) + P(\text{4 heads, 6 tosses})\, P(\text{die} = 6)$$

$$= \left(\frac{1}{2}\right)^4 \frac{1}{6} + \binom{5}{4}\left(\frac{1}{2}\right)^5 \frac{1}{6} + \binom{6}{4}\left(\frac{1}{2}\right)^6 \frac{1}{6}$$

$$= \frac{1}{6}\left(\frac{1}{2}\right)^6 (4 + 10 + 15) = \frac{29}{6} \times \frac{1}{64}$$

6) $$P(A \mid B \wedge C) = \frac{P(A \wedge B \wedge C)}{P(B \wedge C)} = \frac{P(C \mid A \wedge B)\, P(A \wedge B)}{P(B \wedge C)}$$

$$= \frac{P(C \mid B)\, P(A \wedge B)}{P(B \wedge C)} = \frac{P(A \wedge B)}{P(B)} = P(A \mid B)$$

$= 1$ using hypothesis

HW02 - #7

$$P(\text{even \# of sixes in } n \text{ tosses})$$

$$= \sum_{\substack{k=0 \\ k:\text{even}}}^{n} \binom{n}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{n-k}$$

$$= \frac{1}{2}\left[\left(\frac{1}{6}+\frac{5}{6}\right)^n + \left(\frac{5}{6}-\frac{1}{6}\right)^n\right]$$

$$= \frac{1}{2}\left(1+\left(\frac{2}{3}\right)^n\right)$$

HW02 - #8

(a) $\left(\frac{1}{2}\right)^n$

(b) If n is odd then $P = 0$

If n is even then $P = \binom{n}{n/2}\left(\frac{1}{2}\right)^n$

(c) $\binom{n}{2}\left(\frac{1}{2}\right)^n$

(d) $1 - P(\text{exactly one H}) - P(\text{no H})$

$$= 1 - n\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n$$

$$= 1 - (n+1)\left(\frac{1}{2}\right)^n$$