EE 351K – HW3 Solutions

Problem 1

$P(\cdot S) = 2/5$ $\quad P(-R \mid \cdot S) = \frac{1}{6}$

$P(-S) = 3/5$ $\quad P(\cdot R \mid -S) = \frac{1}{5}$

$\cdot S$: Dot sent
$-S$: Dash sent
$\cdot R$: Dot received
$-R$: Dash received

$$P(\cdot S \mid \cdot R) = \frac{P(\cdot S, \cdot R)}{P(\cdot R)} = \frac{(2/5)(5/6)}{(2/5)(5/6) + (3/5)(1/5)} = \frac{25}{34}$$

Problem 2

1 2 . . . . . r . . . . . . n urns

r: $r-1$ red balls, $n-r$ magenta balls

a) $$P(\text{Second ball is magenta}) = \sum_{r=1}^{n} P(\text{Second ball is magenta} \mid \text{urn } r)\underbrace{P(\text{urn } r)}_{1/n}$$

$$= \frac{1}{n}\sum_{r=1}^{n} \underbrace{\frac{r-1}{n-1}}_{\text{1st red}} \cdot \underbrace{\frac{n-r}{n-2}}_{\text{2nd magenta}} + \underbrace{\frac{n-r}{n-1} \cdot \frac{n-r-1}{n-2}}_{\text{both magenta}}$$

$$= \frac{1}{n} \cdot \frac{1}{n-1} \cdot \frac{1}{n-2} \sum_{r=1}^{n} (n-r)(\cancel{r}-1+n-\cancel{r}-1) = \frac{1}{n} \cdot \frac{1}{n-1} \sum_{r=1}^{n} n-r$$

$$= \frac{1}{n} \cdot \frac{1}{n-1} \sum_{u=0}^{n-1} u$$

$$= \frac{1}{n} \cdot \frac{1}{n-1} \cdot \frac{(n-1)n}{2} = \frac{1}{2}$$

b) $P(\text{Second ball is magenta} \mid \text{First ball is magenta})$

$$= \frac{P(\text{both balls are magenta})}{P(\text{First ball is magenta})} = \frac{\cancel{\frac{1}{n}} \sum_{r=1}^{n} \frac{n-r}{\cancel{n-1}} \cdot \frac{n-r-1}{n-2}}{\cancel{\frac{1}{n}} \sum_{r=1}^{n} \frac{n-r}{\cancel{n-1}}}$$

$$= \frac{1}{n-2} \frac{\sum_{u=-1}^{n-2} u}{\sum_{u=0}^{n-1} u} = \frac{1}{n-2} \cdot \frac{\left[\frac{(n-2)(n-1)}{2} - 1\right]}{\frac{(n-1)n}{2}} = \frac{\cancel{2}}{(n-2)(n-1)\cancel{n}} \left[\frac{n^{\cancel{2}} - 3\cancel{n}}{\cancel{2}}\right] = \frac{n-3}{(n-2)(n-1)}$$

Problem 3

25 items → 5 defective

$$P(X=i) = \frac{\binom{5}{i}\binom{20}{4-i}}{\binom{25}{4}} \qquad i = 0,1,2,3,4$$

Problem 4

a) $P(\text{defective}) = \frac{1}{3}\cdot\frac{1}{8} + \frac{1}{3}\cdot\frac{3}{8} + \frac{1}{3}\cdot\frac{1}{4} = \frac{1}{4}$

b) $P(\text{II} \mid \text{Defective}) = \frac{P(\text{II}, \text{Defective})}{P(\text{Defective})} = \frac{\frac{1}{3}\cdot\frac{3}{8}}{\frac{1}{4}} = \frac{1}{2}$

Problem 5

A B C A B C A B C

Prob. that A wins :

H

T T T H

T T T T T T H

⋮

$$P(A\text{ wins}) = p + (1-p)^3 p + (1-p)^6 p + \dots = \frac{p}{1-(1-p)^3}$$

$$P(B\text{ wins}) = (1-p)\cdot P(A\text{ wins})$$

Problem 6

$$P(k \text{ balls in } r_1) = \binom{n}{k}\left(\frac{r_1}{r}\right)^k \left(\frac{r-r_1}{r}\right)^{n-k}$$

P (a ball is in one of the first r_1 boxes) $= \frac{r_1}{r}$

~~P(a ball is not in one of the~~

P(a ball is not in one of the first r_1 boxes) $= 1 - \frac{r_1}{r}$