Quality and Fairness of Online Matching Algorithms for Kidney Exchange

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Problem

One option for patients in need of a kidney with a willing, but incompatible, donor is to enter an exchange pool. Recent research has shown that adding compatible pairs to the pool drastically improves their chances of receiving a match in 2 or 3 cycles. Our research analyzes different algorithms that can be used to optimize online matching in the market. We use primal-dual algorithms to obtain online matching rules which optimize for the sum of expected graft survival (EGS) in the population. The EGS is based on the living kidney donor profile index (LKDPI) given from a data simulator based on parameters from a major transplant center.

Primal

$$\text{maximize} \sum_{t,i,j} w_{t,i,j} x_{t,i,j}$$

subject to

$$\sum_{i,j} x_{t,i,j} \le 1, \qquad \forall t \le T$$

$$\sum_{t,j} x_{t,i,j} + \sum_{t,j} x_{t,j,i} + \sum_{j,j' \in I} x_{(i+T),j,j'} \le 1, \quad \forall i \ge 1$$

$$\begin{aligned} &t \in C = \{1, 2, 3 \dots T, T+1 \dots T+K\} \\ &i \in I = \{0, 1, 2 \dots K\} \\ &j \in I = \{0, 1, 2 \dots K\} \\ &0 < x_{t,i,j} < 1 \quad \forall t, i, j \end{aligned}$$

Dual

$$\text{minimize} \sum_t \alpha_t + \sum_i \beta_i$$

subject to

$$w_{t,i,j} - \alpha_t - \beta_i - \beta_j \le 0, \qquad \forall t \le T, i, j$$

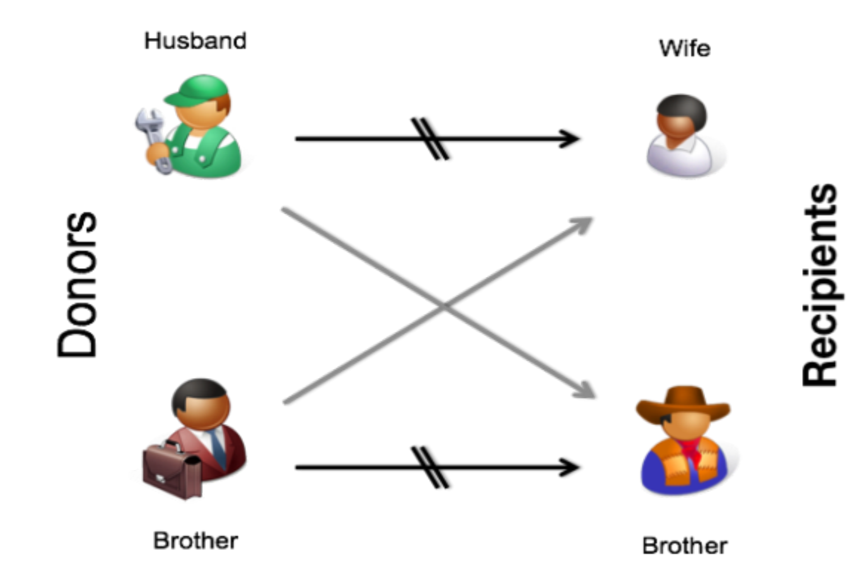
$$w_{t,i,j} - \beta_{t-T} - \beta_i - \beta_j \le 0, \quad \forall t > T, i, j$$

$$\alpha_t > 0 \quad \forall t \le T$$

$$\beta_i > 0 \quad \forall i > 0$$

Online Assignment Rule

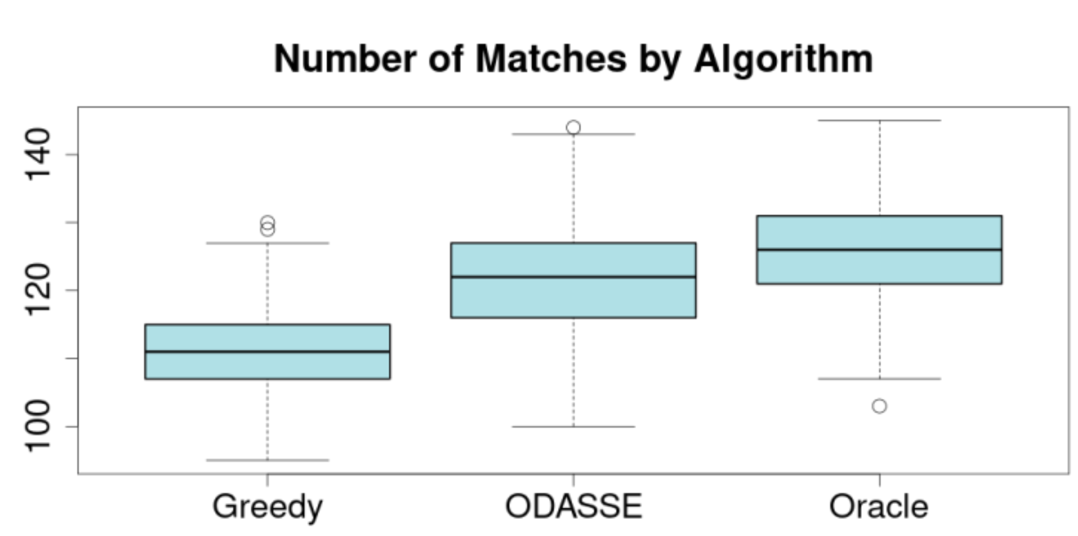
$$\text{Choose } i, j = \arg\max_{i',j'} w_{t,i',j'} - \beta_{i'} - \beta_{j'}$$

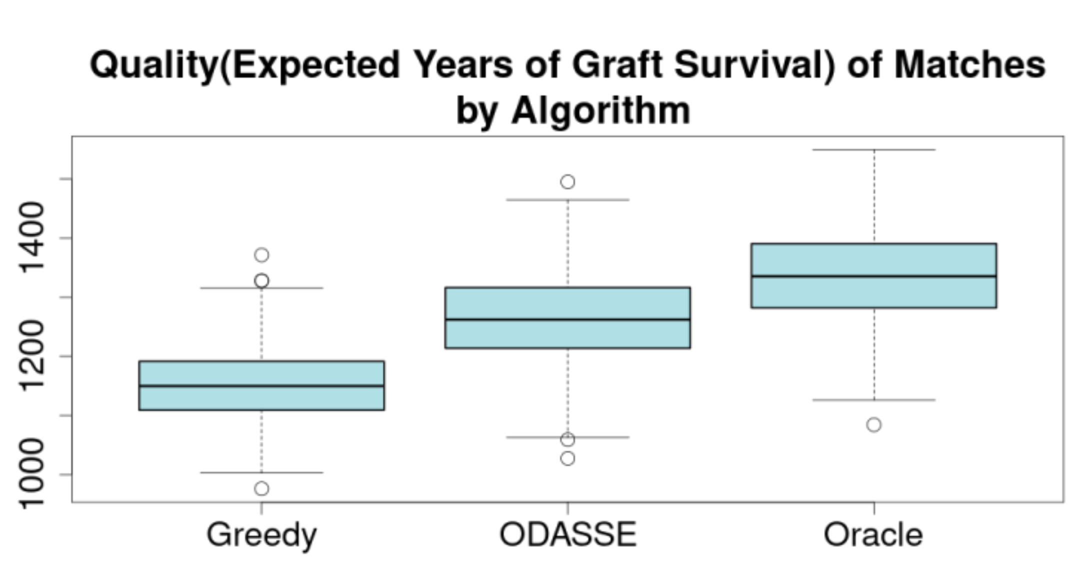


- Pool of K incompatible Pairs
- 1 compatible pair arrives at each of T time steps

- $x_{t,i,j}$: Cycle between pairs t,i and j
- $x_{t,0,0}$ and $x_{t,i,0}$: self and 2 Cycle Matches
- $w_{t,i,j}$: Quality of Cycle (in EGS)

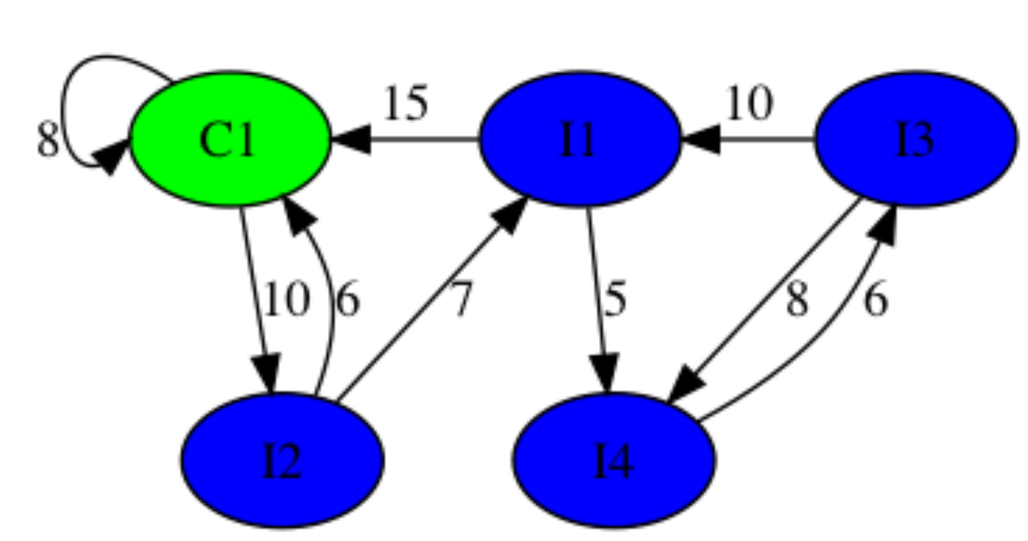
Algorithms

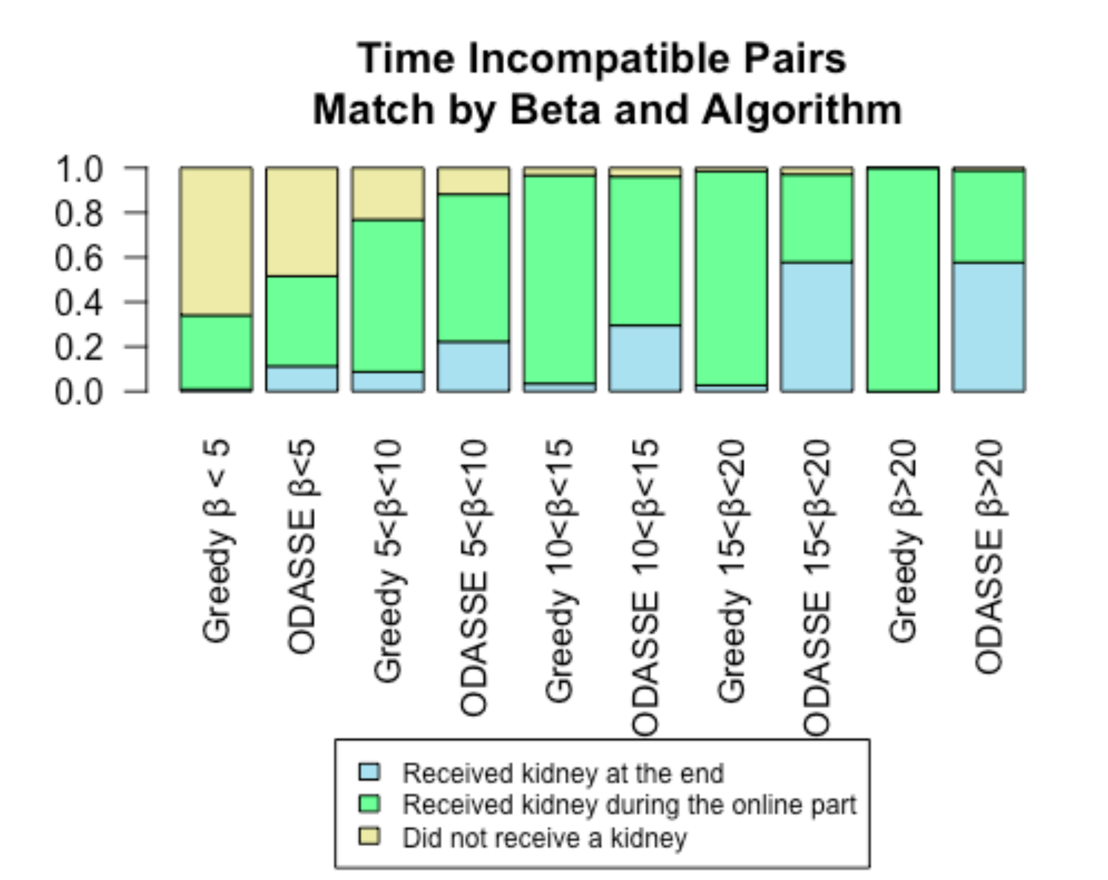




These results were derived from 500 simulations of 50 arriving compatible pairs and starting pool size of 100 incompatible pairs.

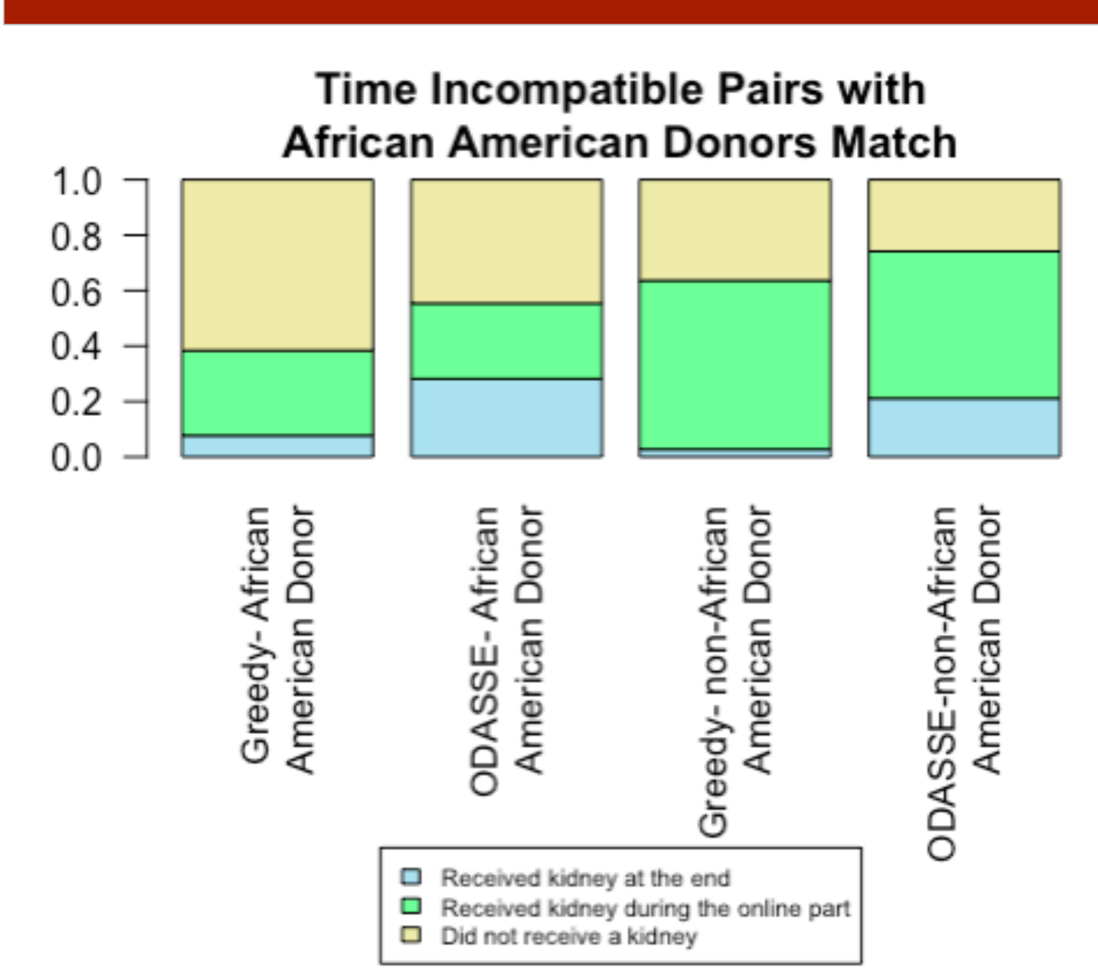
- **Greedy**: Attempts to match each arriving compatible pair to maximize that cycle weight.
- **Online Dual Assignment using Shadow Survival Estimates (ODASSE):**
  - Attempts to predict future based on given information and past experiences.
    - Use Linear Regression based on two sets of data to predict β.
      - Demographics information of pairs.
      - β predicted by LP on incompatible pool only.
    - Performs assignment based on rule given by dual above.
- **Oracle:** Upper bound, we assume we know the future and perform the optimal matching using an integer program.



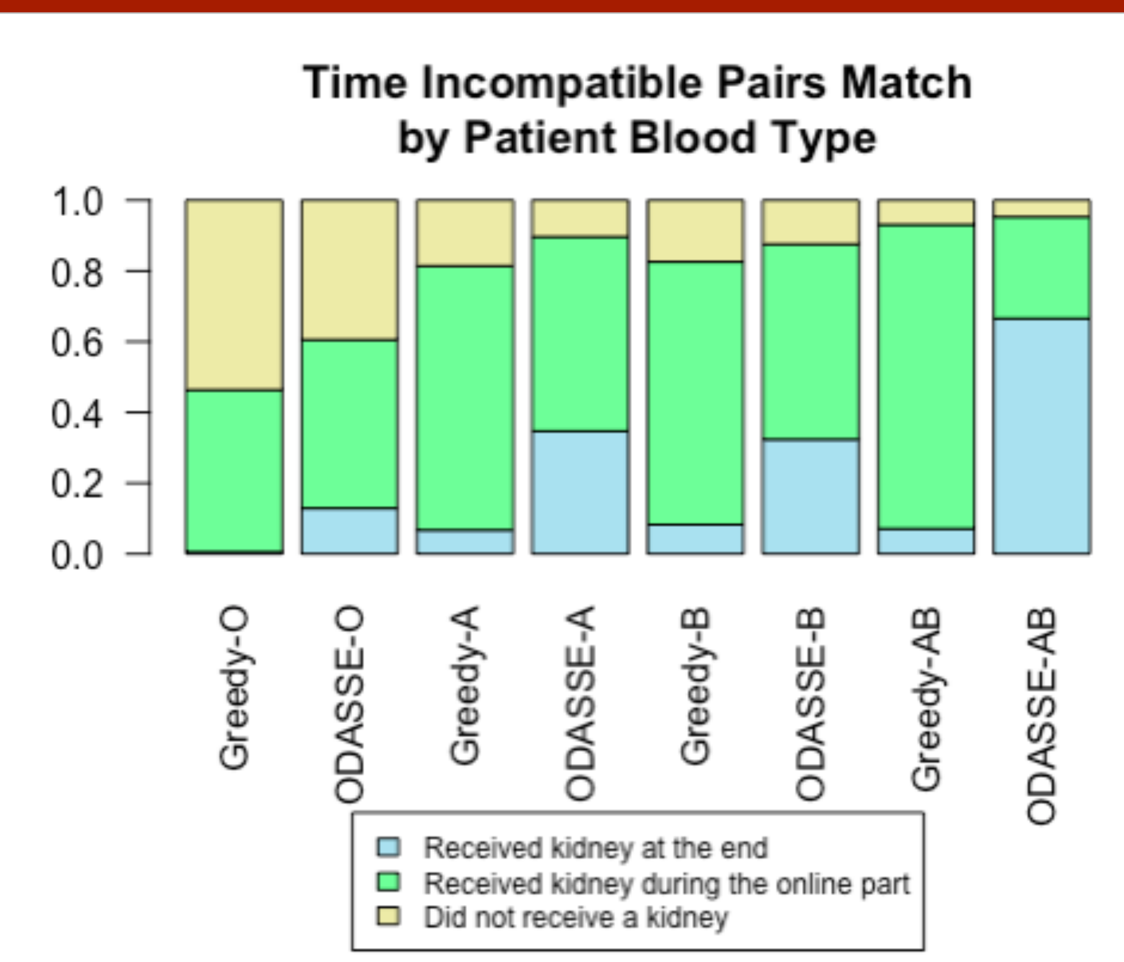


- The ODASSE algorithm holds onto pairs with high betas longer than the greedy algorithm
- This results in 45.5% more matches among incompatible pairs with β < 5 and 14.7% more matches among incompatible pairs with 5 < β < 10

Fairness



- The ODASSE algorithm improved the fairness of outcomes between incompatible pairs with African American donors and those without.
- Pairs with African American donors receive 44.3% more matches in the ODASSE algorithm than the greedy algorithm.
- Similarly, pairs with patients with type O blood receive 30.6% more matches in the ODASSE algorithm than the greedy algorithm.
- These numbers are derived from averages of 500 population simulations of 50 arriving compatible pairs and 100 incompatible pairs.



Policy Implications

- The ODASSE algorithm improves the social welfare (sum of expected survival times) as well as number of transplants performed, compared with the greedy algorithm.
- The ODASSE algorithm also increases the probability that hard-to-match pairs (such as those with patients with type O blood) receive a match.
- Our results also show that the improved outcomes do not come at the cost of reduced fairness.