Quality and Fairness of Online Matching Algorithms for Kidney Exchange

Kelsey Lieberman & William Macke
with Li, Das and Ho

August 2, 2018

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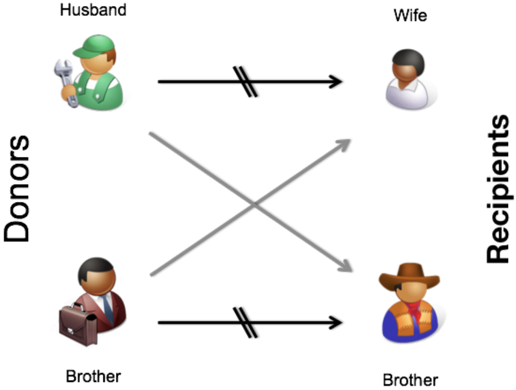
- In 2014, 17,107 kidney transplants took place, about 1/3 from living donors
- Unfortunately, willing living donors are often not medically compatible.
- One option for them is to enter a kidney exchange program

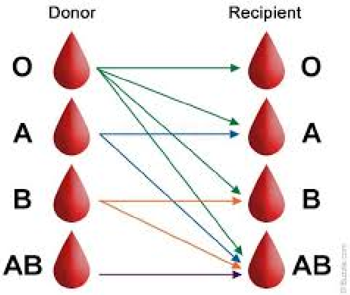
Incompatible Pairs



Compatibility

What makes a pair incompatible?

- Blood type
- Tissue type (measured by PRA)



Quality of Match

LKDPI:

- Blood type of donor
- Blood type of recipient
- Age of donor
- Sex of both
- Weight/BMI of both
- Cigarette use of donor
- If donor is African American
- PRA of recipient

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LKDPI $\rightarrow$ EGS (Expected Graft Survival)

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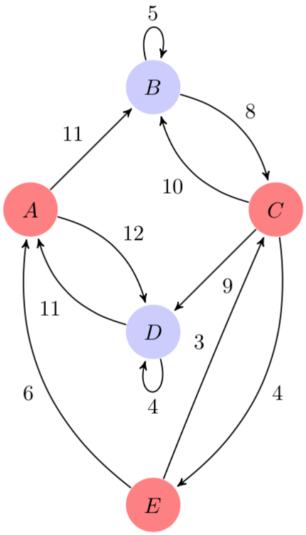

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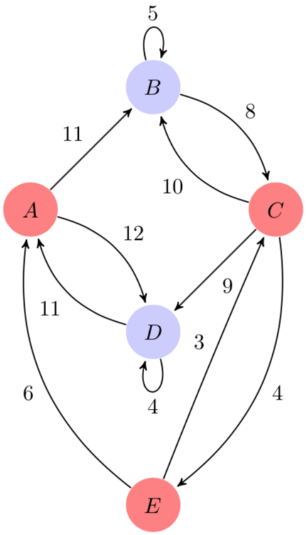

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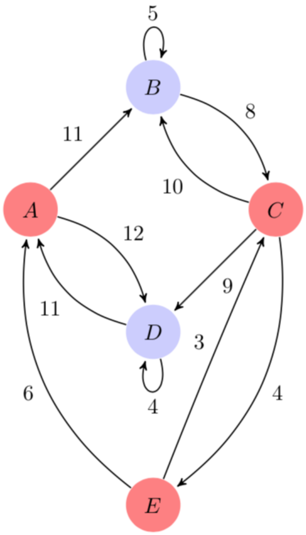

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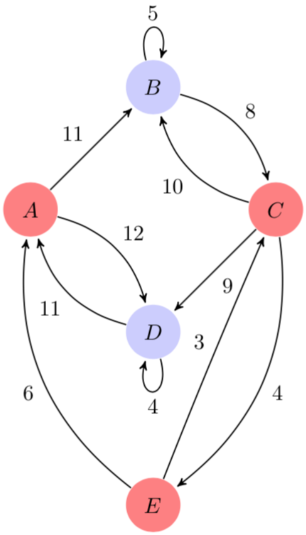

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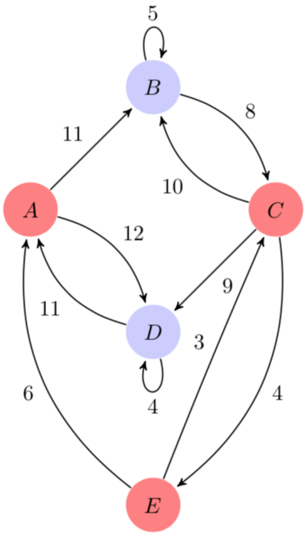

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- Cycles can include up to 3 pairs

Problem Statement

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- Is this method fair?

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  - o Can perform optimization using IP solvers(Gurobi, CBC)

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III Oracle: Upper bound, we assume we know the future and perform the optimal matching using an integer program.

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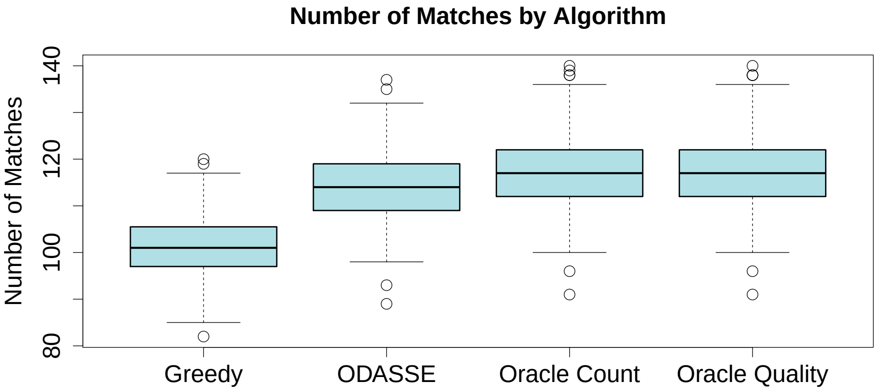
Preliminary Results



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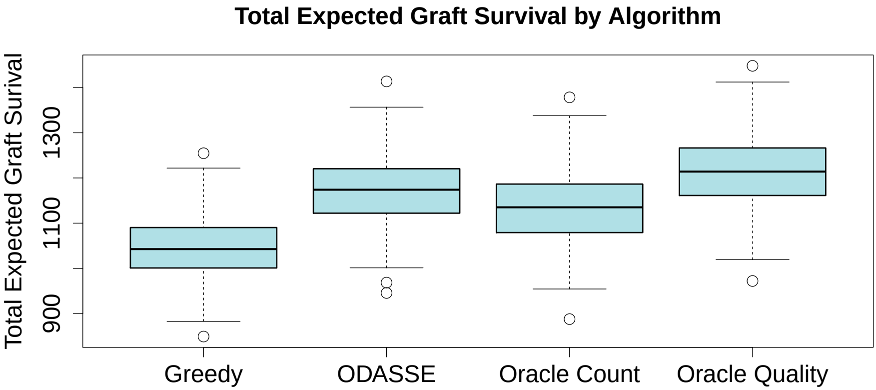
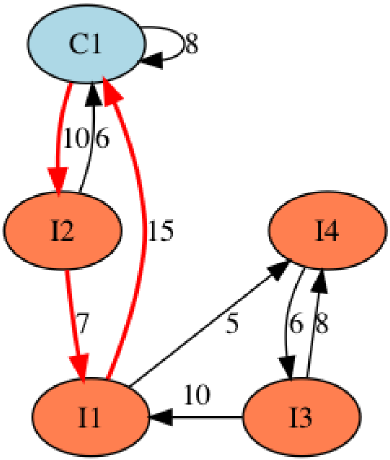


Three Cycles



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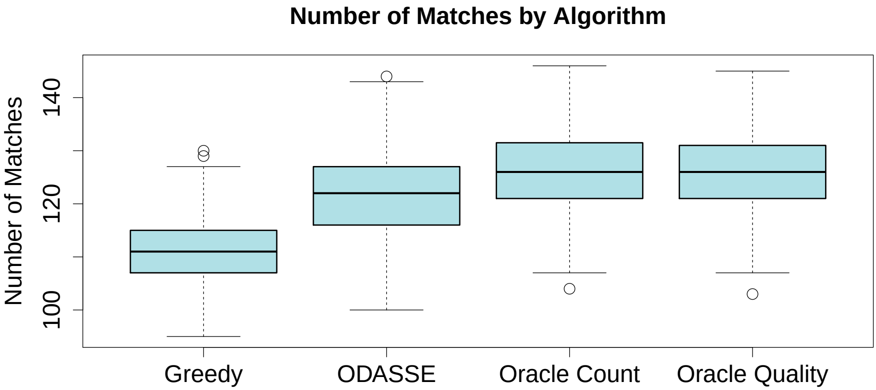

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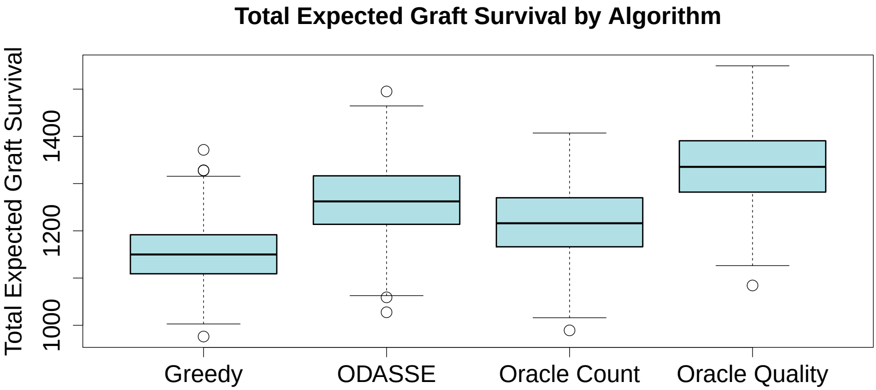

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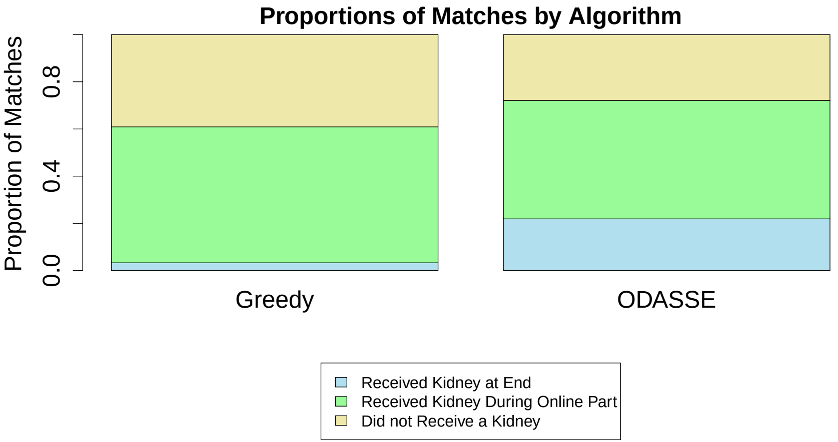
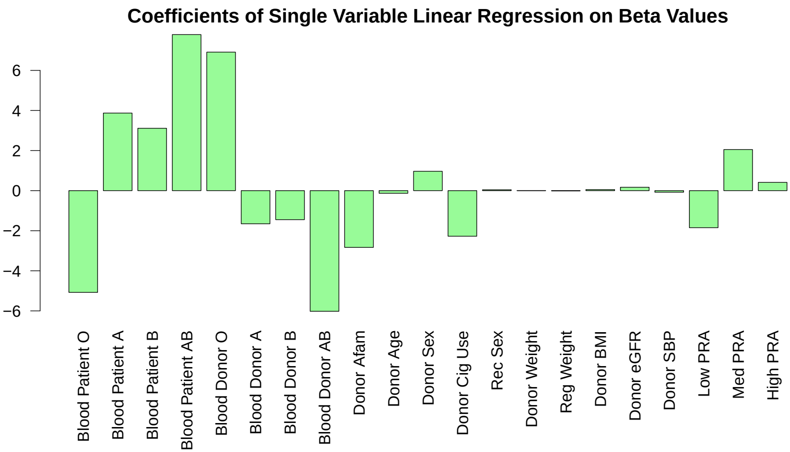
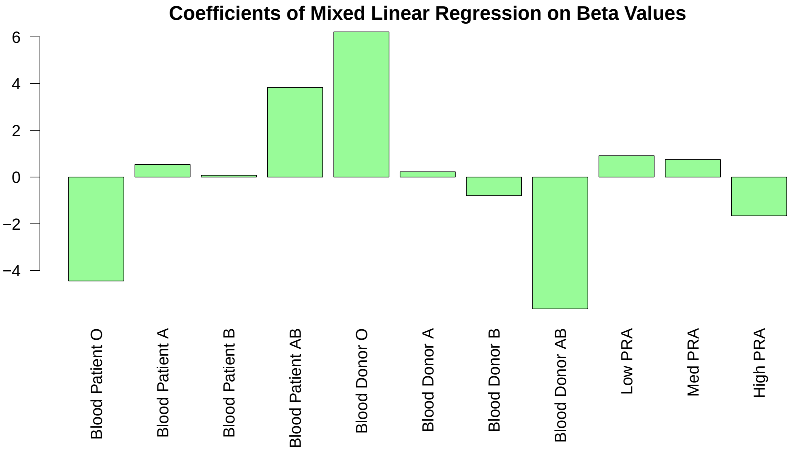


Interpreting Beta



Interpreting Beta



Analyzing Fairness

Do the algorithms result in differential impacts for specific groups?

Groups to consider:

- Demographic groups
- Groups based on beta values

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Groups to consider:

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Outcomes to consider:

- **Proportion** of pairs from that get matched
- **Quality** a pair receives if matched
- **Time** a pair waits to receive a match

Fairness by β

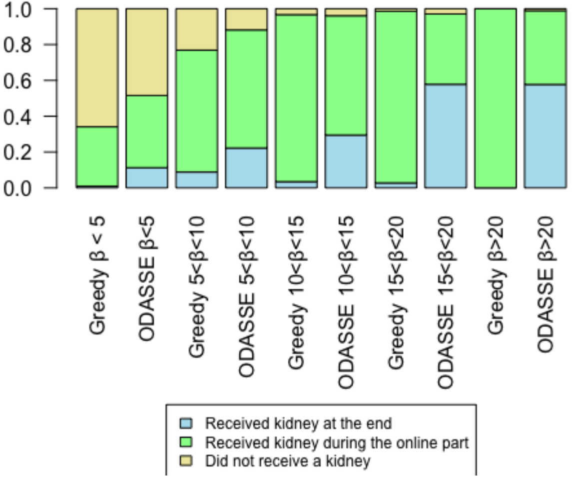

Figure: Incompatible pairs with $\beta < 5$ receive 45.5% more matches with ODASSE algorithm than greedy

Fairness

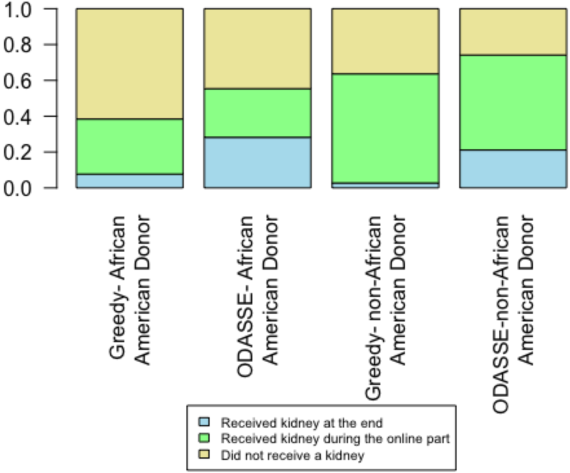

Figure: Incompatible pairs with African American donors receive 44.3% more matches with ODASSE algorithm than greedy

Fairness in Blood Type and Tissue Type

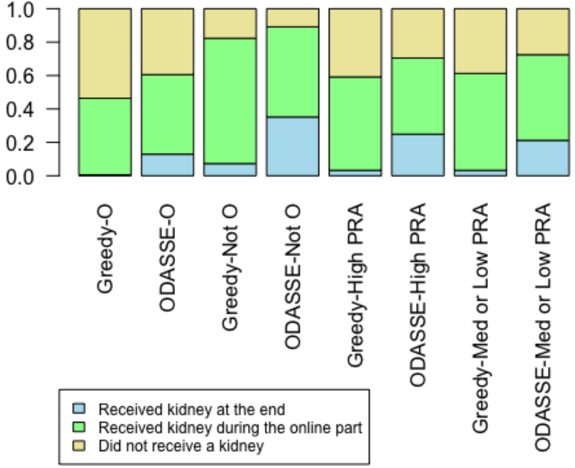

Figure: Incompatible pairs with recipients with type O blood receive 30.6% more matches with ODASSE algorithm than greedy

Received Quality Over Time

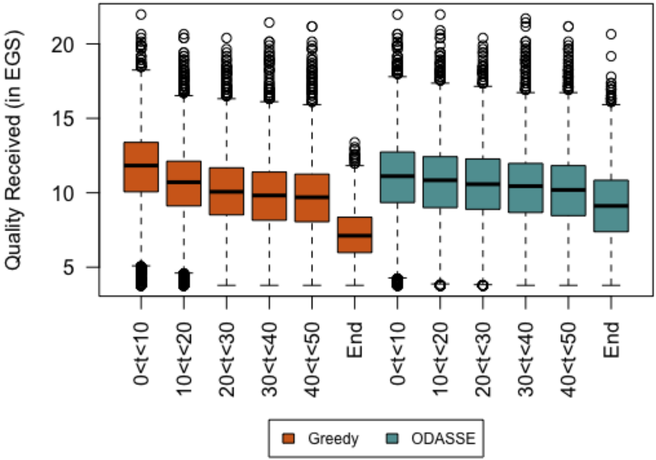

Figure: Quality Recipient Receives by Time Matched

Summary

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- ODASSE improves the social welfare
- ODASSE improves number of transplants performed
- ODASSE improves fairness among hard-to-match pairs
- Improved outcomes do not come at the cost of reduced fairness

Future work

- Expanding the formulation to include arrival and departure protocols for incompatible pairs
- Adding predictions of future *compatible* pairs to improve algorithm