Homework 1

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Problem 1: Radioactive Decay (a) Radioactive decay can be effectively modeled as exponential decay. Let N be the number of atoms remaining, N_0 be the initial number of atoms, r fraction that decays each year, t be the number of years that have passed. N can then be expressed as

$$N = N_0 * e^{-rt}$$

I have implemented a function 'rad_decay' which gives the amount left of a particular quantity, given a particular rate of decay, initial amount, and length of time. This is the function I plotted. I used the range of [0, 10^3] since after 10^3 years the characteristic 'J' curve is visible. This function will look similar for numbers larger than this.

(b) Plugging a starting fraction of 1, the given rate of decay, and the given rates of the time into my function, I computed the requested values. After 500 years, approximately 80.740% remaining. After $5,000$ years, approximately 11.773% is left!

(c) If 60% is remaining then approximately $1,194$ years have passed. Since we have that $N = N_0 * e^{-rt}$, we can manipulate the formula for t

$$N = N_0 * e^{-rt}$$
$$t = \frac{\ln(\frac{N}{N_0})}{-r}$$

and compute for t.

Problem 2: Monte Carlo Simulation for Pi (a) I wrote functions to handle determining whether a point was in the unit circle and approximating Pi. Then, I approximated Pi at 10 different simulation sizes, logarithmically ranging from 1 to 10^6, for 100 times at each one of these sizes. I took the sample mean and sample standard deviation of my tests at each one of these sizes and then graphed the mean and sample standard deviations versus a logarithmic scale for the number of points in each Monte Carlo simulation.

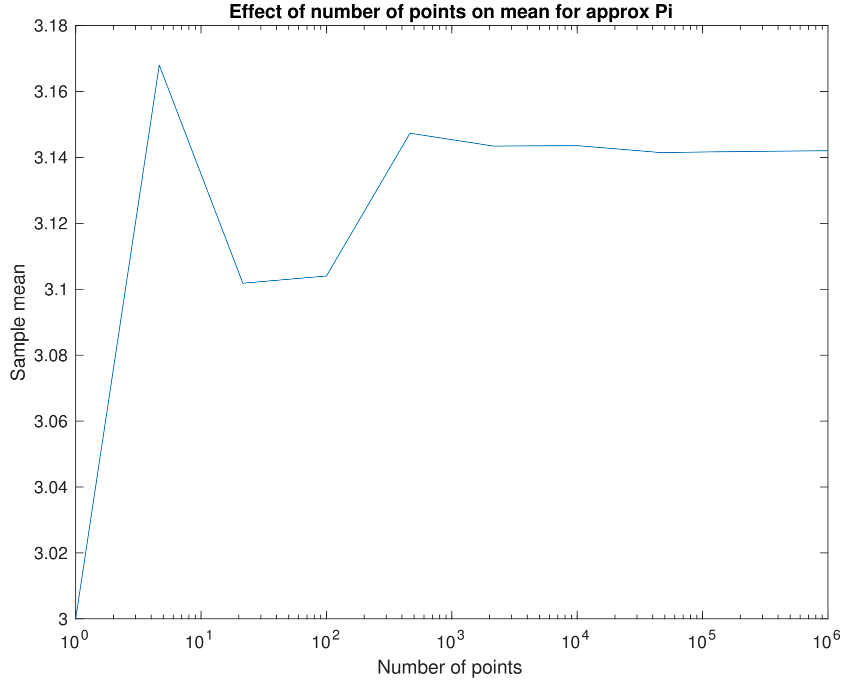
The graph of sample means didn't have much significance except to show a nonperiodic (mostly) dampening oscillation approaching the value of Pi. The population mean for each of each distributions of approximations of Pi at a given sample size is the true value of Pi. This is because the ratio given by the set of all possible points (of which there are infinite) is by definition Pi.

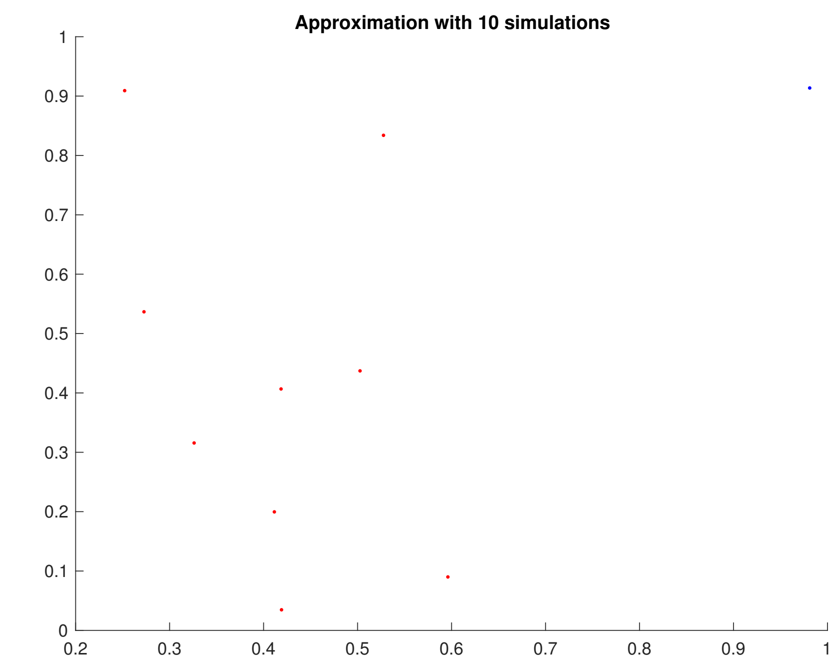
Sample standard deviation (σ) decreased approximately with exponential decay. At 10^4 points, σ fell to approximately 0.01, meaning that 98% of approximations for Pi fell within two standard deviations or 0.02 of the mean. This means that in 98% of cases we would get an approximation of 3.14 ± 0.02 or $[3.12, 3.16]$. I would argue that this is a reasonably good approximation for pi since 3.14 is a popular approximation for Pi and 3 significant figures can be used effectively for most computations that only need 2 digits of accuracy.

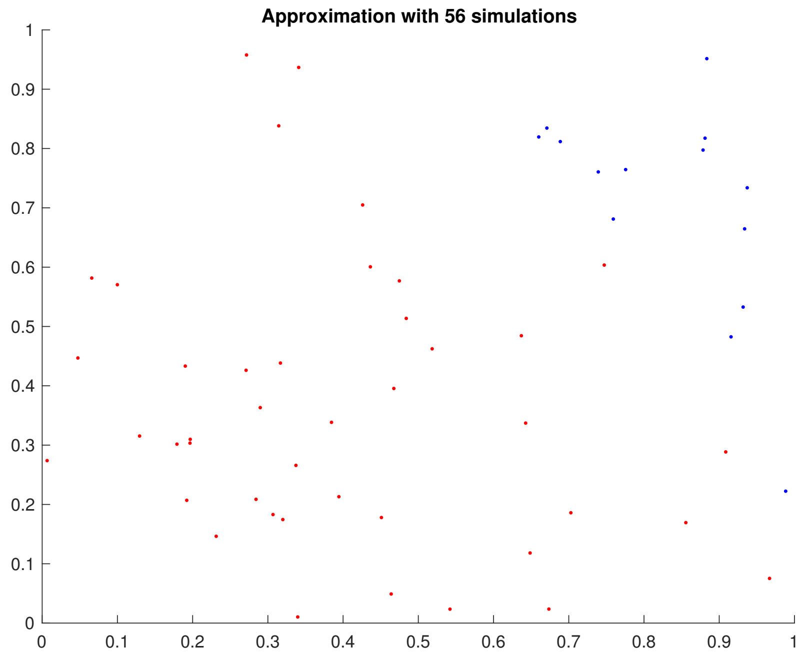
(b) I did not plot all of the values of n I tested since I did 100 simulations at each size and 10 different sizes as this would be unwieldy and take up an unbearable amount of RAM and CPU resources. However, the visualizations I created do effectively illustrate the idea that the rate of accuracy provided by each additional point decreases (Law of Diminishing Returns) and that a relatively small number of points can provide a good approximation.

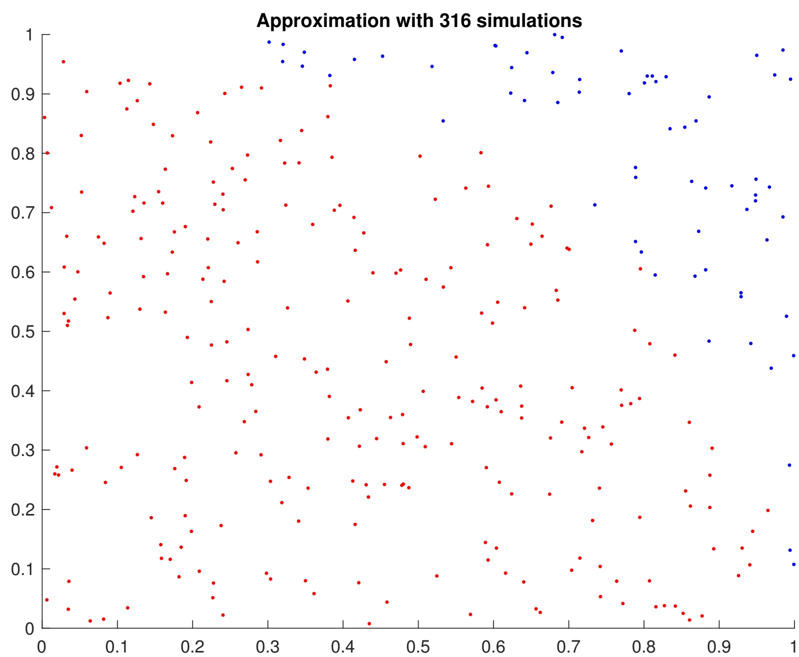
(c) With $1,000$ tests and a chosen number of simulations of $10,000$, we get a min of 3.08880, a max of 3.19440, and an average of 3.14153.

(d) To use a greater r, I would just need to alter two functions. I would need to change my 'in_unit_circle' function to be 'in_circle' and use and replace the 1 in the formula for the y value, $y = \sqrt{1 - x^2}$ by r^2. I would also change any place I sampled numbers to sample them from 0 to r instead of 0 to 1, by multiplying the arrays by r.






Approximation with 56 simulations
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Approximation with 316 simulations
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0.4
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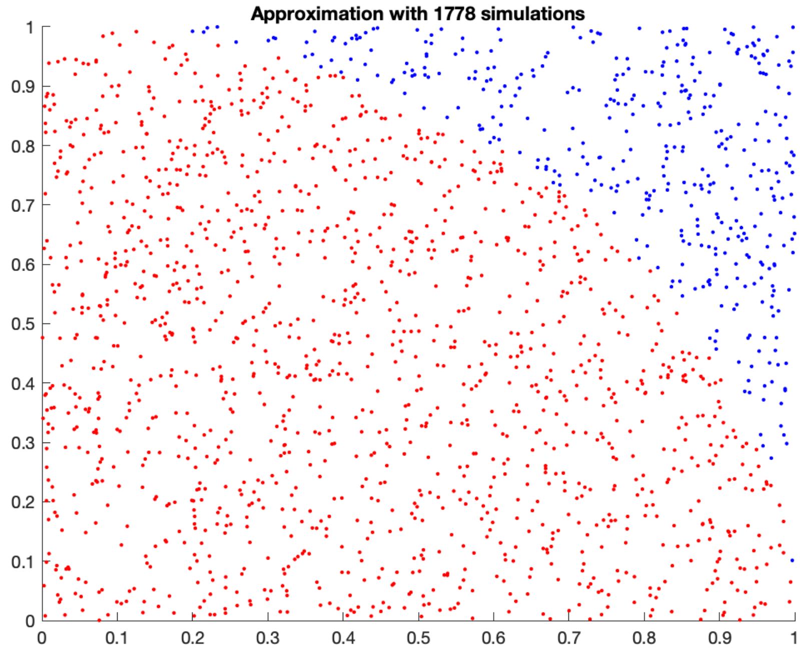
0.6
0.7
0.8
0.9
1


Approximation with 1778 simulations
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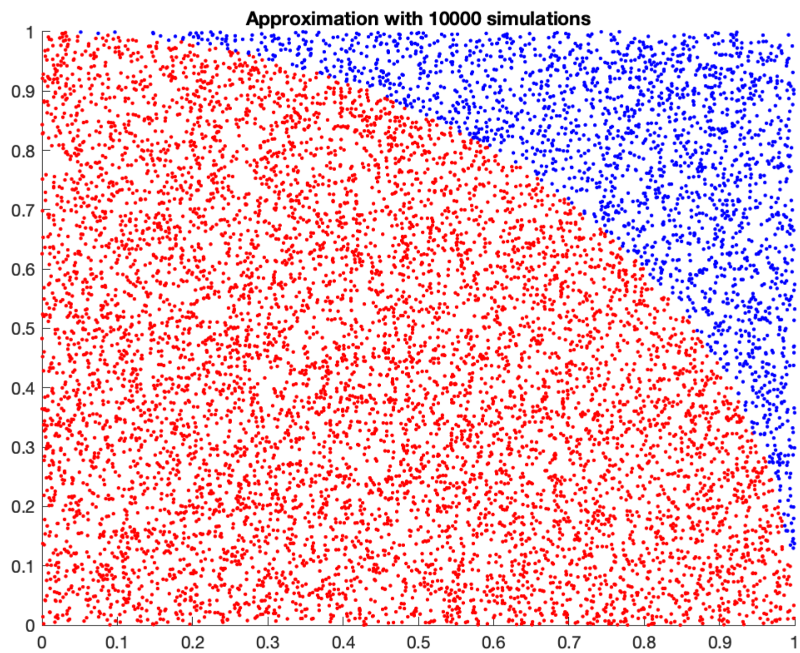
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Approximation with 10000 simulations
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1