Smoothing methods

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What Is Forecasting?

- Process of predicting a future event
- Underlying basis of all business decisions

You'll never get it right. But, you can always get it less wrong.

Realities of Forecasting

- Forecasts are seldom perfect
- Most forecasting methods assume that there is some underlying stability in the system and future will be like the past (causal factors will be the same).
- Accuracy decreases with length of forecast

Forecasting Methods

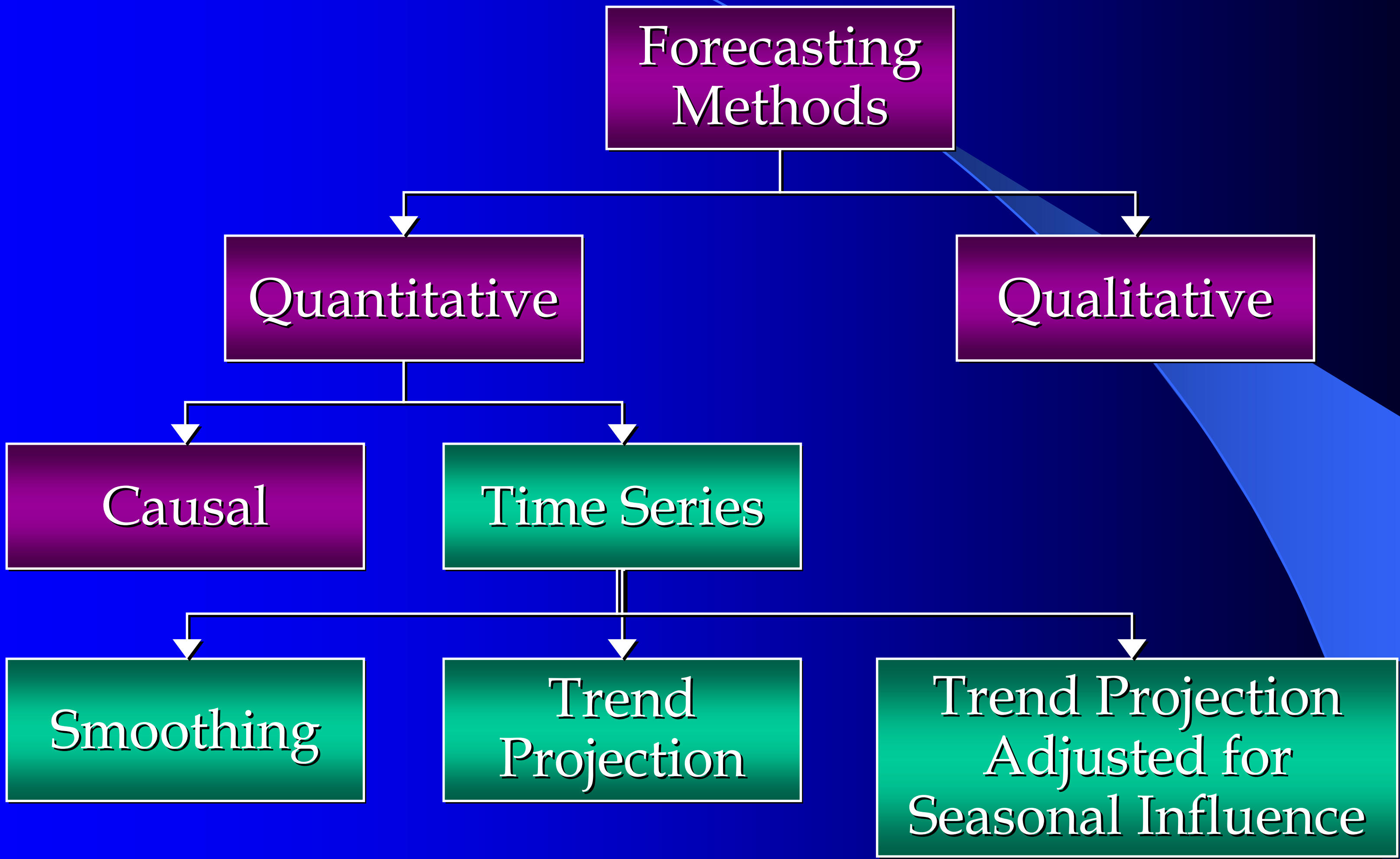
Qualitative Methods are subjective in nature since they rely on human judgment and opinion.

- Used when situation is vague & little data exist
  - New products
  - New technology
- Involve intuition, experience

Quantitative Methods use mathematical or simulation models based on historical demand or relationships between variables.

- Used when situation is 'stable' & historical data exist
  - Existing products
  - Current technology
- Involve mathematical techniques

Forecasting Methods



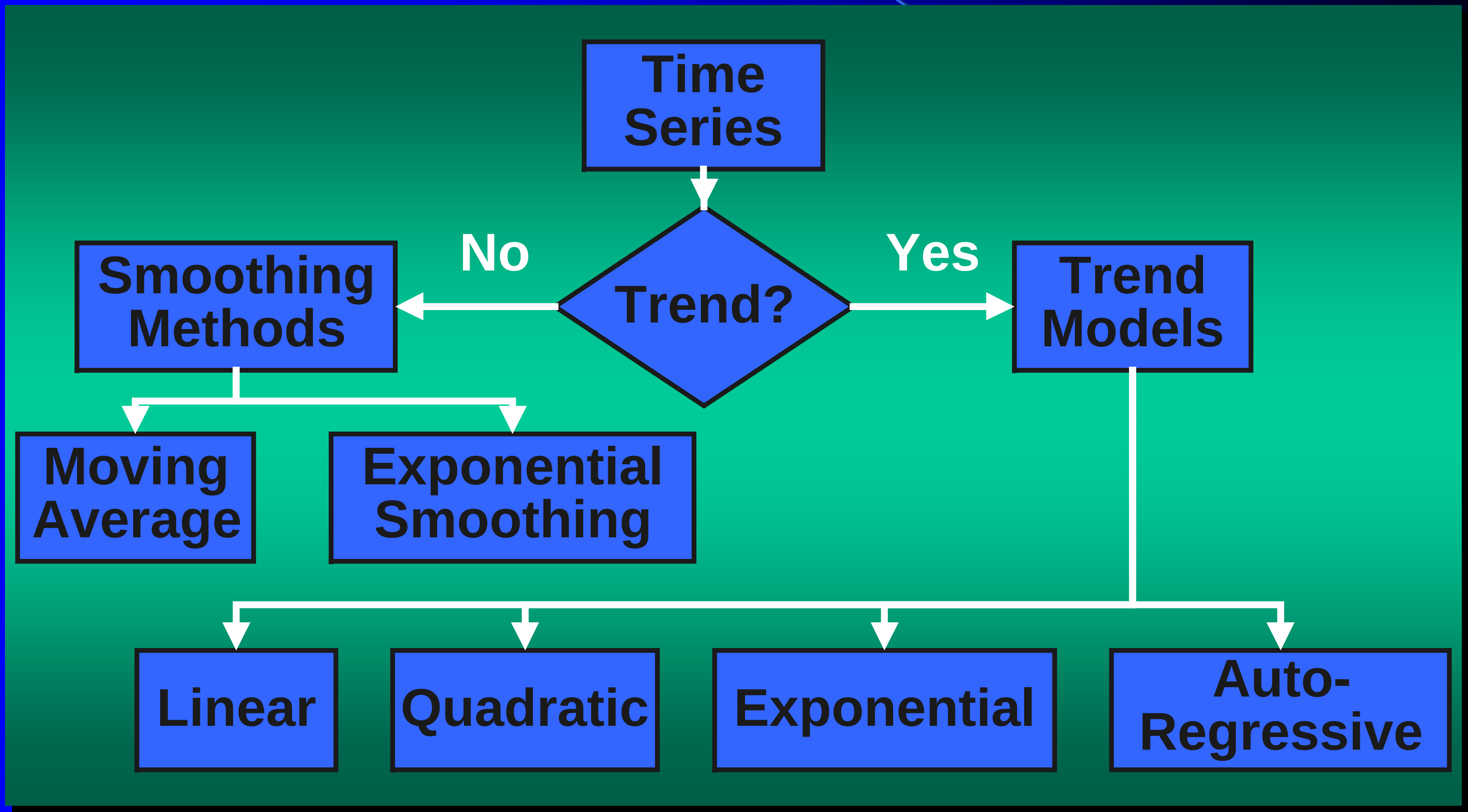
What is a Time Series?

By reviewing historical data over time, we can better understand the pattern of past behavior of a variable and better predict the future behavior.

- Set of evenly spaced numerical data
  - Obtained by observing response variable at regular time periods
- Forecast based only on past values
  - Assumes that factors influencing past, present, & future will continue
- Example

– Year:	2002	2003	2004	2005	2006
– Sales:	78.7	63.5	89.7	93.2	92.1

Time Series Forecasting



Components of a Time Series

- The pattern or behavior of the data in a time series has several components.

Trend

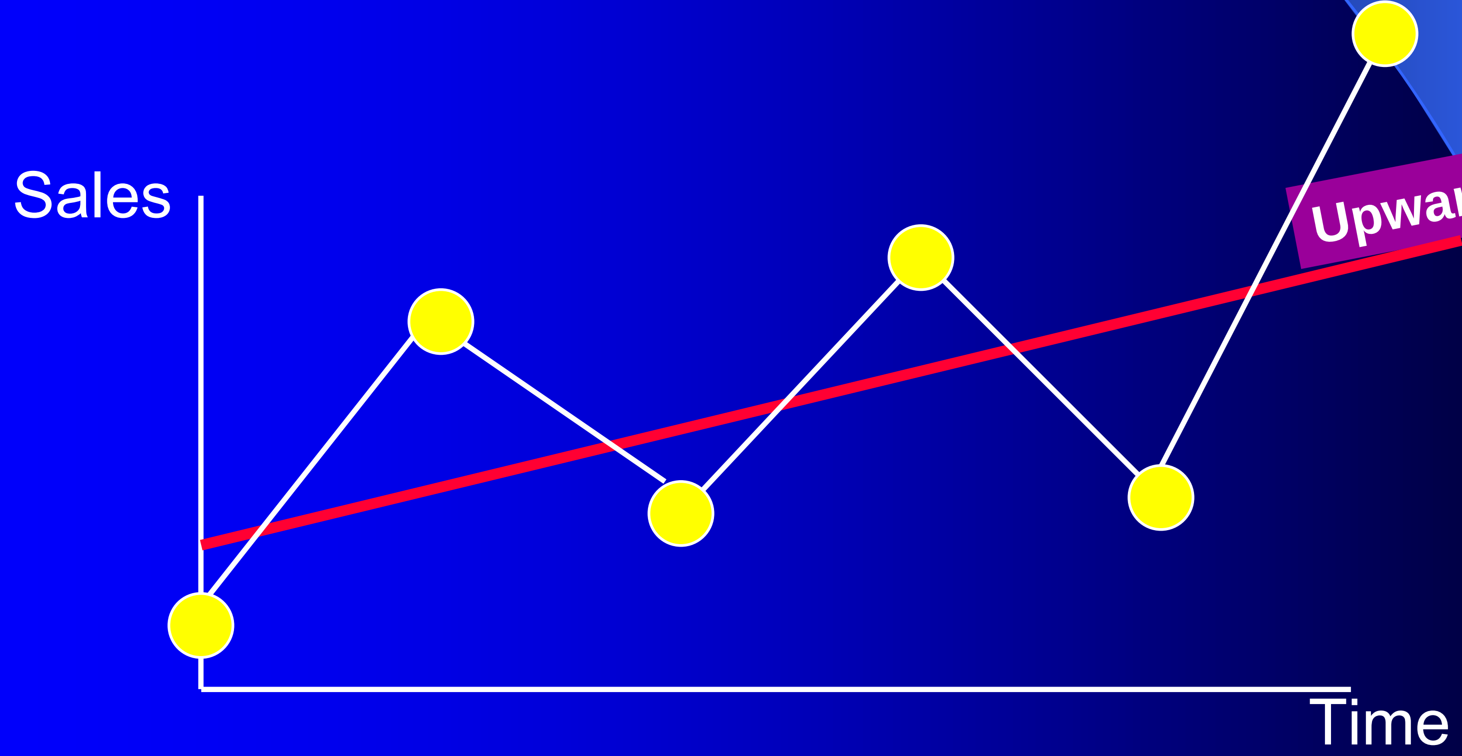
Cyclical

Seasonal

Irregular

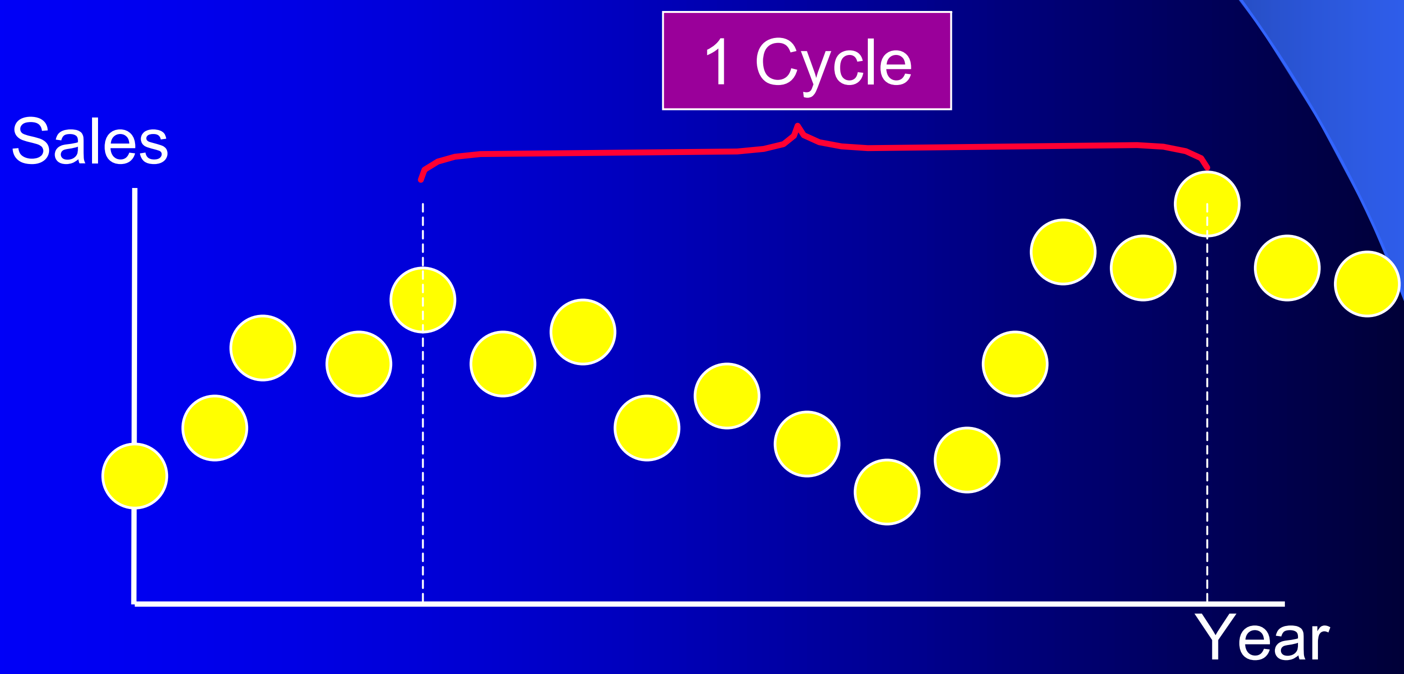
Trend Component

- The trend component accounts for the gradual shifting of the time series to relatively higher or lower values over a long period of time.
- Trend is usually the result of long-term factors such as changes in the population, demographics, technology, or consumer preferences



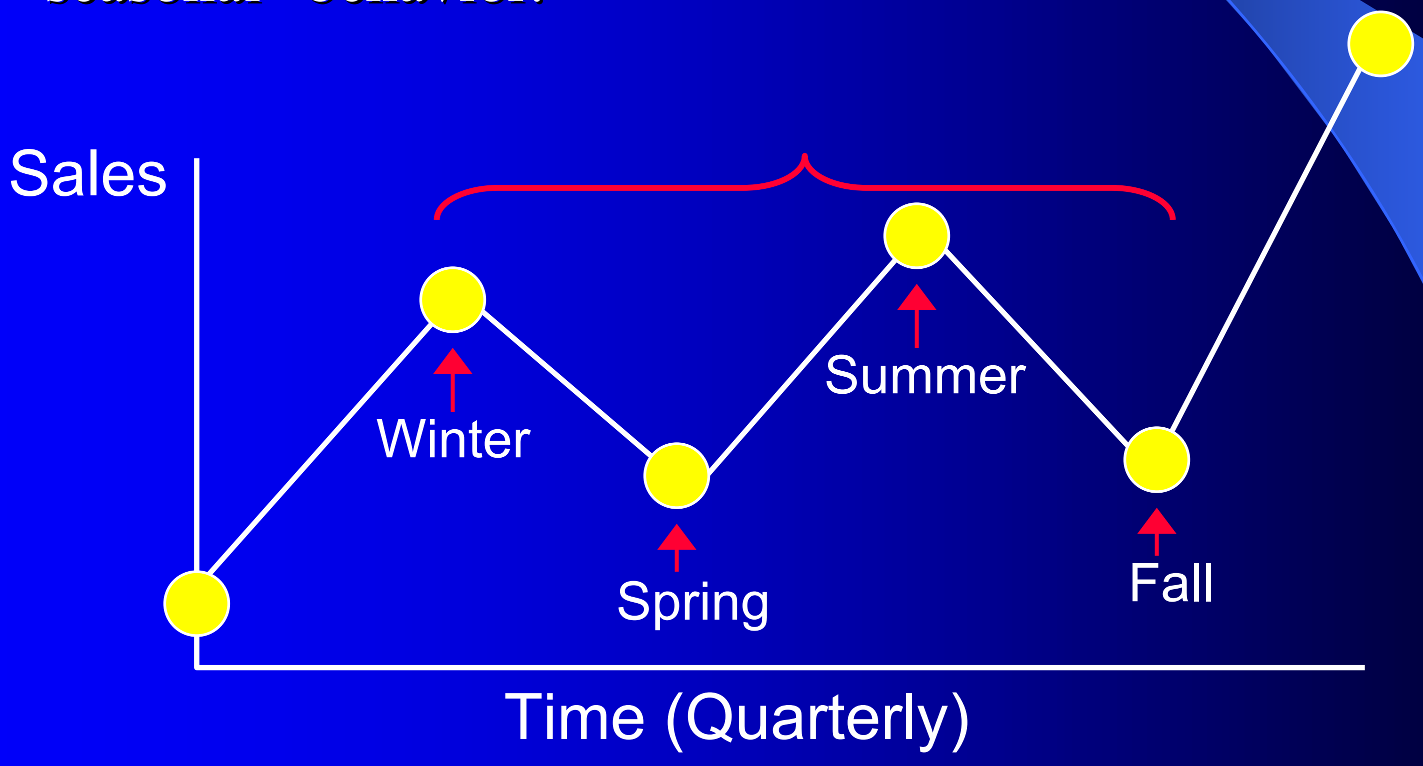
Cyclical Component

- Any regular pattern of sequences of values above and below the trend line lasting more than one year can be attributed to the cyclical component.
- Usually, this component is due to multiyear cyclical movements in the economy.



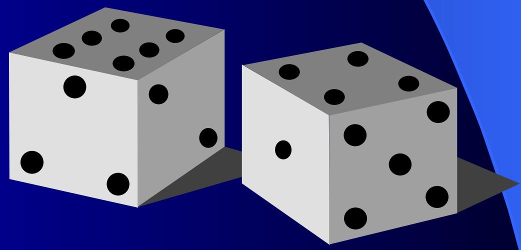
Seasonal Component

- The seasonal component accounts for regular patterns of variability within certain time periods, such as a year.
- The variability does not always correspond with the seasons of the year (i.e. winter, spring, summer, fall).
- There can be, for example, within-week or within-day "seasonal" behavior.

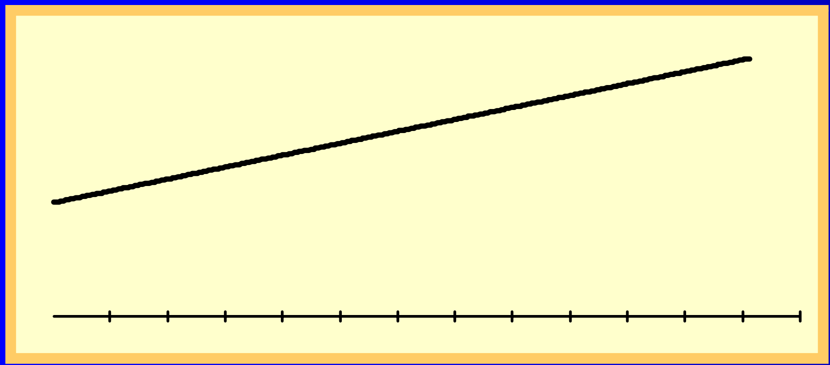


Irregular Component

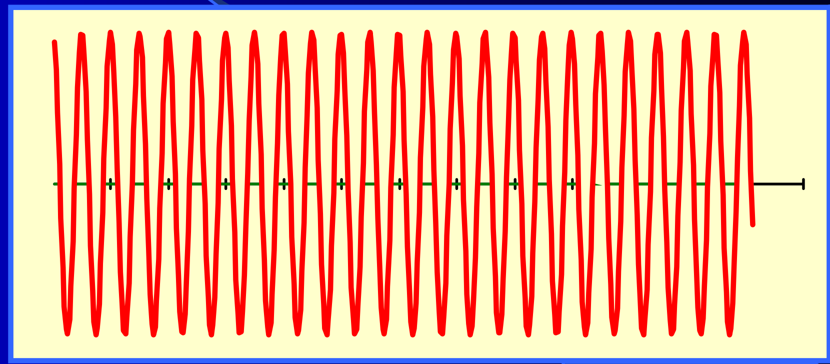
- The irregular component is caused by short-term, unanticipated and non-recurring factors that affect the values of the time series.
- This component is the residual, or “catch-all,” factor that accounts for unexpected data values.
- It is unpredictable.



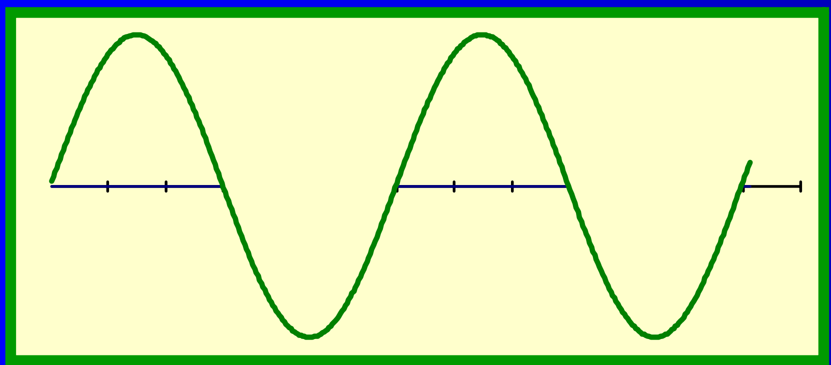
Components of Time Series Data



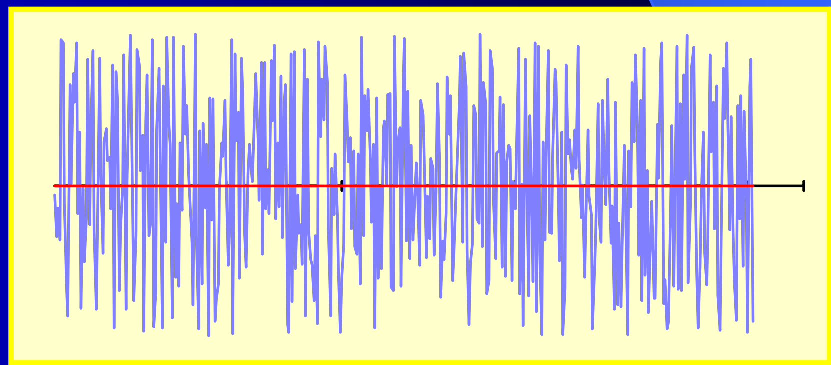
Trend



Seasonal

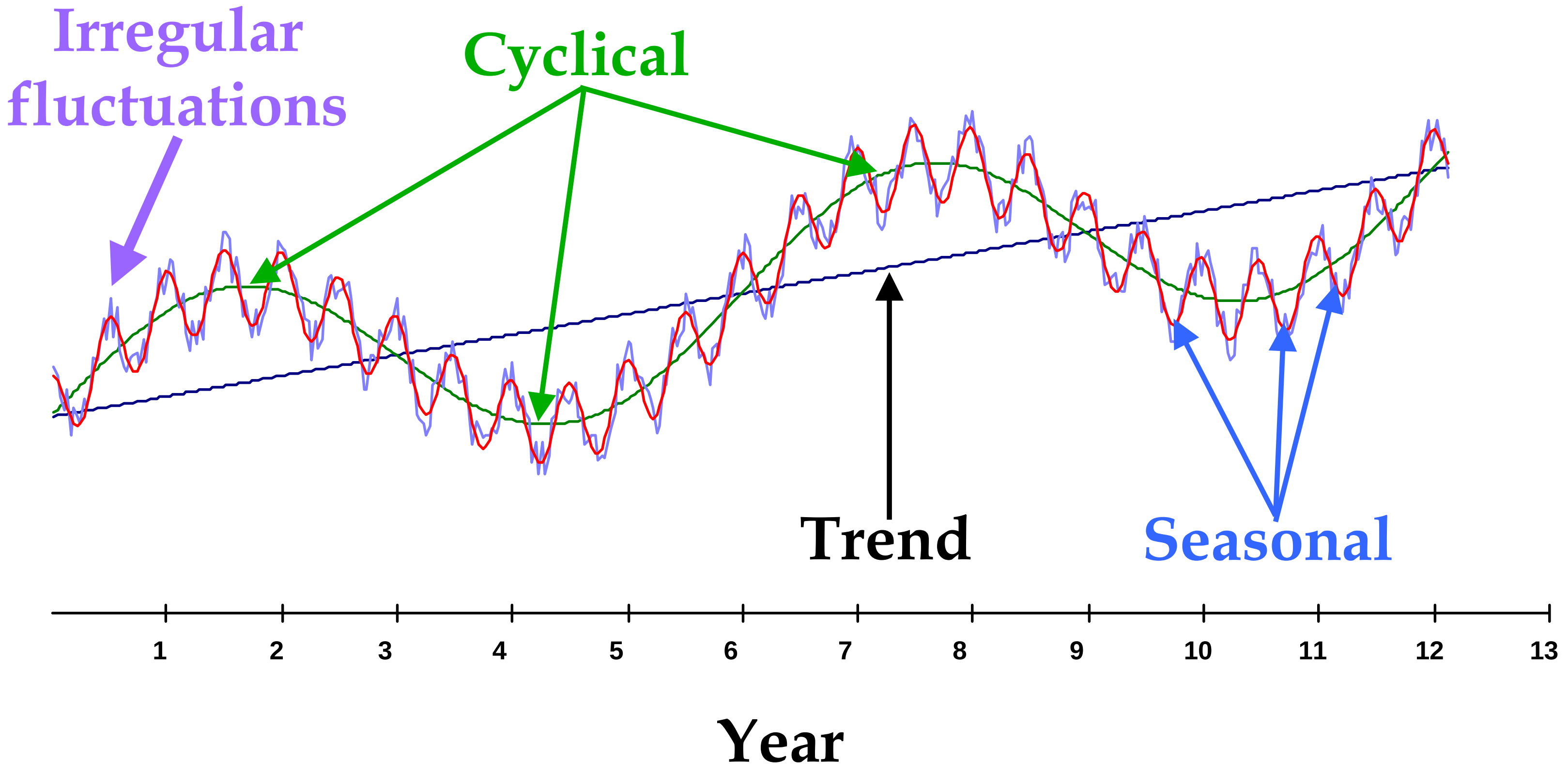


Cyclical



Irregular

Components of Time Series Data



Smoothing Methods

- In cases in which the time series is fairly stable and has no significant trend, seasonal, or cyclical effects, one can use smoothing methods to average out the irregular component of the time series.
- Common smoothing methods are:

Moving Averages
Weighted Moving Averages
Centered Moving Average
Exponential Smoothing

Moving Averages Method

The moving averages method consists of computing an average of the most recent *n* data values for the series and using this average for forecasting the value of the time series for the next period.

Moving averages are useful if one can assume item to be forecast will stay fairly steady over time.

Series of arithmetic means - used only for smoothing, provides overall impression of data over time

$$\text{Moving Average} = \frac{\sum(\text{most recent } n \text{ data values})}{n}$$

Moving Averages

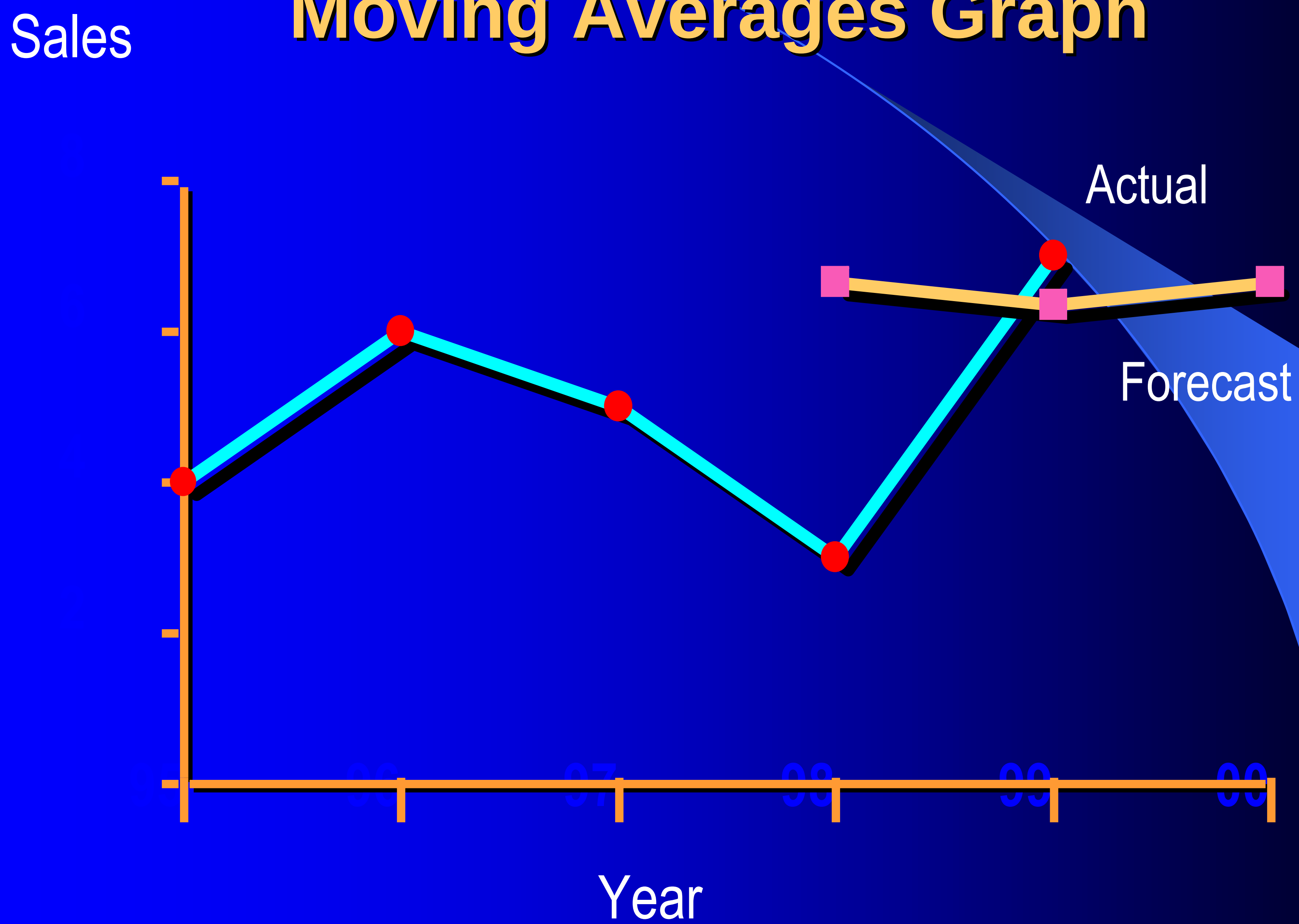
Let us forecast sales for 2007 using a 3-period moving average.

2002	4
2003	6
2004	5
2005	3
2006	7
2007	?

Moving Averages

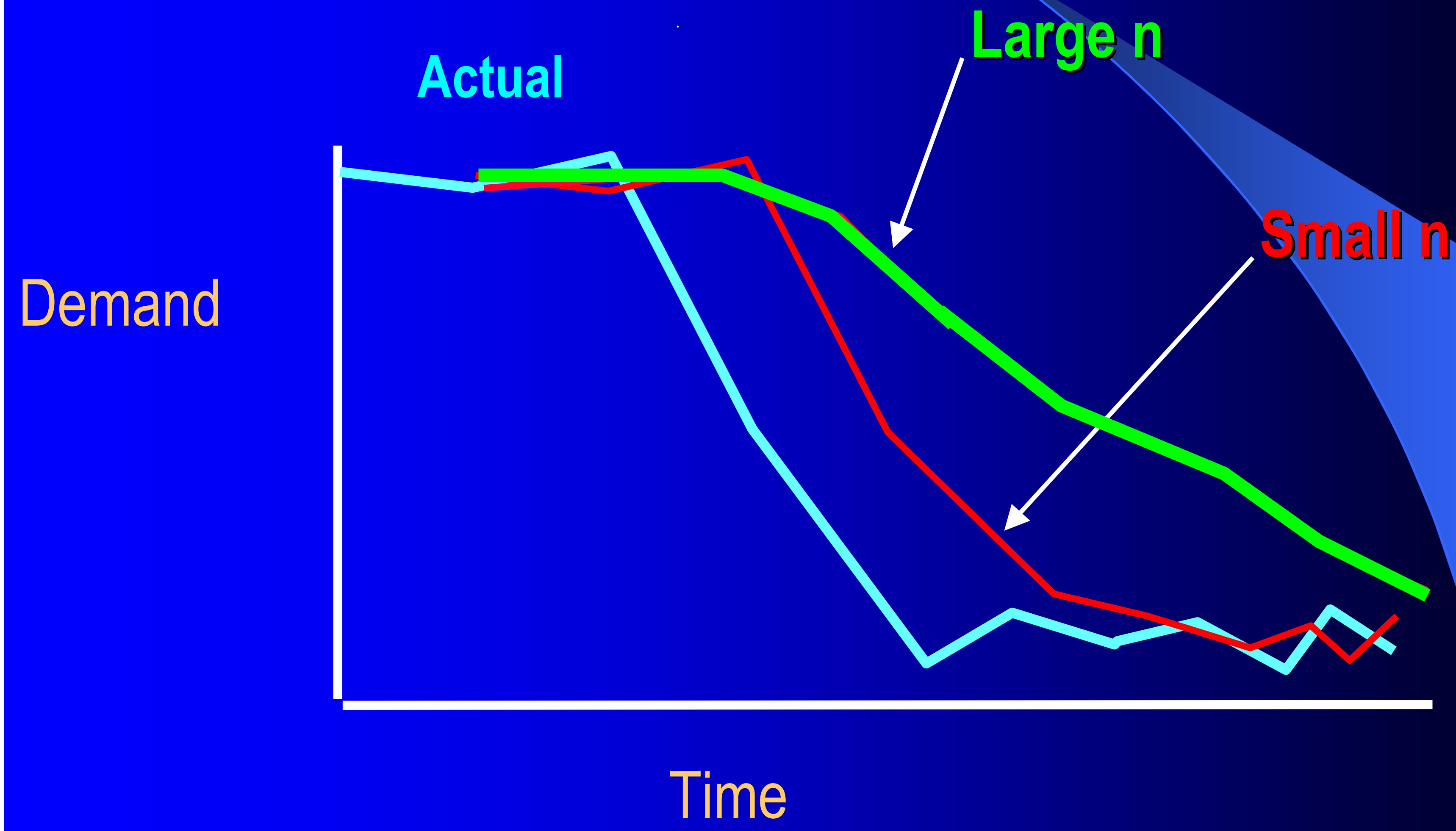
Time	Response Y_i	Moving Total (n=3)	Moving Average (n=3)
2002	4	NA	NA
2003	6	NA	NA
2004	5	NA	NA
2005	3	4+6+5=15	15/3=5.0
2006	7	6+5+3=14	14/3=4.7
2007	NA	5+3+7=15	15/3=5.0

Moving Averages Graph



MONTH	ACTUAL SALES	THREE-MONTH MOVING AVERAGES
January	10	
February	12	
March	16	
April	13	(10 + 12 + 16)/3 = 12.67
May	17	(12 + 16 + 13)/3 = 13.67
June	19	(16 + 13 + 17)/3 = 15.33
July	15	(13 + 17 +19)/3 = 16.33
August	20	(17 + 19 + 15)/3 = 17.00
September	22	(19 + 15 +20)/3 = 18.00
October	19	(15 +20 +22)/3 = 19.00
November	21	(20 + 22+ 19)/3 = 20.33
December	19	(22 + 19 + 21)/3 = 20.67

Moving Averages Graph



Centered Moving Averages Method

The centered moving average method consists of computing an average of *n* periods' data and associating it with the midpoint of the periods. For example, the average for periods 5, 6, and 7 is associated with period 6. This methodology is useful in the process of computing season indexes.

5	10	
6	13	10+13+11: 3= 11.33
7	11	

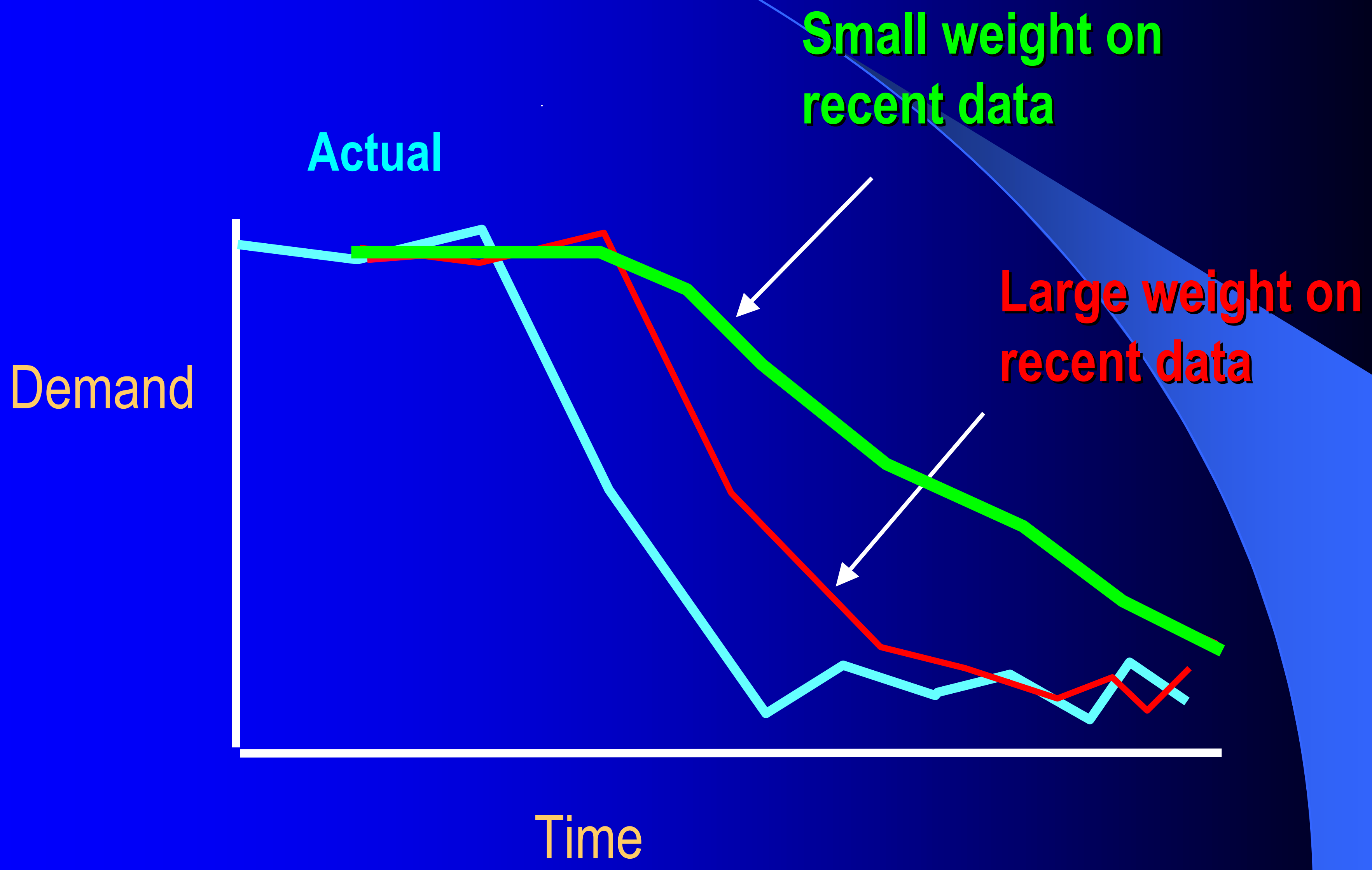
Weighted Moving Averages

- Used when trend is present
  - Older data usually less important
- The more recent observations are typically given more weight than older observations
- Weights based on intuition
  - Often lay between 0 & 1, & sum to 1.0

$$WMA = \frac{\Sigma(\text{Weight for period n}) (\text{Value in period n})}{\Sigma \text{Weights}}$$

MONTH	ACTUAL SALES	WEIGHTED MOVING AVERAGES
January	10	
February	12	
March	16	
April	13	$(1 \times 10 + 2\times 12 + 3 \times 16)/6 = 13.67$
May	17	$(1 \times 12 + 2 \times 16 + 3 \times 13)/6 = 13.83$
June	19	$(1 \times 16 + 2 \times 13 + 3 \times 17)/6 = 15.50$
July	15	$(1 \times 13 + 2 \times 17 + 3 \times 19)/6 = 17.33$
August	20	$(1 \times 17 + 2 \times 19 + 3 \times 15)/6 = 16.67$
September	22	$(1 \times 19 + 2 \times 15 + 3 \times 20)/6 = 18.17$
October	19	$(1 \times 15 + 2 \times 20 + 3 \times 22)/6 = 20.17$
November	21	$(1 \times 20 + 2 \times 22 + 3 \times 19)/6 = 20.17$
December	19	$(1 \times 22 + 2 \times 19 + 3 \times 21)/6 = 20.50$

Weighted Moving Average Graph

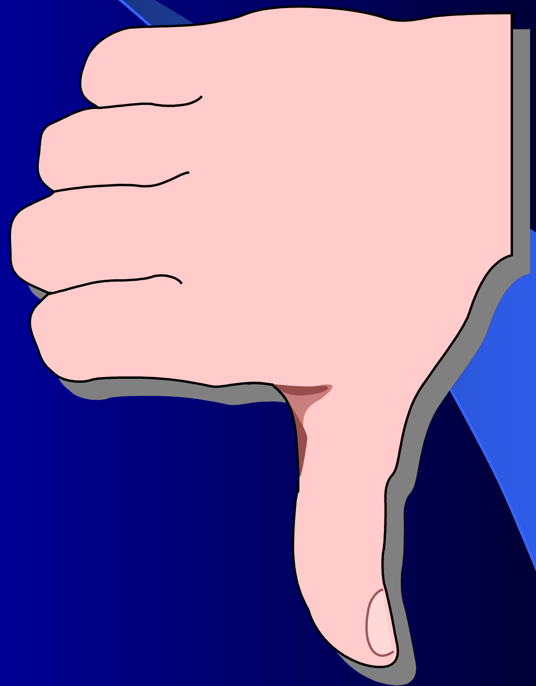


Disadvantages of M.A. Methods

- Increasing n makes forecast less sensitive to changes

- Do not forecast trends well

- Require sufficient historical data

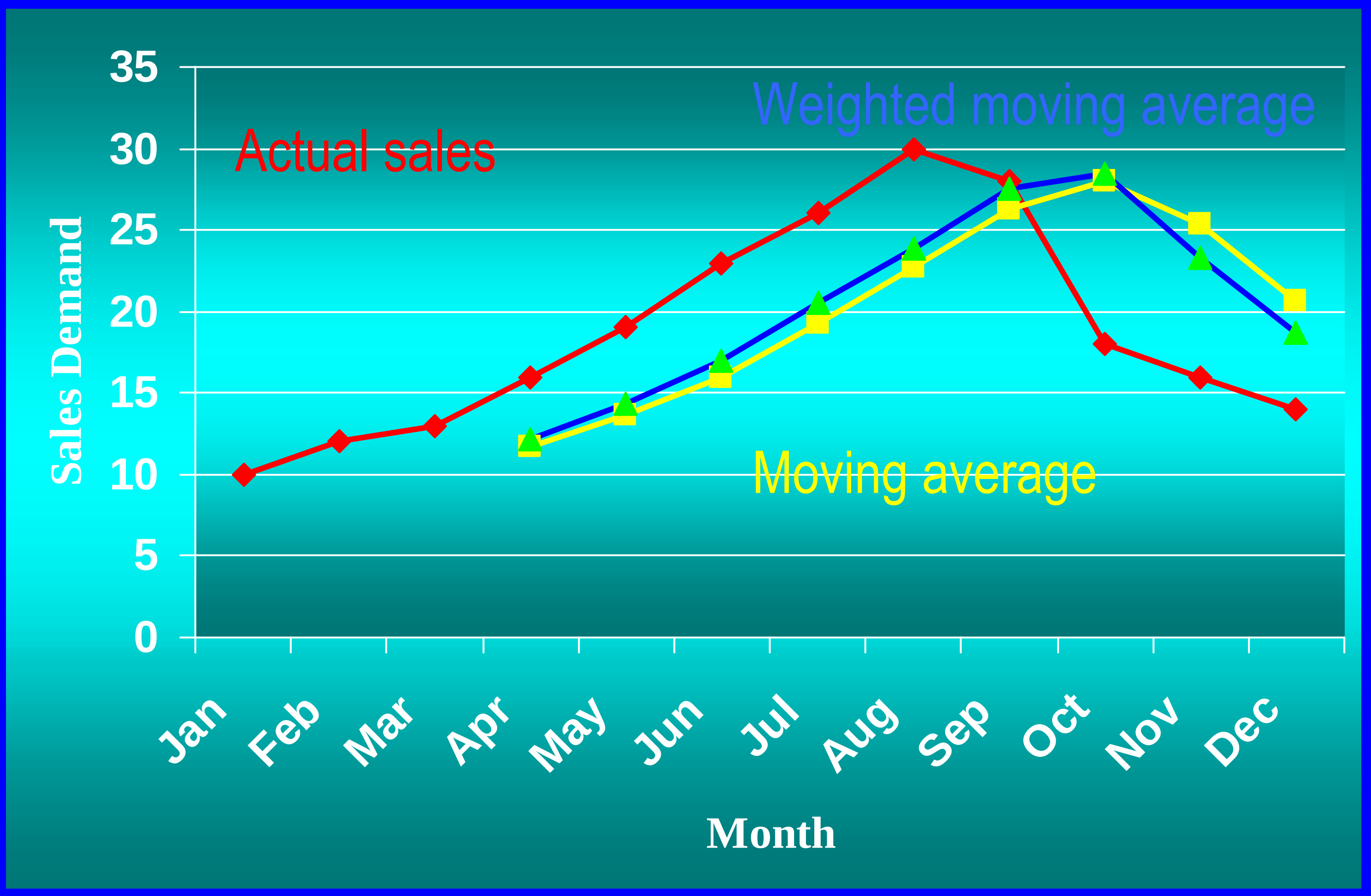


Responsiveness of M.A. Methods

The problem with M.A. Methods :

- Forecast lags with increasing demand
- Forecast leads with decreasing demand

Actual Demand, Moving Average, Weighted Moving Average



All forecasting methods lag ahead of or behind actual demand

M.A. versus Exponential Smoothing

- Moving averages and weighted moving averages are effective in smoothing out sudden fluctuations in demand pattern in order to provide stable estimates.
- Requires maintaining extensive records of past data.
- Exponential smoothing requires little record keeping of past data.
- Form of weighted moving average
  - Weights decline exponentially
  - Most recent data weighted most
- Requires smoothing constant (α)
  - Ranges from 0 to 1
  - Subjectively chosen

Forecasting Using Smoothing Methods

- Exponential Smoothing Methods
  - Single Exponential Smoothing
  - Double (Holt's) Exponential Smoothing
  - Triple (Winter's) Exponential Smoothing

Exponential Smoothing Methods

- Single Exponential Smoothing
  - Similar to single MA
- Double (Holt's) Exponential Smoothing
  - Similar to double MA
  - Estimates trend
- Triple (Winter's) Exponential Smoothing
  - Estimates trend and seasonality

Exponential Smoothing Model

- Single exponential smoothing model

$$F_{t+1} = F_t + \alpha(y_t - F_t)$$

or:

$$F_{t+1} = \alpha y_t + (1-\alpha)F_t$$

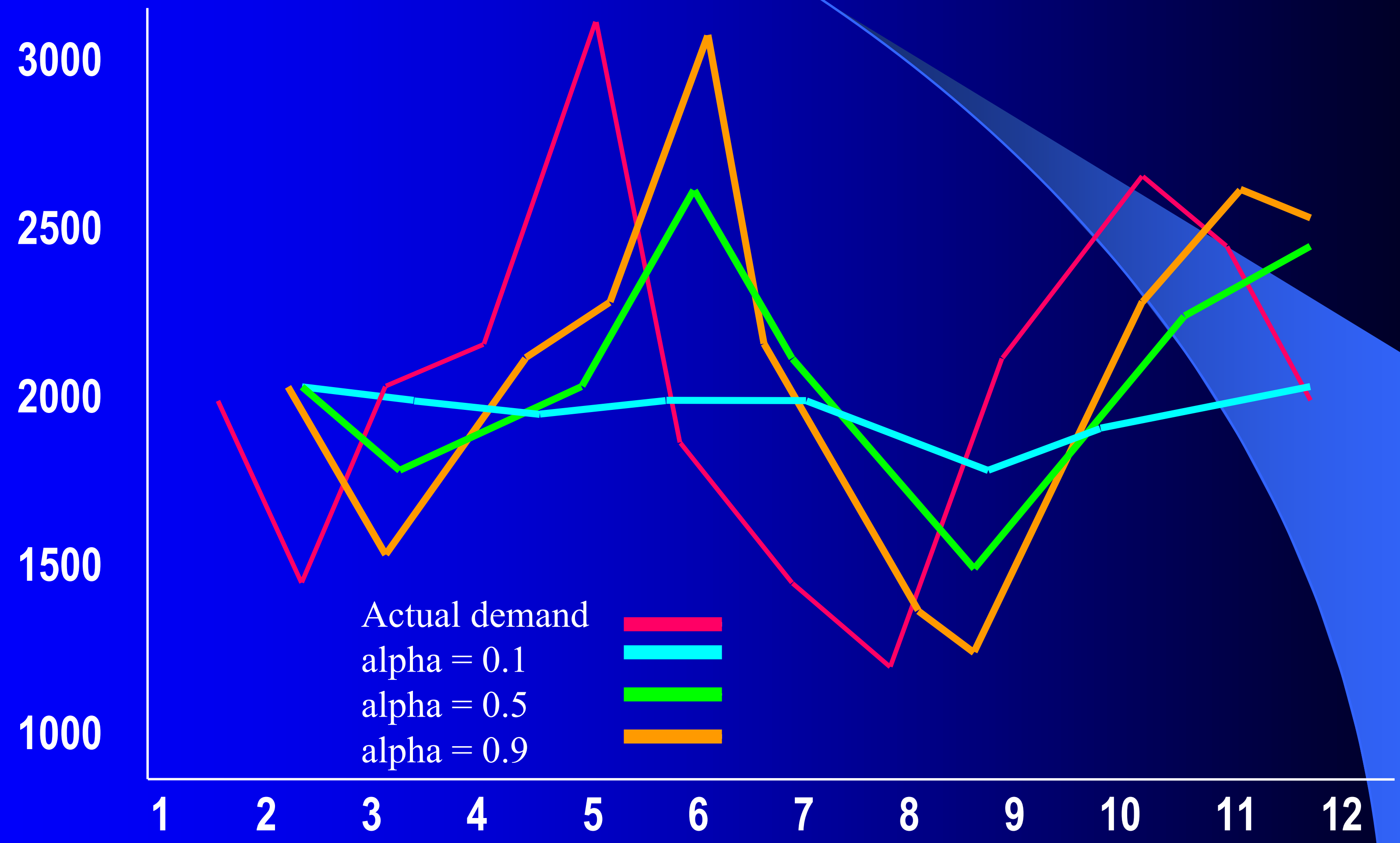
where:

F_{t+1} = forecast value for period t + 1
y_t = actual value for period t
F_t = forecast value for period t
α = alpha (smoothing constant)

Single Exponential Smoothing

- A weighted moving average
- Weights decline exponentially, most recent observation weighted most
- The weighting factor is α
  - Subjectively chosen
  - Range from 0 to 1
  - Smaller α gives more smoothing, larger α gives less smoothing
- The weight is:
  - Close to 0 for smoothing out unwanted cyclical and irregular components
  - Close to 1 for forecasting

Responsiveness to Different Values of α



Exponential Smoothing Example

- Suppose we use weight $\alpha = 0.2$

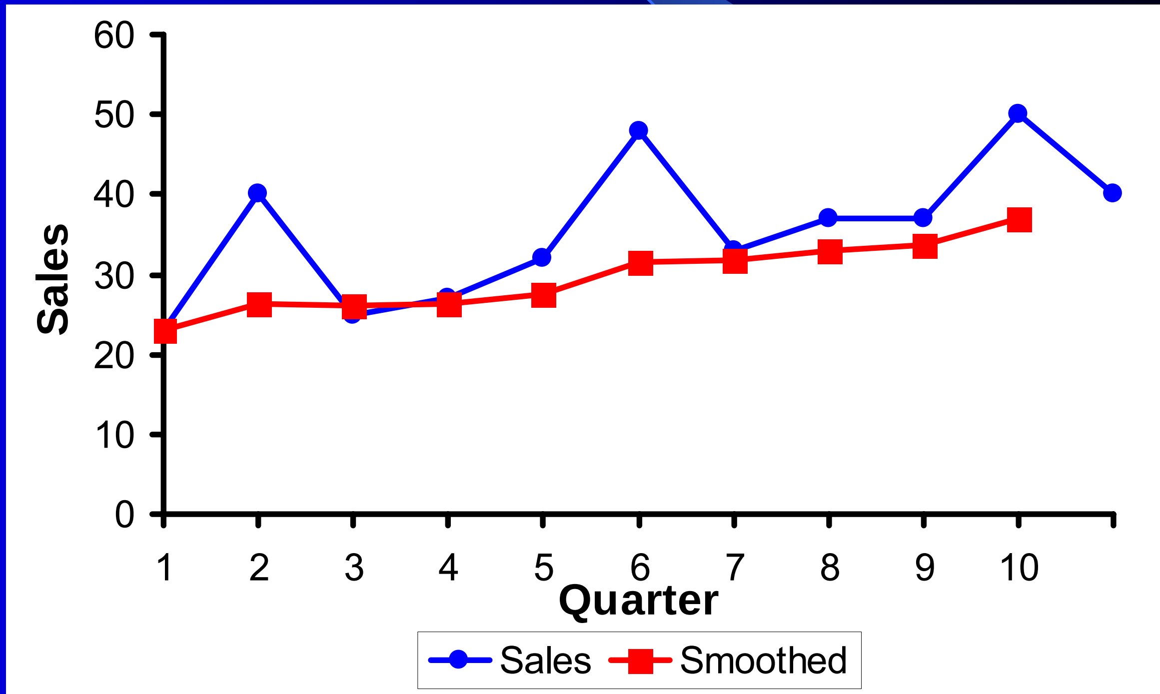
Quarter (t)	Sales (y_t)	Forecast from prior period	Forecast for next period (F_{t+1})
1	23	NA	23
2	40	23	(.2)(40)+(.8)(23)=26.4
3	25	26.4	(.2)(25)+(.8)(26.4)=26.12
4	27	26.12	(.2)(27)+(.8)(26.12)=26.296
5	32	26.296	(.2)(32)+(.8)(26.296)=27.437
6	48	27.437	(.2)(48)+(.8)(27.437)=31.549
7	33	31.549	(.2)(48)+(.8)(31.549)=31.840
8	37	31.840	(.2)(33)+(.8)(31.840)=32.872
9	37	32.872	(.2)(37)+(.8)(32.872)=33.697
10	50	33.697	(.2)(50)+(.8)(33.697)=36.958
etc…	etc…	etc…	etc…

$F_1 = y_1$ since no prior informatio n exists

$$F_{t+1} = \alpha y_t + (1-\alpha)F_t$$

Sales vs. Smoothed Sales

- Seasonal fluctuations have been smoothed
- The smoothed value in this case is generally a little low, since the trend is upward sloping and the weighting factor is only 0.2



Double Exponential Smoothing

- Double exponential smoothing is sometimes called exponential smoothing with trend
- If trend exists, single exponential smoothing may need adjustment
- There is a need to add a second smoothing constant to account for trend

Double Exponential Smoothing Model

$$C_t = \alpha y_t + (1-\alpha)(C_{t-1} + T_{t-1})$$

$$T_t = \beta(C_t - C_{t-1}) + (1-\beta)T_{t-1}$$

$$F_{t+1} = C_t + T_t$$

y_t = actual value in time t

α = constant-process smoothing constant

β = trend-smoothing constant

C_t = smoothed constant-process value for period t

T_t = smoothed trend value for period t

F_{t+1}= forecast value for period t + 1

t = current time period

Double Exponential Smoothing

- Double exponential smoothing is generally done by computer
- One uses larger smoothing constants α and β when less smoothing is desired
- One uses smaller smoothing constants α and β when more smoothing is desired

Exponential Smoothing Method

You want to forecast sales for 2007 using exponential smoothing ($\alpha = 0.10$). The 2001 forecast was 175.

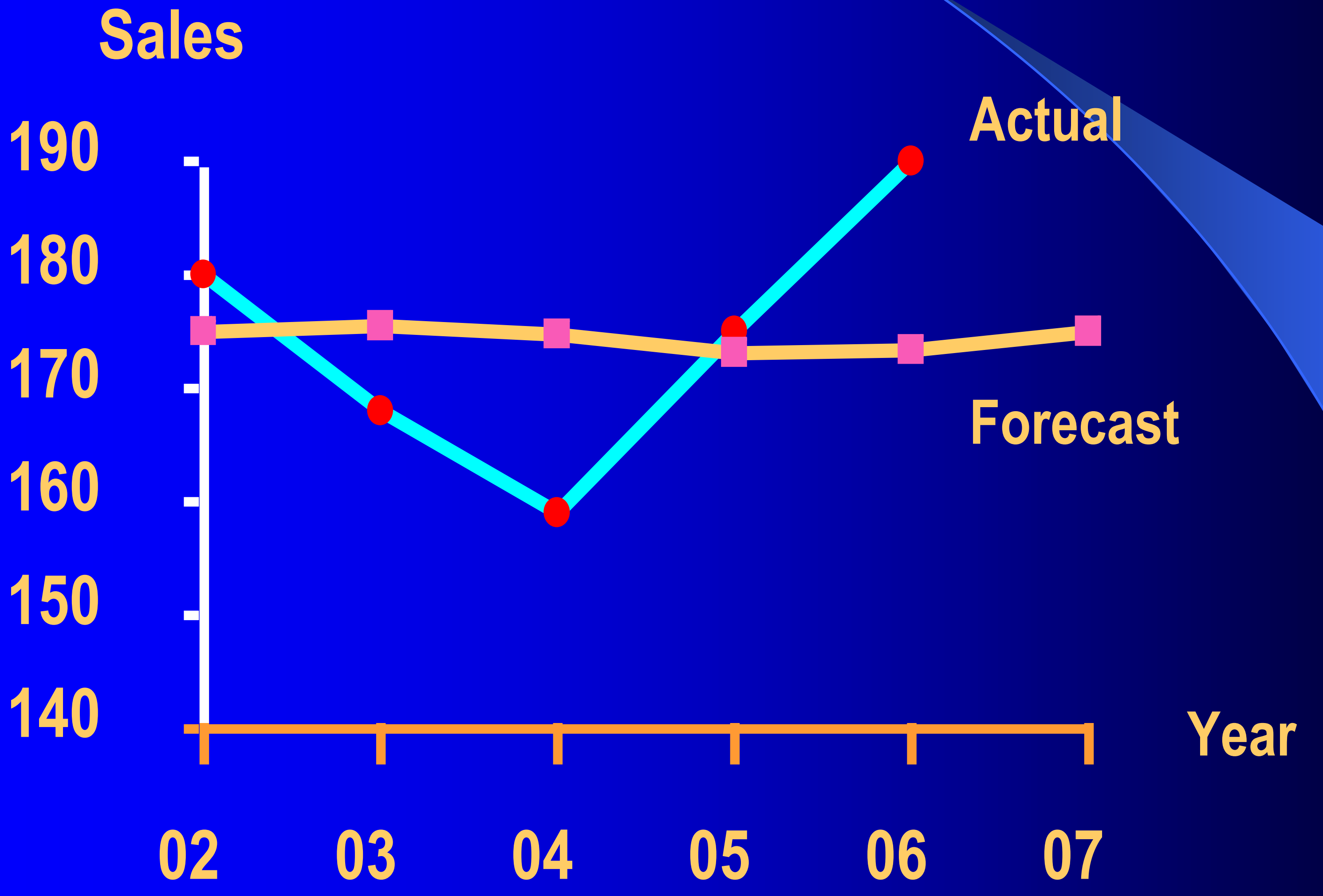
2002	180
2003	168
2004	159
2005	175
2006	190

Exponential Smoothing Method

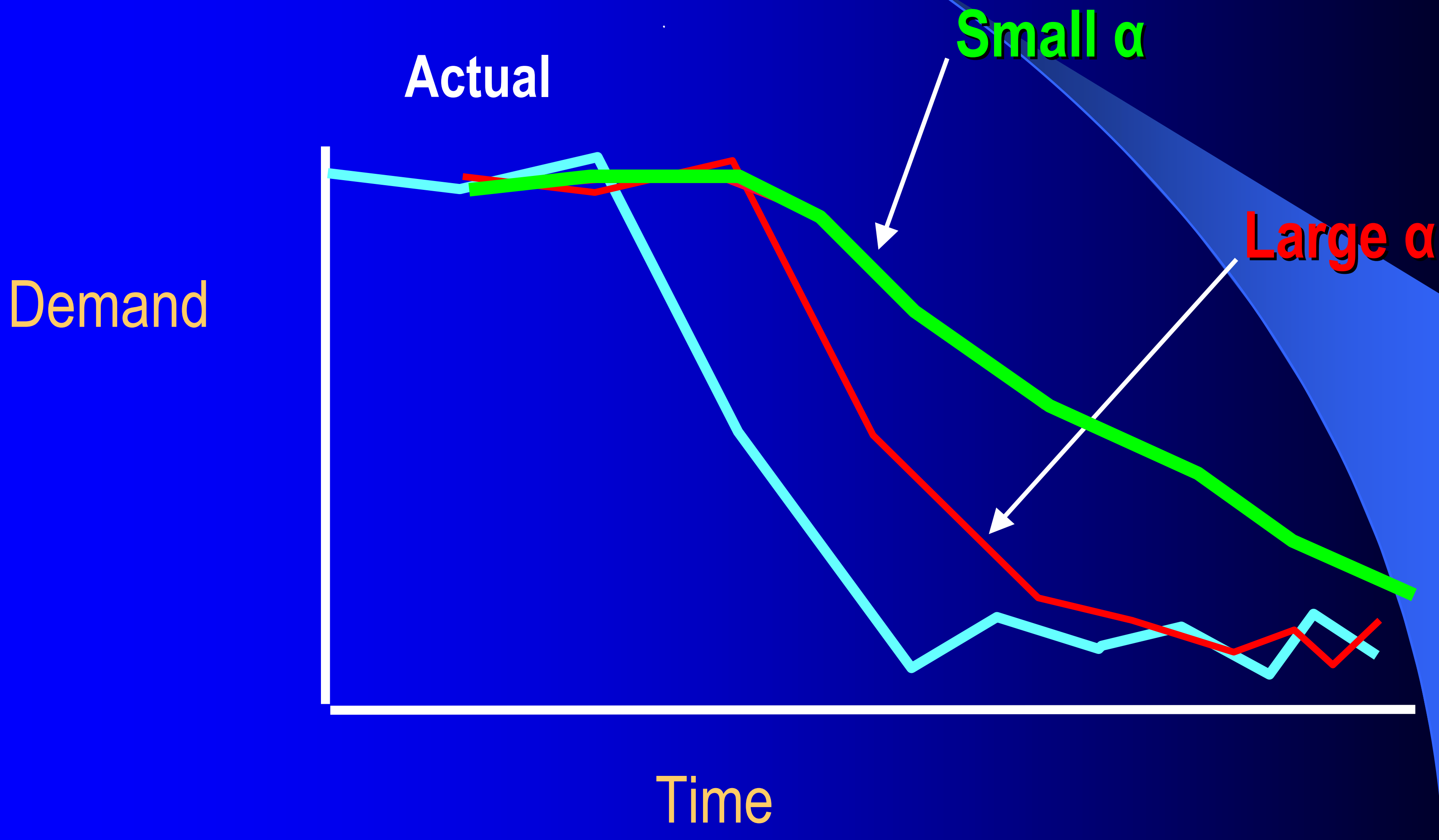
$$F_t = F_{t-1} + \alpha (A_{t-1} - F_{t-1})$$

Time	Actual	Forecast, F_t (α =0.10)
2002	180	175.00 (Given)
2003	168	175.00 + .10(180 - 175.00) = 175.50
2004	159	175.50 + .10(168 - 175.50) = 174.75
2005	175	174.75 + .10(159 - 174.75) = 173.18
2006	190	173.18 + .10(175 - 173.18) = 173.36
2007	NA	173.36 + .10(190 - 173.36) = 175.02

Exponential Smoothing Graph



Exponential Smoothing Graph



Comparing Smoothing Techniques

- Let us determine the smoothing technique that is best for forecasting these sales data : A two period moving average, a three period moving average, exponential smoothing (α=0.1), or exponential smoothing (α=0.2)

Week	Sales	Week	Sales
1	110	6	120
2	115	7	130
3	125	8	115
4	120	9	110
5	125	10	130

Measures of Forecast Accuracy

Mean Squared Error (MSE)

The average of the squared forecast errors for the historical data is calculated. The forecasting method or parameter(s) which minimize this mean squared error is then selected.

Mean Absolute Deviation (MAD)

The mean of the absolute values of all forecast errors is calculated, and the forecasting method or parameter(s) which minimize this measure is selected. The mean absolute deviation measure is less sensitive to individual large forecast errors than the mean squared error measure.

You may choose either of the above criteria for evaluating the accuracy of a method (or parameter).

2 period moving average

	Sales	n=2	Error	
Week (*t*)	Y_t	F_t	$(Y_t - F_t)$	$(Y_t - F_t)^2$
1	110			
2	115	#NV		
3	125	112,5	12,5	156,25
4	120	120	0	0
5	125	122,5	2,5	6,25
6	120	122,5	-2,5	6,25
7	130	122,5	7,5	56,25
8	115	125	-10	100
9	110	122,5	-12,5	156,25
10	130	112,5	17,5	306,25
11		120		
			MSE	98,4375

3 period moving average

	Sales	n=3	Error	
Week (*t*)	Y_t	F_t	$(Y_t - F_t)$	$(Y_t - F_t)^2$
1	110			
2	115	#NV		
3	125	#NV		
4	120	116,6667	3,333333	11,11111
5	125	120	5	25
6	120	123,3333	-3,33333	11,11111
7	130	121,6667	8,333333	69,44444
8	115	125	-10	100
9	110	121,6667	-11,6667	136,1111
10	130	118,3333	11,66667	136,1111
11		118,3333		
			MSE	69,84127

Exponential smoothing (α=0.1)

	Sales	α=0.1	Error	
Week (*t*)	Y_t	F_t	$(Y_t - F_t)$	$(Y_t - F_t)^2$
1	110	#NV		
2	115	110	5	25
3	125	110,5	14,5	210,25
4	120	111,95	8,05	64,8025
5	125	112,755	12,245	149,94
6	120	113,9795	6,0205	36,24642
7	130	114,5816	15,41845	237,7286
8	115	116,1234	-1,1234	1,262016
9	110	116,0111	-6,01106	36,13279
10	130	115,4099	14,59005	212,8696
11				
			MSE	108,248

Exponential smoothing (α=0.2)

	Sales	α=0.2	Error	
Week (*t*)	Y_t	F_t	$(Y_t - F_t)$	$(Y_t - F_t)^2$
1	110	#NV		
2	115	110	5	25
3	125	111	14	196
4	120	113,8	6,2	38,44
5	125	115,04	9,96	99,2016
6	120	117,032	2,968	8,809024
7	130	117,6256	12,3744	153,1258
8	115	120,1005	-5,10048	26,0149
9	110	119,0804	-9,08038	82,45337
10	130	117,2643	12,73569	162,1979
11				
			MSE	87,91584

Comparing Smoothing Techniques

- Since the three period moving average technique (MA_3) provides to lowest MSE value, this is the best smoothing technique to use for forecasting these Sales data in our example.

Thank You for Your Attention

References

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