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Charles Holt's report on exponentially weighted moving averages: an introduction and appreciation

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Abstract

Charles Holt's classic paper on exponentially weighted moving averages appeared as Report ONR 52 from the Office of Naval Research in 1957. Although widely cited, the full version has remained unpublished. This note provides some context for looking back to ONR 52, which then follows in its complete form. We indicate that the paper still has some lessons for modern practice, in terms of both method specification and method choice. The main paper is followed by a "retrospective" written by Professor Holt and reprinted from the *Journal of Economic and Social Measurement*, with kind permission from the editor of that journal.

Keywords: Charles Holt's report; Exponentially weighted moving averages; Introduction

1. Introduction

Over the years, "Holt's Method" has become a standard term of reference for the forecasting method that uses exponentially weighted moving averages to forecast a time series with level and trend components. The term has become so widely recognized that it is often used without qualification or citation. When a citation is given, to Holt (1957), the Office of Naval Research report [ONR Research Memorandum Number 52], there is the suspicion that an author is following custom and has not consulted the original report. Yes, I plead guilty as charged. Indeed, a search of the *Web of Science* (successor to the Science and Social Science Citation Indices) produced only 39 citations for 1985–2003, which does not even begin to hint at the impact that ONR 52 has had.

The *Journal of Economic and Social Measurement* (JESM) is producing a special issue on Econometric Software, part of which includes a retrospective by Professor Holt. This event occasioned the thought that readers of the *International Journal of Forecasting* would be interested not only in the retrospective, but also in the original report. Professor Charles Renfro, editor of the special issue, and JESM have kindly agreed to the reprinting of the retrospective in the *IJF*, and Professor Holt has agreed to the publication of ONR 52 in its original form. Accordingly, the only changes that have been made have been to add an abstract and to correct one or two minor typographical errors.

In the rest of this introduction, we first provide some linkages between ONR 52 and current descriptions of the Holt and Holt–Winters methods (see, for

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example, Makridakis, Wheelwright, & Hyndman, 1998, pp. 158–169), so that the 2003 reader can more readily relate to the original research. We then examine briefly some of the developments that have taken place since ONR 52 appeared.

Throughout this note, α, β, and γ are used as generic smoothing constants to simplify the expressions. The detailed values in any particular case may be obtained from the paper. In addition, the equation numbers cited in this comment are taken from the original paper to facilitate cross-referencing.

2. The level-plus-seasonal model

The basic model introduced in section 2 of ONR 52 contains level and seasonal components in multiplicative form. This combination of components is rarely available in standard forecasting packages, yet it is particularly useful when the series displays little or no trend, and deserves to be implemented more widely.

Following the notation of the report, we may write the updating equations as

$$\bar{S}_t = \alpha P_{t-N} S_t + (1-\alpha)\bar{S}_{t-1}, \tag{4}$$

where $\bar{S}_t$ denotes the smoothed level of the series, S_t denotes the observed values in the series and

$$P_t = \gamma \frac{\bar{S}_{t-1}}{S_t} + (1-\gamma)P_{t-N}, \tag{5}$$

where P_t is a seasonal factor updated once a year (N periods per year). The T-step-ahead forecast, denoted in the paper by ES_{t+T}, is then

$$ES_{t+T} = \bar{S}_t / P_{t+T-N}. \tag{6}$$

By contrast, the "standard" Holt–Winters method for "level + seasonal" may be written in the present notation as

$$\bar{S}_t = \alpha(S_t/Q_t) + (1-\alpha)\bar{S}_{t-1} \tag{4A}$$

$$Q_t = \gamma \frac{S_t}{\bar{S}_t} + (1-\gamma)Q_{t-N}, \tag{5A}$$

$$ES_{t+T} = \bar{S}_t Q_{t+T-N}, \tag{6A}$$

where Q denotes the multiplicative seasonal factor. Comparing these equations to the originals, we see that the principal difference between them lies in the form of the seasonal factor; the term in ONR 52 is the "reciprocal" of the term in Eq. (5A). The reasons for this change are not documented. Whatever the reason, we may speculate that placing the smoothed term in the denominator will achieve greater numerical stability than the original form. A second difference is that Eq. (5) uses $\bar{S}_{t-1}$, whereas Eq. (5A) uses $\bar{S}_t$. The lagged subscript arises naturally from the solution of the original equations (1) and (2) in Holt (1957), which leads to the updating Eqs. (4) and (5). Why then do we use the non-lagged value in "standard" Holt–Winters? The quick answer is that Winters (1960) uses the non-lagged value, but the reasons for that choice are unclear. Interestingly, in their development of an underlying stochastic model for the multiplicative Holt–Winters scheme, Ord, Koehler, and Snyder (1997) argue for the use of the lagged term in order to produce a viable state–space model. These observations suggest that the use of lagged values should become standard practice, although the differences will typically be small.

3. A class of forecasting methods

Overall, the various methods described in the paper match up well with current practice, as can be seen from Table 1, which uses Pegels' (1969) classification of methods; see Makridakis et al. (1998, pages 170–71) for further discussion. Evidently, we should refer to the whole class as Holt's methods, not just the nonseasonal trend scheme. Note that "Holt's method" as defined in the literature appears only implicitly as a special case [5(S)] in the paper.

Table 1
Classification of the Holt methods using Pegels' (1969) scheme

Trend component	Seasonal component		
	None	Additive	Multiplicative
None	1	5 (S)	2
Additive	5 (S)	5	6
Multiplicative	3	–	4

The Arabic numerals in the table refer to the relevant sections of Holt's paper; S denotes a special case.

4. Conclusion

The class of forecasting methods summarized in Table 1 has proved very successful over the years, as seen in various forecasting competitions. See, for example, the M3 Competition in Makridakis and Hibon (2000) and the ensuing discussion in the *International Journal of Forecasting* (2001, vol. 17, no. 4).

Forty-six years is a long time to wait for a key paper to appear in a journal! Nevertheless, we hope that its appearance will provide greater accessibility to a seminal paper, and we are grateful to Professor Holt for allowing us to publish the paper.

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