The Part-Time Parliament

Leslie Lamport, 1998

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Outline

Introduction

Interesting information about 'The Part-Time Parliament':

- the algorithm was first described in 1989 as a technical report [1]
- the paper was submitted in 1990... and accepted in 1998
- several other publications, such as [2, 3, 4], had explaining it as their main purpose...

Some people do use the described **Paxos** protocol:

- Google uses Paxos in their Chubby distributed lock service
- IBM supposedly uses Paxos in their SAN Volume Controller product
- Microsoft uses Paxos in the Autopilot cluster management service
- WANDisco uses Paxos in their replication technology
- Keyspace uses Paxos as its basic replication primitive

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The paper mostly uses an analogy with the fictional ancient island of Paxos, where:

- The parliament's (group's) legislators (processes) and their messengers (communication channels) may be unpredictably absent;
- There may be several ballots for each decree to be passed (rounds in each consensus instance);
- Legislators – and messengers – may be lazy or absent, but not dishonest (non-Byzantine failure model*)

* The algorithm was extended later, however, to tolerate Byzantine failures [5]

Definition

Consensus is the process of agreeing on one single result among a group of participants. Assuming a collection of processes that can propose values, a consensus algorithm ensures that:

- only a single value among the proposed ones may be chosen;
- if no value has been proposed, then no value is chosen;
- a process never learns that a value has been chosen unless it really has been.

Some notations used

A value is chosen after a series of numbered **ballots**. For each ballot B, we have that:

- B_{bal} is its ballot number;
- B_{qrm} is a nonempty finite set of processes (quorum);
- B_{vot} is the set of processes who voted in B – each process may decide to participate (vote) or not in B;
- B_{val} is the value being voted on in that ballot.

A ballot is said to be *successful* when $B_{qrm} \subseteq B_{vot}$ – which means that every quorum member voted.

Some more notations

In each ballot, each process can cast a vote (accept or not a given proposed value). A vote v has the components:

- v_p, which is the process p who cast it;
- v_{bal}, the number of the ballot for which it was cast;
- v_{val}, a value voted on.

For the set β of ballots, the set $Votes(\beta)$ is defined as $\{v : \exists B \in \beta \mid v_p \in B_{vot} \wedge v_{bal} = B_{bal} \wedge v_{val} = B_{val} \}$

Still more notations...

We then define *MaxVote(b, p, β)* as the vote v with highest v_{bal} in $\{v \in Votes(\beta) : (v_p = p) \wedge (v_{bal} < b)\} \cup \{null\}$,
where *null* means that p didn't vote in any $B \in \beta$ ($null_{bal} = -\infty$)

We also define *MaxVote(b, Q, β)* as the vote v, such that v_{bal} is the highest among all *MaxVote(b, p, β)* with $p \in Q$

With such notations, we can specify the conditions which ensure consistency and allow for progress.

Consistency conditions

Consistency is guaranteed if the following conditions are ensured:

- $B1(\beta)$
  $\forall B, B' \in \beta : (B \neq B') \Rightarrow (B_{bal} \neq B'_{bal})$

- $B2(\beta)$
  $\forall B, B' \in \beta : B_{qrm} \cap B'_{qrm} \neq \emptyset)$

- $B3(\beta)$
  $\forall B, B' \in \beta :$
  $(MaxVote(B_{bal}, B_{qrm}, \beta)_{bal} \neq -\infty) \Rightarrow$
  $\Rightarrow (B_{val} = MaxVote(B_{bal}, B_{qrm}, \beta)_{val})$

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Proof sketch

Lemma. If $B1(\beta)$, $B2(\beta)$ and $B3(\beta)$ hold, then

$$((B_{qrm} \subseteq B_{vot}) \wedge (B'_{bal} > B_{bal})) \Rightarrow (B'_{val} = B_{val})$$

for any B, B' in β.

Proof of Lemma. Let $\Psi(B,\beta)$ be defined as:

$$\Psi(B,\beta) = \{B' \in \beta : (B'_{bal} > B_{bal}) \wedge (B'_{val} \neq B_{val})\}$$

Assume that $B_{qrm} \subseteq B_{vot}$. By contradiction, assume that $\Psi(B,\beta) \neq \emptyset$.

(1) As $\Psi(B,\beta) \neq \emptyset$, we pick $C \in \Psi(B,\beta)$, such that $C_{bal} = min\{B'_{bal} : B' \in \Psi(B,\beta)\}$

(2) From (1) and from the definition of $\Psi(B,\beta)$, we have that $C_{bal} > B_{bal}$

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(5) From (4) and the definition of $MaxVote(C_{bal}, C_{qrm}, \beta)$, we know that such vote exists in $Votes(\beta)$ (i.e. it is not *null*).

(6) From (5) and $B3(\beta)$, we have that $MaxVote(C_{bal}, C_{qrm}, \beta)_{val} = C_{val}$

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(9) From (8), and since $MaxVote(C_{bal}, C_{qrm}, \beta)_{val} \neq B_{val}$, we have that $MaxVote(C_{bal}, C_{qrm}, \beta) \in Votes(\Psi(B, \beta))$

(10) By the definition of $MaxVote(C_{bal}, C_{qrm}, \beta)$, we have that $MaxVote(C_{bal}, C_{qrm}, \beta)_{bal} < C_{bal}$

(11) From (10), we have that $\exists B' : B' \in \Psi(B, \beta) \wedge B'_{bal} < C_{bal}$, which means that

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which is a contradiction with (1). $\square$

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With the lemma, we show that, for a given set β of ballots which hold $B1$, $B2$ and $B3$, then any two successful ballots decided the same value.

Theorem 1. If $B1(\beta)$, $B2(\beta)$ and $B3(\beta)$ hold, then

$$((B_{qrm} \subseteq B_{vot}) \wedge (B'_{qrm} \subseteq B'_{vot})) \Rightarrow (B'_{val} = B_{val})$$

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If there are enough correct (non-faulty) processes, then a new ballot may be conducted while $B1$, $B2$ and $B3$ are preserved.

Theorem 2. Let b be a ballot number, and let Q be a set of processes such that $b > B_{bal}$ and $Q \cap B_{qrm} \neq \emptyset$, for all $B \in \beta$. If $B1(\beta)$, $B2(\beta)$ and $B3(\beta)$ hold, then there is a ballot B' with $B'_{bal} = b$ and $B'_{qrm} = B'_{vot} = Q$ such that $B1(\beta \cup \{B'\})$, $B2(\beta \cup \{B'\})$ and $B3(\beta \cup \{B'\})$ hold.

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- $B1(\beta \cup \{B'\})$ follows from $B1(\beta)$ and the fact that $B'_{bal} = b$, and the assumption about b.
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Preliminary protocol

A protocol can be derived directly from $B1(\beta)$, $B2(\beta)$ and $B3(\beta)$, so that all these properties are maintained throughout its execution and a value may* be agreed upon.

- To maintain $B1(\beta)$, whenever p starts a new ballot B, it assigns a sequence number *seq*, which was never used by p. To avoid collisions, p's unique id is appended. Then $B_{bal} = (seq, p_{id})$.
- To maintain $B2(\beta)$, each ballot B has B_{qrm} as the majority set of the processes. As it is impossible to choose two disjoint majority sets from the same group*, $B2(\beta)$ is satisfied.
- To maintain $B3(\beta)$, before initiating a ballot B, p has to find out what is $MaxVote(B, B_{qrm}, \beta)$, which is done by finding the $MaxVote(B, q, \beta)$ for each q in B_{qrm}.
  - If $MaxVote(B, B_{qrm}, \beta) = null$, then B_{val} can be anything; otherwise $B_{val} = MaxVote(B, B_{qrm}, \beta)_{val}$.

* If half of the processes is faulty, consistency is still respected and there is no deadlock, but the ballot B does not succeed

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Preliminary protocol

Phase 1) Find $MaxVote(B, B_{qrm}, \beta)_{val}$

Phase 1a) p picks a ballot number b and sends $NextBallot(b)$ to other processes

Phase 1b) each process q responds with $LastVote(b, v)$ to p, where v is the vote cast by q with the highest ballot number less than b (or $null$)

Phase 2) Execute the ballot B

Phase 2a) after receiving $LastVote(b, v)$ from a majority Q of the processes, p initiates the ballot with number b and value val such that $B3(\beta)$ is maintained, by sending $BeginBallot(b, val)$ to every process in Q

Phase 2b) upon receiving $BeginBallot(b, val)$, q will not vote on it if it has already sent $LastVote(b', v')$, such that $b' > b$; otherwise, it sends $Voted(b, q)$ to p and records this vote in its memory

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Phase 1) Find $MaxVote(B, B_{qrm}, \beta)_{val}$

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Preliminary protocol

Phase 3) Announce the result of consensus

Phase 3a) if p receives $Voted(b, q)$ from a majority Q of processes, then it sends $Success(val)$ to all processes

Phase 3b) upon receiving $Success(val)$, the process learns that the group achieved consensus on val.

Every phase of this protocol is optional. Consistency is still maintained, although progress is not guaranteed.

Each process q must keep *MaxVote(b, q, β)* for each ballot numbered b and remember all $LastVote(b, v)$ messages it has sent.

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Basic protocol

In the preliminary protocol, much information must be kept. A simpler, yet correct, protocol can be derived. Each process p keeps then only:

lastTried$[p]$ – the number of the last ballot p tried to initiate ($-\infty$ if none)

prevVote$[p]$ – the highest ballot for which p ever voted ($-\infty$ if none)

nextBal$[p]$ – the highest ballot number b for which p sent $LastVote(b, v)$ ($-\infty$ if none)

In the preliminary protocol, $LastVote(b, v)$ represented a "promise" from q not to vote in any ballot numbered $b' : LastVote(b, v)_{bal} < b' < b$

Here, $LastVote(b, v)$ represents a stronger "promise" from q not to vote in any ballot numbered $b' < b$

Basic protocol

Phase 1)

Phase 1a) p picks a ballot number $b > lastTried[p]$ and sends $NextBallot(b)$ to other processes

Phase 1b) upon receipt of $NextBallot(b)$ from p, **if** $\boldsymbol{b > nextBal[q]}$, q sets $nextBal[q]$ to b and sends $LastVote(b, v)$ to p, where $v = prevVote[q]$

Phase 2)

Phase 2a) after receiving $LastVote(b, v)$ from a majority Q, **where** $\boldsymbol{b = lastTried[p]}$, p initiates ballot number b with value val such that $B3(\beta)$ is maintained, by sending $BeginBallot(b, val)$ to every process in Q

Phase 2b) upon receiving $BeginBallot(b, val)$ **with** $\boldsymbol{b = nextBal[q]}$, q votes on ballot number b, set $prevVote[q]$ to this vote and sends $Voted(b, q)$ to p

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Consistency is still maintained, and progress is still not guaranteed.

Anyway, it is impossible to guarantee termination of consensus in an asynchronous system [6]...

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Complete protocol

- Same steps of the basic protocol
- Each process performs phases 1b – 3b as soon as they can
- Some process must execute phase 1a
  - Never initiating a ballot prevents termination
  - However, too frequent initiation also does it
  - A single process (leader) should be the only one initiating ballots
- Knowledge of time, and of a maximum message delay Δ, is needed
  - Not exactly an asynchronous system anymore

Complete protocol

- A minimal execution of the basic protocol would take at most 5Δ
- A leader would ensure progress, but that requires leader election
  - when a leader seems to have failed, a new one should be elected (the election takes time T_{el})
  - if not enough responses to (1a) or (2a) arrive after 2Δ, a ballot B had started before T_{el}
    - or too many failures or message losses occurred...
  - in this case, a ballot with number $b' > B_{bal}$ may be started by the leader – *NACK*s to (1a) or (2a) may have carried B_{bal}
  - in the worst case, the leader receives a *NACK* for (2a), 4Δ after T_{el}
  - as no other process initiated any ballot after T_{el}, the ballot numbered b' would succeed 5Δ after the *NACK*
- The complete protocol, with leader election, terminates at most after $T_{el} + 9\Delta$
  - if a majority of processes didnt't fail during that time
  - and if the leader didn't fail for 9Δ after T_{el}
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The **Paxos** Protocol

Multiple consensus instances

- Each instance has an id I_{id} – and there should be a leader
- Phases (1a) and (1b) can be executed for many of them – e.g. p sends $NextBallot(b, n)$, for all instances I with $I_{id} > n$
- The response from q (1b) would contain a $LastVote(I_{id}, b, v)$ message for each instance $I : I_{id} > n$ that q voted in
- Another process can send $NextBallot(b', n')$, with $b' > b$, and p's (2a) messages would be ignored from then on
- This brings the protocol latency, on most cases, down to 3Δ
  - no leader changes, enough correct processes and enough messages
- If each process sent messages (2b) to every other one, we'd have 2Δ
- If instance I^A was passed (phase (3b) concluded) before I^B had a value proposed (phase (2a)), then $I^A_{id} < I^B_{id}$

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The **Paxos** Protocol

Relevance to computer science

- It may be used to implement state-machine replication
  - A system (e.g. a database) may be implemented as a set of states and deterministic state changes
  - The group of processes would be the set of replicas of the system
  - Each state changed would be agreed upon by means of consensus (an ordered instance of Paxos)
  - One of the replicas might be elected as leader, and client requests (state changes) would be sent to it
  - As all replicas would have a consistent set of state changes, so would be their state

- Many derivations from Paxos have been published since it was first described
  - Cheap Paxos, Fast Paxos, Generalized Paxos, Multi-Paxos, Byzantine Paxos, Fast Byzantine Paxos, Fast Byzantine Multi-Paxos, Ring Paxos

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Thank you!

Questions?