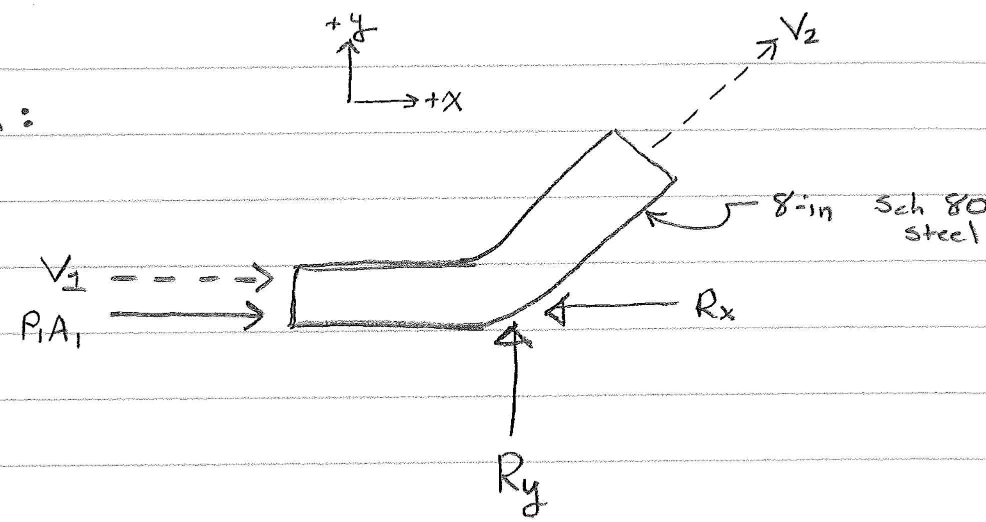
Problem 16.16:



$Q = 6.5\ ft^3/s$

H_2O @ 80°F

Note that since pipe discharges into atm $P_2 = 0$ psig and we will be using P_1 as a gauge pressure.

First find P_1 from the general energy eq

$$\frac{P_1}{\gamma} + \underbrace{\cancel{\frac{V_1^2}{2g}}}_{V_1=V_2} + \underbrace{\cancel{z_1}}_{z_1=z_2} + \underbrace{\cancel{h_A}}_{\text{no pumps}} - \underbrace{\cancel{h_R}}_{\text{no motors}} - h_L = \underbrace{\cancel{\frac{P_2}{\gamma}}}_{P_2 \text{ at atm}} + \cancel{z_2} + \cancel{\frac{V_2^2}{2g}}$$

$$\frac{P_1}{\gamma} - h_L = 0 \Rightarrow P_1 = \gamma h_L$$

h_L is the minor loss in the elbow: $h_L = K\left(\frac{v^2}{2g}\right)$ (10-1)

where

$$K = \frac{L_e}{D} f_T \qquad (10\text{-}8)$$

and therefore $P_1 = \gamma f_T \left(\frac{L_e}{D}\right)\frac{v^2}{2g}$

$$V = \frac{Q}{A} = \frac{6.5\,ft^3/s}{0.3174\,ft^2} = 20.48\,ft/s$$

From table F.2 (pg 501)

$f_T = 0.014$ (table 10.5 pg 242)

$(L_e/D) = 16$ (table 10.4)

$$P_1 = \gamma f_T \left(\frac{L_e}{D}\right) \frac{V^2}{2g}$$

$$P_1 = \left(62.2\,\frac{lb}{ft^3}\right)(0.014)(16)\frac{(20.48\,ft/s)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} = 90.74\,\frac{lb}{ft^2}$$

From table A.1 (pg 488)

Now that we have P_1 we can compute forces:

<u>X-direction:</u>

$$P_1A_1 - R_x = \rho Q(V_{2x} - V_{1x}) = \rho Q(V_2\cos 45^\circ - V_1)$$

using $V_2 = V_1 = V$

$$P_1A_1 - R_x = \rho Q V(\cos 45^\circ - 1) \Rightarrow R_x = P_1A_1 - \rho Q V(\cos 45^\circ - 1)$$

$$R_x = \left(90.74\,\frac{lb}{ft^2}\right)(0.3174\,ft^2) - \left(1.93\,\frac{slugs}{ft^3}\right)\left(6.5\,\frac{ft^3}{s}\right)\left(20.48\,\frac{ft}{s}\right)(-0.293)$$

Table A.2 pg 489

$$R_x = 28.80\,lb + \underbrace{75.28\,\frac{slugs\cdot ft}{s^2}}_{lb} \Rightarrow \boxed{R_x = 104.1\,lb}$$

Y - direction:

$$R_y = \rho Q\,(V_{2y} - V_{1y}) = \rho Q\,(V_2 \sin 45 - 0)$$

using $V_2 = V$

$$R_y = \rho Q V \sin 45^\circ$$

$$R_y = \left(1.93\ \frac{\text{slugs}}{ft^3}\right)\left(6.5\ \frac{ft^3}{s}\right)\left(20.48\ \frac{ft}{s}\right)(0.707) = 181.64\ lb$$

$$\boxed{R_y = 181.6\ lb}$$