Newton's 2nd Law

Newton's 2nd Law can be states as follows. A particle has an acceleration proportional to the resultant force acting on it. The direction of the acceleration is the same as the direction of the resultant force. The proportionality constant is the mass of the particle. This is more simply put by the following equation:

Newton's second law: $\vec{F} = m\vec{a}$

If we have multiple forces acting on the particle we can sum them as vectors:

Newton's second law with multiple forces: $\sum \vec{F} = m\vec{a}$

Note that if no forces are acting on a particle the acceleration must be zero and the velocity a constant. If the particle starts at rest it remains at rest and if it begins in motion it will remain in motion with that same velocity, until it is acted on by a force. So Newton's first law follows from Newton's second law.

SI Units

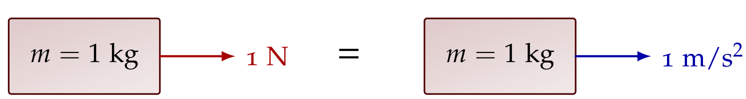


Figure 1: A force of 1 newton gives a 1-kilogram mass an acceleration of 1 $\mathrm{m/s^2}$.

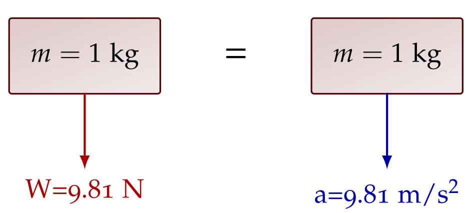


Figure 2: An object with a mass of 1-kilogram has a weight of 9.81 N and and will experience an acceleration of 9.81 $\mathrm{m/s^2}$ during free fall.

USCS Units

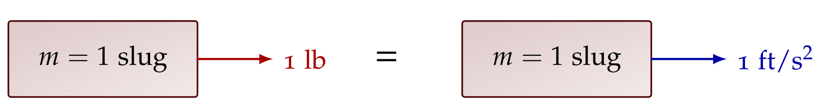


Figure 3: A force of 1 pound applied to a 1-slug mass produces an acceleration of 1 $\mathrm{ft/s^2}$.

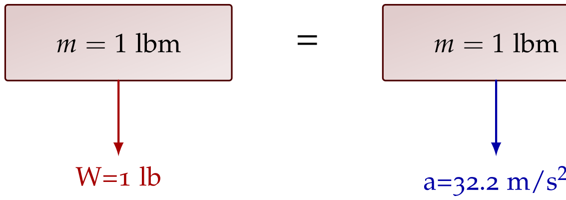


Figure 4: An object with a mass of 1 lbm has a weight of 1 pound and and will experience an acceleration of 32.2 ft/s^2 during free fall. Note that 32.2 lbm = 1 slug.

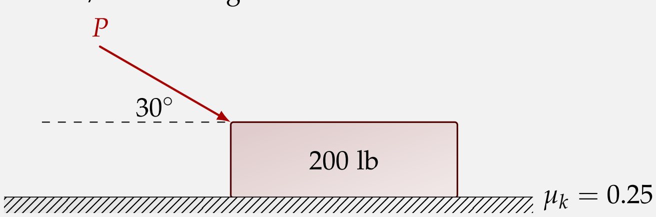
Newton's 2nd law in rectangular coordinates

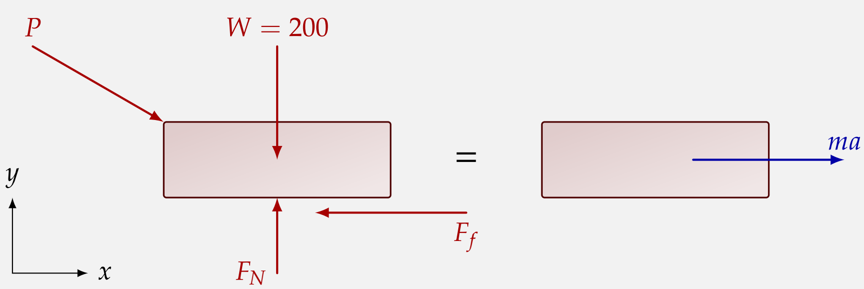
In rectangular coordinates Newton's 2nd law takes on the following form in two dimensions.

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

Sample Problem 3.1

A 200-lb block rests on the floor. The coefficient of kinetic friction between the block and the floor is $\mu_k = 0.25$. Find the magnitude of the force P required to give the block an acceleration of 10 ft/s^2 to the right.





First we find the mass of the block

$$m = \frac{W}{g} = \frac{200 \text{ lb}}{32.2 \text{ m/s}^2} = 6.21 \text{ slug}$$

Newton's 2$^{\text{nd}}$ law in the x-direction is:

$$\sum F_x = ma_x$$
$$P\cos 30 - F_f = (6.21 \text{ slug})(10 \text{ m/s}^2) = 62.1 \text{ lb}$$

3.1 – cont

The force of friction in $F_f = \mu_K F_N = 0.25N$. Substituting this into the previous equation we obtain the following.

$$\boxed{P\cos 30 - 0.25F_N = 62.1 \text{ lb}}$$

Next, in the y-direction:

$$\sum F_y = ma_y$$
$$\boxed{F_N - 200 - P\sin 30 = 0}$$

We now have two equations with the two unknowns, P and F_N. First solve the y-equation for N:

$$F_N = 200 + P\sin 30$$

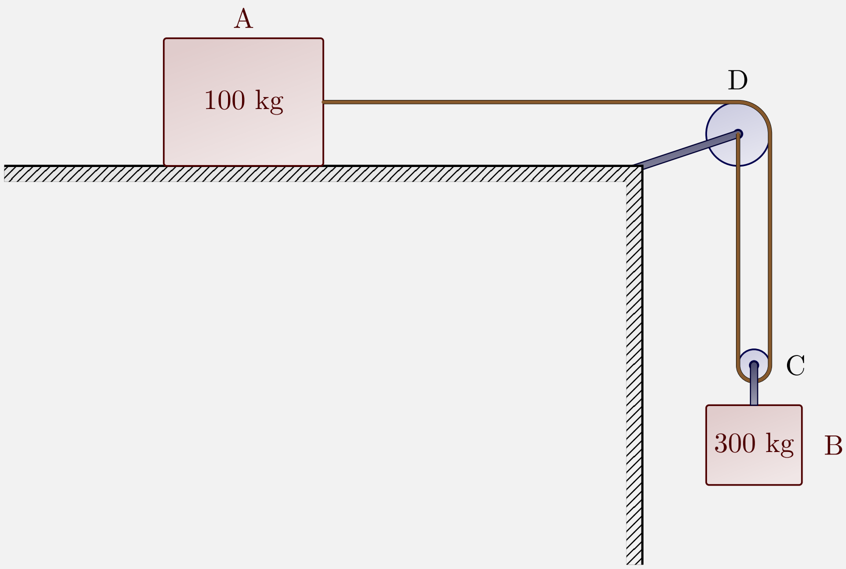
and substitute it back into the x-equation:

$$\begin{aligned}
P\cos 30 - 0.25F_N &= 62.1 \\
P\cos 30 - 0.25\,(200 + P\sin 30) &= 62.1 \\
P\cos 30 - 50 - 0.25P\sin 30 &= 62.1 \\
(\cos 30 - 0.25\sin 30)\,P &= 112.1 \\
(0.741)\,P &= 112.1 \\
\boxed{P = 151.3 \text{ lb}}
\end{aligned}$$

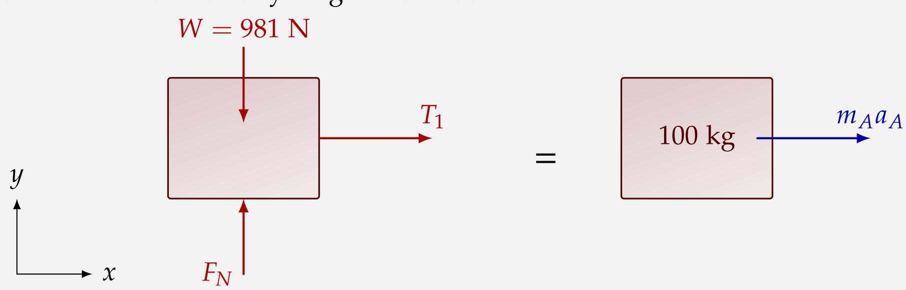
Sample Problem 3.2

The two blocks labeled A and B start from rest. Any friction between block A and the floor can be neglected. The pulleys are frictionless and massless. Find the acceleration of each block and the tension in the cable.

3.2 – cont



Start with the free-body diagram of block A.

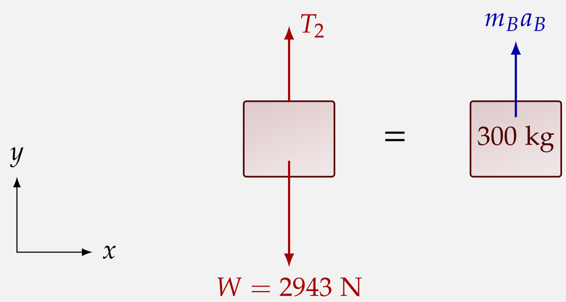


Newton's 2nd law in the x-direction is:

$$\sum F_x = ma_x$$
$$T_1 = m_A a_A$$
$$\boxed{T_1 = 100a_A} \tag{1}$$

Next, draw the free-body diagram of block B. Note that I've draw the acceleration in the positive direction even though I know the block is accelerating downward. In the end I expect to find a negative value for a_B.

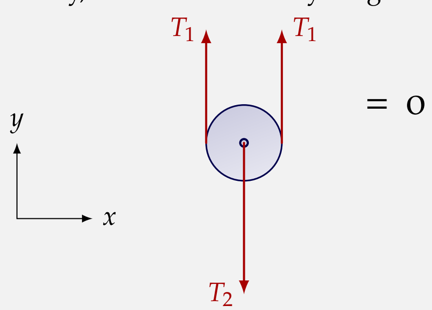
3.2 – cont



Newton's 2nd law in the y-direction is:

$$\sum F_y = ma_y$$
$$T_2 - 2943 = 300a_B$$
$$\boxed{T_2 = 2943 + 300a_B} \tag{2}$$

Finally, draw the free-body diagram of pulley C.



Since the pulley is massless we have set the right-hand side of free-body diagram that represents $m_{\text{pulley}}a_{\text{pulley}}$ to zero. If we sum the forces in the y-direction we have

$$\sum F_y = 0$$
$$2T_1 - T_2 = 0$$
$$\boxed{T_2 = 2T_1} \tag{3}$$

The three equations that are boxed above include the four unknowns, T_1, T_2, a_A, and a_B. To arrive at a fourth equation we have to take into consideration the constraint imposed on the kinematics. If block A moves to the right a certain distance, block B will move down by half that distance.

$$y_B = -\frac{1}{2}x_A$$

Differentiating twice with respect to time we have

$$\boxed{a_B = -\frac{1}{2}a_A} \tag{4}$$

3.2 – cont

The four equations which are boxed above include the four unknowns, a_A, a_B, T_1, T_2. Here is how I solved this. First I substituted eq. 4 into eq. 2

$$\begin{aligned} T_2 &= 2943 + 300a_B \\ T_2 &= 2943 + 300\left(-\frac{1}{2}a_A\right) \\ T_2 &= 2943 - 150a_A \end{aligned}$$

Next substitute eq. 3 into the previous expression.

$$\begin{aligned} T_2 &= 2943 - 150a_A \\ (2T_1) &= 2943 - 150a_A \\ T_1 &= 1471.5 - 75a_A \end{aligned}$$

And finally substitute eq. 1 into the previous one.

$$\begin{aligned} T_1 &= 1471.5 - 75a_A \\ (100a_A) &= 1471.5 - 75a_A \\ 175a_A &= 1471.5 \\ & \boxed{a_A = 8.41 \text{ m/s}^2} \end{aligned}$$

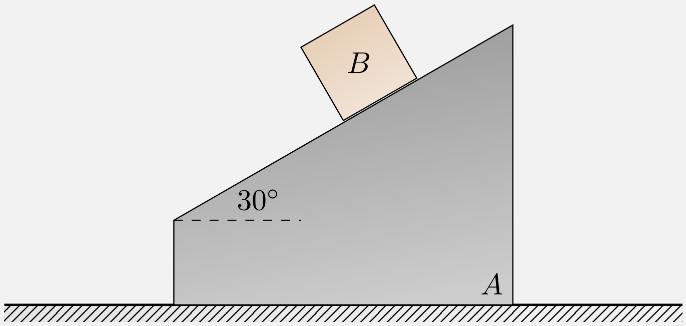
The remaining unknown are then easily found.

$$\boxed{\begin{aligned} a_B &= -4.20 \text{ m/s}^2 \\ T_1 &= 841 \text{ N} \\ T_2 &= 1682 \text{ N} \end{aligned}}$$

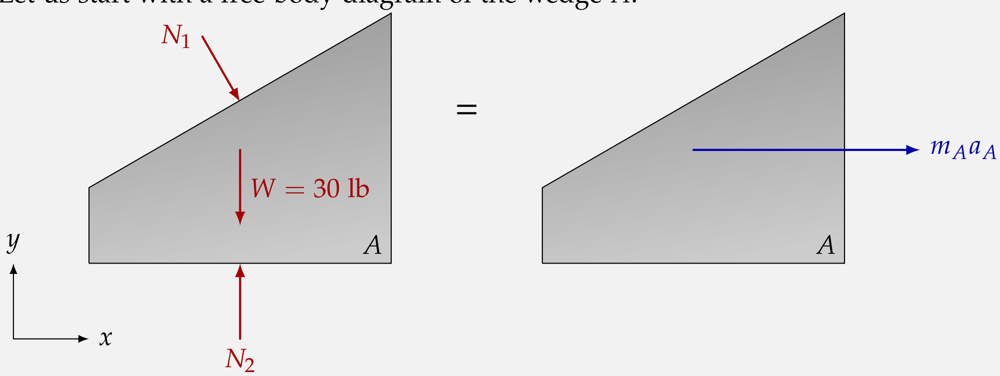
Sample Problem 3.3

The 12-lb block B slides on the 30-lb wedge A. Both the block and wedge start from rest. Friction can be neglected between all surfaces. Find the acceleration of the wedge and the acceleration of the block. [Hint: You will need to make use of the acceleration of the block relative to the wedge.]

3.3 – cont



Let us start with a free-body diagram of the wedge A.



The vector N_1 represents the normal force from block B. The force is perpendicular to the wedge since there is no friction. Applying Newton's 2nd law in the horizontal direction we obtain

$$N_1 \sin 30 = m_A a_A$$

Newton's 2nd law in the vertical direction would give us an expression containing N_2. Since there is no friction we won't need N_2 in what follows.

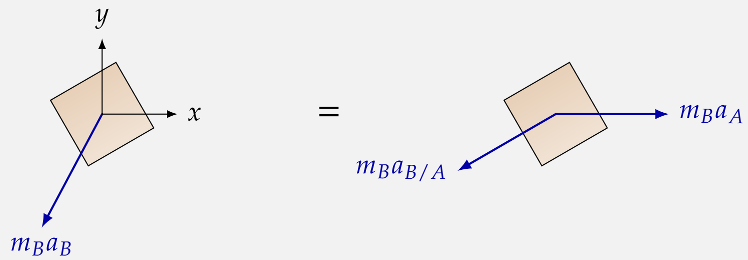
We now discuss block B. This is the tricky part of the problem. We know that the motion of block B is constrained to the top surface of the wedge. If the wedge didn't move that would mean that the acceleration of block B was at an angle of 30°. If the wedge moves as B slides down, the direction of block B's acceleration vector is unknown. But what we do know is the the direction of block B's acceleration vector *with respect to* wedge A is 30°.

Therefore the vector $a_{B/A}$ has a direction of 30°. Now when we draw the kinetic diagram for block B we have to draw the acceleration vector of block B. What this looks like is shown to the left in the figure shown below. The problem is, I don't know the angle that a_B makes. In the

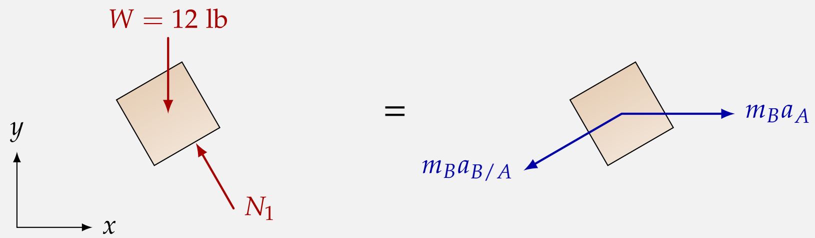
3.3 – cont

picture I just guessed it for the sake of illustration.

Instead I express the vector a_B as the sum of the two vectors $\vec{a}_B = \vec{a}_{B/A} + \vec{a}_A$. But since I know the direction of these two vectors I can accurately represent them on my kinetic diagram. This is shown in the right figure below where $\vec{a}_A$ points to the right and $\vec{a}_{B/A}$ points at 30°.



The free-body and kinetic diagram for block B is



From the above figure we can express Newton's 2nd law in the horizontal direction as

$$-N_1 \sin 30 = m_B a_A - m_B a_{B/A} \cos 30$$

and in the vertical direction

$$N_1 \cos 30 - W_B = -m_B a_{B/A} \sin 30$$

We now have three unknowns $N_1, a_{B/A}, a_A$ and three equations:

$$\begin{aligned} N_1 \sin 30 &= m_A a_A \\ -N_1 \sin 30 &= m_B a_A - m_B a_{B/A} \cos 30 \\ N_1 \cos 30 - W_B &= -m_B a_{B/A} \sin 30 \end{aligned}$$

To solve this I did the following. I solved the first equation for $N_1 = m_A a_A / \sin 30$ and substituted it into the remaining two equations:

$$\begin{aligned} -m_A a_A &= m_B a_A - m_B a_{B/A} \cos 30 \\ m_A a_A \cot 30 - W_B &= -m_B a_{B/A} \sin 30 \end{aligned}$$

I multiple the top equation by $\sin 30$ and the bottom equation by $\cos 30$ and then take the difference of the two to solve for a_A.

3.3 – cont

$$a_A = \frac{W_B \sin 30 \cos 30}{m_A + m_B \sin^2 30} = \frac{12 \sin 30 \cos 30}{0.932 + 0.373 \sin^2 30} = 5.07 \text{ ft/s}^2$$

$$a_{B/A} = \frac{g(m_A + m_B) \sin 30}{m_A + m_B \sin^2 30} = \frac{32.2(0.932 + 0.373) \sin 30}{0.932 + 0.373 \sin^2 30} = 20.5 \text{ ft/s}^2$$

Sample Problems

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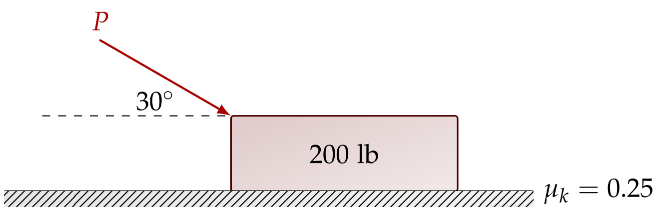

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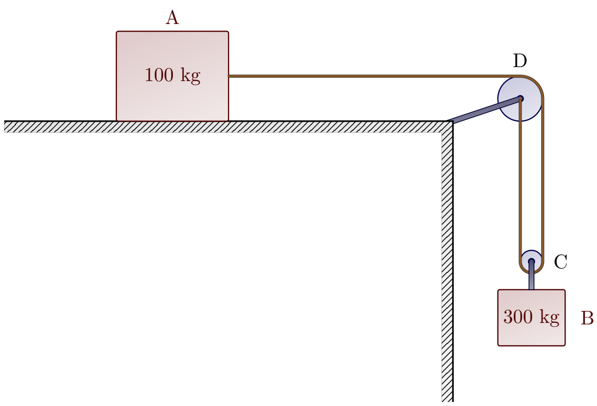

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