Entanglement in N well system

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Let us assume that we have 2 wells A i B or more precisely 2 quantum systems A and B.

In such case the total Hamiltonian of the system is
Ha(t) = Ea(t) |A><A| and Hb(t)= Eb(t)|B><B|.

If we cosnider the interaction betwenne A and B we write it as Hab=Eint(t)(|A><B|+|B><A|).

However in general case we shall consider even more generalized interaction term

Hab1=Eint1(t)[f1(|A><B|)(|B><A|)+(|A>|B>)(<A|<B|)f2+f3(|A><B|)(|B><B|)+
f4(|A><B|)(|A><A|)+f5(|B><A)(|A><A|)+f6(|B><A)(|B><B|) +f71(|A><B|)(|B><A|)+
f72(|B><A|)(|A><B|)].

All functions f1, .. f6, f71, f72 has deeper physical meaning in regards to self fields etc. At first we neglect Hab1 term of Hamiltonian.

Thus total Hamilton of the system (A and B) is H=Ha+Hb+Hab and thus we have

H=Ea(t) |A><A| + Eb(t)|B><B|+Eint(t)(|A><B|+|B><A|). The strength of interaction can be arbitrary so it might happen that Eint>>Ea, Eb so it is non-perturbative interaction or perturbative if Eint <<Ea, Eb or Eint, Ea and Eb can have any values.

At first let us assume for the simplicity that A system has (at least) two eigenstates that are orthogonal so <A1|A2>=0.

In such case |A>=a1 |A1> + a2 |A2>. Normalization condition requires <A|A>=1. (number of particles in the system A is preserved).

In similar fashion we can assume that B system has (at least) two eigenstates that are orthogonal so <B1|B2>=0. Normalization condition requires <B|B>=1 (number of particles in the system B is preserved).

In such case |B>=b1 |B1> + b2 |B2>. Complex conjugate of a1 we denote as a1* , etc ...

Thus we have
|A><B|=(a1 |A1> + a2 |A2>)(b1* <B1| + b2* <B2|)=
=a1 b1* |A1><B1|+ a2 b2* |A2><B2|+a1b2* |A1><B2|+a2b1* |A2><B1|,
and
|B><A|=(b1 |B1> + b2 |B2>)(a1* <A1| + a2* <A2|)=
=b1a1* |B1><A1|+ b2 a2* |B2><A2|+b1 a2* |B1><A2|+b2 a1* |B2><A1|.
and
|A><A|=(a1 |A1> + a2 |A2>)(a1* <A1| + a2* <A2|)=
=a1 a1* |A1><A1|+ a2 a2* |A2><A2|+a1a2* |A1><A2|+a2a1* |A2><A1|,
and
|B><B|=(b1 |B1> + b2 |B2>)(b1* <B1| + b2* <B2|)=
=b1 b1* |B1><B1|+ b2 b2* |B2><B2|+b1b2* |B1><B2|+b2b1* |B2><B1|.

Thus total Hamiltonian can be written as

H=Ea(a1 a1* |A1><A1|+ a2 a2* |A2><A2|+a1a2* |A1><A2|+a2a1* |A2><A1|)+
+Eb (b1 b1* |B1><B1|+ b2 b2* |B2><B2|+b1b2* |B1><B2|+b2b1* |B2><B1|)+
+Eint(a1 b1* |A1><B1|+ a2 b2* |A2><B2|+a1b2* |A1><B2|+a2b1* |A2><B1|)+
+Eint(b1 a1* |B1><A1|+ b2 a2* |B2><A2|+b1 a2* |B1><A2|+b2 a1* |B2><A1|).

It shall be underlined that system A can have more than 2 levels and for example Na eigenstates.
It shall be underlined that system B can have more than 2 levels and for example Nb eigenstates.
In particular |A(B)1>=|1a(b)> (spin-up in A(B) system) and |A(B)2>=|2a(b)> (spin-down in A (B) system). However we do not need to limit ourself to spin.

We can notice that in order to obtain the state close to one of the Bell state
|psi>=(c1 |A1>|B2>+c2 |A2>|B1>) we need to promote terms in total Hamiltonian with operators |A1><B2| and |A2><B1| by increasing certain coefficient and decreasing another coefficients.

In order to obtain entanglement we need to bring systems A and B close and make strong interaction of them and later take them away. Once they are separated and measurement is taken on one of the subsystems A (or B) „spooky action on the distance" might take place on B (A system). In such way get the EPR paradox or violation of Bell inequality. The entanglement take place with proper engineering of interaction by proper biasing of various gates present in the system.

Concrete computations and examples are yet to be given .

In similar way we can conduct the reasoning for A, B and C systems. In priniciple we can treat in this way N interacting systems (N particles) coming from N wells. Thus we can end up with entanglement of N particles.

More rigorous treatment of system dynamics with time includes more detailed description of decoherence and various non-equbilbrium techniques. It then requires the use of Keldysh Green functions .