

MAT 4002 Homework 3

1. Let $f : X \rightarrow Y$ be a continuous map between topological spaces, and let A and B be subsets of X such that $\bar{A} = \bar{B}$. Prove that $f(A) = f(\bar{B})$.
2. (a) Let \mathbf{R}^n be endowed with Euclidean metric. Prove that for any $\mathbf{x} \in \mathbf{R}^n$ and $\epsilon > 0$, there exists $\mathbf{y} \in \mathbf{Q}^n$, $\delta \in \mathbf{Q} \cap (0, \infty)$ such that

$$\mathbf{x} \in B(\mathbf{y}, \delta) \subset B(\mathbf{x}, \epsilon)$$

(b) Prove that the family $\mathcal{B} := \{B(\mathbf{y}, \delta) \mid \mathbf{y} \in \mathbf{Q}^n, \delta \in \mathbf{Q} \cap (0, \infty)\}$ form a basis of the Euclidean topology of \mathbf{R}^n .