MAT 4002 Homework 2

1. Using the discrete metric, give an example such that

$$\overline{B(x,r)} \subset \bar{B}(x,r)$$

is a strict inclusion.

2. Let $(X, \tau_{\text{cofinite}})$ be equipped with cofinite topology. Suppose $\{x_n\}$ is a sequence in X such that $x_m \neq x_n$ whenever $m \neq n$. Show that $\{x_n\}$ is convergent to every $x \in X$.

3. Let $\mathcal{T}_{\text{left}}$ be the family of subsets U of $\mathbb{R}$ with the property that for every $x \in U$ there exists $\varepsilon > 0$ such that $(x - \varepsilon, x] \subseteq U$. Prove that $\mathcal{T}_{\text{left}}$ is a topology on $\mathbb{R}$.

4. Let (X, d) be a metric space and A a non-empty subset of X . For every $x \in X$, define

$$\operatorname{dist}(x, A) = \inf_{a \in A} \mathrm{d}(x, a).$$

Prove that $x \in \bar{A}$ if and only if $\operatorname{dist}(x, A) = 0$.