MAT 4002 Homework 11

1. A retraction of a space X onto a subspace A is a map $r : X \to A$ such that $r \circ i = id$, where $i : A \to X$ is the inclusion map.

   (a) Prove that there is no retraction map $r : D^2 \to S^1$

   (b) Our aim here is to show that any map $r : D^2 \to D^2$ has a fixed point. Suppose that, on the contrary, f has no fixed point; in other words, $f(x) \neq x$ for all $x \in D^2$. Using the pair of points $(x, f(x))$, construct a retraction $D^2 \to S^1$.

   Thus, we deduce that any map $r : D^2 \to D^2$ must have a fixed point. This is called Brouwer's Fixed Point Theorem.

2. Show that the Mobius strip M does not have a retraction onto its boundary circle B .

   (Hint: Recall that M is homotopy equivalent to S^1 . What is the map $i^* : \pi_1(B) \to \pi_1(M)$?)

3. (a) Show that $S^{n-1} = \{\mathbf{x} \in \mathbf{R}^n \mid |\mathbf{x}| = 1\}$ is a homotopy retract of $\mathbf{R}^n \setminus \{\mathbf{0}\}$.

   (b) Hence show that for a fixed $n > 2$, prove that no two of $\mathbf{R}, \mathbf{R}^2$ and $\mathbf{R}^n$ are homeomorphic.