MAT 4002 Homework 9

1. Prove that any continuous map $f : S^m \to S^n$, where $m < n$, is homotopic to a constant map.

   (Hint: use the Simplicial Approximation Theorem to show that f is homotopic to a non-surjective map h , and then use Homework 8 Question 3. Note that the simplicial complex

   $$V = \{1, \ldots, n+2\},\ \Sigma = P(V) \setminus \{V\}$$

   has topological realization $|(V, \Sigma)| \cong S^n$)

2. If $G_1 = \langle X_1 \mid R_1 \rangle, G_2 = \langle X_2 \mid R_2 \rangle$, write down a presentation for the product group $G_1 \times G_2$. Deduce that if G_1, G_2 are finitely presented, then so is $G_1 \times G_2$.

3. Show that $\langle a, b \mid ab = ba, a^5b^2, a^2b \rangle$ is the trivial group.

4. Show that $\langle a, b \mid a^2, b^2, aba = bab \rangle$ is isomorphic to $\langle c, d \mid c^2, d^3, cdcd \rangle$, which is both isomorphic to S_3 .