MAT 4002 Homework 8

1. Let $\alpha : S^n \to S^n$ be the antipodal map (defined by $\alpha(x) = -x$). Prove that α is homotopic to the identity if n is odd.

   (Hint: Do the case for $n = 1$ first. The other odd cases shall follow from there)

2. Let X be a contractible space and let Y be any space. Show that

   (a) X is path-connected;

   (b) $X \times Y$ is homotopy equivalent to Y ;

   (c) any two maps from Y to X are homotopic;

   (d) if Y is path-connected, any two maps from X to Y are homotopic.

3. For any two maps $f, g : X \to S^n$ such that $f(x) \neq -g(x)$ for all $x \in X$, show that f and g are homotopic.

4. The wedge $X \vee Y$ of two spaces X and Y , containing basepoints x and y , is the space obtained from the disjoint union of X and Y by identifying x and y . Often, the resulting space is independent of the choice of basepoints, in which case there is no need to specify them.

   Prove that the following spaces are homotopy equivalent:

   (a) $S^1 \vee S^1$,

   (b) the torus with one point removed,

   (c) $\mathbb{R}^2$ minus two points.

   (You can draw pictures to describe the homotopy between the spaces.)