Text as Data

Justin Grimmer

Associate Professor
Department of Political Science
Stanford University

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Structured Topic Models

1) Task:
    - Examine how document attention, topic content varies $\rightsquigarrow$ over time, across authors, or with general set of covariates
2) Objective Function

$$f(\boldsymbol{X}, \boldsymbol{\pi}, \boldsymbol{\Theta}, \boldsymbol{\alpha}, \boldsymbol{W})$$

Where:
    - $\boldsymbol{W}$ condition on information in document $\rightsquigarrow$ other potential modifications to objective function. Meta-data.
    - $f(\boldsymbol{X}, \boldsymbol{\pi}, \boldsymbol{\Theta}, \boldsymbol{\alpha}, \boldsymbol{W})$ may encode additional information $\rightsquigarrow$ layers of clustering, layers of topics, etc
3) Optimization
    - EM, Variational Approximation, Gibbs Sampling, ...
4) Validation $\rightsquigarrow$ many of the same methods from clustering
    - Semantic, Convergent, Discriminant, Predictive, Hypothesis validity
    - How do we avoid the electric machine critique?

LDA Revisited

$$
\begin{aligned}
\boldsymbol{\theta}_k &\sim \text{Dirichlet}(\mathbf{1}) \\
\boldsymbol{\pi}_i|\boldsymbol{\alpha} &\sim \text{Dirichlet}(\boldsymbol{\alpha}) \\
\boldsymbol{\tau}_{im}|\boldsymbol{\pi}_i &\sim \text{Multinomial}(1, \boldsymbol{\pi}_i) \\
x_{im}|\boldsymbol{\theta}_k, \tau_{imk} = 1 &\sim \text{Multinomial}(1, \boldsymbol{\theta}_k)
\end{aligned}
$$

LDA Revisited

$$
\begin{aligned}
\textbf{Unigram Model}_k &\sim \text{Dirichlet}(\mathbf{1}) \\
\textbf{Doc. Prop}_i &\sim \text{Dirichlet}(\textbf{Pop. Proportion}) \\
\textbf{Word Topic}_{im} &\sim \text{Multinomial}(1, \textbf{Doc. Prop}_i) \\
\text{Word}_{im} &\sim \text{Multinomial}(1, \textbf{Unigram Model}_k)
\end{aligned}
$$

A General Hierarchical Structure

LDA:

Pop. Proportion

A General Hierarchical Structure

LDA:

Pop. Proportion

Doc. Prop$_1$ **Doc. Prop**$_2$ $\cdots$ **Doc. Prop**$_N$

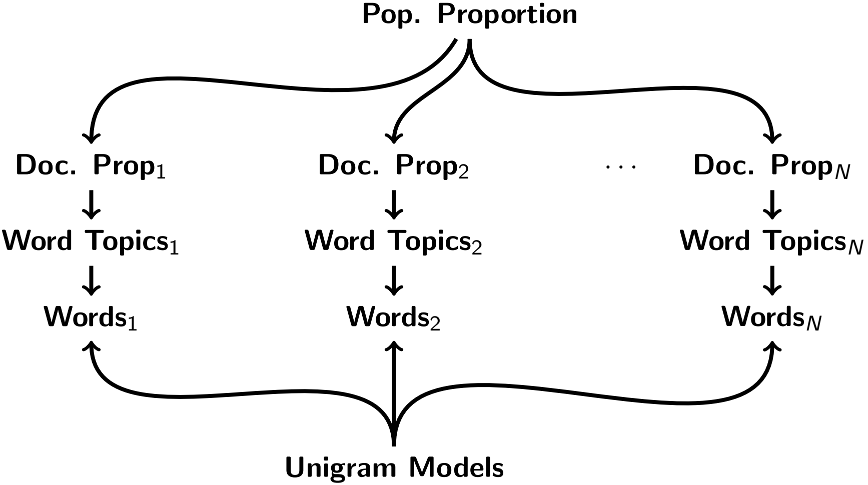
A General Hierarchical Structure

LDA:

$$
\begin{tikzcd}[column sep=small]
 & & \textbf{Pop. Proportion} \arrow[dll, bend right] \arrow[dl, bend right] \arrow[drr, bend left] & & \\
\textbf{Doc. Prop}_1 \arrow[d] & \textbf{Doc. Prop}_2 \arrow[d] & & \cdots & \textbf{Doc. Prop}_N \arrow[d] \\
\textbf{Word Topics}_1 & \textbf{Word Topics}_2 & & & \textbf{Word Topics}_N
\end{tikzcd}
$$

A General Hierarchical Structure

LDA:



A General Hierarchical Structure

Dynamic Topic Model (Quinn et al 2010)

Dynamic Prior Across Days

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Day Prop$_1$ **Day Prop**$_2$ $\cdots$ **Day Prop**$_N$

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Dynamic Topic Model (Quinn et al 2010)

Dynamic Prior Across Days

Day Prop$_1$ → **Speech Topics$_1$**

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$\cdots$

Day Prop$_N$ → **Speech Topics$_N$**

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Dynamic Topic Model (Quinn et al 2010)

Dynamic Prior Across Days

Day Prop$_1$ → **Speech Topics$_1$** → **Speeches$_1$**

Day Prop$_2$ → **Speech Topics$_2$** → **Speeches$_2$**

$\cdots$

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Unigram Models

A General Hierarchical Structure

Expressed Agenda Model (Grimmer 2010)

Average Attention Across Authors

A General Hierarchical Structure

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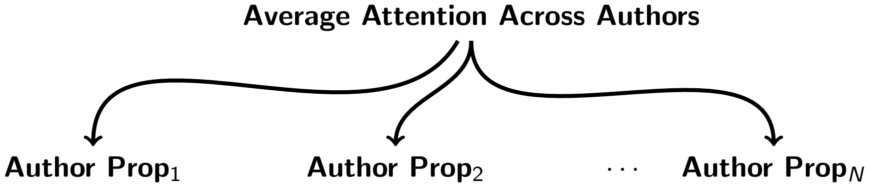

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Expressed Agenda Model (Grimmer 2010)

Average Attention Across Authors

Author Prop$_1$	**Author Prop**$_2$	$\cdots$	**Author Prop**$_N$
$\downarrow$	$\downarrow$		$\downarrow$
Text Topics$_1$	**Text Topics**$_2$		**Text Topics**$_N$

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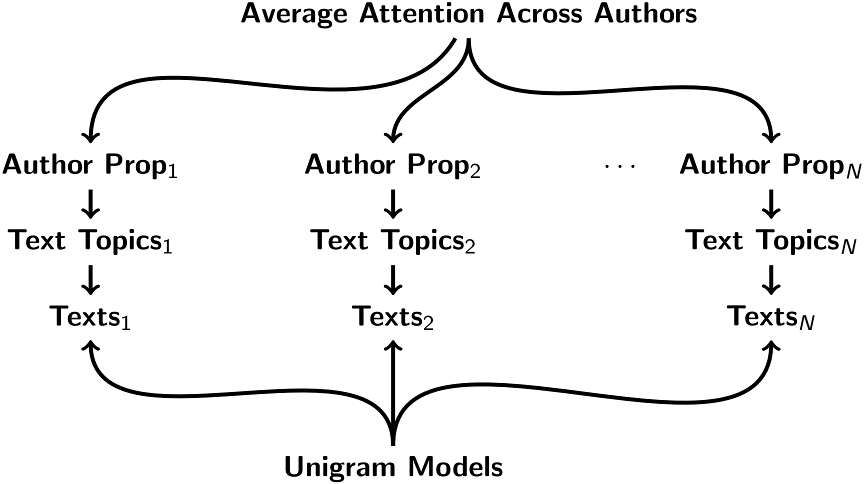

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Structural Topic Model (Roberts, Stewart, Airoldi 2014)

$$g(\boldsymbol{W}, \gamma)$$

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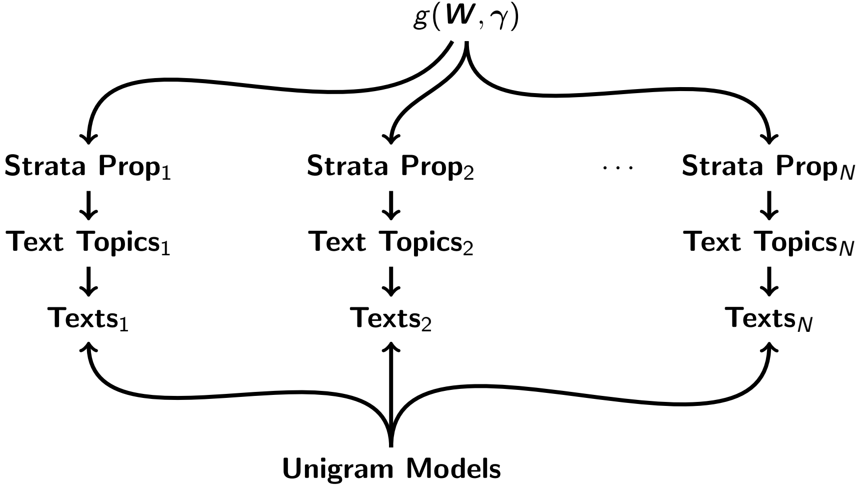

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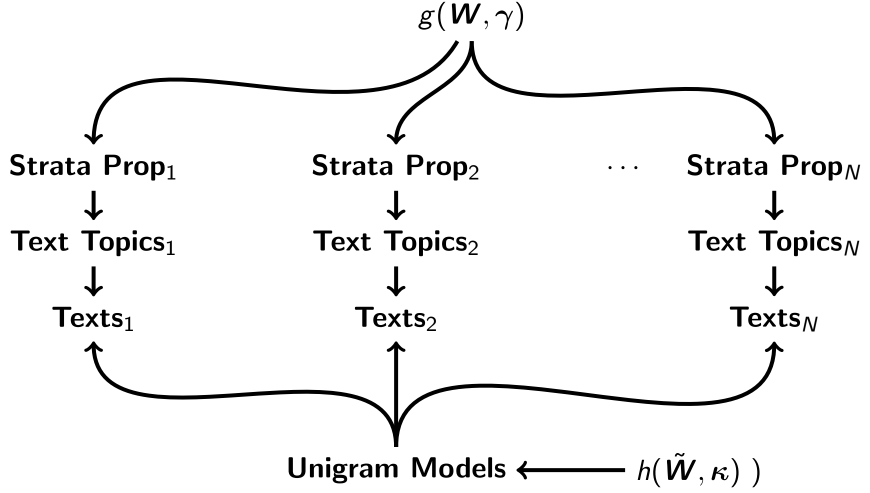

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Conditioning on Unknown Covariates$\rightsquigarrow$ levels of mixtures at proportions (Grimmer 2013; Wallach 2008)

Mixture of Top. Attn. Models

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Strata Indic. **Topic Attention Models**

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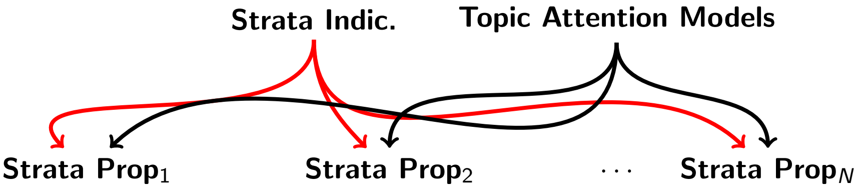

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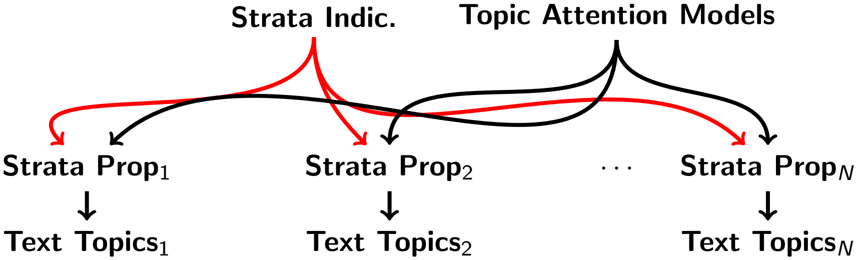

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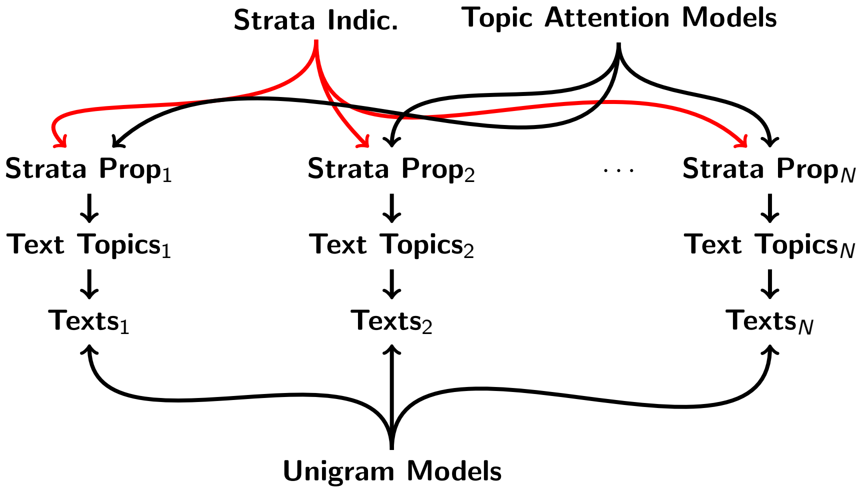

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Conditioning on Unknown Covariates for Topics $\rightsquigarrow$ hierarchy of topics (Li and McCallum 2006; Blaydes, Grimmer, and McQueen 2014)

Population attention

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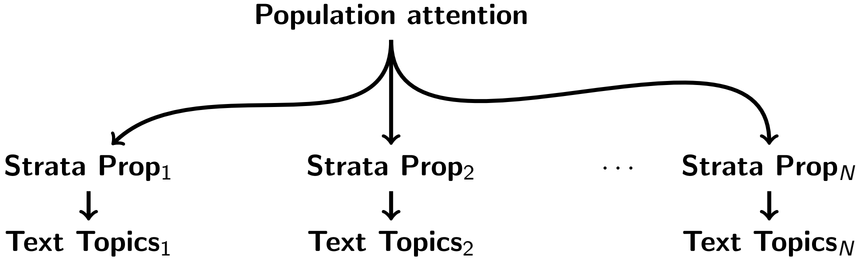

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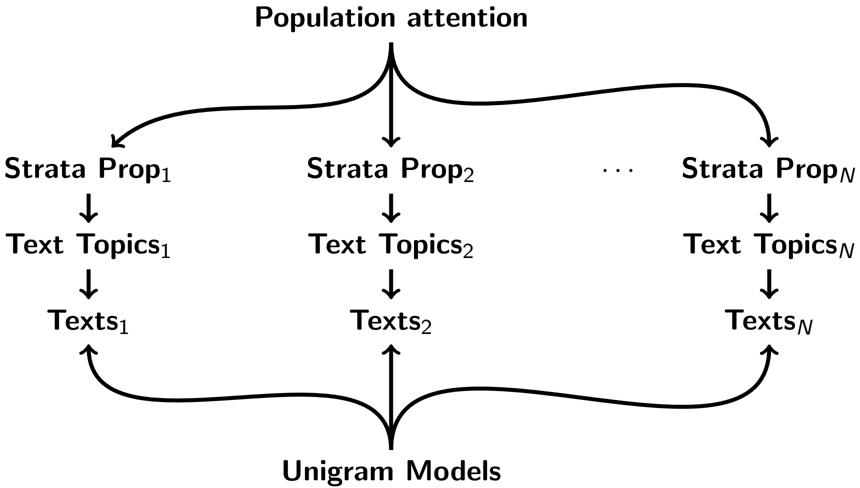

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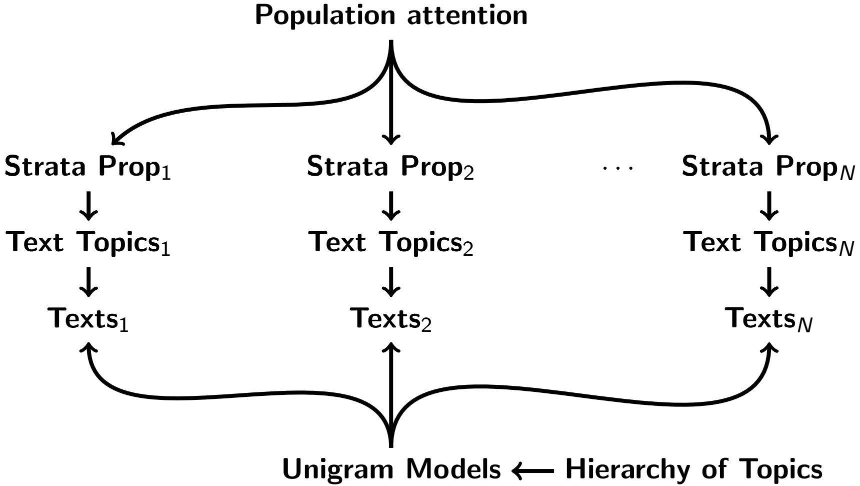

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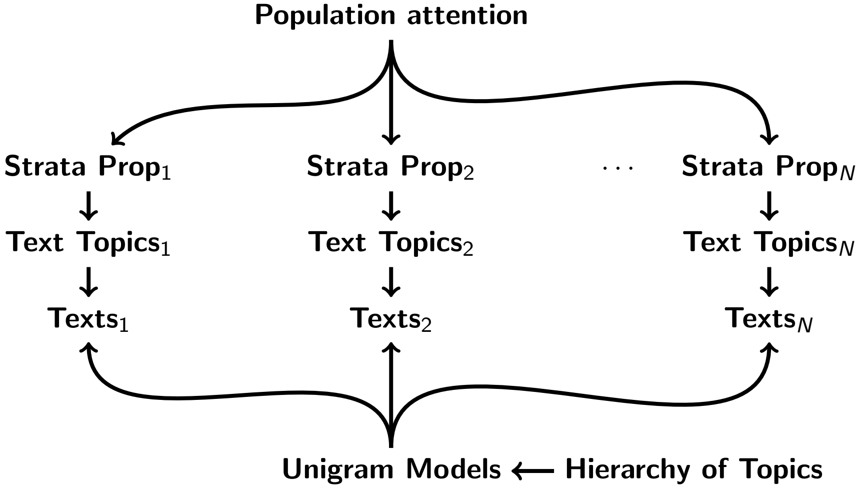

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  - Improved topics $\rightsquigarrow$ small word conditions, structure could help

Plan for the Class

1) Discuss model with unknown covariates for strata proportions $\rightsquigarrow$ presentational style
2) Discuss model with hierarchy of topics $\rightsquigarrow$ mirrors genre

Unknown Covariates for Issue Attention: Measuring Attention in Senate Press Releases

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Describe model that facilitates estimation of presentational styles in Senate press releases

- Characterize representation provided to constituents
- Divide attention over a set of topics
- Given attention to topics, write press releases

Presentational Styles$\rightsquigarrow$ Objective Function

- $\pi_{itk} \equiv$ Attention senator i allocates to issue k in year t
- $\pi_{itk} \equiv$ Probability press release is about issue k
- $\boldsymbol{\pi}_{it} = (\pi_{it1}, \ldots, \pi_{it44})$

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- If $\tau_{ijtk} = 1$ then

$$\boldsymbol{x}_{ijt} \sim \text{Multinomial}(n_{ijt}, \boldsymbol{\theta}_k).$$

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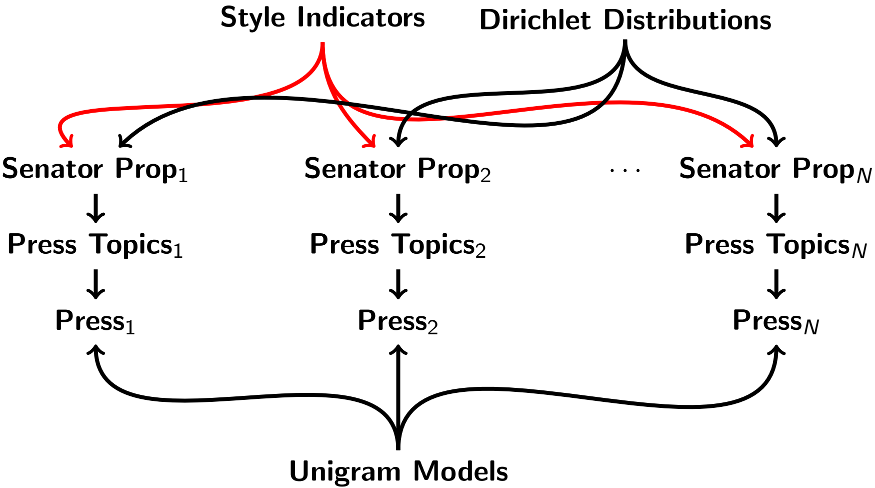
Mixture of Styles, Mixture of Topics



Posterior:

$$
\begin{aligned}
p(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau} | \boldsymbol{X}) \quad \propto \quad & \prod_{k=1}^{K} \prod_{s=1}^{S} \frac{\exp(-\frac{\alpha_{ks}}{1/4})}{1/4} \times \frac{\Gamma(\sum_{w=1}^{W} \lambda_w)}{\prod_{w=1}^{W} \Gamma(\lambda_w)} \prod_{w=1}^{W} \theta_{k,w}^{\lambda_w - 1} \times \\
& \prod_{i=1}^{N} \prod_{t=2005}^{2007} \prod_{s=1}^{S} \left[\beta_s \frac{\Gamma(\sum_{k=1}^{K} \alpha_{ks})}{\prod_{k=1}^{K} \Gamma(\alpha_{ks})} \prod_{k=1}^{K} \pi_{itk}^{\alpha_{ks}-1} \prod_{j=1}^{D_{it}} \prod_{k=1}^{K} \left[\pi_{itk} \prod_{w=1}^{W} \theta_{kw}^{x_{ijtw}} \right]^{\tau_{ijtk}} \right]^{\sigma_{its}}
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1) Estimate with Variational Approximation

Posterior:

$$p(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau} | \boldsymbol{X}) \propto \prod_{k=1}^{K} \prod_{s=1}^{S} \frac{\exp(-\frac{\alpha_{ks}}{1/4})}{1/4} \times \frac{\Gamma(\sum_{w=1}^{W} \lambda_w)}{\prod_{w=1}^{W} \Gamma(\lambda_w)} \prod_{w=1}^{W} \theta_{k,w}^{\lambda_w - 1} \times$$
$$\prod_{i=1}^{N} \prod_{t=2005}^{2007} \prod_{s=1}^{S} \left[\beta_s \frac{\Gamma(\sum_{k=1}^{K} \alpha_{ks})}{\prod_{k=1}^{K} \Gamma(\alpha_{ks})} \prod_{k=1}^{K} \pi_{itk}^{\alpha_{ks} - 1} \prod_{j=1}^{D_{it}} \prod_{k=1}^{K} \left[\pi_{itk} \prod_{w=1}^{W} \theta_{kw}^{x_{ijtw}} \right]^{\tau_{ijtk}} \right]^{\sigma_{its}}$$

1) Estimate with Variational Approximation
2) Determining number of clusters at top? (Grimmer, Shorey, Wallach, and Zlotnick, In Progress)

Posterior:

$$p(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau}|\boldsymbol{X}) \propto \prod_{k=1}^{K}\prod_{s=1}^{S}\frac{\exp(-\frac{\alpha_{ks}}{1/4})}{1/4} \times \frac{\Gamma(\sum_{w=1}^{W}\lambda_w)}{\prod_{w=1}^{W}\Gamma(\lambda_w)}\prod_{w=1}^{W}\theta_{k,w}^{\lambda_w - 1} \times$$
$$\prod_{i=1}^{N}\prod_{t=2005}^{2007}\prod_{s=1}^{S}\left[\beta_s\frac{\Gamma(\sum_{k=1}^{K}\alpha_{ks})}{\prod_{k=1}^{K}\Gamma(\alpha_{ks})}\prod_{k=1}^{K}\pi_{itk}^{\alpha_{ks}-1}\prod_{j=1}^{D_{it}}\prod_{k=1}^{K}\left[\pi_{itk}\prod_{w=1}^{W}\theta_{kw}^{x_{ijtw}}\right]^{\tau_{ijtk}}\right]^{\sigma_{its}}$$

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$$p(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau} | \boldsymbol{X}) \propto \prod_{k=1}^{K} \prod_{s=1}^{S} \frac{\exp(-\frac{\alpha_{ks}}{1/4})}{1/4} \times \frac{\Gamma(\sum_{w=1}^{W} \lambda_w)}{\prod_{w=1}^{W} \Gamma(\lambda_w)} \prod_{w=1}^{W} \theta_{k,w}^{\lambda_w - 1} \times$$
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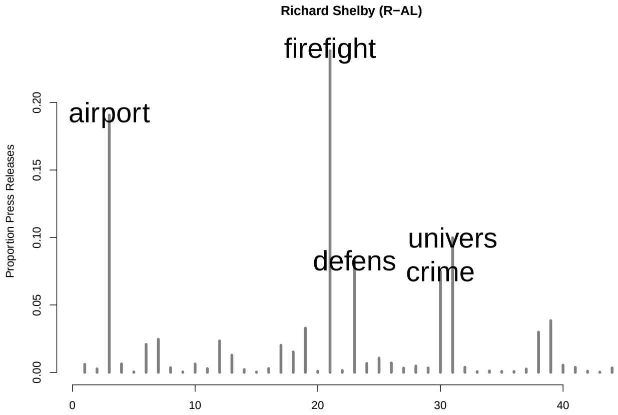
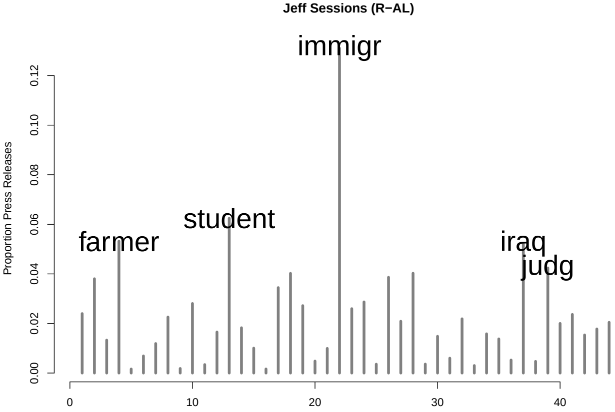
1) Estimate with Variational Approximation
2) Determining number of clusters at top? (Grimmer, Shorey, Wallach, and Zlotnick, In Progress)
    - Non-parametric model $\rightsquigarrow$ statistical selection
    - Experiments/Coding Exercises to assess


Richard Shelby (R−AL)
firefight
airport
univers
defens
crime
Proportion Press Releases
0.00
0.05
0.10
0.15
0.20
0
10
20
30
40


Jeff Sessions (R−AL)
immigr
student
farmer
iraq
judg
Proportion Press Releases
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0
10
20
30
40

Notions of validity: From Quinn, Monroe, et al (2010)

- Semantic Validity: All categories are coherent and meaningful
- Convergent Construct Validity: Measures concur with existing measures in critical details.
- Discriminant Construct Validity: Measures differ from existing measures in productive ways.
- Predictive Measure: Measures from the model corresponds to external events in expected ways.
- Hypothesis Validity: Measures generated from the model can be used to test substantive hypotheses.

To establish utility of new measures, demonstrate variety of validations
None of these validations are performed using a canned statistic
All: require substantive knowledge on areas (and what we expect!) [

Home Style Measures, Semantic Validity

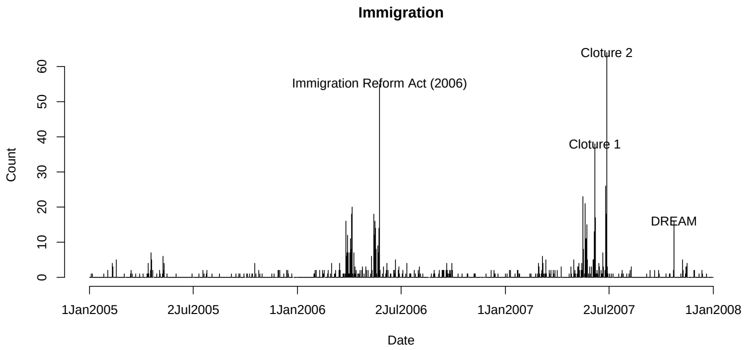
Must: Demonstrate to reader that topics are coherent and semantically meaningful

Description	Stems	%
Honorary	honor,prayer,rememb,fund,tribut	5.0
Transp. Grants	airport,transport,announc,urban,hud	4.8
Iraq	iraq,iraqi,troop,war,sectarian	4.7
DHS Policy	homeland,port,terrorist,dh,fema	4.1
Judicial Nom.	judg,court,suprem,nomin,nomine	3.8
Fire Dept. Grant	firefight,homeland,afgp,award,equip	3.7

How: examples in text are also useful.

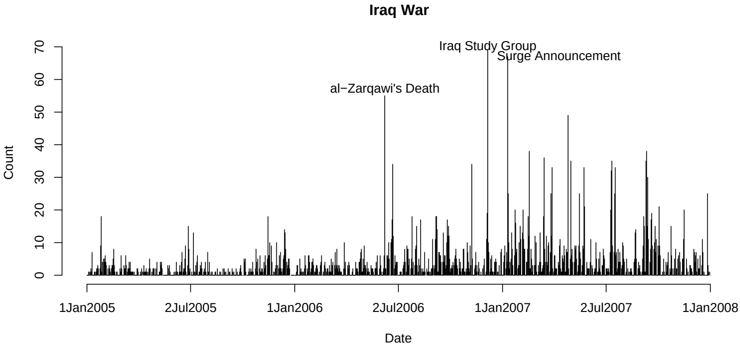
Home Style Measures, Convergent Validity

Over time variation



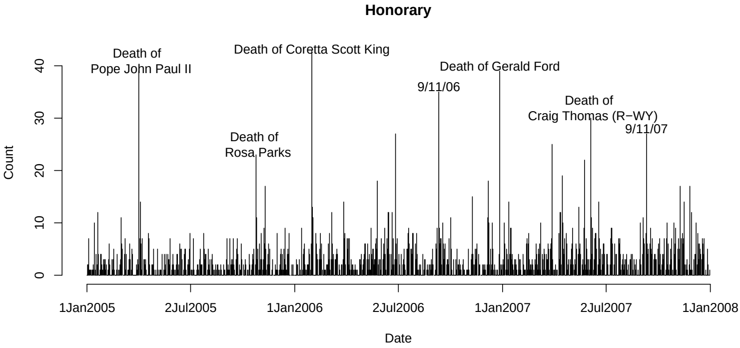
Home Style Measures, Convergent Validity

Over time variation



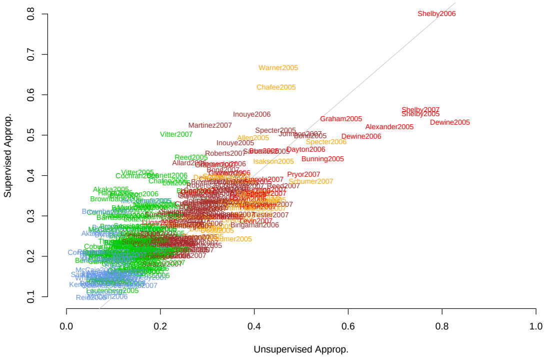
Home Style Measures, Convergent Validity

Over time variation

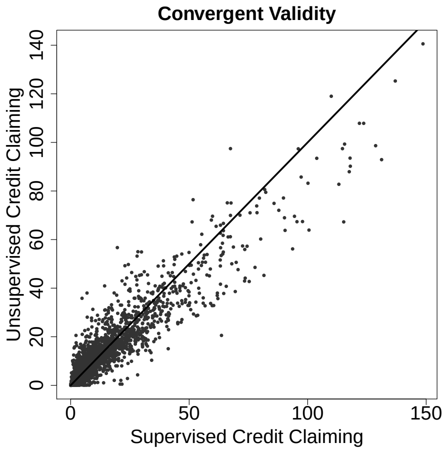


Home Style Measures, Convergent Validity

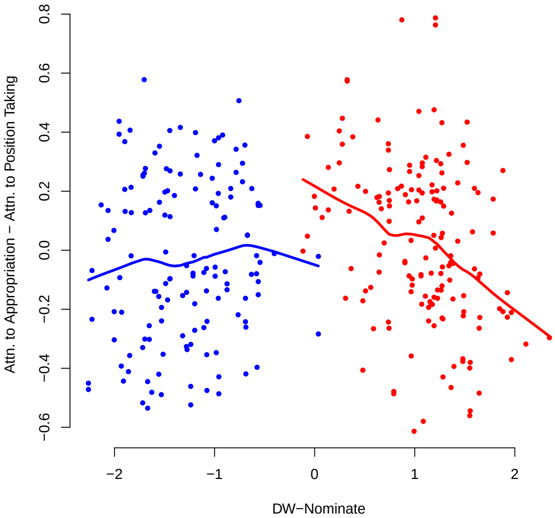
Supervised/Unsupervised Convergence



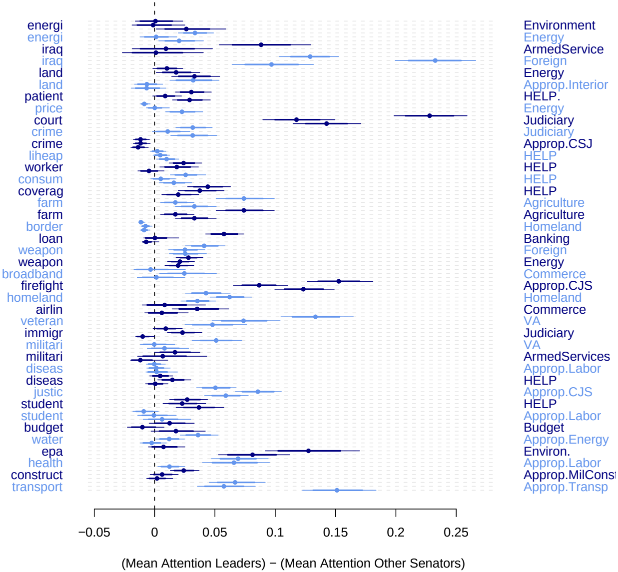
Home Style Measures, Convergent Validity



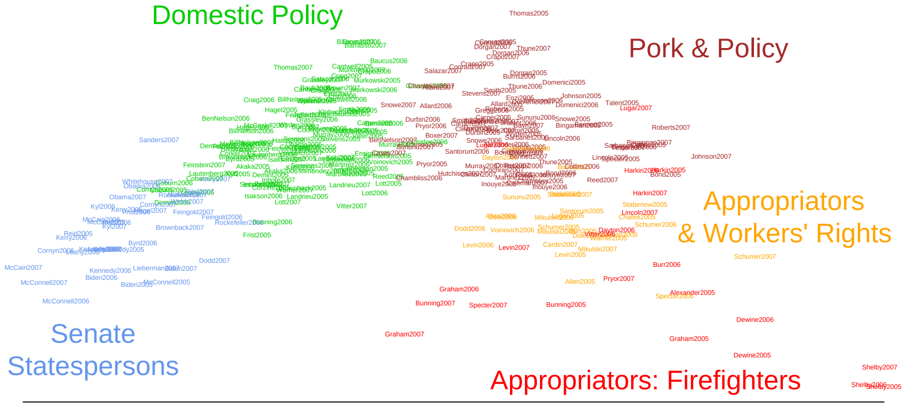
Discriminant Construct Validity



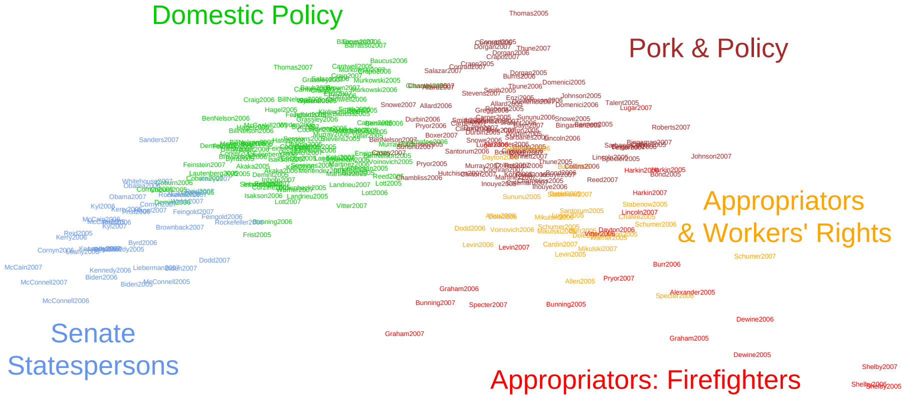
Predictive Validity



Hypothesis Validity



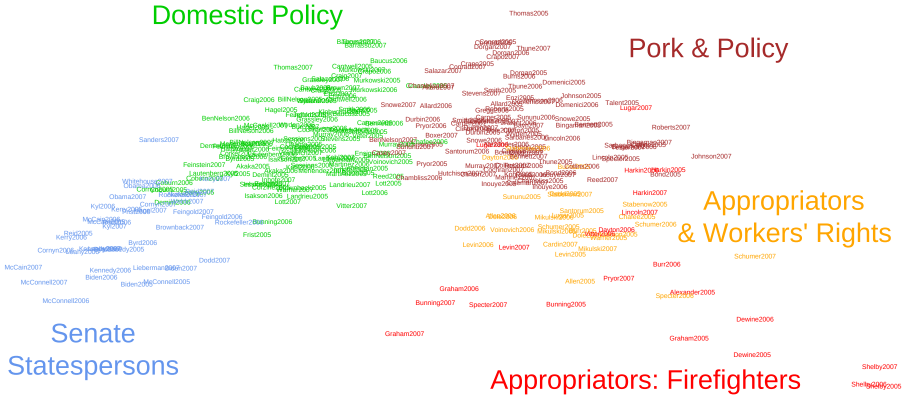
Hypothesis Validity



Senate Statesperson

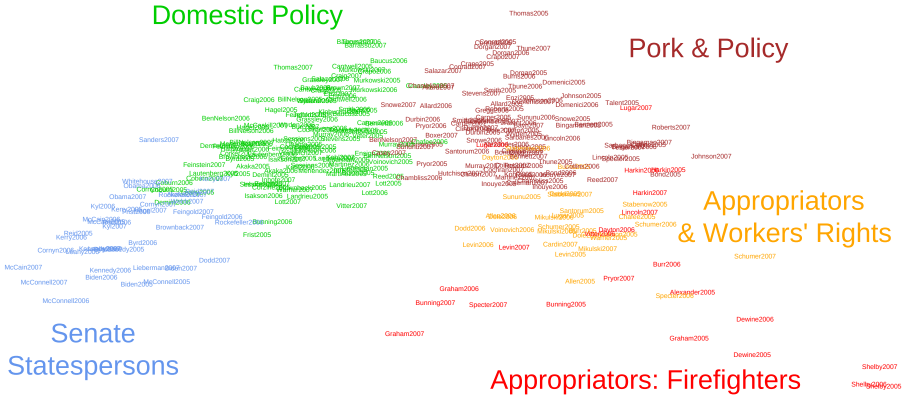
- Iraq War
- Intelligence
- Intl. Relations

Hypothesis Validity



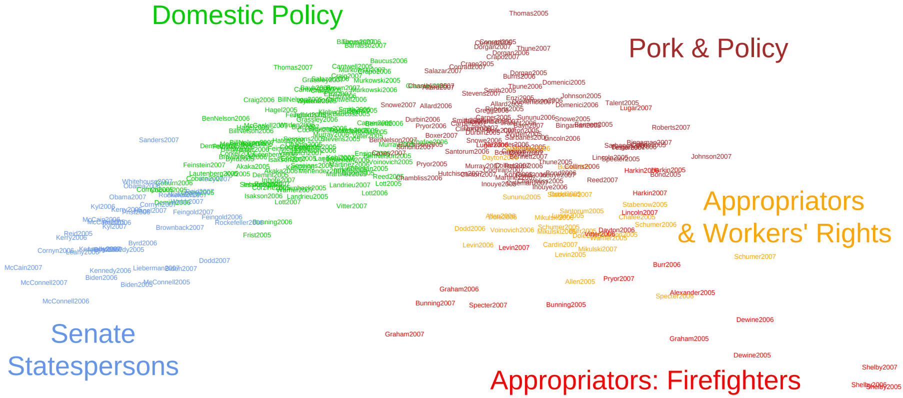
Senate Statesperson	Domestic Policy
- Iraq War	- Environment
- Intelligence	- Gas prices
- Intl. Relations	- DHS
	- Consumer

Hypothesis Validity



Senate Statesperson	Domestic Policy	Pork & Policy
- Iraq War	- Environment	- WRDA grants
- Intelligence	- Gas prices	- Farming
- Intl. Relations	- DHS	- Health Care
	- Consumer	- Education

Hypothesis Validity



Senate Statesperson
- Iraq War
- Intelligence
- Intl. Relations

Domestic Policy
- Environment
- Gas prices
- DHS
- Consumer

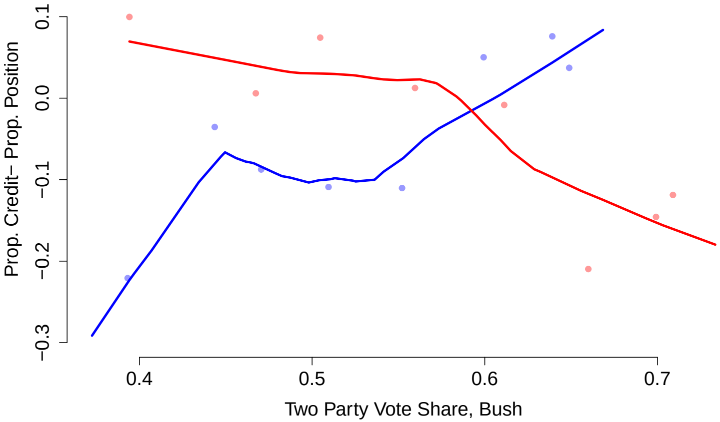
Pork & Policy
- WRDA grants
- Farming
- Health Care
- Education

Appropriators
- Fire Grants
- Airport Grants
- University Money

Hypothesis Validity

Why do senators adopt different styles?

District Fit



What are the right number of topics?

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Blaydes, Grimmer, and McQueen [In Progress]$\rightsquigarrow$ estimate nested topics to explore the Mirros for Princes

The Mirrors Genre

26 Christian mirrors

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Work with translations

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Work with translations $\rightsquigarrow$ little evidence of selection

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- Collect data on collection of 98 (51 Christian, 47 Islamic, some not translated)

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Work with translations $\rightsquigarrow$ little evidence of selection

- Collect data on collection of 98 (51 Christian, 47 Islamic, some not translated)
- No difference on Year/Region

Preprocessing Texts

47 books

Preprocessing Texts

47 books $\rightsquigarrow$ Each divided into paragraphs

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Create feature space

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$$\mathbf{x}_{ij} = (x_{ij1}, x_{ij2}, \ldots, x_{ij2124})$$

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We work with a normalized version of the documents,

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We work with a normalized version of the documents,

$$\mathbf{x}_{ij}^{*} = \frac{\mathbf{x}_{ij}}{\sqrt{\mathbf{x}_{ij}^{'}\mathbf{x}_{ij}}}$$

Measuring Themes in the Mirrors

Model built around two hierarchies:

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1) Books $\rightsquigarrow$ paragraphs (Blei, Ng, Jordan 2003; Wallach, 2008; Quinn et al 2010; Grimmer 2010; Roberts et al 2014)

Measuring Themes in the Mirrors

Model built around two hierarchies:

1) Books $\rightsquigarrow$ paragraphs (Blei, Ng, Jordan 2003; Wallach, 2008; Quinn et al 2010; Grimmer 2010; Roberts et al 2014)
2) Coarse topics $\rightsquigarrow$ granular topics (Li and McCallum 2006; Gopal and Yang 2014)

Measuring Themes in the Mirrors

Estimate four quantities of interest

Measuring Themes in the Mirrors

Estimate four quantities of interest

1) Granular topics (60)

Measuring Themes in the Mirrors

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3) Each book i's $\textbf{themes}_i$

$$\textbf{themes}_i \quad = \quad (\text{theme}_{i1}, \text{theme}_{i2}, \ldots, \text{theme}_{i60})$$

Measuring Themes in the Mirrors

Estimate four quantities of interest

1) Granular topics (60)
2) Coarse (broad) topics (3)
    - Each granular topic classified into one coarse topic
3) Each book i's $\textbf{themes}_i$
4) Each short segment's granular (and coarse) topic

A Hierarchy of Topics

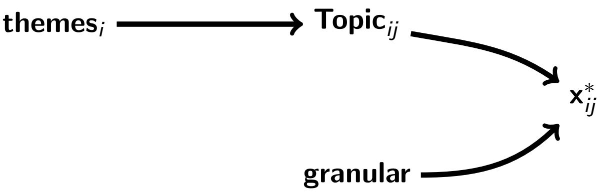
themes$_i$

A Hierarchy of Topics

$$\textbf{themes}_i \longrightarrow \textbf{Topic}_{ij}$$

$$\textbf{Topic}_{ij} \sim \text{Multinomial}(1, \textbf{themes}_i)$$

A Hierarchy of Topics



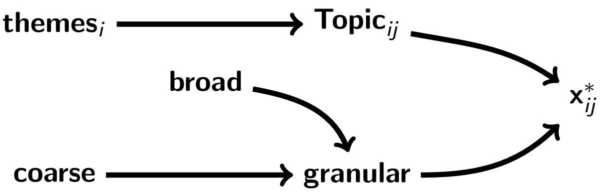
$$
\begin{aligned}
\textbf{Topic}_{ij} &\sim \text{Multinomial}(1, \textbf{themes}_i) \\
\mathbf{x}^*_{ij} | \text{Topic}_{ijk} = 1 &\sim \text{vMF}(\kappa, \textbf{granular}_k)
\end{aligned}
$$

A Hierarchy of Topics

$$
\begin{tikzcd}
\textbf{themes}_i \arrow[rr] & & \textbf{Topic}_{ij} \arrow[dr, bend left] & \\
& \textbf{broad} \arrow[dr, bend left] & & \mathbf{x}^*_{ij} \\
\textbf{coarse} \arrow[rr] & & \textbf{granular} \arrow[ur, bend right] &
\end{tikzcd}
$$

$$
\begin{aligned}
\textbf{Topic}_{ij} &\sim \text{Multinomial}(1, \textbf{themes}_i) \\
\mathbf{x}^*_{ij}|\text{Topic}_{ijk} = 1 &\sim \text{vMF}(\kappa, \textbf{granular}_k) \\
\textbf{broad}_k &\sim \text{Multinomial}(1, \textbf{Broad Theme Prior}) \\
\textbf{granular}_k|\text{broad}_{km} = 1 &\sim \text{vMF}(\kappa, \textbf{coarse}_m)
\end{aligned}
$$

A Hierarchy of Topics



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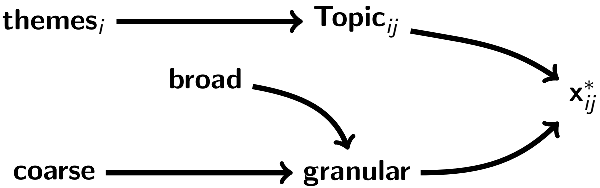
\end{aligned}
$$

Estimate model with Variational Approximation

A Hierarchy of Topics



$$\begin{aligned}
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\end{aligned}$$

Estimate model with Variational Approximation
Model selection: automatic model fit, qualitative evaluation

Interpreting Unsupervised Models

Two approaches to labeling output

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1) Computational: identify discriminating words

Interpreting Unsupervised Models

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2) Manual: Segments classified to coarse, granular topics. Read, discuss, and label

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Unsupervised models structure and guide our reading

Art of Rulership

Practices and ideals of political rule

Art of Rulership

Practices and ideals of political rule

king

Art of Rulership

Practices and ideals of political rule

king,princ

Art of Rulership

Practices and ideals of political rule

king,princ,citi

Art of Rulership

Practices and ideals of political rule

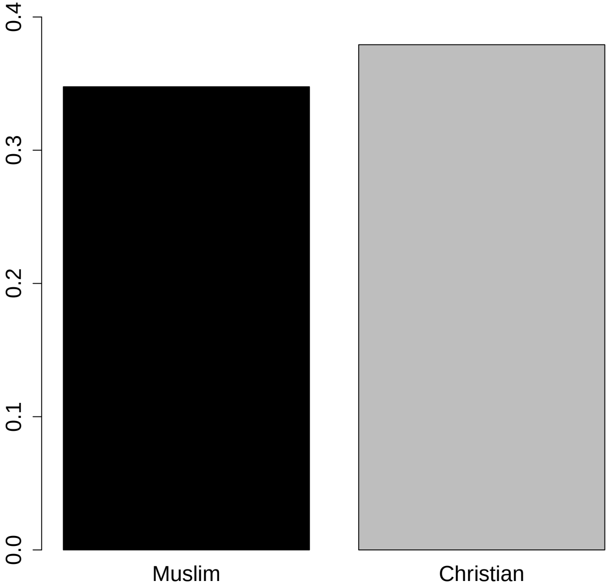
king,princ,citi,great,place,work,emperor,enemi,armi,letter

Art of Rulership

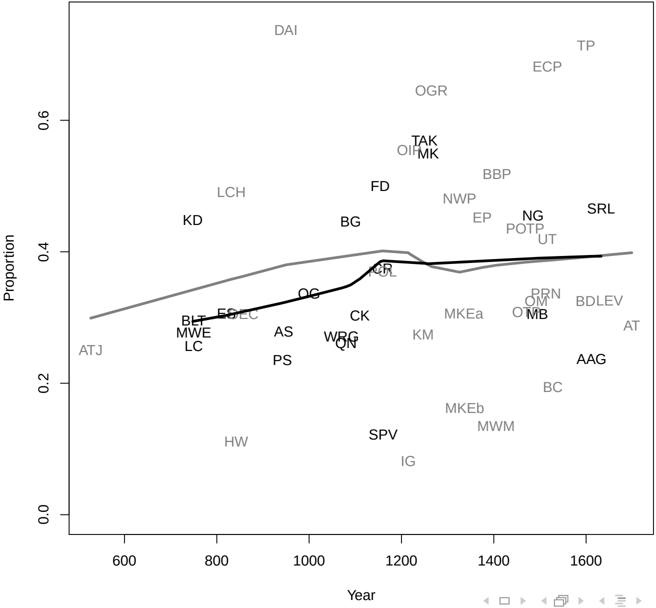
Practices and ideals of political rule

king,princ,citi,great,place,work,emperor,enemi,armi,letter

36.5% of paragraphs


0.0
0.1
0.2
0.3
0.4
Muslim
Christian

Coarse Topic 1



Religion and Virtue

Connection between religion, virtue, justice and political rule

Religion and Virtue

Connection between religion, virtue, justice and political rule

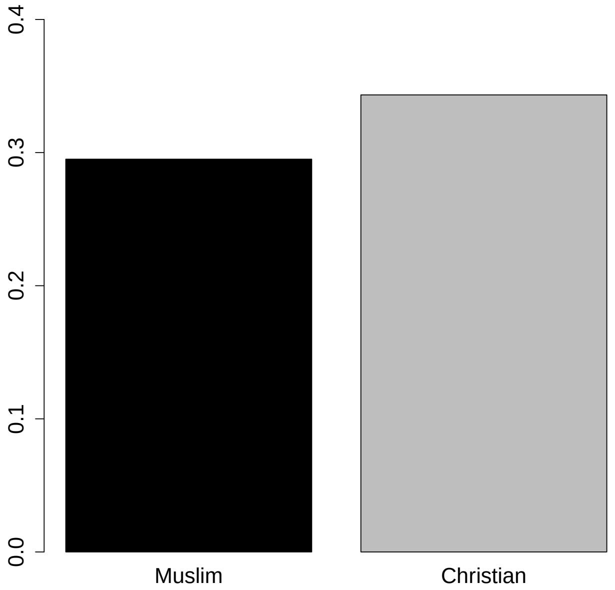
almighti,good,virtu,power,ruler,justic,prayer,rule,prophet,mena

Religion and Virtue

Connection between religion, virtue, justice and political rule

almighti,good,virtu,power,ruler,justic,prayer,rule,prophet,mena

32.2% of pargraphs


0.0
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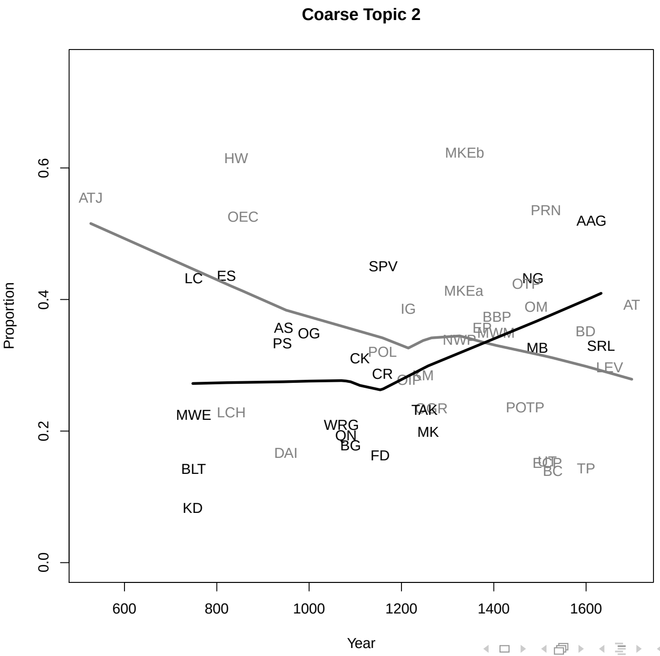
Coarse Topic 2
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MWM
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PS
NWP
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Inner Life of the Ruler

Personal relationships, care for and practices of the self, and ultimate fate of the soul

Inner Life of the Ruler

Personal relationships, care for and practices of the self, and ultimate fate of the soul

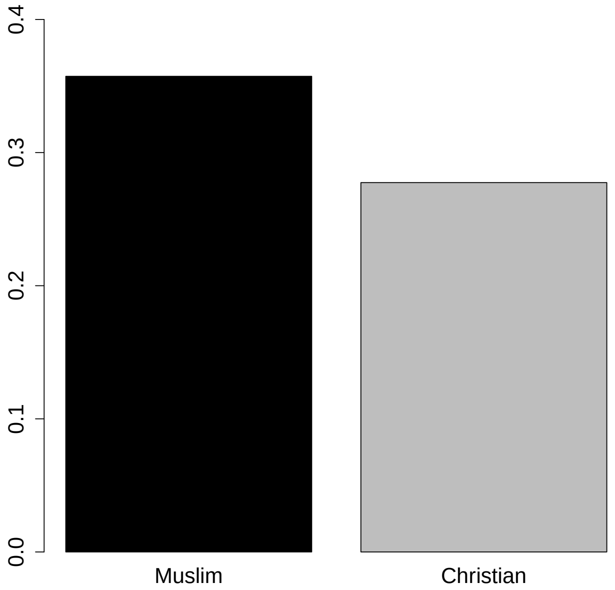
man,land,woman,know,bodi,eye,ladi,love,faculti,old

Inner Life of the Ruler

Personal relationships, care for and practices of the self, and ultimate fate of the soul

man,land,woman,know,bodi,eye,ladi,love,faculti,old

31.2% of paragraphs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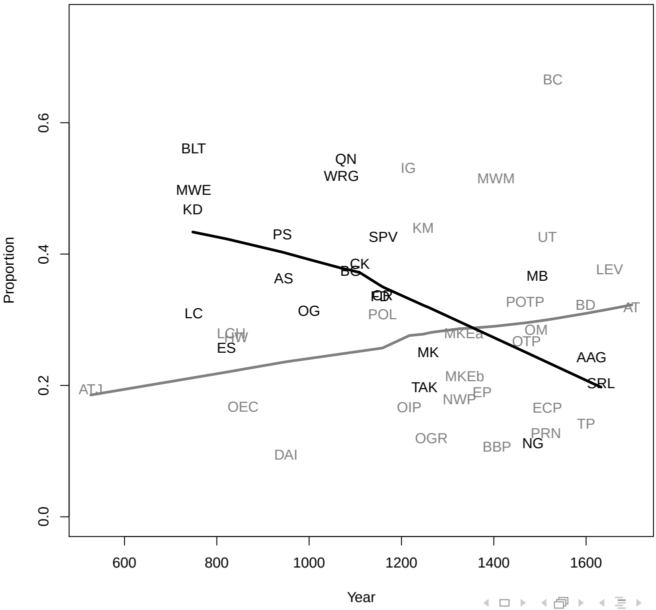
Coarse Topic 3

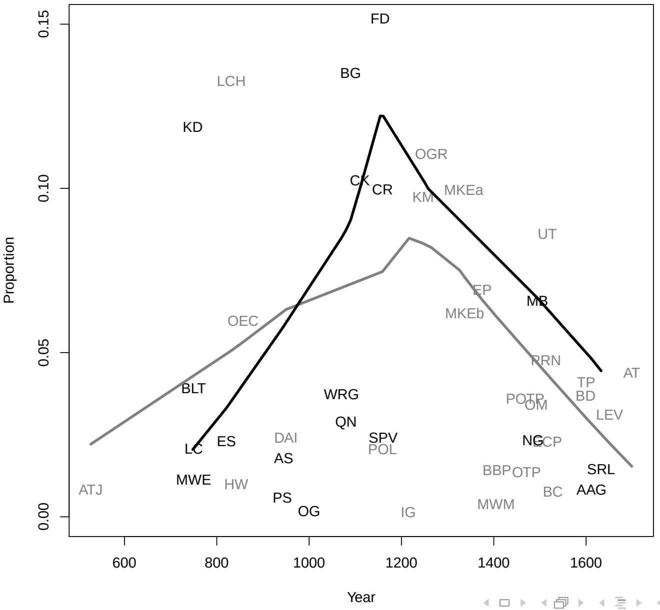


Granular: Best Practices for Ruling

king,princ,citi,great,place,work,emperor,enemi,armi,letter
king,kingdom,royal,minist,reign,father,court,majesti,presenc,war

6.2% of paragraphs

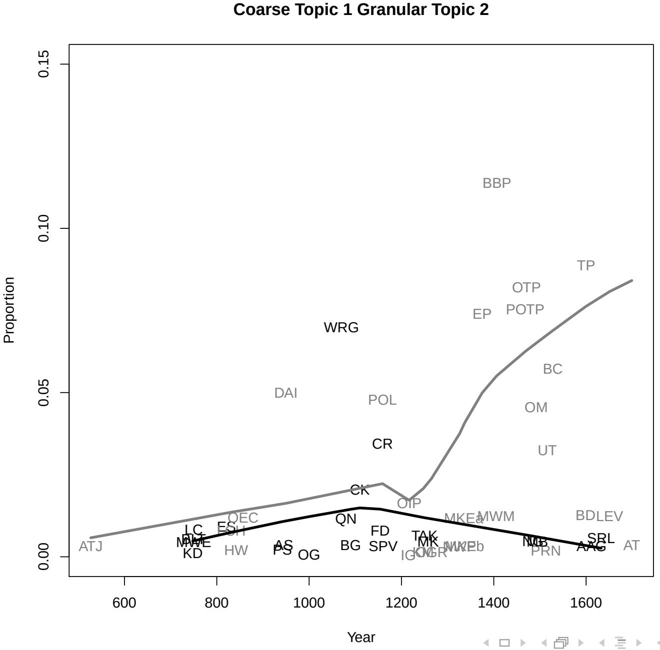
Coarse Topic 1 Granular Topic 1



Granular: Characteristics that distinguish Just Ruler from Tyrant

king,princ,citi,great,place,work,emperor,enemi,armi,letter
king,kingdom,royal,minist,reign,father,court,majesti,presenc,war
princ,good,peopl,christian,tyranni,war,mind,ought,state,public

3.1% of paragraphs


Coarse Topic 1 Granular Topic 2
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BDLEV
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FS
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TAK
SRL
MK
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BG
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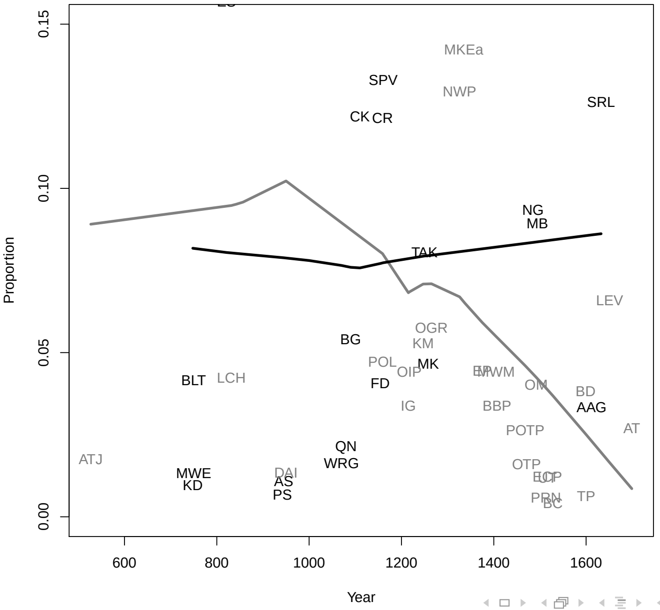
HW
PS
OG
IG
PRN
AAC
ATJ
AT
KD

Granular: Religious Virtues and Political Ideals

almighti,good,virtu,power,ruler,justic,prayer,rule,prophet,mena
almighti,bless,grant,peac,messeng,prophet,merci,holi,command,grace

6.9% of paragraphs

Coarse Topic 2 Granular Topic 1



Structural Topic Models

- Encode observed and unobserved meta data
- ?Improve substantive inferences

Next week:

1) Hanna Wallach on canonical topic models
2) Introduction to supervised learning

Work on your problem sets!

Appendix: Inference for both Models

Inference

Invariance in posterior makes it difficult (impossible) to approximate with sampling based methods (relabeling, aliasing problem).
Deterministic alternative: variational approximations.
Intuition: approximate posterior with simpler (still very general) approximating distribution.
Make approximation as "close" as possible

Approximate posterior with:

$$
\begin{aligned}
q(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau}) &= q(\boldsymbol{\alpha})q(\boldsymbol{\beta})q(\boldsymbol{\theta})q(\boldsymbol{\sigma})q(\boldsymbol{\pi})q(\boldsymbol{\tau}) \\
&= q(\boldsymbol{\beta}) \prod_{k=1}^{K} q(\boldsymbol{\theta})_k \prod_{i=1}^{n} \prod_{t=2005}^{2007} \left[q(\boldsymbol{\sigma})_{it} q(\boldsymbol{\pi})_{it} \prod_{j=1}^{J} q(\boldsymbol{\tau})_{ijt} \right] \prod_{s=1}^{S} q(\boldsymbol{\alpha})_s
\end{aligned}
$$

Variational Approximation

Optimization goal:

- Minimize the Kullback-Leibler divergence between approximating distribution q and true posterior p
  - KL-divergence is a functional: takes functions as an input, returns a positive scalar
  - Measures "divergence" between two measures
- Use calculus of variations and theory from exponential models to derive iterative algorithm
- See "An Introduce to Bayesian Inference via Variational Approximations" for extended introduction (Grimmer, 2011)

Minimize KL-divergence by Maximizing a Lower Bound

$$\log p(\boldsymbol{Y}) = \log \sum_{\boldsymbol{\sigma}} \sum_{\boldsymbol{\tau}} \iiiint p(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau} | \boldsymbol{Y}) d\boldsymbol{\theta} d\boldsymbol{\alpha} d\boldsymbol{\beta} d\boldsymbol{\pi}$$

$$\log p(\boldsymbol{Y}) = \log \sum_{\boldsymbol{\sigma}} \sum_{\boldsymbol{\tau}} \iiiint p(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau} | \boldsymbol{Y}) \frac{q(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau})}{q(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau})} d\boldsymbol{\theta} d\boldsymbol{\alpha} d\boldsymbol{\beta} d\boldsymbol{\pi}$$

$$\log p(\boldsymbol{Y}) \geq \underbrace{\sum_{\boldsymbol{\sigma}} \sum_{\boldsymbol{\tau}} \iiiint q(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau}) \log \frac{p(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau} | \boldsymbol{Y})}{q(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\sigma}, \boldsymbol{\pi}, \boldsymbol{\tau})} d\boldsymbol{\theta} d\boldsymbol{\alpha} d\boldsymbol{\beta} d\boldsymbol{\pi}}_{\mathcal{L}(q)}.$$

Minimize KL-divergence by Maximizing a Lower Bound

$$\log p(\boldsymbol{Y}) = \mathcal{L}(q) + \text{KL}(q||p)$$

Minimize KL-divergence by Maximizing a Lower Bound

$$
\begin{aligned}
\log p(\boldsymbol{Y}) &= \mathcal{L}(q) + \mathrm{KL}(q||p) \\
\underbrace{\log p(\boldsymbol{Y})}_{\text{fixed number}} &= \mathcal{L}(q) + \underbrace{\mathrm{KL}(q||p)}_{\text{Positive}}
\end{aligned}
$$

Minimize KL-divergence by Maximizing a Lower Bound

$$
\begin{aligned}
\log p(\boldsymbol{Y}) &= \mathcal{L}(q) + \mathrm{KL}(q||p) \\
\underbrace{\log p(\boldsymbol{Y})}_{\text{fixed number}} &= \mathcal{L}(q) + \underbrace{\mathrm{KL}(q||p)}_{\text{Positive}}
\end{aligned}
$$

If $\mathcal{L}(q)$ get bigger, $\mathrm{KL}(q||p)$ get smaller. $\Rightarrow$

Minimize KL-divergence by Maximizing a Lower Bound

$$
\begin{aligned}
\log p(\boldsymbol{Y}) &= \mathcal{L}(q) + \text{KL}(q||p) \\
\underbrace{\log p(\boldsymbol{Y})}_{\text{fixed number}} &= \mathcal{L}(q) + \underbrace{\text{KL}(q||p)}_{\text{Positive}}
\end{aligned}
$$

If $\mathcal{L}(q)$ get bigger, $\text{KL}(q||p)$ get smaller. $\Rightarrow$
If $\mathcal{L}(q)$ is at a maximum $\text{KL}(q||p)$ is at a minimum (duals).

Maximizing $\mathcal{L}(q)$

Goal: choose q to maximize $\mathcal{L}(q)$.

$$q(\boldsymbol{\alpha})^{\text{old}}, q(\boldsymbol{\beta})^{\text{old}}, q(\boldsymbol{\theta})^{\text{old}}, q(\boldsymbol{\sigma})^{\text{old}}, q(\boldsymbol{\pi})^{\text{old}}, q(\boldsymbol{\tau})^{\text{old}}.$$

Iterative Algorithm:
Choose, $q(\boldsymbol{\pi})^{\text{new}}$ to max $\mathcal{L}(q)$–holding $q(\boldsymbol{\theta})^{\text{old}}, q(\boldsymbol{\tau})^{\text{old}}, q(\boldsymbol{\alpha})^{\text{old}}, q(\boldsymbol{\beta})^{\text{old}}, q(\boldsymbol{\sigma})^{\text{old}}$ constant.
Choose, $q(\boldsymbol{\theta})^{\text{new}}$ to max $\mathcal{L}(q)$–holding $q(\boldsymbol{\pi})^{\text{new}}, q(\boldsymbol{\tau})^{\text{old}}, q(\boldsymbol{\alpha})^{\text{old}}, q(\boldsymbol{\beta})^{\text{old}}, q(\boldsymbol{\sigma})^{\text{old}}$ constant.
Choose, $q(\boldsymbol{\tau})^{\text{new}}$ to max $\mathcal{L}(q)$–holding $q(\boldsymbol{\theta})^{\text{new}}, q(\boldsymbol{\pi})^{\text{new}}, q(\boldsymbol{\alpha})^{\text{old}}, q(\boldsymbol{\beta})^{\text{old}}, q(\boldsymbol{\sigma})^{\text{old}}$ constant.
Choose, $q(\boldsymbol{\alpha})^{\text{new}}$ to max $\mathcal{L}(q)$–holding $q(\boldsymbol{\theta})^{\text{new}}, q(\boldsymbol{\tau})^{\text{new}}, q(\boldsymbol{\pi})^{\text{new}}, q(\boldsymbol{\beta})^{\text{old}}, q(\boldsymbol{\sigma})^{\text{old}}$ constant.
Choose, $q(\boldsymbol{\beta})^{\text{new}}$ to max $\mathcal{L}(q)$–holding $q(\boldsymbol{\theta})^{\text{new}}, q(\boldsymbol{\tau})^{\text{new}}, q(\boldsymbol{\pi})^{\text{new}}, q(\boldsymbol{\alpha})^{\text{new}}, q(\boldsymbol{\sigma})^{\text{old}}$ constant.
Choose, $q(\boldsymbol{\sigma})^{\text{new}}$ to max $\mathcal{L}(q)$–holding $q(\boldsymbol{\theta})^{\text{new}}, q(\boldsymbol{\tau})^{\text{new}}, q(\boldsymbol{\pi})^{\text{new}}, q(\boldsymbol{\alpha})^{\text{new}}, q(\boldsymbol{\beta})^{\text{new}}$ constant.

Example for $q(\boldsymbol{\pi})^{\text{new}}$

$$\mathcal{L}(q) = \int q(\boldsymbol{\pi})^{\text{new}} \underbrace{\left\{ \sum_{\boldsymbol{\sigma}} \sum_{\boldsymbol{\tau}} \iiint \log p(\boldsymbol{Y}, \boldsymbol{\pi}, \boldsymbol{\alpha}, \boldsymbol{\theta}, \boldsymbol{\tau}) q(\boldsymbol{\sigma})^{\text{old}} q(\boldsymbol{\tau})^{\text{old}} q(\boldsymbol{\theta})^{\text{old}} q(\boldsymbol{\alpha})^{\text{old}} q(\boldsymbol{\beta})^{\text{old}} d\boldsymbol{\theta} d\boldsymbol{\alpha} d\boldsymbol{\beta} \right\}}_{\mathrm{E}_{\boldsymbol{\alpha},\boldsymbol{\tau},\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\beta}}[\log p(\boldsymbol{Y},\boldsymbol{\pi},\boldsymbol{\alpha},\boldsymbol{\theta},\boldsymbol{\tau},\boldsymbol{\beta},\boldsymbol{\sigma})]} d\boldsymbol{\pi}$$
$$- q(\boldsymbol{\pi})^{\text{new}} \log q(\boldsymbol{\pi})^{\text{new}} + \text{constants}$$

Define

$$\log \tilde{p}(\boldsymbol{\pi}) = \mathrm{E}_{\boldsymbol{\alpha},\boldsymbol{\tau},\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\beta}}[\log p(\boldsymbol{Y}, \boldsymbol{\pi}, \boldsymbol{\alpha}, \boldsymbol{\theta}, \boldsymbol{\tau}, \boldsymbol{\beta}, \boldsymbol{\sigma})] + \text{constants}$$

Example for $q(\boldsymbol{\pi})^{\text{new}}$

Substituting $\log \tilde{p}(\boldsymbol{\pi})$,

$$\begin{aligned} &= \int q(\boldsymbol{\pi})^{\text{new}} \log \left(\frac{\tilde{p}(\boldsymbol{\pi})}{q(\boldsymbol{\pi})^{\text{new}}} \right) d\boldsymbol{\pi} \\ &= -\text{KL}(q(\boldsymbol{\pi})^{\text{new}} || \tilde{p}(\boldsymbol{\pi})) \end{aligned}$$

$\Rightarrow$ At a maximum when $q(\boldsymbol{\pi})^{\text{new}} = \tilde{p}(\boldsymbol{\pi})$.
Equivalently,

$$\begin{aligned} \log q(\boldsymbol{\pi})^{\text{new}} &= \log \tilde{p}(\boldsymbol{\pi}) \\ &= \text{E}_{\boldsymbol{\alpha},\boldsymbol{\tau},\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\beta}}[\log p(\boldsymbol{Y}, \boldsymbol{\pi}, \boldsymbol{\alpha}, \boldsymbol{\theta}, \boldsymbol{\tau}, \boldsymbol{\beta}, \boldsymbol{\sigma})] + \text{constants} \end{aligned}$$

Or,

$$q(\boldsymbol{\pi})^{\text{new}} = \frac{\exp(\text{E}_{\boldsymbol{\alpha},\boldsymbol{\tau},\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\beta}}[\log p(\boldsymbol{Y}, \boldsymbol{\pi}, \boldsymbol{\alpha}, \boldsymbol{\theta}, \boldsymbol{\tau}, \boldsymbol{\beta}, \boldsymbol{\sigma})])}{\int \exp(\text{E}_{\boldsymbol{\alpha},\boldsymbol{\tau},\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\beta}}[\log p(\boldsymbol{Y}, \boldsymbol{\pi}, \boldsymbol{\alpha}, \boldsymbol{\theta}, \boldsymbol{\tau}, \boldsymbol{\beta}, \boldsymbol{\sigma})]) d\boldsymbol{\pi}}$$

To maximize $\mathcal{L}(q)$ we use the following iterative algorithm

$$
\begin{aligned}
q(\boldsymbol{\sigma})^{\text{new}} & \propto \exp\left(\mathrm{E}_{\boldsymbol{\tau},\boldsymbol{\theta},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\pi}}[\log p(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\pi},\boldsymbol{\tau},\boldsymbol{Y})]\right) \\
q(\boldsymbol{\tau})^{\text{new}} & \propto \exp\left(\mathrm{E}_{\boldsymbol{\sigma},\boldsymbol{\theta},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\pi}}[\log p(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\pi},\boldsymbol{\tau},\boldsymbol{Y})]\right) \\
q(\boldsymbol{\theta})^{\text{new}} & \propto \exp\left(\mathrm{E}_{\boldsymbol{\sigma},\boldsymbol{\tau},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\pi}}[\log p(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\pi},\boldsymbol{\tau},\boldsymbol{Y})]\right) \\
q(\boldsymbol{\alpha})^{\text{new}} & \propto \exp\left(\mathrm{E}_{\boldsymbol{\sigma},\boldsymbol{\tau},\boldsymbol{\theta},\boldsymbol{\beta},\boldsymbol{\pi}}[\log p(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\pi},\boldsymbol{\tau},\boldsymbol{Y})]\right) \\
q(\boldsymbol{\beta})^{\text{new}} & \propto \exp\left(\mathrm{E}_{\boldsymbol{\sigma},\boldsymbol{\tau},\boldsymbol{\theta},\boldsymbol{\alpha},\boldsymbol{\pi}}[\log p(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\pi},\boldsymbol{\tau},\boldsymbol{Y})]\right) \\
q(\boldsymbol{\pi})^{\text{new}} & \propto \exp\left(\mathrm{E}_{\boldsymbol{\sigma},\boldsymbol{\tau},\boldsymbol{\theta},\boldsymbol{\alpha},\boldsymbol{\beta}}[\log p(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\theta},\boldsymbol{\sigma},\boldsymbol{\pi},\boldsymbol{\tau},\boldsymbol{Y})]\right)
\end{aligned}
$$

Update for $q(\boldsymbol{\sigma})_{it}$

$q(\boldsymbol{\sigma})_{it}$ is a Multinomial$(1, \boldsymbol{c}_{it})$ distribution, with typical parameter c_{its}

$$\boldsymbol{c}_{its} \quad \propto \quad \exp\left\{ \mathrm{E}[\log \beta_s] + \log \Gamma(\sum_{k=1}^{K} \alpha_{ks}) - \sum_{k=1}^{K} \log \Gamma(\alpha_{ks}) + \sum_{k=1}^{K} (\alpha_{ks} - 1) \mathrm{E}[\log \pi_{itk}] \right\}.$$

Update for $q(\boldsymbol{\tau})_{ijt}$

$q(\boldsymbol{\tau})_{ijt}$ is a Multinomial$(1, \boldsymbol{r}_{ijt})$ distribution with typical parameter,

$$r_{ijtk} \quad \propto \quad \exp\left\{ \mathrm{E}[\log \pi_{itk}] + \sum_{w=1}^{W} y_{ijtw} \mathrm{E}[\log \theta_{kw}] \right\}.$$

Update for $q(\boldsymbol{\pi})_{it}$

$q(\boldsymbol{\pi})_{it}$ is a Dirichlet(γ_{it}) distribution, with typical parameter γ_{itk} equal to

$$\gamma_{itk} = \sum_{s=1}^{S} c_{its}\alpha^{*}_{sk} + \sum_{j=1}^{D_{it}} r_{ijtk}$$

Update for $q(\boldsymbol{\theta})_k$

$q(\boldsymbol{\theta})_k$ is a Dirichlet($\boldsymbol{\eta}_k$) distribution, with typical parameter equal to,

$$\eta_{kw} = \lambda_w + \sum_{i=1}^{n} \sum_{t=2005}^{2007} \sum_{j=1}^{D_{it}} r_{itjk} y_{itw}$$

Update for $q(\boldsymbol{\beta})$

$q(\boldsymbol{\beta})$ is a Dirichlet(ϕ) distribution, with typical parameter ϕ_s equal to,

$$\phi_s = 1 + \sum_{i=1}^{n} \sum_{t=2005}^{2007} c_{its}$$

Completing $q(\boldsymbol{\sigma})_{it}$ and $q(\boldsymbol{\tau})_{ijt}$

Finishing $q(\boldsymbol{\sigma})_{it}$:

- $\text{E}[\log \beta_s] = \Psi(\phi_s) - \Psi(\sum_{z=1}^{S} \phi_z)$ where $\Psi(\cdot)$ is the digamma function (the derivative of the gamma function)
- $\text{E}[\log \pi_{itk}] = \Psi(\gamma_{itk}) - \Psi(\sum_{z=1}^{K} \gamma_{itz})$

Finishing $q(\boldsymbol{\tau})_{ijt}$

- $\text{E}[\log \theta_{kw}] = \Psi(\eta_{kw}) - \Psi(\sum_{z=1}^{w} \eta_{kz})$.

Update Steps for $\boldsymbol{\alpha}_s$

(Newton-Raphson, Minka 2000)

- Define $N_s = \sum_{i=1}^{n} \sum_{t=2005}^{2007} c_{its}$.

Differentiating with respect to α_{ks} shows that

$$\frac{\partial \log q(\boldsymbol{\alpha})_k^{\text{new}}}{\partial \alpha_{ks}} = -\frac{1}{\lambda} + N_s \Psi(\sum_{k=1}^{K} \alpha_{ks}) - N_s \Psi(\alpha_{ks}) + \sum_{i=1}^{n} \sum_{t=2005}^{2007} c_{its} \frac{\left(\Psi(\gamma_{itk}) - \Psi(\sum_{z=1}^{K} \gamma_{itz})\right)}{Ns}$$

- Call Gradient $\frac{\partial \log q(\boldsymbol{\alpha})_k^{\text{new}}}{\partial \boldsymbol{\alpha}_k}$.
- Define H as the Hessian (matrix of second derivatives).
- Diagonal element $h_{jj} = N_s \Psi^{'}(\sum_{k=1}^{K} \alpha_{ks}) - N_s \Psi^{'}(\alpha_{js})$ where $\Psi^{'}(\cdot)$ is the trigamma function
- Off-diagonal element $(a \neq b)$ $h_{ab} = N_z \Psi^{'}(\sum_{k=1}^{K} \alpha_{ks})$.

For each s we iterate,

$$\boldsymbol{\alpha}_s^{\text{new}} = \boldsymbol{\alpha}_s^{\text{old}} - \mathrm{H}^{-1} \frac{\partial \log q(\boldsymbol{\alpha})_k^{\text{new}}}{\partial \boldsymbol{\alpha}_k}$$

until convergence

Initialize $\boldsymbol{\gamma}_{it}^{\text{old}}$ (for all i and t), $\boldsymbol{\eta}_k^{\text{old}}$ (for all k), $\boldsymbol{\phi}^{\text{old}}$, $\boldsymbol{\alpha}_s^{\text{old}}$ (for all s).
Do until convergence in lower-bound.
- for all i, t, j and k, set

$$r_{ijtk}^{\text{new}} \propto \exp\left(\Psi(\gamma_{itk}^{\text{old}}) - \Psi(\sum_{z=1}^{K} \gamma_{itz}^{\text{old}}) + \sum_{w=1}^{W} y_{ijtw}\left[\Psi(\eta_{kw}^{\text{old}}) - \Psi(\sum_{z=1}^{W} \eta_{kz}^{\text{old}})\right]\right)$$

-for all i,t, and s set

$$\begin{aligned} c_{its}^{\text{new}} \propto \exp(\Psi(\phi_s^{\text{old}}) - \Psi(\sum_{z=1}^{S} \phi_s^{\text{old}}) + \log\Gamma(\sum_{k=1}^{K} \alpha_{ks}^{\text{old}}) - \sum_{k=1}^{K} \log\Gamma(\alpha_{ks}^{\text{old}}) \\ + \sum_{k=1}^{K}(\alpha_{ks}^{\text{old}} - 1)[\Psi(\gamma_{itk}^{\text{old}}) - \Psi(\sum_{z=1}^{K} \gamma_{itz}^{\text{old}})]) \end{aligned}$$

- for all i, t , and k set

$$\gamma_{itk}^{\text{new}} = \sum_{s=1}^{S} c_{its}^{\text{new}} \alpha_{ks}^{\text{old}} + \sum_{j=1}^{D_{it}} r_{ijtk}^{\text{new}}$$

- for all k and w set

$$\eta_{kw}^{\text{new}} = \lambda_w + \sum_{i=1}^{n} \sum_{t=2005}^{2007} \sum_{j=1}^{D_{it}} r_{ijtk}^{\text{new}} y_{ijtw}$$

-for all s set

$$\phi_s^{\text{new}} = 1 + \sum_{i=1}^{n} \sum_{t=2005}^{2007} c_{its}^{\text{new}}$$

- For all s obtain $\boldsymbol{\alpha}_s^{\text{new}}$ using Newton-Raphson algorithm.
- Evaluate lower-bound.

If converged:
Return posterior approximation.

$<$ / Variational Approximation $>$

< Model Selection >

Number of Topics

1) Substantive search (about 40-50)
2) 10-fold cross-validation. Loss function, approximate predictive posterior

$$p(\hat{\boldsymbol{y}}|\boldsymbol{Y}) \approx \sum_{\hat{\boldsymbol{\tau}}} \int\int p(\hat{\boldsymbol{y}}|\hat{\boldsymbol{\tau}}, \boldsymbol{\theta}) p(\hat{\boldsymbol{\tau}}|\boldsymbol{\pi}) q(\boldsymbol{\theta}|\boldsymbol{\eta}) q(\boldsymbol{\pi}) d\boldsymbol{\theta} d\boldsymbol{\pi}$$

3) Convergence with Nonparametric Bayesian model (Dirichlet process prior)

All converge on about 44 topics

Number of Styles

$$\text{BIC} = 2 \log p(\boldsymbol{Y})$$

1) $\text{BIC} \approx 2(\mathcal{L}(q) + \log K! + \log S!) - (K \times S)(n)$
2) $\text{BIC} \approx 2 \log p(\boldsymbol{Y} | \bar{\boldsymbol{\pi}}, \bar{\theta}, \bar{\boldsymbol{\tau}}) - (K \times S)(n)$

< / Model Selection >

Hierarchy of Topics

Posterior Distribution

$$p(\boldsymbol{\alpha}, \boldsymbol{\pi}, \boldsymbol{\eta}, \boldsymbol{\beta}, \boldsymbol{\sigma}, \boldsymbol{\mu}, \boldsymbol{\tau} | \boldsymbol{X}) \propto \prod_{m=1}^{M} c(\kappa) \exp\left(\kappa \boldsymbol{\eta}_m^{'} \frac{\mathbf{1}}{\sqrt{\boldsymbol{J}}}\right) \times \prod_{m=1}^{M} \prod_{k=1}^{K} \left[\beta_m c(\kappa) \exp\left(\kappa \boldsymbol{\mu}_k^{'} \boldsymbol{\eta}_m\right)\right]^{\sigma_{mk}} \prod_{k=1}^{K} \exp(-\alpha_k) \times$$

$$\prod_{i=1}^{48} \left[\frac{\Gamma(\sum_{k=1}^{K} \alpha_k)}{\prod_{k=1}^{K} (\alpha_k)} \prod_{k=1}^{K} \pi_{ik}^{\alpha_k - 1} \times \prod_{j=1}^{D_i} \prod_{k=1}^{K} \left[\pi_{ik} c(\kappa) \exp(\kappa \boldsymbol{x}_{ij}^{*} \boldsymbol{\mu}_k\right]^{\tau_{ijk}} \right] \quad (0.1)$$

Which we approximate with:

$$q(\boldsymbol{\alpha}, \boldsymbol{\pi}, \boldsymbol{\eta}, \boldsymbol{\beta}, \boldsymbol{\sigma}, \boldsymbol{\mu}, \boldsymbol{\tau}) = q(\boldsymbol{\alpha}) q(\boldsymbol{\pi}) q(\boldsymbol{\eta}) q(\boldsymbol{\beta}) q(\boldsymbol{\sigma}) q(\boldsymbol{\mu}) q(\boldsymbol{\tau}) \quad (0.2)$$

$$= q(\boldsymbol{\alpha}) \prod_{i=1}^{48} q(\boldsymbol{\pi})_i \prod_{m=1}^{M} q(\boldsymbol{\eta})_m q(\boldsymbol{\beta}) \prod_{k=1}^{K} q(\boldsymbol{\sigma})_k \prod_{k=1}^{K} q(\boldsymbol{\mu})_k \prod_{i=1}^{48} \prod_{j=1}^{D_i} q(\boldsymbol{\tau})_{ij}$$

Update for $q(\boldsymbol{\sigma})_k$

$q(\boldsymbol{\sigma})_k$ is a Multinomial(1, $\boldsymbol{c}_k$) where typical element c_{mk} is equal to

$$c_{mk} \quad \propto \quad \exp\left(\mathrm{E}[\log \beta_m] + \mathrm{E}[\kappa \boldsymbol{\mu}_k \boldsymbol{\eta}_m]\right).$$

We will complete the update step when we have determined the remaining forms of the distribution

Update for $q(\boldsymbol{\tau})_{ij}$

$q(\boldsymbol{\tau})_{ij}$ is a Multinomial(1, $\boldsymbol{r}_{ij}$, with typical element of r_{ijk} equal to

$$r_{ijk} \quad \propto \quad \exp\left(\mathrm{E}[\log \pi_{ik}] + \mathrm{E}[\kappa \boldsymbol{y}_{ij}^{*} \boldsymbol{\mu}_k]\right)$$

Again, as we complete the parametric forms of the other update steps we can complete this update equation.

Update for $q(\boldsymbol{\pi})_i$

$q(\boldsymbol{\pi})_i$ is a Dirichlet($\boldsymbol{\gamma}_i$) distribution, where typical element γ_{ik} is equal to

$$\gamma_{ik} = \alpha_k + \sum_{j=1}^{D_i} r_{ijk}$$

Update for $q(\boldsymbol{\beta})$

$q(\boldsymbol{\beta})$ is a Dirichlet($\boldsymbol{\phi}$) distribution with typical parameter ϕ_m equal to

$$\phi_m = 1 + \sum_{k=1}^{K} c_{mk}$$

Update for $q(\boldsymbol{\eta})_m$

Given the complications of taking expectations with the vMF distribution, we instead provide maximization steps for the vMF parameters. To obtain the form of the updates we follow the derivation outlined in Banerjee et al (2005). To do this, we take the log of the posterior distribution and identify the parameters that depend upon $\boldsymbol{\eta}_m$.

$$\log(p(\boldsymbol{\eta}_m) \quad = \quad \sum_{k=1}^{K} c_{km}\kappa\boldsymbol{\mu}_k\boldsymbol{\eta}_m + \kappa\boldsymbol{\eta}_m\frac{\mathbf{1}}{\sqrt{\boldsymbol{J}}} + \text{constants}$$

Update for $q(\boldsymbol{\eta})_m$

To set up the constrained optimization we also introduce the Langragian λ, with the constraint that $\boldsymbol{\eta}_m^{'}\boldsymbol{\eta}_m = 1$,

$$\log(p(\boldsymbol{\eta}_m) \quad \propto \quad \sum_{k=1}^{K} c_{km}\kappa\boldsymbol{\mu}_k\boldsymbol{\eta}_m + \kappa\boldsymbol{\eta}_m\frac{\mathbf{1}}{\sqrt{\boldsymbol{J}}} - \lambda(\boldsymbol{\eta}_m^{'}\boldsymbol{\eta}_m - 1).$$

Differentiating with respect to $\boldsymbol{\eta}_m$, setting equal to zero and solving yields

$$\frac{\kappa\left(\sum_{k=1}^{K} c_{mk}\boldsymbol{\mu}_k + \frac{\mathbf{1}}{\sqrt{\boldsymbol{J}}}\right)}{2\lambda} \quad = \quad \boldsymbol{\eta}_m \qquad (0.3)$$

Update for $q(\boldsymbol{\eta})_m$

If we differentiate with respect to λ and solve we see that $\boldsymbol{\eta}_m^{'}\boldsymbol{\eta}_m = 1$ or that $||\boldsymbol{\eta}_m^{'}\boldsymbol{\eta}_m|| = 1$. Substituting this into Equation 0.3 we have,

$$
\begin{aligned}
\frac{\kappa}{2\lambda}\left(\left(\sum_{k=1}^{K} c_{mk}\boldsymbol{\mu}_k + \frac{\mathbf{1}}{\sqrt{J}}\right)^{'}\left(\sum_{k=1}^{K} c_{mk}\boldsymbol{\mu}_k + \frac{\mathbf{1}}{\sqrt{J}}\right)\right)^{1/2} &= 1 \\
\frac{\kappa||\sum_{k=1}^{K} c_{mk}\boldsymbol{\mu}_k + \frac{\mathbf{1}}{\sqrt{J}}||}{2} &= \lambda
\end{aligned}
$$

Doing a final substitution we have

$$
\boldsymbol{\eta}_m^{*} = \frac{\sum_{k=1}^{K} c_{mk}\boldsymbol{\mu}_k + \frac{\mathbf{1}}{\sqrt{J}}}{||\sum_{k=1}^{K} c_{mk}\boldsymbol{\mu}_k + \frac{\mathbf{1}}{\sqrt{J}}||}
$$

Update for $q(\boldsymbol{\mu})_k$

Following a very similar set of derivations, the update step for $\boldsymbol{\mu}_k$ is

$$\boldsymbol{\mu}_k^* = \frac{\sum_{i=1}^{48}\sum_{j=1}^{D_i} r_{ijk}\boldsymbol{x}_{ij}^* + \sum_{m=1}^{M} c_{mk}\boldsymbol{\eta}_m^*}{||\sum_{i=1}^{48}\sum_{j=1}^{D_i} r_{ijk}\boldsymbol{x}_{ij}^* + \sum_{m=1}^{M} c_{mk}\boldsymbol{\eta}_m^*||}$$

Completing updates for $q(\boldsymbol{\sigma})_k$ and $q(\boldsymbol{\tau})_{ij}$

Given the forms $\text{E}[\log \beta_m] = \Psi(\phi_m) - \Psi(\sum_{m=1}^{M} \phi_m)$ and $\text{E}[\log \pi_{ik}] = \Psi(\gamma_{ik}) - \Psi(\sum_{k=1}^{K} \gamma_{ik})$ where $\Psi(\cdot)$ is the Digamma function.

Update for q(α)

A closed form update for the α parameters is unavailable. So we use the Newton-Raphson algorithm outlined in Minka (2000) and Blei, Ng, and Jordan (2003).