Imperfect Monitoring

- punishments at off-path histories
- in practice, monitoring is often imperfect (Green & Porter)

$\boxed{\text{take } \delta \to 1}$

- Prisoner's Dilemma. with imperfect public monitoring.

	E	S
E	2, 2	−1, 3
S	3, −1	0, 0

→ these payoffs are unobservable. but is concerned by players

$y \in \{\underline{y}, \bar{y}\}$

$$\Pr(y = \bar{y} \mid a) = \begin{cases} p, & \text{if } a = EE \\ q. & \text{if } a \in \{SE, ES\} \\ r. & \text{if } a = SS \end{cases}$$

where $0 < q < p < 1$ and $r < p$

$\underline{p < 1}$: can't perfectly infer whether partner shirked.

$\underline{q < p}$: conditional on partner working, my working increases chance of positive signal.

⇒ ex-post payoffs. [this part is mainly of interpretation concern. many researchers may only care about the ex-ante payoffs]

	$\bar{y}$	$\underline{y}$	
E	a	b	→ i'm working
S	c	d	→ i'm shirking

$$2 = pa + (1-p)b$$

$$-1 = q_e a + (1-q_e) b$$

$$3 = q_e c + (1-q_e) d$$

$$0 = rc + (1-r) d$$

Equilibrium:

• $\boxed{\text{perfect public eqa.}}$

take h^t, $\hat{h}^t$ s.t. y_τ is constant in h^t, $\tilde{h}^t$

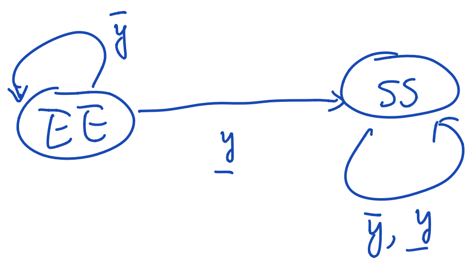
→ histories.

for all $\tau \le t$, $y_\tau = \tilde{y}_\tau$

then $\sigma(h^t) = \sigma(\hat{h}^t)$

[no restriction on player's strategies (they can deviate to private-history based strategies), but they just don't have the incentive to do so. So this solution only contains public observable history (because every other player is doing so)

Grim Trigger:



$$V(w_{EE}) = (1-\delta)\cdot 2 + \delta\left(p V(w_{EE}) + (1-p) V(w_{SS})\right)$$

why here we just put the "ex-ante strategy state", instead of the ex-post realization of world state here?

Because everything is linear. so only expectation matters

$$V(w_{SS}) = (1-\delta)\cdot 0 + \delta V(w_{SS}) = 0$$

Grim-Trigger IC:

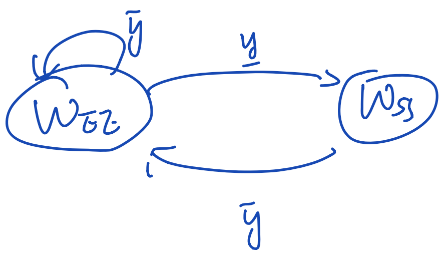
$$V(w_{EE}) \geq (1-\delta) 3 + \delta\cdot\left(q\cdot V(w_{EE}) + (1-q) V(w_{SS})\right)$$

$$V(w_{zz}) = \frac{c(1-\delta)\cdot 2}{1-\delta p}$$

If take p fixed. $\delta \to 1 \Rightarrow V(w_{zz}) \to 0$.
[intuition: only care infinite case]

By IC: $$\frac{2c(1-\delta)}{1-\delta p} \geq \frac{3c(1-\delta)}{1-\delta q}$$

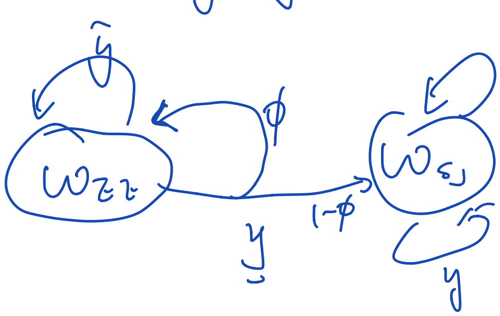
Add Forgiveness to Grim-Trigger.



$$V(w_{EE}) = c(1-\delta)\cdot 2 + \delta\cdot\left(pV(w_{zz}) + (1-p)V(w_{sj})\right)$$

$$V(w_{sj}) = \delta\cdot\left[rV(w_{zz}) + (1-r)V(w_{sj})\right]$$

Another way of adding forgiveness is:



$V(W_{SS}) = 0$

$$V(W_{ZZ}) = (1-\delta)\cdot 2 + \delta\cdot\{p\,V(W_{ZZ}) + (1-p)(\phi\,V(W_{ZZ}) + (1-\phi)\,V(W_{SS})\}$$

$$\Rightarrow V(W_{ZZ}) = \frac{(1-\delta)\cdot 2}{1-\delta\cdot(p+(1-p)\phi)}$$

Intuitively, these two ways should be equivalent (in expectation sense)

But does the second way has the renegotiation problem? Say standing at W_{SS}, would they agree to cooperate again?

$$IC: \quad V(W_{ZZ}) \geq (1-\delta)\cdot 3 + \delta\{(q+(1-q)\phi)\,V(W_{ZZ}) + (1-q)(1-\phi)\,V(W_{SS})\}$$

choose ϕ to maximize $V(W_{ZZ})$ s.t. IC

If we pick $\delta \to 1$ and ϕ being ~~optimal~~.

$$V(W_{ZZ}) = 2 - \frac{(1-p)}{p-q} < 2$$