

# Trade with Heterogeneous Prior Beliefs and Asymmetric Information

Morris 1994

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## Milgrom and Stokey

If an economic environment has the following properties:

- Agents are weakly risk-averse,
- Initial allocation is Pareto efficient relative to  $\theta$ -trades,
- Prior beliefs are concordant,
- It is common knowledge that everyone prefers a  $\theta$ -trade  $t$  to 0-trade,

then, everyone is indifferent between making and not making this trade. If utility functions are strictly concave,  $t$  is the 0-trade.

# Motivation

No Trade Theorem: Concordant Priors (and other assumptions)  
 $\implies$  No Trade.

This Paper: What differences in priors are needed to generate trade?

- Depends on the trading environment.

# Example

## Example

e.g.  $\begin{matrix} \text{ex-ante IC} \\ \text{OR} \\ \text{interim IC} \\ \text{OR} \\ \text{ex-post IC} \end{matrix}$  et.

This is what I mean by 'trading environment' matters;

Due to IC restriction, the differences in beliefs did not lead to trade.

# Road Map

In general, occurrence of trade depends on *trading rules* as well as *informational environment*.

So, no trade results in different environments require different belief structures.

Indirect approach: belief structure  $\Longleftrightarrow$  efficiency  $\implies$  No Trade.

# Efficiency Notions

Two ambiguities in notions of efficiency:

- Timing of welfare evaluation: Ex-ante, Interim, Ex-Post.
- Class of feasible reallocations/trades: Unconstrained, incentive compatible (IC), public.

## Summary of Results

C \ T	Ex-Ante	Interim	Ex-Post
Unconst.	concordant	(w) consistent conc.	revelation cons. conc.
IC	noisy conc.	noisy (w) cons. conc.	-
Public	Public conc.	P. (w) cons. conc.	P. rev. cons. conc.

“An initially efficient allocation is T C efficient iff beliefs are (C,T).”



# The Model

Exchange economy w/ asymmetric information.

Finite set of payoff-relevant states,  $S$ .

Finte set of agents,  $H = \{1, \dots, H\}$ .

$L$  commodities.

Each agent,  $h$  has a concave, strictly increasing and  $C^2$  utility function,  $u_h$ .

For each agent,  $h$ , there is a finite set of possible signals,  $T_h$ .

Let  $T = T_1 \times \cdots \times T_H$  and  $\Omega = T \times S$ .

For each agent,  $h$ , there is a prior belief  $\pi_h \in \Delta(\Omega)$  with  $\pi = (\pi_1, \dots, \pi_H)$ .

An allocation is a vector,  $e = (e_1, \dots, e_H)$  where  $e_h : \Omega \rightarrow \mathbb{R}^L$ .

## The Ex-Ante Efficiency

**$T^*$ -Support:** People agree on set of possible signal profiles,  
 $T^* \subset T$ .

**Full Marginal Support (FMS):** Each agent considers (i) each payoff-relevant state possible, (ii) each signal of each agent possible, after observing his signal.

An allocation  $e$  is **payoff-relevant** if it only depends on payoff-relevant states (but not signals).

$y$  **ex-ante dominates**  $e \iff y$  ex-ante Pareto dominates  $e$ .

An allocation  $x$  is a **feasible trade**, if  $\sum_{i=1}^H x_h(t, s) \leq 0$  for each  $t$  and  $s$ .

An allocation  $e$  is **initially efficient** if it is payoff-relevant and there does not exist a payoff-relevant feasible trade  $x$  s.t.  $e + x$  ex-ante dominates  $e$ .

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# Ex-ante Unconstrained Efficiency

Beliefs are **concordant** if they satisfy (FMS) and people's posteriors agree, conditional on payoff-relevant states.

## Theorem

*An initially efficient allocation is ex-ante efficient if and only if beliefs are concordant.*

An allocation  $e$  is **ex-ante incentive efficient** if  $e$  is IC and there does not exist a feasible trade  $x$  s.t.  $e + x$  is IC and ex-ante dominates  $e$ .

Beliefs  $\psi$  are **a noisy version** of some beliefs  $\theta$  if, each agent's belief is as if,

$\theta$  is their underlying belief but they consider their signals noisy so they take an affine combination of  $\theta$  and their beliefs under each of their signals.

An allocation  $e$  is **ex-ante incentive efficient** if  $e$  is IC and there does not exist a feasible trade  $x$  s.t.  $e + x$  is IC and ex-ante dominates  $e$ .

Beliefs  $\psi$  are **a noisy version** of some beliefs  $\theta$  if, for each agent,

$$\psi_h(t, s) = \alpha_h(t_h)\theta_h(t, s) + \sum_{t'_h \in T_h} \beta_h(t'_h, t_h)\psi_h((t'_h, t_{-h}), s) \quad (1)$$

for some  $\alpha_h : T_h \rightarrow (0, 1]$  and  $\beta_h : T_h^2 \rightarrow \mathbb{R}_+$ , for each  $t, s$  and  $h$ .



## Ex-Ante IC Efficiency

Beliefs are **noisy concordant** if they are a noisy version of concordant beliefs.

### Theorem

*An initially efficient allocation is ex-ante incentive efficient if and only if beliefs are noisy concordant.*

An allocation  $e$  is **public** if it is measurable w.r.t. public events so that  $e_h(t, s) = e_h(t', s)$  for each  $s$  and  $t' \in P(t)$ .

- $P(t)$  is a set of signal profiles that is self-evident at signal profile  $t$ .

An allocation  $e$  is **ex-ante public efficient** if  $e$  is public and there does not exist a feasible trade  $x$  s.t.  $e + x$  is public and ex-ante dominates  $e$ .

Beliefs  $\psi$  are a **public version** of beliefs  $\theta$  if, for each agent,

$$\sum_{t \in E} \psi_h(t, s) = \sum_{t \in E} \theta_h(t, s) \quad (2)$$

for each  $s$  and each public events  $E$ .

Beliefs are **public concordant** if they are a public version of concordant beliefs.

# Ex-Ante Public Efficiency

## Theorem

*An initially efficient allocation is ex-ante public efficient if and only if beliefs are public concordant.*

# Interim Efficiency and No Trade Theorems

So far, we considered the stage where agents know they would receive some signals in the future.

Now we study when the information will lead to trade after agents observe their signals.

$y$  **interim dominates**  $e \iff y$  interim Pareto dominates  $e$ .

An allocation  $e$  is **interim (incentive/public) efficient** if ( $e$  is IC/public and) there does not exist a feasible trade  $x$  s.t.  $e + x$  interim dominates  $e$  (and  $e + x$  is IC/public).

Beliefs have **public support** if everyone's prior assigns a positive probability to each (nonempty) public event.

Beliefs  $\psi$  are **weakly consistent version** of beliefs  $\theta$  if (i) they both have public support and (ii) whenever they are well-defined,  $\psi$  and  $\theta$  agree on posteriors conditioned on signals.

Beliefs  $\psi$  are a **consistent version** of  $\theta$  if they both satisfy (FMS) and  $\psi$  are a weakly consistent version of  $\theta$ .

Beliefs  $\psi$  are **noisy consistent concordant** if

$$\begin{array}{ccccc} \theta & \implies & \gamma & \implies & \psi \\ \text{concordant} & & \text{consistent with } \theta & & \text{noisy version of } \gamma \end{array} .$$

## Theorem

*An initially efficient allocation is interim (incentive/public) efficient if beliefs are (noisy/public) consistent concordant.*

*If an initially efficient allocation is interim (incentive/public) efficient, then the beliefs are (noisy/public) weakly consistent concordant.*



# Ex-Post Efficiency and Rational Expectations Equilibrium

Generically, all private information is revealed in an REE.

So, ex-post efficiency concepts will be relevant for evaluating an allocation as REE after arrival of information (signal profile).

Ex-post domination and efficiencies are defined as before; evaluated before  $s$  has unveiled.

Beliefs  $\psi$  are a **revelation consistent version** of  $\theta$  if they both satisfy (FMS) and for each agent, beliefs agree on the probabilities of payoff-relevant states conditional on *signal profiles*.

## Theorem

*An initially efficient allocation is ex-post (**public**) efficient if and only if beliefs are (**public**) revelation consistent concordant.*

# REE

An allocation is REE if there is a price vector s.t.

- ① Prices are independent of payoff-relevant states,
- ② Allocation of  $h$  only depends on payoff-relevant state and  $t_h$ ,
- ③ There is no other allocation  $x$  such that (i) (2) holds for  $x$ ;  
(ii)  $x$  is feasible and (iii)  $e + x$  ex-ante dominates  $e$ .

## Corollary

*An initially efficient allocation is REE if beliefs are consistent concordant.*

*If an initially efficient allocation is an REE, then the beliefs are public revelation consistent concordant.*

## Back to the Summary

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# Conclusions

- Not 'anything goes' if we relax the common prior assumption.
- "Assorted ad hoc ploys to get around no trade theorems -such as assuming unmodelled 'noise traders'- have been used in the literature."