ORIE 5380: Optimization Methods
Final Project

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Executive Summary

In this project, we are solving a aircraft cargo delivery problem. Our goal is to figure out the weekly schedule that minimize the operating cost. In the problem setting, there are 3 airports, everyday a different amount of cargo is imported into each airport. We can allocate the aircraft to deliver the cargo, or just transfer empty aircraft among airports. The cargo do not need to be delivered in the same day as it enters the system, but left it on ground would incur some cost per day. The cost we are trying to minimize thus comes from the reallocation of empty aircraft and the cost of left cargo on the ground. We ran a integer programming optimization model to solve this original problem. We also tweak the fleet size and cargo amount to get more insights of this problem.

Problem Overview

A company called Express Air runs a cargo delivery business by operating the aircraft fleet among 3 airports, airport A, airport B and airport C. There is a different amount of cargo that is imported into each airport on a daily basis, it should be delivered from the origin airport to a destination airport (the destination airport can't be the same as the origin airport). Notice that the cargo do not need to be delivered exactly the same day as it arrives into the system. It can be left on the ground for an arbitrary amount of days before it is delivered, but there is a cost for each one aircraft load of cargo that is left on the ground per day.

The operator of the Express Air looks at the amount of cargo that was not delivered the day before, the amount of cargo that arrives into the system on the current day, and decide how much cargo should be delivered for each pair of origin-destination in the current day. Naturally, the operator has to decide how many aircraft fleet should carry the cargo between origin-destination pairs. He also has the option of relocating the empty aircraft among different airports, if the operator thinks the relocation serves to balance the amount of inbound and outbound cargo in the airport. There might be some aircraft that is left in the same location too.

The cost is composed of three parts. The first is the cost of cargo delivery with loaded aircraft, but the total amount of loaded movements that the aircraft have to do is fixed, since all cargo in the system needed to be delivered. Thus, this portion of cost can't be optimized. The second part is the cost of relocation of the empty aircraft. The third part is the cost of holding cargo on the ground. Our objective here is to minimize the total cost of empty aircraft relocation and the cargo holding cost. We are aimed to figure out the weekly aircraft and cargo movement pattern.

Data Description

Express Air currently operates 1200 aircraft. There are 3 airports, airport A, airport B, airport C. In order to make it easier to record, we use 0 to denote airport A, while 1 stands for airport B, 2 for airport C. The set of locations is $N = \{0, 1, 2\}$. We consider the 5-days weekly cycle, that is, the total amount of days in the planning horizon is $T = 5$. The cost of relocating empty aircraft among different pairs of origin-destination airport is listed in Figure 1. Let C_{ij} denotes the cost of relocate a empty craft from airport i to airport j. The cost of holding one full aircraft of cargo on the ground is $C_h = 10$ per day. We assume that the time to travel between each airport is a full day, since the airport are far enough from each other.

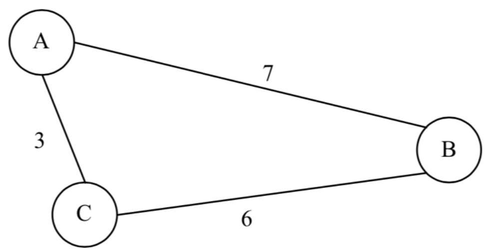

Figure 1: Empty aircraft relocation cost among different airports

The cargo that needs to be delivered has a consistent pattern over weeks. Table 1 shows the amount of cargo importing into the system on each day of the week that needs to be delivered between each origin-destination pair. The unit of quantities in the table is one aircraft load, each aircraft can carry exactly one aircraft load of cargo. Here we denotes:
A_{ijt} = Number of cargo that is arriving into the system on day t that needed to be delivered from airport i to airport j.

	M	T	W	T	F
A-B	100	200	100	400	300
A-C	50	50	50	50	50
B-A	25	25	25	25	25
B-C	25	25	25	25	25
C-A	40	40	40	40	40
C-B	400	200	300	200	400

Table 1: Amounts of cargo (in aircraft loads) arriving into the system on each day that need to be carried between each origin-destination airport

Model Overview

Decision Variables for aircraft:
X_{ijt} = Number of aircraft that carry a load of cargo from airport i to airport j on day t

Y_{ijt} =Number of empty aircraft that is relocated from airport i to airport j on day t
Z_{it} = Number of aircraft that stay at airport i on day t
Decision Variables for cargo:
W_{ijt} = Number of cargo that is holding on ground on day t that should be delivered from airport i to airport j(Note that W_{ijt} is cumulative, and represents the value in the "morning" rather than end of the day)
In this case, the Integer Programming is given by:

$$min \sum_{t=1}^{5} \sum_{i \in N} \sum_{j \in N} C_{ij} Y_{ijt} + \sum_{t=1}^{5} \sum_{i \in N} \sum_{j \in N} C_h W_{ijt} (i \neq j)$$

$st. \sum_{j \in N} X_{ijt} + \sum_{j \in N} Y_{ijt} + Z_{it} = \sum_{j \in N} X_{ji,t-1} + \sum_{j \in N} Y_{ji,t-1} + Z_{i,t-1} (\forall t = 2, ..., 5, i \in N, i \neq j)$

$\sum_{j \in N} X_{ij1} + \sum_{j \in N} Y_{ij1} + Z_{i1} = \sum_{j \in N} X_{ji5} + \sum_{j \in N} Y_{ji5} + Z_{i5} (\forall i \in N, i \neq j)$

$\sum_{i \in N} \sum_{j \in N} (X_{ijt} + Y_{ijt}) + \sum_{i \in N} Z_{it} = 1200 (\forall t = 1, ..., 5)$

$W_{ijt} = -X_{ij,t-1} + A_{ij,t-1} + W_{ij,t-1} (\forall t = 2, ..., 5, i \in N, j \in N, i \neq j)$

$W_{ij1} = -X_{ij5} + A_{ij5} + W_{ij5} (\forall i \in N, j \in N, i \neq j)$

$X_{ijt}, Y_{ijt}, Z_{it}, W_{ijt}$ are all integers, $\forall t = 1, ..., 5, i \in N, j \in N$

Mathematical Details of Optimization Model

In the optimization above. The objective function accounts for the total cost from the empty aircraft relocation and the cargo holding on ground.
We considered two sets of constraints. One for aircraft flow, the other for cargo flow. The travel time between each origin-destination pair is a full day, thus, if a certain number of aircraft is used between a particular origin-destination airport during day t, then these aircraft become available at the destination airport at the beginning of day t+1, no matter it carries the cargo or not. In the first constraint, $\sum_{j \in N} X_{ijt}$ corresponds to the number of loaded aircraft that leaves airport i on day t, $\sum_{j \in N} Y_{ijt}$ corresponds to the number of empty aircraft that leaves airport i on day t. Thus, $\sum_{j \in N} X_{ijt} + \sum_{j \in N} Y_{ijt} + Z_{it}$ on the left hand side of the first constraint corresponds to the total number of aircraft leaving airport i or staying at airport i on day t. On the other hand, $\sum_{j \in N} X_{ji,t-1}$ corresponds to the number of loaded aircraft that starts moving towards airport i on day t-1, $\sum_{j \in N} Y_{ji,t-1}$ corresponds to the number of empty aircraft that starts moving towards airport i on day t-1. Similarly, $Z_{i,t-1}$ is the number of aircraft that stays at airport i on day t-1. Thus, $\sum_{j \in N} X_{ji,t-1} + \sum_{j \in N} Y_{ji,t-1} + Z_{i,t-1}$ on the right hand side of the first constraint corresponds to the total number of aircraft that are available at airport i at the beginning of day t. In this case, the first constraint ensures that the total number of aircraft leaving airport i or staying at airport i on day t should be equal to the total number of aircraft available at airport i at the beginning of day t. Similarly, in the second constraint, we ensures that the total number of aircraft that flies

into a particular airport or stays at the airport at the last day of the week should be equal to the total number of aircraft that leaves this airport or stays at the airport at the first day of the next week.
The third constraint makes sure that the total number of empty or loaded aircraft that move between different origin-destination pairs and the aircraft that stays at the same airport for a particular day does not exceed the fleet size limit 1200.
In the 4th constraint, W_{ijt} on the left hand side corresponds to the number of cargo that is holding on ground on day t that should be delivered from airport i to airport j in the "morning". On the right hand side, $X_{ij,t-1}$ corresponds to the number of cargo that is delivered from airport i to airport j during the day of day t-1, $A_{ij,t-1}$ corresponds to the number of cargo that is added into the system that need to be carried from airport i to airport j during the day of day t-1, and W_{ijt} corresponds to the number of cargo that is holding on ground on day t-1 that should be delivered from airport i to airport j in the "morning". We ensure that the two parts in different sides of the equation is equal, since the number of cargo that is holding on ground on day t in the "morning" should be equal to day t-1's holding cargo in the "morning" plus the added cargo, then minus the cargo that is delivered on day t-1 for a particular origin-destination pair. Similarly, the 5th equation is used for make sure the weekly loop is closed considering the cargo relationship of the last day of this week and the first day of the next week.
We have two more equations for checking the model's correctness. The first is $\sum_{t=1}^{5} X_{ijt} = \sum_{t=1}^{5} A_{ijt}(\forall i \in N, j \in N, i \neq j)$, this equation makes sure that all added cargo are delivered for a certain origin-destination pair. The second is $X_{ijt} < W_{ijt} + A_{ijt}(\forall i \in N, j \in N, i \neq j, t = 1, ..., 5,$ which ensures that the number of cargo delivered on day t is less than the cargo holding on ground in that morning plus the number of cargo added into the system that day for a certain origin-destination pair.
One thing to notice is that, we considered Monday to Friday as a complete 5-day cycle and ignore the existence of weekend. That is, we assume that the cargo left at each airport didn't incur any cost during weekend. We will follow this setup in the following discussions. However, if taking the weekend cost into consideration, the objective function will change to

$$min \sum_{t=1}^{5} \sum_{i \in N} \sum_{j \in N} C_{ij} Y_{ijt} + \sum_{t=1}^{5} \sum_{i \in N} \sum_{j \in N} C_h W_{ijt} + 2C_h W_{ij5}(i \neq j)$$

while the constraints do not change.

Schedule Interpretation

The full output of the model can be found in Appendix 3. The optimal objective cost is 17925.

The optimal solution returned by the script should certainly satisfy the constraint. However, we still produced the following tables in order to make intuitive sense out of the result. Take the routine from Airport A to Airport B as an example, we verified the in and out flow of the cargo.
Note that number of units of cargo left at the end of the day should be and is equal to number of units of cargo left in the morning of the next day for each day of the weekly repeated schedule, thus the solution verifies the correctness of the LP.
The in and out flow of the aircrafts is even easier to verify, by adding up X, Y and Z. That is,

Cargo	Left in the morning (W)	Arrival (a)	Transported (X)	Left at the end of the day
M	5	100	15	90
T	90	200	290	0
W	0	100	100	0
T	0	400	400	0
F	0	300	295	5

Table 2: Cargo dynamics from Airport A to B

$\sum_{j=0}^{2} X_{ijt} + \sum_{j=0}^{2} Y_{ijt} + Z_{it} = \sum_{j=0}^{2} X_{jit-1} + \sum_{j=0}^{2} Y_{jit-1} + Z_{it-1}$.

For Airport A (i or j index 0), the result is as follows.
Monday (t index 0) out = Friday (index 4) in: $X_{001} + X_{002} + Y_{001} + Y_{002} + Z_{00} = X_{410} + X_{420} + Y_{410} + Y_{420} + Z_{40}$. According to the solution, $15 + 50 + 0 + 0 + 0 = 25 + 40 + 0 + 0 + 0$, which verifies the correctness of the LP.
Tuesday (t index 1) out = Monday (index 1) in: $X_{101} + X_{102} + Y_{101} + Y_{102} + Z_{10} = X_{010} + X_{020} + Y_{010} + Y_{020} + Z_{00}$. According to the solution, $290 + 50 + 0 + 0 + 0 = 25 + 40 + 85 + 0 + 0$, which verifies the correctness of the LP.
Wednesday (t index 2) out = Tuesday (index 1) in: $X_{201} + X_{202} + Y_{201} + Y_{202} + Z_{20} = X_{110} + X_{120} + Y_{110} + Y_{120} + Z_{10}$. According to the solution, $100 + 50 + 0 + 0 + 0 = 25 + 40 + 85 + 0 + 0$, which verifies the correctness of the LP.
Thursday (t index 3) out = Wednesday (index 2) in: $X_{301} + X_{302} + Y_{301} + Y_{302} + Z_{30} = X_{210} + X_{220} + Y_{210} + Y_{220} + Z_{20}$. According to the solution, $400 + 50 + 0 + 0 + 0 = 25 + 40 + 385 + 0 + 0$, which verifies the correctness of the LP.
Friday (t index 4) out = Thursday (index 3) in: $X_{401} + X_{402} + Y_{401} + Y_{402} + Z_{40} = X_{310} + X_{320} + Y_{310} + Y_{320} + Z_{30}$. According to the solution, $295 + 50 + 0 + 0 + 0 = 25 + 40 + 280 + 0 + 0$, which verifies the correctness of the LP.

As shown in the output, there are in total 0 node explored during branch-and-bound algorithm. This means that there's already integer solutions at the root. Thus, we relaxed the constraint that variables should be integer and changed them to be continuous values and ran the program again. It turns out they have the same optimal value with a minimal cost of 17925.

Additional Analysis and Findings

We further did a sequence of experiments, including 1) changing the fleet size, 2) changing the units of cargo arriving at each airport i and needed to be transported to airport j, to see their influence on the fleet allocation dynamics and the total cost. Please refer to the script in Appendix 1. All figures mentioned in this section are in Appendix 4.

Fleet size. As observed in the initial solution, there are quite a lot Y variables with non-zero values, which means that there are considerable number of empty aircrafts transporting every day and this might incur unnecessary cost. Therefore, we want to replace the fleet size to be smaller to see if there's still feasible solutions and thus can reduce the total cost. On the other hand, if we increase the fleet size, there is possibility that there will be more full aircrafts transporting between airports to reduce the cost incurred by units of cargo that are stranded. Starting from these two intuitions, we created a sequence of fleet size, namely 1000, 1100, 1200, 1300, 1400, 1500 and ran the integer program with these values. For fleet size we picked that is smaller than 1200, there is no feasible solutions.

For larger fleet sizes, we plotted the X, Y, Z, W values for each origin and destination airport pair along the days of the week as fleet size increases. The X-axis represents day of the week and the Y-axis represents the value of the variables; different color of lines represent the schedule under different fleet sizes. Red, green, blue, yellow lines correspond to fleetsize 1200, 1300, 1400, 1500. The labels are the sum of the variable over a week. Several observations are as follows:

- Sum of full aircrafts transporting between an origin-destination pair (X) for the week does not change, but the numbers are more balanced between days; considering different fleet size day by day, the days with larger X remedy the days with smaller X. Taking Figure 1: 2-1 (representing full aircrafts from C to B) as an example, on Day 0 (Monday), as fleet size increases from 1200 to 1500, X decreases; on Day 4 (Friday), as fleet size increase, X in contrast increases. The resulting line becomes "smoother" as Monday and Friday "balance off". This itself is beneficial as a more balanced schedule would be more efficient and convenient from operational perspective.
- Sum of empty aircrafts transporting between an origin-destination pair (Y) for the week does not change. Figure 2
- On certain day, cumulative leftover cargo (W) generally decreases as fleet in creases. This can be interpreted in combination with the first observation that if there's more leftover cargo, it needs more full aircrafts to transport the cargo; it also incurs more cost because more leftover are not transported; but if the lines of X are more smooth, it means there observes less extreme W incurring less cost. Thus, this also proves it is beneficial to increase fleet size. Figure 3
- Sum of remaining aircrafts at each airport (Z) generally increases, but there are no patterns along the days. We consider the change is directly due to the increase of fleet size. Figure 4

To sum up, under this particular problem setting, it is generally beneficial to increase fleet size. If considering the purchase and maintenance cost of the fleet, we might want to define a convex cost function to find out an optimal fleet size.

Arrival of cargo. An observation on the table of cargo arriving is that, for the origin-destination pair A to B and C to B, the number of units of arrival cargo is greatly larger than those of the other origin-destination pairs. It is very intuitive to continue with the "balancing off" idea to make the arrival at each airport less disparate and hypothesize to reduce cost.

We created a new arrival table based on the initial setting, by controlling the total of arrival at all airport on each day as the same then relocating some of the units from A to B and from C to B to other pairs.

For the plots, red, blue, green lines correspond original arrival arrangement, slightly more extreme allocations, and balanced allocations. Several observations are as follows:

	M	T	W	T	F
A-B	110	130	90	130	140
A-C	100	120	90	120	140
B-A	110	120	90	120	140
B-C	110	120	90	120	140
C-A	100	120	90	120	140
C-B	110	130	90	130	140

Table 3: Balanced amounts of cargo (in aircraft loads) arriving into the system on each day that need to be carried between each origin-destination airport

- Number of full aircrafts transporting between an origin-destination pair (X) becomes more balanced between days as well as locations, as the amount of cargo to be transported is more balanced. Figure 5
- Number of empty aircrafts transporting between an origin-destination pair (Y) decreases a great deal, as the full-loaded aircraftd flying into each destination already almost fulfilled the outgoing transporting demand for the next day. This will reduce the cost incurred by empty aircrafts. Figure 6
- Cumulative leftover cargo decreases and becomes 0 at each airport, which is ideal. Figure 7
- Number of remaining aircrafts at each airport (Z) increases because number of empty aircrafts decreases. Figure 8

We also tried to create more extreme arrival allocations, but the integer program did not have a feasible solution.
To sum up, as arrivals become more balanced, the aircrafts traveling the day before will have a higher utilization rate on the next day, and the number of travelling aircraft "fits" tighter to the amount of cargo needed to be transported. Balancing the arrival could probably save much cost. However, this manipulate might be artificial, as in real life we might not be able to change the amount of cargo at each airport at will due to requirements. Nevertheless, this experiment could offer some insights considering the arrival arrangements; if there is possibility to design a more balanced workload for each airport, it would be beneficial in reducing cost.