Errors for near-inertial wave calculations

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Abstract

1. Background

In order to find some of the basic descriptors of a near-inertial wave,

$$\Psi = Re(\Psi(z)exp(2pi(\omega_0 t - kx - ly - m_0 z' + \phi\psi))) \tag{1}$$

we use a best fit plane-wave to describe the observations. From this plane-wave we have the variables: m, ω, and ϕ. Figure **??** shows the χ^2 value for varying values of ω_1. With a red vertical line at what we are using for the best fit value ($0.35 = 1.04f$). The next step is to calculate the error on each

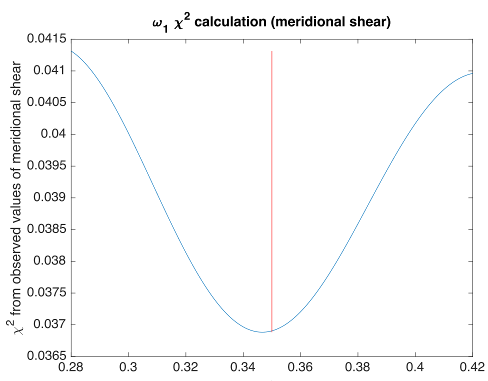

Figure 1:

of these variables so that we are able to determine the error in the various subsequent calculations. This is especially important in the assessment of the validity of our descriptions of the observed wave.

2. Initial error–test statistic

Hypothesis testing:
$H_0 : \bar{V_a} = \bar{V_b}$
$H_1 : \bar{V_a} \neq \bar{V_b}$

$$Z_{95} = \frac{\bar{V_a} - \bar{V_b}}{(\frac{s_a^2}{N_a} - \frac{s_b^2}{N_b})^{\frac{1}{2}}} \tag{2}$$

Where:
$t_{95} = 1.646$ =The 95% significance level for a one-tailed confidence interval.
$\bar{V_a}$ = average difference between the observations and the best fit plane-wave squared
$\bar{V_b}$ = average difference between the observations and the plane-wave, with various values for m, ω, and ϕ, independently while holding the other two variables constant, squared
$s_a^2 = \frac{\Sigma(V_a - \bar{V_a})^2}{N-1}$ =variance of the expected plane-wave solution
$s_b^2 = \frac{\Sigma(V_b - \bar{V_b})^2}{N-1}$ =variance of the varying plane-wave solution
N_a =number of independent measurements
N_b =number of independent measurements

The t-score for over 1000 independent measurements at 95% confidence interval is 1.646. We calculate the Z-score for the plane-wave across varying values of ω_1 ([.28 .42] by 0.00005).

2.1. Questions

- How many rows should I use for independent measurements?
- Do I round to the nearest hundreth? Down on the lower bound and up on the upper bound?

- Should it be symmetric?
- If I divide by 4 is that saying every fourth row is independent or every third row is independent?
- if it is not symmetric do I need to calculate the upper and lower bounds independently in each error calculation?

3. Errors

variable	**error**	**method**
salinity	$\pm 2 \times 10^{-3}$	(Voet 2015)
temperature	$\pm 5 \times 10^{-4}$ °C	(Voet 2015)
u, v	2 cm s^{-1}	(Voet 2015)
r_I	$\pm.06$	RMS
ω_1	± 0.02	t-score
ω_2	± 0.01	t-score
m_1		t-score
m_2		t-score
ϕ_1	± 20	t-score
ϕ_2	± 20	t-score

3.1. questions

- What is the error of the lat and lon? I know it's very small but it would play a role in the f_{cor} and f_{eff} calculation.
- Matthew uses the RMS of the r_I for the error, we can also calculate this using the measured errors because $r_I = \sqrt{\frac{\frac{KE(z)}{PE(z)}+1}{\frac{KE(z)}{PE(z)}-1}}$

4. Propagation of errors

How to propagate the errors through various calculations (Mandel 1984, Bevington and Robinson 1992):

equation	**error**
$z = x + y + ...$ or $z = x - y - ...$	$\sigma_z = \sqrt{(\sigma_x)^2 + (\sigma_y)^2 + ...}$
$z = cx$	$\sigma_z = c\sigma_x$
$z = \frac{x*y}{d}$	$\frac{\sigma_z}{z} = \sqrt{(\frac{\sigma_x}{x})^2 + (\frac{\sigma_y}{y})^2 + (\frac{\sigma_d}{d})^2 + ...}$
$z = x^m y^n$	$\frac{\sigma_z}{z} = \sqrt{(\frac{m\sigma_x}{x})^2 + (\frac{n\sigma_y}{y})^2}$

List of variables to calculate error:

variable	**equation**	**error**
ρ		
N		
Θ		
$\lVert U \rVert$	$\sqrt{u^2 + v^2}$	$\frac{1}{2}\sqrt{(u^2 + v^2)((2u\sigma_u)^2 + (2v\sigma_v)^2)}$
KE	$\rho\langle u^2 + v^2 \rangle$	$\rho(u^2 + v^2)\sqrt{(\frac{(2u\sigma_u)^2+(2v\sigma_v)^2}{u^2+v^2})^2 + (\frac{\sigma_\rho}{\rho})^2}$
PE	$\frac{1}{2}\rho N^2 \langle \eta^2 \rangle$	$\frac{1}{4}\rho N^2 \langle \eta^2 \rangle \sqrt{(\frac{2N\sigma_N}{N^2})^2 + (\frac{2\eta\sigma_\eta}{\eta^2})^2 + (\frac{\sigma_\rho}{\rho})^2}$
f_{eff}	$\sqrt{\frac{\omega_0^2}{r_I^2 + \frac{m^2U^2cos^2(\theta-\alpha)}{N^2}(r_1^2-1)}}$	See Eqn. **??** below
K	$\sqrt{k^2 + l^2} = \sqrt{\frac{m^2 f_{eff}^2 (r_1^2-1)}{N^2}}$	$\frac{1}{2}\sqrt{\frac{m^2 f_{eff}^2 (r_I^2-1)}{N^2}\left[(\frac{2\sigma_m}{m})^2 + (\frac{2\sigma_{f_{eff}}}{f})^2 + (\frac{2\sigma_{r_I} r_I}{r_I^2-1})^2 + (\frac{2\sigma_N}{N})^2\right]}$
γ	$\frac{\partial\eta}{\partial z} = \frac{N^2(z)}{N^2(t,z)} - 1$	
$u_z,\ v_z$		
Fr	$\frac{S}{N}$	

$$\sqrt{\frac{r_I^2 + \frac{m^2U^2cos^2(\theta-\alpha)}{N^2}(r_1^2-1)}{\omega_0^2}\left[(\frac{2\sigma_{\omega_0}}{\omega_0})^2 + \frac{(2\sigma_{r_I} r_I)^2 + (\frac{m^2U^2cos^2(\theta-\alpha)}{N^2}(r_1^2-1))((\frac{2\sigma_m}{m})^2 + (\frac{2\sigma_U}{U})^2 + (\frac{2\sigma_{r_I} r_I}{r_I^2-1})^2 + (\frac{2\sigma_N}{N})^2)}{(r_I^2 + \frac{m^2U^2cos^2(\theta-\alpha)}{N^2}(r_1^2-1))^2}\right]} \tag{3}$$

4.1. questions

- for where I plug in the measured value do I plug in the average? Max?

Example:

$$\|U\| = \sqrt{u^2 + v^2}$$

$$\text{error } u^2 \rightarrow \frac{\sigma_{u^2}}{u^2} = \frac{2\sigma_u}{u} \rightarrow \sigma_{u^2} = 2u\sigma_u$$

$$\text{error } u^2 + v^2 \rightarrow \sigma_{u^2+v^2} = \sqrt{(2u\sigma_u)^2 + (2v\sigma_v)^2}$$

$$\text{error } \sqrt{u^2 + v^2} \rightarrow \frac{\sigma_{\sqrt{u^2+v^2}}}{\sqrt{u^2+v^2}} = \frac{1}{2}\sqrt{(2u\sigma_u)^2 + (2v\sigma_v)^2}$$

$$\rightarrow \sigma_{\|U\|} = \frac{1}{2}\sqrt{(u^2 + v^2)((2u\sigma_u)^2 + (2v\sigma_v)^2)}$$

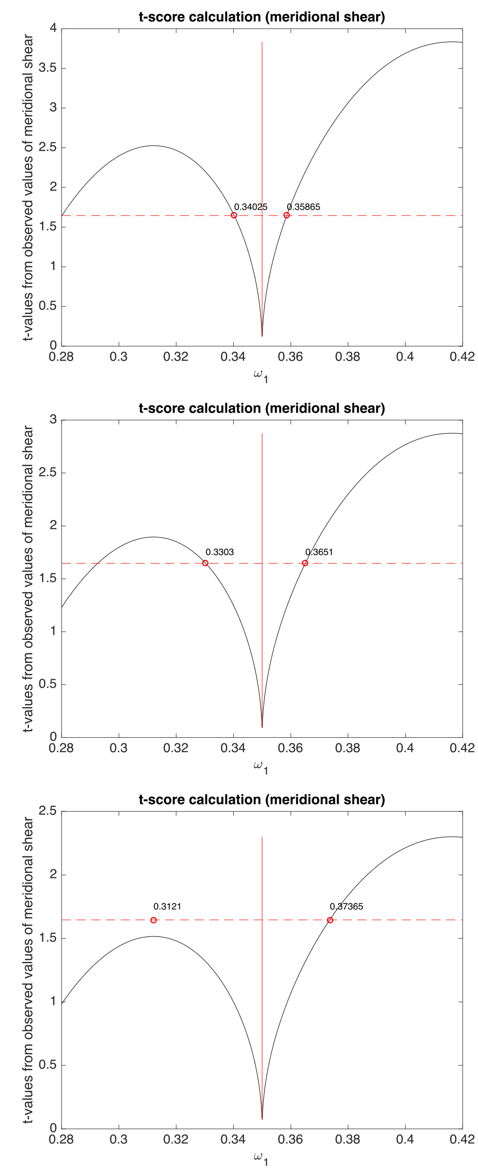

Figure 2: Top - every third row independent, Middle - every fourth row independent, Bottom - every fifth row independent. I've been using 4 because it was the widest possible before the intersection became imaginary. The solid red line is the best fit value. The dashed line is the t-score for a 95% confidence interval. The black line is the plot of the t-score for each value of the variable, in this case ω_1.