Relevant Theorems and misc:
Proposition 6.1: The minimal generating polynomial $D_{\bar{Y}}$ for the generic sequence $\{A^i\bar{Y}\}$ has determinantal degree $\bar{\nu}$.

Theorem 8.1 (Villard): Let A be a $N \times N$ matrix over K with minimal polynomial $\pi_A(\lambda)$, and let Y with $n \geq \phi$ columns chosen at random. If $K = GF(q)$ then $Prob\{dim\langle Y\rangle = \bar{\nu}\} \geq \Phi(\pi_A, \phi)$.

Proposition 8.2 Let A be a $N \times N$ matrix over K with minimal polynomial $\pi_A(\lambda)$, and let X and Y be chosen at random with m rows and n columns. If $m \geq min\{\phi, n\}$, then $D_Y(\lambda) = D_W(\lambda)$ with probability no less than $\Phi(\pi_A, min\{\phi, n\})$.

2.6 of KaVi04: $rank(Hk_{e,d}) = deg(det(F_X^{A,Y}))$ for $d \geq deg(F_X^{A,Y})$ and $e \geq n$

2.7 of KaVi04: $\nu = max\{rank(Hk_{e,d}(A, X, Y))\}$ over all possible e, d, X, Y. Moreover, ν is equal to the sum of the degrees of the first M invariant factors of $\lambda I - A$ (where M is the size of X, Y)

2.12 of KaVi04: Let $s_i, \ldots, s_\phi$ be all the invariant factors of $\lambda I - A$. This i^{th} invariant factor of $F_X^{A,Y}$ divides s_i. Furthermore, there exist matrices W, Z st $\forall i$, $1 \leq i \leq min(M, \phi)$, the i^{th} invariant factor of $F_W^{A,Z}$ is equal to s_i (all other remaining ones are equal to 1).

Proposition 1: If we choose random matrices X, Y, then $deg(det(F_X^{A,Y})) = \bar{\nu}$ with high probability.

Proof: Directly follows from the theorems 8.1 and 8.2 and proposition 6.1

Proposition 2: For random choice of blocking matrices W, Z, the i^{th} invariant factor of $F_W^{A,Z}$ is equal to the i^{th} invariant factor of A for $1 \leq i \leq min(M, \phi)$ (with all other remaining factors equal to 1) with high probability.

Proof: We choose $\mathbb{X}, \mathbb{Y}$ as the specialization of $\bar{X}, \bar{Y}$ given in (Villard, corollary 6.4). From equation (2.17) of theorem 2.12, we have that

$$deg(det(F_{\mathbb{X}}^{A,\mathbb{Y}})) = \bar{\nu} = max_{X,Y}(deg(det(F_X^{A,Y}))) = \nu$$

Thus, by proposition 1, for any random matrices W, Z, with high probability,

$$deg(det(F_W^{A,Z})) = \nu$$

Now, assume $deg(det(F_W^{A,Z})) = \nu$ and let $\bar{s_i}$ be the i^{th} invariant factor $F_W^{A,Z}$. Then, by the first assertion of theorem 2.12, $\bar{s_i}$ divides s_i. Since ν is the sum of the degrees of the invariant factors of $\lambda I - A$, this can only happen if $s_i = \bar{s_i}$ by the same reasoning as the end of theorem 2.12.