# SOME USEFUL FORMULAS

*(The section number indicates the principal reference in the text.)*

**Perpetuity** (3-2)

The value of a perpetuity of $1 per year is:

$$PV = \frac{1}{r}$$

**Annuity** (3-2)

The value of an annuity of $1 per period for $t$ years ($t$-year annuity factor) is:

$$PV = \frac{1}{r} - \frac{1}{r(1+r)^t}$$

**A Growing Perpetuity (the "Gordon" model)** (3-2)

If the first period's cash flow is $1 at year 1 and if cash flows thereafter grow at a constant rate of $g$ in perpetuity,

$$PV = \frac{1}{r-g}$$

**Continuous Compounding** (3-3)

If $r$ is the continuously compounded rate of interest, the present value of $1 received in year $t$ is:

$$PV = \frac{1}{e^{rt}}$$

**Equivalent Annual Cost** (6-3)

If an asset has a life of $t$ years, the equivalent annual cost is:

$$\frac{\text{PV (costs)}}{t\text{-year annuity factor}}$$

**Measures of Risk** (7-2 to 7-4)

Variance of returns = $\sigma^2$
= expected value of $(\tilde{r} - r)^2$

Standard deviation of returns = $\sqrt{\text{variance}} = \sigma$

Covariance between returns of stocks 1 and 2 = $\sigma_{12}$ = expected value of $[(\tilde{r}_1 - r_1)(\tilde{r}_2 - r_2)]$

Correlation between returns of stocks 1 and 2:

$$\rho_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2}$$

Beta of stock $i = \beta_i = \dfrac{\sigma_{im}}{\sigma_m^2}$

The variance of returns on a portfolio with proportion $x_i$ invested in stock $i$ is:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}$$

**Capital Asset Pricing Model** (8-2)

The expected risk premium on a risky investment is:

$$r - r_f = \beta(r_m - r_f)$$

**Unlevering Betas** (9-3 and 19-1)

$$\beta_{\text{assets}} = \beta_{\text{debt}}\left(\frac{D}{V}\right) + \beta_{\text{equity}}\left(\frac{E}{V}\right)$$

where

$D$ and $E$ = the market values of debt and equity ($V = D + E$)

**Weighted Average Cost of Capital** (9-3 and 19-1)

$$\text{WACC} = r_D(1 - T_c)D/V + r_E E/V$$

where

$r_D$ and $r_E$ = the expected returns on debt and equity

$T_c$ = the marginal rate of corporate tax

$D$ and $E$ = the market values of debt and equity ($V = D + E$)

**MM's Proposition II** (17-2 and 19-3)

The required return on equity ($r_E$) increases in line with the debt–equity ratio calculated using market values ($D/E$):

$$r_E = r + (r - r_D)D/E$$

where r is the opportunity cost of capital.

### Relationship between the Value of a Call and a Put (20-2)

The relationship between the value of a European call and a European put is:

Value of call + present value of exercise price = value of put + share price

### Black–Scholes Formula for Value of a Call (21-3)

Value of a call = $[N(d_1) \times P] - [N(d_2) \times \text{PV(EX)}]$

where

$$d_1 = \log[P/\text{PV(EX)}]/\sigma\sqrt{t} + \sigma\sqrt{t}/2$$
$$d_2 = d_1 - \sigma\sqrt{t}$$

$N(d)$ = cumulative normal probability function
PV(EX) = present value of exercise price
$t$ = number of periods to exercise date
$P$ = current price of stock
$\sigma$ = standard deviation per period of continuously compounded rate of return on stock
log = natural logarithm

### Inputs to Binomial Option Valuation Model (21-1 and 21-2)

$$\text{Probability of upside change in risk-neutral world} = p = \frac{r_f - d}{u - d}$$

$$1 + \text{Upside change} = u = e^{\sigma\sqrt{h}}$$

$$1 + \text{Downside change} = d = 1/u$$

where $\sigma$ = standard deviation of price changes per year
$h$ = interval as fraction of a year

$$\text{Option delta} = \frac{\text{spread of possible option prices}}{\text{spread of possible stock prices}}$$

### Bond Duration and Volatility (24.3)

$$\text{Duration of T-period bond} = \frac{\sum_{t=1}^{T} t \times C_t/(1+y)^t}{\sum_{t=1}^{T} C_t/(1+y)^t}$$

Volatility (modified duration) = Duration/(1 + y)

### Value of Lease (26-4)

If $\text{LCF}_t$ is the lease's cash outflow in period $t$, the value of an $N$-period lease of an asset costing INV is:

$$\text{INV} - \sum_{t=0}^{N} \frac{\text{LCF}_t}{[1 + r(1 - T_c)]^t}$$

### Value of a Future (27-2)

$$\frac{\text{Futures price}}{(1+r_f)^t} = \text{spot price} + \text{PV}\begin{pmatrix}\text{storage}\\ \text{costs}\end{pmatrix} - \text{PV}\begin{pmatrix}\text{convenience}\\ \text{yield}\end{pmatrix}$$

### Interest Rate Parity (28-2)

$$\frac{1 + r_¥}{1 + r_\$} = \frac{f_{¥/\$}}{s_{¥/\$}}$$

### Growth Rates (29-5)

Sustainable growth rate (no increase in leverage) = plowback ratio × return on equity

Internal growth rate (no external finance)
= retained earnings/net assets
= plowback ratio × return on equity × equity/net assets