- Convection/Diffusion
- Hyperbolic
- Numerical Integration
- Interpolation
- Parallel computing

$$\theta_t + +u\theta_x = \Gamma\theta_{xx} \tag{1}$$

$$\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} = \Gamma\frac{\partial^2\theta}{\partial x^2} \tag{2}$$

$$\text{let } \varphi = \rho Y_i \tag{3}$$

$$\frac{\partial\rho Y_i}{\partial t} + \vec{v}\cdot\nabla\rho Y_i = \rho\Gamma\nabla^2 Y_i \nabla\cdot\vec{v} = 0 \tag{4}$$

FTCS

$$\varphi^{n+1} = \varphi^n - \frac{u\Delta t}{z\Delta x}\left(\varphi_{i+1} - \varphi_{i-1}\right)^n + \frac{\Gamma\Delta t}{\Delta x^2}\left(\varphi_{i+1} - 2\varphi_i + \varphi_{i-1}\right) \tag{5}$$

SS.

$$\frac{\partial u\phi}{\partial x} = \Gamma\frac{\partial^2\phi}{\partial x^2} \tag{6}$$

$$(u\phi)_e - (u\phi)_w = \Gamma\left(\frac{\partial\phi}{\partial x}\right)_e - \Gamma\left(\frac{\partial\phi}{\partial x}\right)_w \tag{7}$$

$$\frac{1}{2}u_e\left(\phi_E + \phi_P\right) - \frac{1}{2}u_w\left(\phi_P + \phi_W\right) = \frac{\Gamma}{\delta}\left(\phi_E - \phi_P\right) - \frac{\Gamma}{\delta}\left(\phi_P - \phi_W\right) \tag{8}$$

$$Re = \frac{u\delta}{\Gamma} \tag{9}$$

$$\phi_W\left[-\frac{Re}{2} - 1\right] + \phi_P\left[2\right] + \phi_E\left[\frac{Re}{2} - 1\right] = 0 \tag{10}$$

$$\phi_P = \frac{1}{2}\left[\phi_w\left(\frac{Re}{2} + 1\right) - \phi_E\left(\frac{Re}{2} - 1\right)\right] \tag{11}$$

FTCS is wrong for this example for two reasons:

- Unphysical results can occur
- Dependent on Reynold's number

$c_p\phi_p = c_E\phi_E + c_W\phi_W$ Mass balance is

$$\frac{\partial\rho}{\partial t} + \frac{\partial u}{\partial x} = 0 \tag{12}$$

$$\frac{\partial u}{\partial x} = 0 \tag{13}$$

Add Upwinding let $u > 0$

$$u\phi_e - u\phi_w = \Gamma(\frac{\partial \phi}{\partial x})_e - \Gamma(\frac{\partial \phi}{\partial x})_w \tag{14}$$

$$u\phi_p - u\phi_W = \Gamma(\phi_E - \phi_P) - \frac{\Gamma}{\delta}(\phi_p - \phi_w) \tag{15}$$

$$\phi_p = \phi_E \left[(\frac{\Gamma}{\delta})/(u + 2\frac{\Gamma}{\delta}) \right] + \phi_W \left[(u + \frac{\Gamma}{\delta})/(u + 2\frac{\Gamma}{\delta}) \right] \tag{16}$$