Problem 4.37

Solution:

Known quantities:
Functions.

Find:
The phasor form.

Analysis:
In phasor form:

a) $V(jw) = 155\angle\text{-}25°$ V

b) $V(jw) = 5\angle\text{-}130°$ V

c) $I(jw) = 10\angle 63° + 15\angle\text{-}42° = (4.54 + j8.91) + (11.15 - j\,10.04) = 15.69 - j1.13 = 15.73\angle\text{-}4.12°$ A

d) $I(jw) = 460\angle\text{-}25° - 220\angle 75° = (416.90 - j194.40) - (56.94\text{-}j212.50) = 359.96 + j18.10 = 360.4\angle 2.88°$ A

Problem 4.38

Solution:

Known quantities:
Complex number.

Find:
The polar form.

Analysis:

a) $4 + j4 = 4\sqrt{2}\angle 45° = 5.66\angle 45°$

b) $\text{-}3 + j\,4 = 5\ \angle 126.9°$

c) $j + 2 - j4 - 3 = \text{-}1 \text{-}j\,3 = 3.16\angle\text{-}108.4°$

Problem 4.39

Solution:

Known quantities:
Complex number.

Find:
The polar form.

Analysis:

a) $(50 + j10)(4 + j8) = (50.99\angle 11.30°)(8.94\angle 63.43°) = 456.1\angle 74.7°$

$(50 + j10)(4 + j8) = 200 + j400 + j40 + j^2\,80 = 120 + j440 = 456.1\angle 74.7°$

b) $(j2 - 2)(4 + j5)(2 + j7) = (2.82\angle 135°)(6.40\angle 51.34°)(7.28\angle 74.05°) = 131.8\angle 260.4° = 131.8\angle\text{-}99.6°$

$(j2 - 2)(4 + j5)(2 + j7) = \text{-}36 \text{-} j126 \text{-} j4 \text{-} j^2\,14 = \text{-}22 \text{-} j130 = 131.8\angle\text{-}99.6°$

Problem 4.40

Solution:

Known quantities:
Complex number.

Find:
a) Complex conjugate
b) Polar form, by first multiplying numerator and denominator by the complex conjugate.
c) Polar form, by converting into polar coordinates.

Analysis:

$A = 4 + j\,4,\ A^* = 4 - j\,4$

a) $B = 2 - j\,8,\ B^* = 2 + j\,8$

$C = -5 + j\,2,\ C^* = -5 - j\,2$

b)

$$\frac{1+j7}{4+j4} = \frac{(1+j7)(4-j4)}{(4+j4)(4-j4)} = \frac{4 - j4 + j28 - j^2\,28}{16+16} = \frac{32+j24}{32} = 1 + j\,0.75 = 1.25\angle 36.87°$$

$$\frac{j4}{2-j8} = \frac{j4(2+j8)}{(2-j8)(2+j8)} = \frac{-32+j8}{4+64} = -\frac{32}{68} + j\frac{8}{68} = 0.485\angle 165.96°$$

$$\frac{1}{-5+j2} = \frac{1(-5-j2)}{(-5+j2)(-5-j2)} = \frac{-5-j2}{25+4} = -\frac{5}{29} - j\frac{2}{29} = 0.1857\angle -158.2°$$

c) Repeat b) converting to polar form first:

$$\frac{1+j7}{4+j4} = \frac{7.071\angle 81.87°}{4\sqrt{2}\angle 45°} = 1.25\angle 36.87°$$

$$\frac{j4}{2-j8} = \frac{4\angle 90°}{8.246\angle 75.96°} = 0.485\angle 165.96°$$

$$\frac{1}{-5+j2} = \frac{1\angle 0°}{5.385\angle 158.2°} = 0.1857\angle -158.2°$$

Problem 4.41

Solution:

Known quantities:
Complex number.

Find:
Real-imaginary form

Analysis:

$$j^j = e^{-\pi/2} = 0.2079$$

$$e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1 + j0 = -1$$

Section 4.4: Phasor Solution of Circuits with Sinusoidal Excitation

Focus on Methodology: Phasors

1. Any sinusoidal signal may be mathematically represented in one of two ways: a **time domain form**: $v(t) = A\cos(\omega t + \theta)$, and a frequency domain form: $\mathbf{V}(j\omega) = Ae^{j\theta} = A\angle\theta$. Note the $j\omega$ in the notation $\mathbf{V}(j\omega)$, indicating the $e^{j\omega t}$ dependence of the phasor. In the remainder of this chapter, bold uppercase quantities indicate phasor voltages and currents.
2. A phasor is a complex number, expressed in polar form, consisting of a *magnitude* equal to the peak amplitude of the sinusoidal signal and a *phase angle* equalto the phase shift of the sinusoidal signal *referenced to a cosine signal.*
3. When using phasor notation, it is important to note the specific frequency ω of the sinusoidal input.

Problem 4.43

Solution:

Known quantities:
The current through and the voltage across a component.

Find:
a) Whether the component is a resistor, capacitor, inductor
b) The value of the component in ohms, farads, or henrys.

Analysis:
a) The current and the voltage can be expressed in phasor form:

$$\mathbf{I} = 17\angle -15^o\ \mathbf{mA},\ \mathbf{V} = 3.5\angle 75^o\ \mathbf{V}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{3.5\angle 75^o\ \mathbf{V}}{17\angle -15^o\ \mathbf{mA}} = 205.9\angle 90^o\ \Omega = 0 + j\cdot 205.9\ \Omega$$

The impedance has a positive imaginary or reactive component and a positive angle of 90 degree indicating that this is an inductor (see Fig. 4.39).

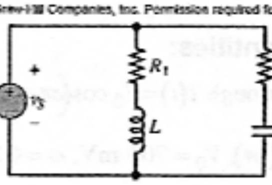
b) $Z_L = j\cdot X_L = j\cdot\omega L = j\cdot 205.9\ \Omega \Rightarrow L = \dfrac{205.9\ \Omega}{628.3\ \dfrac{\mathbf{rad}}{\mathbf{s}}} = 327.7\ \mathrm{m}\dfrac{\mathbf{Vs}}{\mathbf{A}} = 327.7\ \mathbf{mH}$

Problem 4.47

Solution:

Known quantities:

The values of the impedance, $R_1 = 2.3\ \mathbf{k\Omega}$, $R_2 = 1.1\ \mathbf{k\Omega}$, $L = 190\ \mathbf{mH}$, $C = 55\ \mathbf{nF}$ and the voltage applied to the circuit shown in Figure P4.47, $v_s(t) = 7\cos\left(3000t + 30^o\right)\mathbf{V}$.



Find:

The equivalent impedance of the circuit.

Analysis:

$$X_L = \omega L = \left(3\ \mathbf{k}\frac{\mathbf{rad}}{\mathbf{s}}\right)(190\ \mathbf{mH}) = 0.57\ \mathbf{k\Omega} \Rightarrow Z_L = +j\cdot X_L = +j\cdot 0.57\ \mathbf{k\Omega}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(3\ \mathbf{k}\frac{\mathbf{rad}}{\mathbf{s}}\right)(55\ \mathbf{nF})} = 6.061\ \mathbf{k\Omega} \Rightarrow Z_C = -j\cdot X_C = -j\cdot 6.061\ \mathbf{k\Omega}$$

$$Z_{eq1} = Z_{R1} + Z_L = R_1 + jX_L = 2.3 + j\cdot 0.57\ \mathbf{k\Omega} = 2.37\angle 13.92^o\ \mathbf{k\Omega}$$

$$Z_{eq2} = Z_{R1} + Z_C = R_1 - jX_C = 1.1 - j\cdot 6.061\ \mathbf{k\Omega} = 6.16\angle -79.71^o\ \mathbf{k\Omega}$$

$$Z_{eq} = \frac{Z_{eq1}\cdot Z_{eq2}}{Z_{eq1} + Z_{eq2}} = \frac{\left(2.37\angle 13.92^o\ \mathbf{k\Omega}\right)\cdot\left(6.16\angle -79.71^o\ \mathbf{k\Omega}\right)}{\left(2.3 + j\cdot 0.57\ \mathbf{k\Omega}\right) + \left(1.1 - j\cdot 6.061\ \mathbf{k\Omega}\right)} =$$

$$= \frac{14.60\angle -65.79^o\ \mathbf{k\Omega}^2}{3.4 - j\cdot 5.491\ \mathbf{k\Omega}} = \frac{14.60\angle -65.79^o\ \mathbf{k\Omega}^2}{6.458\angle -58.23^o\ \mathbf{k\Omega}} = 2.261\angle -7.56^o\ \mathbf{k\Omega}$$

Problem 4.48

Solution:

Known quantities:

The values of the impedance, $R_1 = 3.3\ \mathbf{k\Omega}$, $R_2 = 22\ \mathbf{k\Omega}$, $L = 1.90\ \mathbf{H}$, $C = 6.8\ \mathbf{nF}$ and the voltage applied to the circuit shown in Figure P4.47, $v_s(t) = 636\cos\left(3000t + 15^o\right)\mathbf{V}$.

Find:

The equivalent impedance of the circuit.

Analysis:

$$X_L = \omega L = \left(3\ \mathbf{k}\frac{\mathbf{rad}}{\mathbf{s}}\right)(1.90\ \mathbf{H}) = 5.7\ \mathbf{k\Omega} \Rightarrow Z_L = +j\cdot X_L = +j\cdot 5.7\ \mathbf{k\Omega}$$

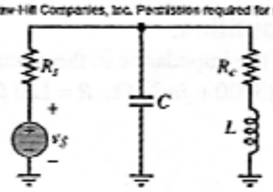
$$X_C = \frac{1}{\omega C} = \frac{1}{\left(3\ \mathbf{k}\frac{\mathbf{rad}}{\mathbf{s}}\right)(6.8\ \mathbf{nF})} = 49.02\ \mathbf{k\Omega} \Rightarrow Z_C = -j\cdot X_C = -j\cdot 49.02\ \mathbf{k\Omega}$$

$$Z_{eq1} = Z_{R1} + Z_L = R_1 + jX_L = 3.3 + j\cdot 5.7\ \mathbf{k\Omega} = 6.59\angle 59.93^o\ \mathbf{k\Omega}$$

$$Z_{eq2} = Z_{R1} + Z_C = R_1 - jX_C = 22 - j\cdot 49.02\ \mathbf{k\Omega} = 53.73\angle -65.83^o\ \mathbf{k\Omega}$$

Problem 4.53

Solution:



Known quantities:

The values of the impedance, $R_s = 50\Omega$, $R_c = 40\Omega$, $L = 20\ \mu\text{H}$, $C = 1.25\ \text{nF}$, and the voltage applied to the circuit shown in Figure P4.53,

$v_s(t) = V_0 \cos(\omega t + 0^o)\ V_0 = 10\ \text{V},\ \omega = 6\ \text{M}\dfrac{\text{rad}}{\text{s}}$.

Find:

The current supplied by the source.

Analysis:

Assume clockwise currents:

$$X_L = \omega L = \left(6\ \text{M}\frac{\text{rad}}{\text{s}}\right)(20\ \mu\text{H}) = 1203\Omega \Rightarrow Z_L = 0 + j120\ \Omega = 120\angle 90^o\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(6\ \text{M}\frac{\text{rad}}{\text{s}}\right)(1.25\ \text{nF})} = 133.3\ \Omega \Rightarrow Z_C = 0 - j133.3\ \Omega = 133.3\angle -90^o\ \Omega$$

$$Z_{R_c} = 40 - j\Omega = 40\angle 0^o\ \Omega,\ Z_{R_s} = 50 - j\ \Omega = 50\angle 0^o\ \Omega$$

Equivalent impedances:

$$Z_{eq1} = Z_{R_c} + Z_L = 40 + j120\ \Omega = 126.5\angle 71.56^o\ \Omega$$

$$Z_{eq} = Z_{R_s} + \frac{Z_C \cdot Z_{eq1}}{Z_C + Z_{eq1}} = 50 + j0\ \Omega + \frac{(133.3\angle -90^o\ \Omega)\cdot(126.5\angle 71.56^o\ \Omega)}{133.3\angle -90^o\ \Omega + 126.5\angle 71.56^o\ \Omega} =$$

$$= 50 + j0\ \Omega + \frac{16.87\angle -18.44^o\ \text{k}\Omega^2}{42.161\angle -18.44^o\ \Omega} = 50\angle 0^o\ \Omega + 400\angle 0^o\ \Omega = 450\angle 0^o\ \Omega$$

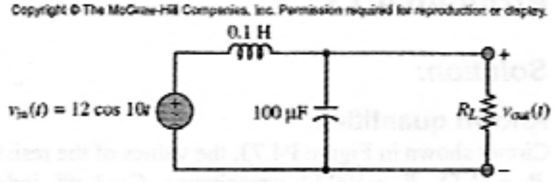
OL: $$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_{eq}} = \frac{10\angle 0^o\ \text{V}}{450\angle 0^o\ \Omega} = 22.22\angle 0^o\ \text{mA} \Rightarrow i_s(t) = 22.22\cos(\omega t + 0^o)\ \text{mA}$$

Note:

The equivalent impedance of the parallel combination is purely resistive; therefore, the frequency given is the resonant frequency of this network.

Problem 4.71

Solution:



Known quantities:

Circuit shown in Figure P4.71 the values of the impedance, $L = 0.1$ H, capacitance, $C = 100\ \mu F$, and the voltage source $v_{in}(t) = 12\cos(10t)$ V.

Find:

The Thèvenin equivalent of the circuit as seen by the load resistor R_L.

Analysis:

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j10\ \frac{\text{rad}}{\text{s}} \cdot 100\ \mu F} = -j1000\ \Omega$$

$$Z_L = j\omega L = j10\ \frac{\text{rad}}{\text{s}} \cdot 0.1\ \text{H} = j1\ \Omega$$

The equivalent impedance is:

$$Z_T = Z_L \| Z_C = \frac{Z_L \cdot Z_C}{Z_L + Z_C} = \frac{j(-j1000)}{j - j1000} = \frac{1000}{-j999} = 1.001\angle 90^o\ \Omega = j1.001\ \Omega$$

The Thèvenin voltage is:

$$V_T = \frac{Z_C}{Z_L + Z_C} V_{in} = \frac{-j1000}{j - j1000} \cdot 12\angle 0^o = \frac{1000}{999} \cdot 12\angle 0^o = 12.012\angle 0^o\ \text{V}$$

Problem 4.72

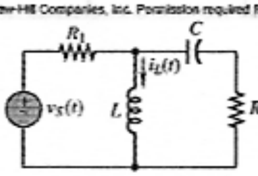
Solution:

Known quantities:

Circuit shown in Figure P4.72 the values of the resistance, $R_1 = 4\ \Omega$, $R_2 = 4\Omega$, capacitance, $C = 1/4$ F, inductance, $L = 2$ **H**, and the voltage source $v_s(t) = 2\cos(2t)$ **V**.

Find:

The current in the circuit $i_L(t)$ using phasor techniques.



Analysis:

$$\mathbf{V}_S(t) = 2\angle 0^o\ \mathbf{V}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j2\,\frac{1}{4}} = -j2\ \Omega$$

$$Z_L = j\omega L = j2 \cdot 2 = j4\ \Omega$$

Applying the voltage divider rule:

$$V_L = \frac{(Z_L \| (Z_C + Z_2))}{Z_1 + (Z_L \| (Z_C + Z_2))} V_S = \frac{4\angle 36.8°}{4\angle 0° + 4\angle 36.8°} 2\angle 0^o = 1.05\angle 18.4°\text{V}$$

Therefore, the current is:

$$I_L = \frac{V_L}{Z_L} = \frac{1.05\angle 18.4°}{4\angle 90°} = 0.2635\angle -71.6^o\ \text{A}$$

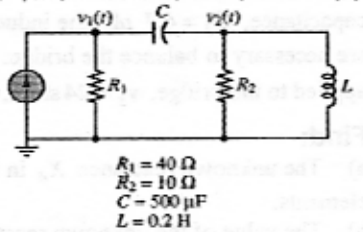
$$i_L(t) = 0.2635\cos\left(2t - 71.6^o\right)\ \text{A}$$

Problem 4.74

Solution:

Known quantities:

Circuit shown in Figure P4.74, the values of the resistance, $R_1 = 40\ \Omega$, $R_2 = 10\ \Omega$, capacitance, $C = 500\ \mu\text{F}$, inductance, $L = 0.2$ H, and the current source $i_s(t) = 40\cos(100t)$ A.



Find:

The voltages in the circuit $v_1(t)$ and $v_2(t)$.

Analysis:

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{100 \cdot 500 \cdot 10^{-6}} = -j20\ \Omega\ ,$$

$$Z_L = j\omega L = j100 \cdot 0.2 = j20\ \Omega$$

Applying KCL at node 1, we have:

$$I_S = \frac{V_1}{R_1} + \frac{V_1 - V_2}{Z_C} \Rightarrow I_S = \left(\frac{1}{R_1} + \frac{1}{Z_C}\right)V_1 - \frac{1}{Z_C}V_2 \Rightarrow 40\angle 0^o = \left(\frac{1}{40} + \frac{j}{20}\right)V_1 - \frac{j}{20}V_2$$

Applying KCL at node 2, we have

$$\frac{V_1 - V_2}{Z_C} = \frac{V_2}{R_2} + \frac{V_2}{Z_L} \Rightarrow \frac{V_1}{Z_C} = \left(\frac{1}{R_2} + \frac{1}{Z_L} + \frac{1}{Z_C}\right)V_2 \Rightarrow j\frac{V_1}{20} = \left(\frac{1}{10} - j\frac{1}{20} + j\frac{1}{20}\right)V_2$$

Therefore:

$$\begin{cases} 40\angle 0^o = \left(\frac{1}{40} + \frac{j}{20}\right)V_1 - \frac{j}{20}V_2 \\ j\frac{V_1}{20} = \left(\frac{1}{10}\right)V_2 \end{cases} \Rightarrow \begin{cases} 40\angle 0^o = \left(\frac{1}{40} + \frac{j}{20}\right)(-j2V_2) - \frac{j}{20}V_2 \\ V_1 = -j2V_2 \end{cases} \Rightarrow$$

$$\begin{cases} 40\angle 0^o = -\frac{j}{20}V_2 + \frac{1}{10}V_2 - \frac{j}{20}V_2 = \left(\frac{1}{10} - \frac{j}{10}\right)V_2 \\ V_1 = -j2V_2 \end{cases}$$

$$V_2 = \frac{40\angle 0^o}{\left(\frac{1}{10} - \frac{j}{10}\right)} = 282.84\angle 45^o\ \text{V},\ V_1 = -j2V_2 = 565.68\angle -45^o\ \text{V}$$

$$v_2(t) = 282.84\cos\left(100t + 45^o\right)\text{V},\ v_1(t) = 568.68\cos\left(100t - 45^o\right)\text{V}$$

Problem 4.76

Solution:

Known quantities:
Circuit shown in Figure P4.72, the values of the resistance, $R_1 = 4\ \Omega$, $R_2 = 4\Omega$, capacitance, $C = 1/4$ **F**, inductance, $L = 2$ **H**, and the voltage source $v_s(t) = 2\cos(2t)$ **V**.

Find:
The Thévenin impedance seen by resistor R_2.

Analysis:

$$Z_T = (R_1 \| Z_L) + (Z_C) = (4 \| j4) + (-j2) = j2(1-j) - j2 = 2 + j2 + (-j2) = 2\ \Omega$$

Problem 4.77

Solution:

Known quantities:
Circuit shown in Figure P4.74, the values of the resistance, $R_1 = 10\ \Omega$, $R_2 = 40\Omega$, capacitance, $C = 500\ \mu F$, inductance, $L = 0.2$ **H**, and the current source $i_s(t) = 40\cos(100t)$ **A**.

Find:
The Thévenin voltage seen by inductance L.

Analysis:
The Thévenin equivalent voltage source is the open-circuit voltage at the load terminals:

$$\mathbf{V}_T = R_2\mathbf{I}_2 = 40\mathbf{I}_2$$

From the current division, we have

$$\mathbf{I}_2 = \frac{R_1}{(R_2 + Z_C) + R_1}\mathbf{I}_S = \frac{10}{(40\text{-}j20) + 10} 40\angle 0^o = 7.43\angle 21.8^o\ \mathbf{A}$$

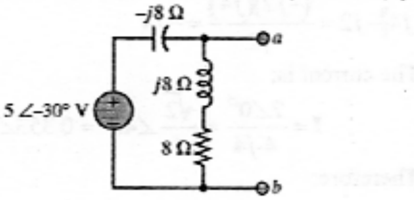
$$\mathbf{V}_T = R_2\mathbf{I}_2 = 40 \cdot 7.43\angle 21.8^o = 297\angle 21.8^o\ \mathbf{V}$$

$$v_T(t) = 297\cos\left(100t + 21.8^o\right)\mathbf{V}$$

Problem 4.78

Solution:

Known quantities:
Circuit shown in Figure P4.78, the values of the impedance, $R = 8\Omega$, $Z_C = -j8\ \Omega$, $Z_L = j8\ \Omega$, and the voltage source $\mathbf{V}_s = 5\angle -30^o$ **V**.



Find:
The Thévenin equivalent circuit seen from the terminals a-b.

Analysis:
The Thévenin equivalent circuit is given by:

$$V_{TH} = \left(\frac{8 + j8}{8 + j8 - j8}\right) 5\angle -30^\circ = (1 + j) 5\angle -30^o = 7.07\angle 15\ \text{V}$$

$$Z_{TH} = \frac{(8 + j8)(-j8)}{8 + j8 - j8} = (8 - j8) = 8\sqrt{2}\angle -45^o\ \Omega$$

rad = -.785
$8\sqrt{2} = 11.3$