Chapter 14

Free-Form Deformation (FFD)

Free-form deformation (FFD) is a technique for manipulating any shape in a free-form manner. Pierre Bézier used this idea to manipulate large numbers of control points for Béziersurface patches [B́74, B́78], and the power of FFD as a modeling tool was more fully explored in [SP86b]. This chapter discusses the 2D case of FFD.

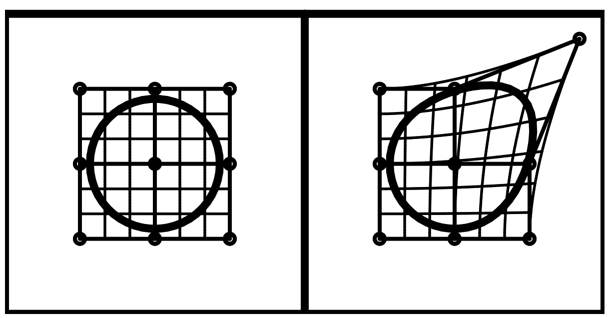


Figure 14.1: FFD example

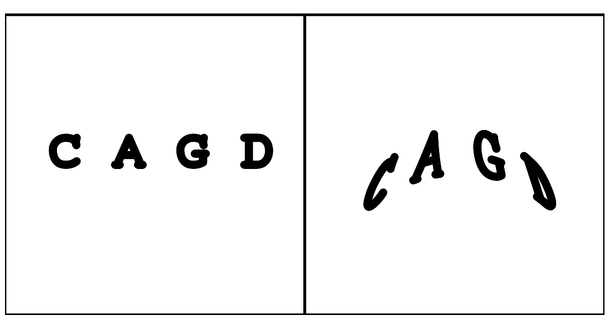


Figure 14.2: FFD example

2D FFD is a map from $R^2 \rightarrow R^2$; that is, it defines a new position for every point in a given rectangular region. In Figure 14.1, the FFD is specified using the nine control points. The undeformed scene appears at the left, and the right shows what happens the the grid and circle after the control points are moved. The grid helps to visualize how FFD works. FFD is a powerful modeling tool because anything drawn inside of the initial, undeformed rectangle will experience the distortion, such as the text in Figure 14.2.

Denote by (X_{min}, Y_{min}) and (X_{max}, Y_{max}) the corners of a deformation region, and by m and n the degrees of the FFD function (there are $m+1$ vertical columns and $n+1$ horizontal rows of control points).

FFD is a two-step process:

1. Compute the (s,t) coordinates for each point to be deformed. The s and t coordinates of a point in the deformation region range between 0 and 1 (see Figure 14.3). For a point in the

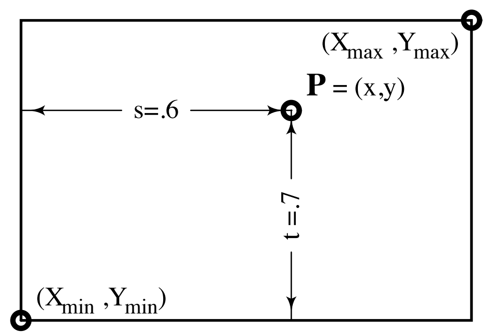


Figure 14.3: FFD local coordinates

   rectangular region whose Cartesian coordinates are (x,y),

$$s = \frac{x - X_{min}}{X_{max} - X_{min}}, \qquad t = \frac{y - Y_{min}}{Y_{max} - Y_{min}}. \tag{14.1}$$

2. Compute the homogeneous coordinates $\mathbf{X}(s,t) = (X,Y,W)$ of the deformed point using the **rational bivariate tensor product Bernstein polynomial** equation

$$\mathbf{X}(s,t) = \sum_{j=0}^{n} \sum_{i=0}^{m} B_i^m(s) B_j^n(t) \mathbf{P}_{ij} \tag{14.2}$$

   where $B_i^n(t)$ and $B_j^m(s)$ are Bernstein polynomials and $\mathbf{P}_{ij} = w_{ij}(x_{ij}, y_{ij}, 1)$ are the homogeneous coordinates of the displaced control point i,j. Note that weights can be assigned to the FFD control points.

If all weights $w_{ij} = 1$ and if the control points form a rectangular lattice

$$\mathbf{P}_{i,j} = \left(X_{min} + \frac{i}{m}(X_{max} - X_{min}), Y_{min} + \frac{j}{n}(Y_{max} - Y_{min}) \right) \tag{14.3}$$

as shown in Figure 14.4, FFD is the identity transformation: all points end up where they started from.

Points outside of the rectangle are not moved. If a shape is only partially inside the FFD region, one can control the degree of continuity between the deformed and undeformed portions of the shape by "freezing" rows of control points, as shown in Figure 14.5.

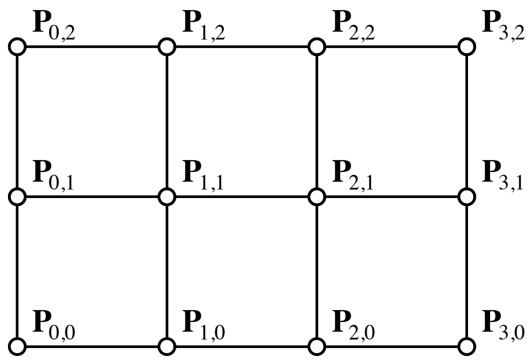


Figure 14.4: FFD undisplaced control points

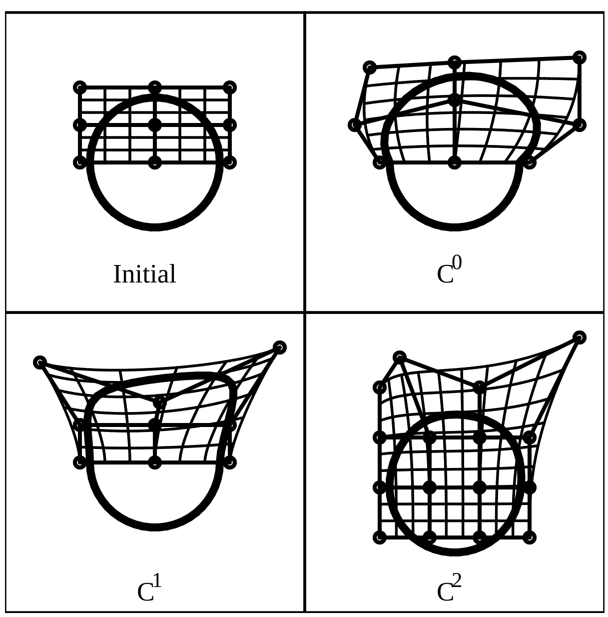
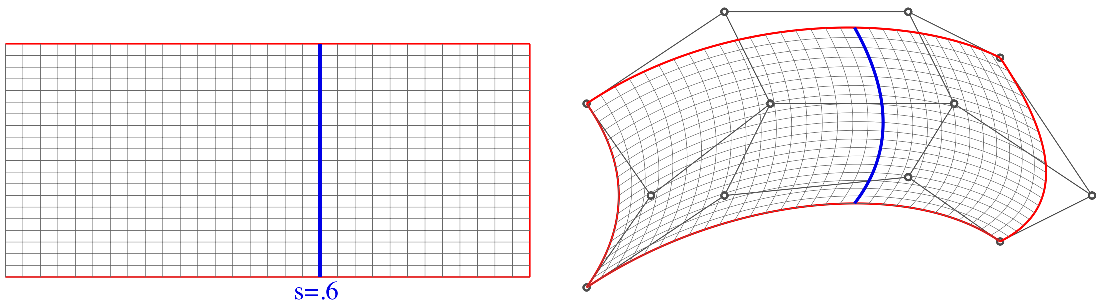


Figure 14.5: Continuity control

14.0.1 Deformed Lines

Each vertical line that undergoes FFD maps to a Bézier curve of degree n, and each horizontal line maps to a Bézier curve of degree m. Thus, the "vertical" curves in Figure 14.6.b are degree-two Bézier curves and the "horizontal" curves are degree three. In particular, we have highlighted the



(a) Undeformed Grid with un-displaced control points. (b) Deformed Grid.

Figure 14.6: FFD Applied to a Grid of Lines.

line $s = .6$ in Figure 14.6.a and its deformation is the degree-two Bézier curve shown in Figure 14.6.b. The control points for this Bézier curves are determined as follows.

Referring to Figure 14.7, consider each horizontal row of control points in the FFD control grid to be a control polygon for a cubic Bézier curve. Call these curves $\mathbf{H}_0(s)$, $\mathbf{H}_1(s)$, and $\mathbf{H}_2(s)$ where the control points for $\mathbf{H}_i(s)$ are $\mathbf{P}_{0,i}$, $\mathbf{P}_{1,i}$, $\mathbf{P}_{2,i}$, $\mathbf{P}_{3,i}$. Then, the three control points for the

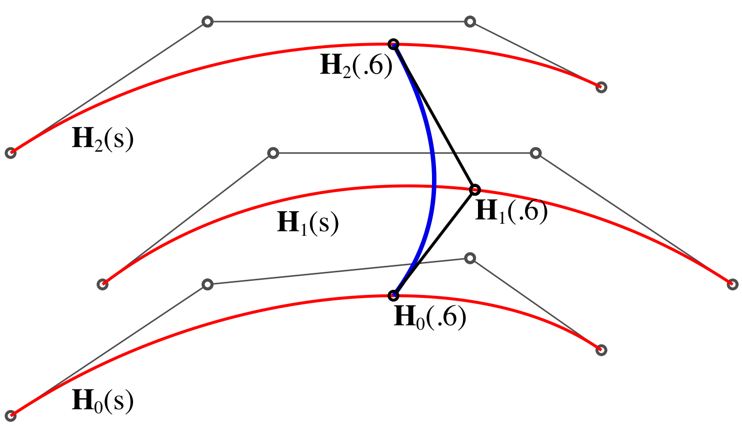
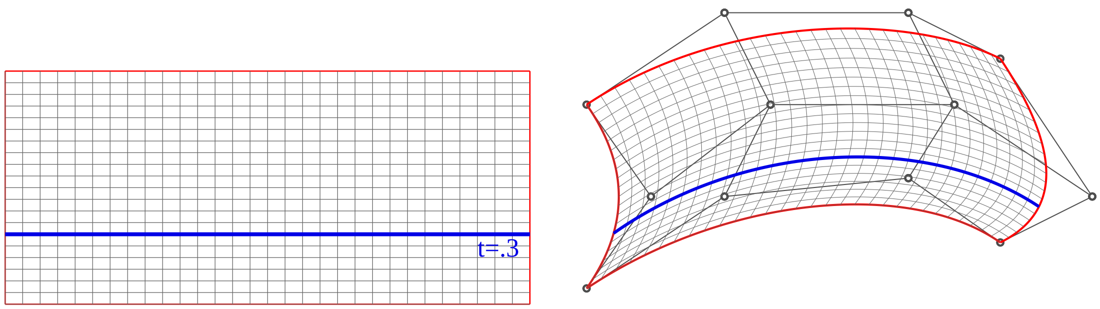


Figure 14.7: Control points of deformed line.

degree-two Bézier curve in Figure 14.6.b are $\mathbf{H}_0(.6)$, $\mathbf{H}_1(.6)$, and $\mathbf{H}_2(.6)$.



(a) Undeformed Grid with horizontal line at $t = .3$.

(b) Deformed Grid.

Figure 14.8: FFD Applied to a horizontal line $t = .3$

The horizontal line $t = .3$ is highlighted in Figure 14.8.a and its deformation is the degree-three Bézier curve shown in Figure 14.8.b. The control points for this Bézier curves are determined as follows.

Referring to Figure 14.9, consider each vertical column of control points in the FFD control grid to be a control polygon for a quadratic Bézier curve. Call these curves $\mathbf{V}_0(s)$, $\mathbf{V}_1(s)$, $\mathbf{V}_2(s)$, and $\mathbf{V}_3(s)$ where the control points for $\mathbf{V}_i(s)$ are $\mathbf{P}_{i,0}$, $\mathbf{P}_{i,1}$, $\mathbf{P}_{i,2}$. Then, the four control points for

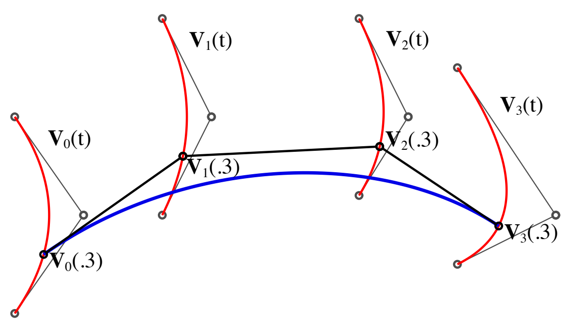


Figure 14.9: Control points of deformed line.

the degree-three Bézier curve in Figure 14.9.b are $\mathbf{V}_0(.3)$, $\mathbf{V}_1(.3)$, $\mathbf{V}_2(.3)$, and $\mathbf{V}_3(.3)$.