STRATHMORE UNIVERSITY
School of Computing and Engineering Sciences
DISCRETE MATHEMATICS
TAKEAWAY ASSIGNMENT
DUE DATE: FRIDAY, 12th July, 2024
NO SUBMISSION BY EMAIL

1. Given 3 subsets A, B and C of a universal set U, suppose $n(U) = 68$, $n(A \cup B \cup C) = 64$, $n(A) = 50$, $n(B) = 49$, $n(C) = 46$. Further, $n(A \cap B) = 39$, $n(C \cap B) = 33$ and $n(A \cap C) = 36$. Find $n(A \cap B \cap C)$ and represent this information on a Venn diagram.　　　　[3 Marks]

2. Is it true that all complex numbers are real numbers? Justify your answer.　　　　[2 marks]

3. Determine the range and domain of the function:　　　　[2 Marks]

$$B(b) = -3(b + 1)^2 + 6.$$

4. Represent the following on a Venn diagram:

   (a) $B - (C \cap A)$.　　　　[1 Mark]
   (b) $(C - A^c) \cap (A \cap B)$.　　　　[2 Marks]

5. Given that $f(x) = x^2 + 4$, $g(x) = x - 9$ and $f \circ g = g \circ f$, find the value of x.　　　　[3 Marks]

6. Given that $p = Daisy\ is\ smart$ and $q = Daisy\ is\ honest$, translate the following logical expression into an English statement; $p \vee (\sim p \wedge q)$.
.　　　　[2 marks]

7. Is the compound statement $(p \rightarrow q) \wedge (q \rightarrow p)$ a tautology, a fallacy or a contigency? show.　　　　[3 Marks]

8. Prove by induction that $9^n + 3$ is divisible by 4 for $n \in \mathbb{N}$.　　　　[3 Marks]

9. Find the middle term in the binomial expansion of $(2x^2 - \frac{1}{x})^{10}$.
.　　　　[3 Marks]

10. Are the following functions or relations, justify your answer:

    (a) $R_1 = \{(2,3), (0.5,0), (2,7), (-4,6)\}$　　　　[2 Marks]
    (b) $R_2 = \{x : |x| \div x \in \mathbb{R}\}$　　　　[2 Marks]

11. Show by induction that $w(w+1)(2w+1)$ is a multiple of 6. Where w is a natural number. [6 Marks]

12. In the expansion of $\left(2x^2 - \frac{1}{x}\right)$, find the term whose simplified form involves $\frac{1}{x}$. [3 Marks]

13. Expand $(1 - 2x)^{\frac{1}{3}}$ and use the first four terms to evaluate $0.8^{\frac{1}{3}}$

14. Find the first four terms of the expansion $(-5x + 8)^{\frac{-2}{3}}$. [3 Marks]

15. Use binomial theorem to approximate $\sqrt{30}$ correct to $3dp$. p [3 Marks]

16. Let c be the proposition *it is cold*, s be the proposition *it is snowing* and h be *I am staying home*. Give the English interpretation of the following logical expressions:

    (a) $(c \wedge s) \to h$. [1 Mark]
    (b) $(c \vee s) \to h$. [1 Mark]
    (c) $\sim (h \to c)$. [1 Mark]
    (d) $s \wedge (c \vee h)$. [1 Mark]

17. Give the converse, inverse and contrapositive of the the statement:
    It is raining is a sufficient condition for my not going to town.
    . [3 Marks]

18. If a and b are nearly equal, show that $\left(\frac{5a-2b}{3b}\right)^{\frac{1}{3}}$. [4 Marks]

19. Determine whether the following relation defined
    on $A = \{1, 2, 3, 4, \cdots, 13, 14\}$ is reflexive, symmetric and transitive:
    $R = \{(x, y) : 3x - y = 0\}$. [3 Marks]

20. Four different pens and 5 different books are to be arranged on a row. Find the number of possible arrangements if the four pens must be kept together. [2 Marks]

21. How many four digit numbers can be made from the
    digits $1, 2, 3, 4, 5, 6, 7, 8, 9$ if no digit is repeated. How many of these numbers are greater than 3000. [3 Marks]

22. Construct a truth table for the compound statement: [4 Marks]

$$[p \to (q \vee r)] \wedge [\sim (p \leftrightarrow \sim r)]$$

23. Show that the relation R defined on $\mathbb{R}$ defined as $R = \{(x, y) : y^2 \geq x\}$ is neither reflexive, transitive nor symmetric. [3 Marks]