Appendix A
Proof of Theorem 1

We define $\delta_n = \frac{1}{v_s} - \frac{1}{v_n}, \forall n \in \mathcal{N}$.

A. $\alpha = 1$:

The first-order partial derivative of the utility function is:

$$\frac{\partial U_n}{\partial l_n} = r\frac{\tau_n a\delta_n}{T_n - t(l_n)} - \tau_n a\beta v_n. \tag{1}$$

The second-order partial derivative of the utility function is:

$$\frac{\partial U^2}{\partial^2 l_n} = -r\frac{(\tau_n a\delta_n)^2}{(T_n - t(l_n))^2} < 0. \tag{2}$$

Since the second-order derivative is always negative, the utility function is strictly concave with respect to l_n, so it has a unique maximum.

- When $v_n \le v_s$: $\delta_n \le 0$, the first-order derivative is always less than zero, so the optimal decision is $l_n^*(r) = l_n^{\min}$.
- When $v_n > v_s$: $\delta_n > 0$. Set the first-order derivative to zero and solve for the optimal cut layer:

$$l_n^{\alpha=1}(r) = \frac{1}{a\beta v_n\tau_n}r - \frac{T_n}{a\tau_n\delta_n} + \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} - \frac{b}{a}. \tag{3}$$

Since $l_n^{\min} \le l_n \le l_n^{\max}$, we give the following solution:

$$l_n^*(r) = \begin{cases} l_n^{\min}, & r_{min} \le r < \xi_{\text{left}}^{n,\alpha=1}, \\ l_n^{\alpha=1}(r), & \xi_{\text{left}}^{n} \le r \le \xi_{\text{right}}^{n,\alpha=1}, \\ l_n^{\max}, & \xi_{\text{right}}^{n,\alpha=1} < r \le r_{max}, \end{cases} \tag{4}$$

where $\xi_{\text{left}}^{n,\alpha=1} = a\beta v_n\tau_n\left(l_n^{\min} + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t+\frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a}\right)$, $\xi_{\text{right}}^{n,\alpha=1} = a\beta v_n\tau_n\left(l_n^{\max} + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t+\frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a}\right)$.

When $\alpha = 1$, the optimal solution function for client n is:

$$l_n^*(r) = \begin{cases} l_n^{\min}, & \text{if } v_n \le v_s, \\ \begin{cases} l_n^{\min}, & r_{min} \le r < \xi_{\text{left}}^{n}, \\ l_n^{\alpha=1}(r), & \xi_{\text{left}}^{n} \le r \le \xi_{\text{right}}^{n}, \\ l_n^{\max}, & \xi_{\text{right}}^{n} < r \le r_{max}, \end{cases}, & \text{if } v_n > v_s. \end{cases} \tag{5}$$

B. $0 < \alpha < 1$:

The first-order partial derivative of the utility function is:

$$\frac{\partial U_n}{\partial l_n} = r\frac{\tau_n a\delta_n}{(T_n - t(l_n))^\alpha} - \tau_n a\beta v_n. \tag{6}$$

The second-order partial derivative of the utility function is:

$$\frac{\partial U^2}{\partial^2 l_n} = -r\alpha\frac{(\tau_n a\delta_n)^2}{(T_n - t(l_n))^{\alpha+1}} < 0. \tag{7}$$

Since the second-order derivative is always negative, the utility function is strictly concave with respect to l_n, so it has a unique maximum.

- When $v_n \le v_s$: $\delta_n \le 0$. The first-order derivative is always less than zero, so the optimal solution is $l_n^*(r) = l_n^{\min}$.
- When $v_n > v_s$: Set the first-order derivative to zero and solve for the optimal cut layer:

$$l_n^{0<\alpha<1}(r) = \frac{{\delta_n}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n}r^{\frac{1}{\alpha}} - \frac{T_n}{a\tau_n\delta_n} + \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} - \frac{b}{a}. \tag{8}$$

Since $l_n^{\min} \le l_n \le l_n^{\max}$, we give the following solution:

$$l_n^*(r) = \begin{cases} l_n^{\min}, & r_{min} \le r < \xi_{\text{left}}^{n,0<\alpha<1}, \\ l_n^{0<\alpha<1}(r), & \xi_{\text{left}}^{n} \le r \le \xi_{\text{right}}^{n,0<\alpha<1}, \\ l_n^{\max}, & \xi_{\text{right}}^{n,0<\alpha<1} < r \le r_{max}, \end{cases} \tag{9}$$

where $\xi_{\text{left}}^{n,0<\alpha<1} = (\frac{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n}{{\delta_n}^{\frac{1}{\alpha}-1}})^\alpha(l_n^{\min} + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t+\frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a})^\alpha$, $\xi_{\text{right}}^{n,0<\alpha<1} = (\frac{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n}{{\delta_n}^{\frac{1}{\alpha}-1}})^\alpha(l_n^{\max} + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t+\frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a})^\alpha$.

When $0 < \alpha < 1$, the optimal solution function for client n is:

if $v_n \le v_s$, $l_n^*(r) = l_n^{\min}$;

if $v_n > v_s$,

$$l_n^*(r) = \begin{cases} l_n^{\min}, & r_{min} \le r < \xi_{\text{left}}^{n,0<\alpha<1}, \\ l_n^{0<\alpha<1}(r), & \xi_{\text{left}}^{n,0<\alpha<1} \le r \le \xi_{\text{right}}^{n,0<\alpha<1} \\ l_n^{\max}, & \xi_{\text{right}}^{n,0<\alpha<1} < r \le r_{max}, \end{cases}. \tag{10}$$

We integrate the solution forms of two cases and find that when $0 < \alpha \le 1$, $\xi_{\text{left}}^{n}(\alpha) = (\frac{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n}{{\delta_n}^{\frac{1}{\alpha}-1}})^\alpha(l_n^{\min} + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t+\frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a})^\alpha$, $\xi_{\text{right}}^{n}(\alpha) = (\frac{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n}{{\delta_n}^{\frac{1}{\alpha}-1}})^\alpha(l_n^{\max} + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t+\frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a})^\alpha$ and $l_n^{\alpha}(r) = \frac{{\delta_n}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n}r^{\frac{1}{\alpha}} - \frac{T_n}{a\tau_n\delta_n} + \frac{S_t+\frac{aL+b}{v_s}}{a\delta_n} - \frac{b}{a}$.

In summary, the optimal solution function for client n is:

$$l_n^*(r) = \begin{cases} l_n^{\min}, & \text{if } v_n \le v_s, \\ \begin{cases} l_n^{\min}, & r_{min} \le r < \xi_{\text{left}}^{n}(\alpha), \\ l_n^{\alpha}(r), & \xi_{\text{left}}^{n}(\alpha) \le r \le \xi_{\text{right}}^{n}(\alpha), \\ l_n^{\max}, & \xi_{\text{right}}^{n}(\alpha) < r \le r_{max}, \end{cases}, & \text{if } v_n > v_s. \end{cases} \tag{11}$$

Appendix B
Proof of Lemma 1

Under assumption 1, we have $T_n = t_n^{max}$, so when $v_s < v_n$:

$$\begin{aligned}
l_n^{\alpha}(r) &= \frac{{\delta_n}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}} \tau_n} r^{\frac{1}{\alpha}} - \frac{t_n^{max}}{a\tau_n\delta_n} + \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} - \frac{b}{a} \\
&= \frac{{\delta_n}^{\frac{1}{\alpha}-1} r^{\frac{1}{\alpha}}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}} \tau_n} - \frac{\tau_n(-b\delta_n + S_t + \frac{f(L)}{v_s})}{a\tau_n\delta_n} + \frac{S_t + \frac{f(L)}{v_s}}{a\delta_n} - \frac{b}{a} \\
&= \frac{{\delta_n}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}} \tau_n} r^{\frac{1}{\alpha}} \geq 0.
\end{aligned} \tag{12}$$

Therefore, the Assumption 1 yields a candidate cut-layer that is always nonnegative. Under Assumption 1, where the maximum allowable cut layer approaches positive infinity, any such value for users with r and $v_n > v_s$ automatically lies in the feasible interval $[l_n^{\min}, l_n^{\max}]$.

Therefore, we revised the conclusion in the general case and provided the solution under assumption 1. In summary, the optimal solution function for client n under assumption 1 is:

$$l_n^*(r) = \begin{cases} l_n^{\min}, & \text{if } v_s \geq v_n, \\ l_n^{\alpha}(r), & \text{if } v_s < v_n. \end{cases} \tag{13}$$

Appendix C
Proof of Theorem 2

A. $v_s < v_{min}$

The server's computational capability is inferior to that of the clients and under the setting ($\alpha = 1$), the server's cost function can be derived based on the clients' optimal strategies. For simplicity, we define $Z = \sum_{n\in\mathcal{N}} a\tau_n v_s \beta(L + \frac{T_n}{a\tau_n\delta_n} - \frac{(S_t + \frac{aL+b}{v_s})}{a\delta_n} + \frac{b}{a})$, representing all terms in the equation that are independent of the variable r. After simplification, the cost function takes the form:

$$\begin{aligned}
C_s(r) &= \sum_{n\in\mathcal{N}} r\cdot\ln(r\cdot\frac{\delta_n}{\beta v_n}) - \sum_{n\in\mathcal{N}} a\tau_n v_s\beta\frac{1}{a\beta v_n\tau_n} r \\
&\quad + \sum_{n\in\mathcal{N}} a\tau_n v_s\beta(L + \frac{T_n}{a\tau_n\delta_n} - \frac{\left(S_t + \frac{aL+b}{v_s}\right)}{a\delta_n} + \frac{b}{a}) \\
&= Nr\ln r + r\sum_{n\in\mathcal{N}}(\ln(\frac{\delta_n}{\beta v_n}) - \frac{v_s}{v_n}) + Z.
\end{aligned} \tag{14}$$

To determine the optimal reward r^*, we analyze the first and second derivatives of $C_s(r)$:

$$\frac{\partial C_s}{\partial r} = N(1 + \ln r) + \sum_{n\in\mathcal{N}}(\ln\frac{\delta_n}{\beta v_n} - \frac{v_s}{v_n}), \tag{15}$$

$$\frac{\partial^2 C_s}{\partial r^2} = \frac{N}{r} > 0 \quad \text{for} \quad r > 0. \tag{16}$$

The positive second derivative implies that $C_s(r)$ is strictly convex in r, ensuring that any critical point found is the global minimum. Setting the first derivative to zero yields the unique optimal reward:

$$r_1 = \exp\{\frac{\sum_{n\in\mathcal{N}}(\frac{v_s}{v_n} - ln\frac{\delta_n}{\beta v_n})}{|N|} - 1\}. \tag{17}$$

B. $v_{min} \leq v_s \leq v_{max}$

We define $Z = \sum_{n\in\mathcal{N}_1} a\tau_n v_s\beta(L - (-\frac{T_n}{a\tau_n\delta_n} + \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} - \frac{b}{a})) + \sum_{n\in\mathcal{N}_2} a\tau_n v_s\beta(L - l_n^{min})$, representing all terms in the equation that are independent of the variable r. The server's cost function under the proportional fairness setting ($\alpha = 1$) is as follows:

$$\begin{aligned}
&C_s(r) = \\
&\sum_{n\in\mathcal{N}_1} r\cdot\ln\left(r\cdot\frac{\delta_n}{\beta v_n}\right) + \sum_{n\in\mathcal{N}_2} r\cdot\ln\left(T_n - t_n(l_n^{min})\right) \\
&+ \sum_{n\in\mathcal{N}_1} a\tau_n v_s\beta(L - (\frac{r}{a\beta v_n\tau_n} - \frac{T_n}{a\tau_n\delta_n} + \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} - \frac{b}{a})) \\
&+ \sum_{n\in\mathcal{N}_2} a\tau_n v_s\beta(L - l_n^{min}) \\
=& r(\sum_{n\in\mathcal{N}_1}(\ln\frac{\delta_n}{\beta v_n} - \frac{v_s}{v_n}) + \sum_{n\in\mathcal{N}_2}\ln(T_n - t_n(l_n^{min}))) \\
&+ |\mathcal{N}_1| r\ln r + Z,
\end{aligned} \tag{18}$$

The first-order and second-order derivatives of $C_s(r)$ with respect to r are:

$$\begin{aligned}
&\frac{\partial C_s}{\partial r} = \\
&\mathcal{N}_1(1 + \ln r) + \sum_{n\in\mathcal{N}_1}(\ln\frac{\delta_n}{\beta v_n} - \frac{v_s}{v_n}) + \sum_{n\in\mathcal{N}_2}\ln(T_n - t_n(l_n^{min})).
\end{aligned} \tag{19}$$

$$\frac{\partial^2 C_s}{\partial r^2} = \frac{|\mathcal{N}_1|}{r} \geq 0. \tag{20}$$

The positive second derivative implies strong convexity, ensuring the existence of a unique global minimum, given by:

$$r_2 = \exp\{\frac{\sum\limits_{n\in\mathcal{N}_1}(\frac{v_s}{v_n} - \ln\frac{\delta_n}{\beta v_n}) - \sum\limits_{n\in\mathcal{N}_2} ln(T_n - t_n(l_n^{\min}))}{|\mathcal{N}_1|} - 1\}. \tag{21}$$

C. $v_s > v_{max}$

In this case, all user decisions are fixed, so $C_s(r)$ is an increasing function of r, so $r^* = r_{min}$.

In summary, the server's optimal reward coefficient r^* is given by:

$$r^* = \begin{cases} r_1, & \text{if } v_s < v_{\min}, \\ r_2, & \text{if } v_{\min} \leq v_s \leq v_{\max}, \\ r_{min}, & \text{if } v_s > v_{\max}, \end{cases} \tag{22}$$

Appendix D
Proof of Theorem 3

A. $v_s < v_{min}$

The server's computational capability is inferior to that of the clients. We define $Z = \sum_{n\in\mathcal{N}} a\tau_n v_s \beta(L + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a})$, representing all terms in the equation that are independent of the variable r. Under the setting ($0 < \alpha < 1$), the cost function takes the form:

$$\begin{aligned}
C_s(r) = &\sum_{n\in\mathcal{N}} r\frac{\left(\frac{r\delta_n}{\beta v_n}\right)^{\frac{1-\alpha}{\alpha}}}{1-\alpha} - \sum_{n\in\mathcal{N}} a\tau_n v_s\beta\frac{{\delta_n}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n} r^{\frac{1}{\alpha}} \\
&+ \sum_{n\in\mathcal{N}} a\tau_n v_s\beta(L + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a}) \\
=&r^{\frac{1}{\alpha}}\sum_{n\in\mathcal{N}}(\frac{\left(\frac{\delta_n}{\beta v_n}\right)^{\frac{1-\alpha}{\alpha}}}{1-\alpha} - a\tau_n v_s\beta\frac{{\delta_n}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n}) + Z \\
=&r^{\frac{1}{\alpha}}\sum_{n\in\mathcal{N}}(\frac{\delta_n^{\frac{1}{\alpha}-1}}{\beta_n^{\frac{1}{\alpha}-1} v_n^{\frac{1}{\alpha}}})(\frac{v_n}{1-\alpha} - v_s) + Z.
\end{aligned} \tag{23}$$

To determine the optimal r^*, we analyze the first and second derivatives of $C_s(r)$:

$$\frac{\partial C_s}{\partial r} = r^{\frac{1}{\alpha}-1}\frac{1}{\alpha}\sum_{n\in\mathcal{N}}(\frac{\delta_n^{\frac{1}{\alpha}-1}}{\beta_n^{\frac{1}{\alpha}-1} v_n^{\frac{1}{\alpha}}})(\frac{v_n}{1-\alpha} - v_s), \tag{24}$$

At this time, the server's cost function is a power function. Since $v_n \geq v_{min} > v_s$, so $\frac{v_n}{1-\alpha} > v_n > v_s$, so $\sum_{n\in\mathcal{N}}(\frac{v_n}{1-\alpha} - v_s) > 0$. At this point, $C_s(r)$ is a monotonically increasing function , then $r^* = r_{min}$.

B. $v_{min} \leq v_s \leq v_{max}$

For similarity, we define: $\gamma_1 = \sum_{n\in\mathcal{N}_1}(\frac{\delta_n^{\frac{1}{\alpha}-1}}{\beta_n^{\frac{1}{\alpha}-1} v_n^{\frac{1}{\alpha}}})(\frac{v_n}{1-\alpha} - v_s)$ and $\gamma_2 = \sum\limits_{n\in\mathcal{N}_2}\frac{(T_n - t_n(l_n^{min}))^{1-\alpha}}{1-\alpha}$. We also define $Z = \sum_{n\in\mathcal{N}_1} a\tau_n v_s\beta(L + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a}) + \sum_{n\in\mathcal{N}_2} a\tau_n v_s\beta(L - l_n^{min})$, representing all terms in the equation that are independent of the variable r.

When $v_{min} < v_s < v_{max}$, the server's cost function under the setting ($0 < \alpha < 1$) is as follows:

$$\begin{aligned}
C_s(r) = &\sum_{n\in\mathcal{N}_1} r\frac{\left(\frac{r\delta_n}{\beta v_n}\right)^{\frac{1-\alpha}{\alpha}}}{1-\alpha} - \sum_{n\in\mathcal{N}_1} a\tau_n v_s\beta\frac{{\delta_n}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n} r^{\frac{1}{\alpha}} \\
&+ \sum_{n\in\mathcal{N}_1} a\tau_n v_s\beta(L + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a}) \\
&+ \sum_{n\in\mathcal{N}_2}\frac{r(T_n - t_n(l_n^{min}))^{1-\alpha}}{1-\alpha} + \sum_{n\in\mathcal{N}_2} a\tau_n v_s\beta(L - l_n^{min}) \\
&= \gamma_1 r^{\frac{1}{\alpha}} + \gamma_2 r + Z
\end{aligned} \tag{25}$$

In $\mathcal{N}_1$, $v_n > v_s$, so we have $\gamma_1 > 0$ and $\gamma_2 > 0$. $C_s'(r) = \gamma_1\frac{1}{\alpha}r^{\frac{1}{\alpha}-1} + \gamma_2$ and $C_s^{''}(r) = \gamma_1\frac{1}{\alpha}(\frac{1}{\alpha}-1)r^{\frac{1}{\alpha}-2}$. So $C_s^{'}(r) > 0$, then $r^* = r_{min}$.

C. $v_s > v_{max}$

In this case, all user decisions are fixed, so $C_s(r)$ is an increasing function of r, so $r^* = r_{min}$.

In summary, the optimal solution r^* of the server is $r^* = r_{min}$.

Appendix E
Proof of Proposition 1

A. *When* $v_s > v_{max}$

$$\begin{aligned}
&C_s^*(0 < \alpha < 1) = \\
&\sum_{n\in\mathcal{N}} r_{min}\frac{(T_n - t_n(l_n^{min}))^{1-\alpha}}{1-\alpha} + E_s(f(L) - f(l_n(r_{min}))).
\end{aligned} \tag{26}$$

$$\begin{aligned}
&C_s^*(\alpha = 1) = \\
&\sum_{n\in\mathcal{N}} r_{min} ln(T_n - t_n(l_n^{min})) + E_s(f(L) - f(l_n(r_{min}))).
\end{aligned} \tag{27}$$

It can be proved that for any $x > 0$ and $0 < \alpha < 1$, there is $lnx < \frac{x^{1-\alpha}}{1-\alpha}$. According to this formula, we have for $n \in \mathcal{N}$, $\frac{(T_n - t_n(l_n^{min}))^{1-\alpha}}{1-\alpha} > ln(T_n - t_n(l_n^{min}))$.

According to the conclusion, we have

$$\begin{gathered}
\sum_{n\in\mathcal{N}} r_{min}\frac{(T_n - t_n(l_n^{min}))^{1-\alpha}}{1-\alpha} > \sum_{n\in\mathcal{N}} r_{min} ln(T_n - t_n(l_n^{min})), \\
C_s^*(0 < \alpha < 1) > C_s^*(\alpha = 1).
\end{gathered} \tag{28}$$

B. *When* $v_s < v_{min}$

We define $Z = \sum_{n\in\mathcal{N}} a\tau_n v_s\beta(L + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a})$ and $W = -\frac{\sum_{n\in\mathcal{N}}(\ln\frac{\delta_n}{\beta v_n} - \frac{v_s}{v_n})}{|N|} - 1$.

The server cost under $0 < \alpha < 1$ is

$$\begin{aligned}
&C_s^*(0 < \alpha < 1) \\
&= \sum_{n\in\mathcal{N}}(\frac{\left(\frac{\delta_n}{\beta v_n}\right)^{\frac{1-\alpha}{\alpha}}}{1-\alpha} - a\tau_n v_s\beta\frac{{\delta_n}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n})r^{\frac{1}{\alpha}} \\
&+ \sum_{n\in\mathcal{N}} a\tau_n v_s\beta(L + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a}) \\
&= \sum_{n\in\mathcal{N}}\frac{{\delta_n}^{\frac{1}{\alpha}-1}}{\beta^{\frac{1}{\alpha}-1} v_n^{\frac{1}{\alpha}}}(\frac{v_n}{1-\alpha} - v_s) r_{min}^{\frac{1}{\alpha}} + Z
\end{aligned} \tag{29}$$

The cost under $\alpha=1$ is

$$\begin{aligned}
&C_s^*(\alpha=1)\\
&=\sum_{n\in\mathcal{N}} r^*\ln\left(r^*\frac{\delta_n}{\beta v_n}\right)-\sum_{n\in\mathcal{N}} a\tau_n v_s\beta\frac{1}{a\beta v_n\tau_n}r^*\\
&+\sum_{n\in\mathcal{N}} a\tau_n v_s\beta(L+\frac{T_n}{a\tau_n\delta_n}-\frac{\left(S_t+\frac{aL+b}{v_s}\right)}{a\delta_n}+\frac{b}{a})\\
&=\sum_{n\in\mathcal{N}} e^W\ln(e^W\frac{\delta_n}{\beta v_n})-\sum_{n\in\mathcal{N}} a\tau_n v_s\beta\frac{1}{a\beta v_n\tau_n}e^W+Z\\
&=\sum_{n\in\mathcal{N}} e^W W+\sum_{n\in\mathcal{N}} e^W(\ln\frac{\delta_n}{\beta v_n}-\frac{v_s}{v_n})+Z\\
&=e^W W|N|-e^W(W+1)|N|+Z\\
&=-e^W|N|+Z.
\end{aligned}\tag{30}$$

Based on (29) and (30), we have

$$\begin{aligned}
&C_s^*(0<\alpha<1)-C_s^*(\alpha=1)=\\
&\sum_{n\in\mathcal{N}}\frac{\delta_n{}^{\frac{1}{\alpha}-1}}{\beta^{\frac{1}{\alpha}-1}v_n^{\frac{1}{\alpha}}}(\frac{v_n}{1-\alpha}-v_s)r_{min}^{\frac{1}{\alpha}}+Z-(-e^W|N|+Z)\\
&=\sum_{n\in\mathcal{N}}\frac{\delta_n{}^{\frac{1}{\alpha}-1}}{\beta^{\frac{1}{\alpha}-1}v_n^{\frac{1}{\alpha}}}(\frac{v_n}{1-\alpha}-v_s)r_{min}^{\frac{1}{\alpha}}+e^W|N|>0
\end{aligned}\tag{31}$$

So $C_s^*(0<\alpha<1)>C_s^*(\alpha=1)$.

C. when $v_{min}<v_s<v_{max}$

We define $X=\sum_{n\in\mathcal{N}_1} a\tau_n v_s\beta(L+\frac{T_n}{a\tau_n\delta_n}-\frac{S_t+\frac{aL+b}{v_s}}{a\delta_n}+\frac{b}{a})++\sum_{n\in\mathcal{N}_2} a\tau_n v_s\beta(L-l_n^{min})$ and $G=\sum\limits_{n\in\mathcal{N}_1}(\ln\frac{\delta_n}{\beta v_n}-\frac{v_s}{v_n})+\sum\limits_{n\in\mathcal{N}_2}\ln(T_n-t_n(l_n^{\min}))$.

The server cost under $0<\alpha<1$ is

$$\begin{aligned}
&C_s^*(0<\alpha<1)=\\
&\sum_{n\in\mathcal{N}_1} r\frac{(\frac{r\delta_n}{\beta v_n})^{\frac{1-\alpha}{\alpha}}}{1-\alpha}-\sum_{n\in\mathcal{N}_1} a\tau_n v_s\beta\frac{\delta_n{}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}}v_n^{\frac{1}{\alpha}}\tau_n}r^{\frac{1}{\alpha}}\\
&+\sum_{n\in\mathcal{N}_1} a\tau_n v_s\beta\left[L+\frac{T_n}{a\tau_n\delta_n}-\frac{S_t+\frac{aL+b}{v_s}}{a\delta_n}+\frac{b}{a}\right]\\
&+\sum_{n\in\mathcal{N}_2} r\cdot\frac{\left(T_n-t_n(l_n^{min})\right)^{1-\alpha}}{1-\alpha}+\sum_{n\in\mathcal{N}_2} a\tau_n v_s\beta(L-l_n^{min})\\
&=\sum_{n\in\mathcal{N}_1}(\frac{\delta_n^{\frac{1}{\alpha}-1}}{\beta_n^{\frac{1}{\alpha}-1}v_n^{\frac{1}{\alpha}}})(\frac{v_n}{1-\alpha}-v_s)r^{\frac{1}{\alpha}}\\
&+\sum_{n\in\mathcal{N}_2} r\cdot\frac{\left(T_n-t_n(l_n^{min})\right)^{1-\alpha}}{1-\alpha}+X.
\end{aligned}\tag{32}$$

The cost under $\alpha=1$ is

$$\begin{aligned}
&C_s^*(\alpha=1)=\\
&\sum_{n\in\mathcal{N}_1} r^*\cdot\ln\left(r^*\cdot\frac{\delta_n}{\beta v_n}\right)+\sum_{n\in\mathcal{N}_2} r^*\cdot\ln\left(T_n-t_n(l_n^{min})\right)\\
&+\sum_{n\in\mathcal{N}_2} a\tau_n v_s\beta(L-l_n^{min})\\
&+\sum_{n\in\mathcal{N}_1} a\tau_n v_s\beta(L-(\frac{r}{a\beta v_n\tau_n}-\frac{T_n}{a\tau_n\delta_n}+\frac{S_t+\frac{aL+b}{v_s}}{a\delta_n}-\frac{b}{a}))\\
&=\sum_{n\in\mathcal{N}_1} r^*\cdot\ln\left(r^*\cdot\frac{\delta_n}{\beta v_n}\right)+\sum_{n\in\mathcal{N}_2} r^*\cdot\ln\left(T_n-t_n(l_n^{min})\right)+X\\
&=\sum_{n\in\mathcal{N}_1} e^{-\frac{G}{|\mathcal{N}_1|}-1}\cdot\ln\left(e^{-\frac{G}{|\mathcal{N}_1|}-1}\cdot\frac{\delta_n}{\beta v_n}\right)\\
&+\sum_{n\in\mathcal{N}_2} e^{-\frac{G}{|\mathcal{N}_1|}-1}\cdot\ln\left(T_n-t_n(l_n^{min})\right)+X\\
&=\sum_{n\in\mathcal{N}_1} e^{-\frac{G}{|\mathcal{N}_1|}-1}(-\frac{G}{|\mathcal{N}_1|}-1)+\sum_{n\in\mathcal{N}_1} e^{-\frac{G}{|\mathcal{N}_1|}-1}\ln(\frac{\delta_n}{\beta v_n})\\
&+\sum_{n\in\mathcal{N}_2} e^{-\frac{G}{|\mathcal{N}_1|}-1}\cdot\ln\left(T_n-t_n(l_n^{min})\right)+X\\
&=-e^{-\frac{G}{|\mathcal{N}_1|}-1}|\mathcal{N}_1|+X.
\end{aligned}\tag{33}$$

Based on (32) and (33), we have

$$\begin{aligned}
&C_s^*(0<\alpha<1)-C_s^*(\alpha=1)=\\
&\sum_{n\in\mathcal{N}_1}(\frac{\delta_n^{\frac{1}{\alpha}-1}}{\beta_n^{\frac{1}{\alpha}-1}v_n^{\frac{1}{\alpha}}})(\frac{v_n}{1-\alpha}-v_s)r^{\frac{1}{\alpha}}+\sum_{n\in\mathcal{N}_2} r\cdot\frac{\left(T_n-t_n(l_n^{min})\right)^{1-\alpha}}{1-\alpha}+X\\
&-(-e^{-\frac{G}{|\mathcal{N}_1|}-1}|\mathcal{N}_1|+X)\\
&=\sum_{n\in\mathcal{N}_1}(\frac{\delta_n^{\frac{1}{\alpha}-1}}{\beta_n^{\frac{1}{\alpha}-1}v_n^{\frac{1}{\alpha}}})(\frac{v_n}{1-\alpha}-v_s)r^{\frac{1}{\alpha}}+\sum_{n\in\mathcal{N}_2} r\cdot\frac{\left(T_n-t_n(l_n^{min})\right)^{1-\alpha}}{1-\alpha}\\
&+e^{-\frac{G}{|\mathcal{N}_1|}-1}|\mathcal{N}_1|>0
\end{aligned}\tag{34}$$

So $C_s^*(0<\alpha<1)>C_s^*(\alpha=1)$.

In summary, we have $C_s^*(0<\alpha<1)>C_s^*(\alpha=1)$.

Appendix F
Discussion on the Excluded Case of $\alpha=0$

A. Client Utility

We define $\delta_n=\frac{1}{v_s}-\frac{1}{v_n},\forall n\in\mathcal{N}$.

The payment received by the client $n\in\mathcal{N}$ is

$$p_n(l_n)=r(T_n-t_n(l_n))\tag{35}$$

The utility of the client $n\in\mathcal{N}$ is

$$U_n(l_n)=p_n(l_n)-c_n(l_n).\tag{36}$$

The first-order partial derivative of the utility function is:

$$\frac{\partial U_n}{\partial l_n}=r\delta_n\tau_n a-\tau_n a\beta v_n=\tau_n a(r\delta_n-\beta v_n).\tag{37}$$

- When $v_n\le v_s$: $\delta_n\le 0$, the first-order derivative is always less than zero, so the optimal decision is $l_n^*(r)=l_n^{\min}$.

- When $v_n > v_s$: $\delta_n > 0$. we give the following solution:

$$l_n^*(r) = \begin{cases} l_n^{\min}, & r_{min} < r < \frac{\beta v_n}{\delta_n}, \\ l_n^{\max}, & \frac{\beta v_n}{\delta_n} \le r \le r_{max}, \end{cases} \tag{38}$$

B. Server Cost

The cost of the server is

$$C_s(r) = \sum_{n\in\mathcal{N}} r(T_n - t_n(l_n(r))) + \beta_s v_s \sum_{n\in\mathcal{N}} (f(L) - f(l_n(r))). \tag{39}$$

- When $v_s > v_{max}$: $C_s(r) = \sum_{n\in\mathcal{N}} r(T_n - t_n(l_n^{\min})) + \beta_s v_s \sum_{n\in\mathcal{N}} (f(L) - f(l_n^{\min}))$, so $r^* = r_{min}$.
- When $v_s < v_{min}$: Due to the fact that $l_n(r)$ in (38) is a piecewise function and only takes two extreme values, we divide the entire domain into multiple intervals. Specifically, we sort based on the critical values $\frac{\beta v_n}{\delta_n}$ of N users, resulting in $N+1$ consecutive intervals, i.e., $[r_{min}, \frac{\beta v_1}{\delta_1}), [\frac{\beta v_1}{\delta_1}, \frac{\beta v_2}{\delta_2}), ..., [\frac{\beta v_N}{\delta_N}, r_{max}]$. In a interval $[\frac{\beta v_k}{\delta_k}, \frac{\beta v_{k+1}}{\delta_{k+1}}), k \in \mathcal{N} \& k \neq N$, the l_n choices of all users remain unchanged, so the cost function $C_s(r)$ is

$$\begin{aligned} C_s(r) = r(&\sum_{n\ge k} (T_n - t_n(l_n^{\min})) + \sum_{n<k} (T_n - t_n(l_n^{\max}))) \\ +\beta_s v_s(&\sum_{n\ge k} (f(L) - f(l_n^{\min})) + \sum_{n<k} (f(L) - f(l_n^{\max}))), \end{aligned} \tag{40}$$

which is a linear function with respect to r in the interval $[\frac{\beta v_k}{\delta_k}, \frac{\beta v_{k+1}}{\delta_{k+1}})$. It shows that the function increases monotonically within this interval, therefore its optimal value appears at the left endpoint of the interval $\frac{\beta v_k}{\delta_k}$. Based on this, it can be concluded that within each interval, the decisions of all users are fixed, so the optimal value is always taken at the left boundary of the interval. So $r^* = argmin_{r\in\{r_{min}, \frac{\beta v_n}{\delta_n}\}, n\in\mathcal{N}} C_s(r)$.
- When $v_{min} \le v_s \le v_{max}$: We define $\mathcal{N}_1 = \{n \in \mathcal{N} \mid v_n > v_s\}$ and $\mathcal{N}_2 = \{n \in \mathcal{N} \mid v_n \le v_s\}$. We sort based on the critical values $\frac{\beta v_n}{\delta_n}$ of $|\mathcal{N}_1|$ users, resulting in $|\mathcal{N}_1| + 1$ intervals, i.e., $[r_{min}, \frac{\beta v_1}{\delta_1}), [\frac{\beta v_1}{\delta_1}, \frac{\beta v_2}{\delta_2}), ..., [\frac{\beta v_{|\mathcal{N}_1|}}{\delta_{|\mathcal{N}_1|}}, r_{max}]$. In the interval $[\frac{\beta v_k}{\delta_k}, \frac{\beta v_{k+1}}{\delta_{k+1}}), k \in \mathcal{N}_1 \& k \neq |\mathcal{N}_1|$, the cost function $C_s(r)$:

$$\begin{aligned} &C_s(r) = \\ &\sum_{n\in\mathcal{N}_2} r(T_n - t_n(l_n^{\min})) + \beta_s v_s \sum_{n\in\mathcal{N}_2} (f(L) - f(l_n^{\min})) \\ &+ r(\sum_{n\ge k \& n\in\mathcal{N}_1} (T_n - t_n(l_n^{\min})) + \sum_{n<k \& n\in\mathcal{N}_1} (T_n - t_n(l_n^{\max}))) \\ &+ \beta_s v_s(\sum_{n\ge k \& n\in\mathcal{N}_1} (f(L) - f(l_n^{\min})) + \sum_{n<k \& n\in\mathcal{N}_1} (f(L) - f(l_n^{\max}))), \end{aligned} \tag{41}$$

which is still a linear function with respect to r in the interval $[\frac{\beta v_k}{\delta_k}, \frac{\beta v_{k+1}}{\delta_{k+1}})$, so $r^* = argmin_{r\in\{r_{min}, \frac{\beta v_n}{\delta_n}\}, n\in\mathcal{N}_1} C_s(r)$.

In summary, the optimal solution r^* of the server is

$$r^* = \begin{cases} r_{min}, & v_s > v_{max}, \\ argmin_{r\in\{r_{min}, \frac{\beta v_n}{\delta_n}\}, n\in\mathcal{N}_1} C_s(r), & v_{min} \le v_s \le v_{max}, \\ argmin_{r\in\{r_{min}, \frac{\beta v_n}{\delta_n}\}, n\in\mathcal{N}} C_s(r), & v_s < v_{min} \end{cases} \tag{42}$$