Appendix A
Proof of Theorem 1

When $\alpha = 1$ *:*

The first-order partial derivative of the utility function is:

$$\frac{\partial U_n}{\partial l_n} = r\frac{\tau_n a\delta_n}{T - t(l_n)} - \tau_n a\beta v_n. \tag{1}$$

The second-order partial derivative of the utility function is:

$$\frac{\partial U^2}{\partial^2 l_n} = -r\frac{(\tau_n a\delta_n)^2}{(T - t(l_n))^2} < 0. \tag{2}$$

Since the second-order derivative is always negative, the utility function is strictly concave with respect to l_n, so it has a unique maximum.

- When $v_n \le v_s$: $\delta_n \le 0$, the first-order derivative is always less than zero, so the optimal decision is $l_n^*(r) = l_n^{\min}$.
- When $v_n > v_s$: $\delta_n > 0$. Set the first-order derivative to zero and solve for the optimal cut layer:

$$l_n^{\alpha=1}(r) = \frac{1}{a\beta v_n \tau_n} r - \frac{T_n}{a\tau_n\delta_n} + \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} - \frac{b}{a}. \tag{3}$$

Since $l_n^{\min} \le l_n \le l_n^{\max}$, we give the following solution:

$$l_n^*(r) = \begin{cases} l_n^{\min}, & 0 < r < \xi_{\text{left}}^{n,\alpha=1}, \\ l_n^{\alpha=1}(r), & \xi_{\text{left}}^{n} \le r \le \xi_{\text{right}}^{n,\alpha=1}, \\ l_n^{\max}, & \xi_{\text{right}}^{n,\alpha=1} < r \le r_{max}, \end{cases} \tag{4}$$

where $\xi_{\text{left}}^{n,\alpha=1} = a\beta v_n \tau_n \left(l_n^{\min} + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a} \right)$, $\xi_{\text{right}}^{n,\alpha=1} = a\beta v_n \tau_n \left(l_n^{\max} + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a} \right)$, $\delta_n = \frac{1}{v_s} - \frac{1}{v_n}$.

When $\alpha = 1$, the optimal solution function for client n is:

$$l_n^*(r) = \begin{cases} l_n^{\min}, & \text{if } v_n \le v_s, \\ \begin{cases} l_n^{\min}, & 0 < r < \xi_{\text{left}}^{n}, \\ l_n^{\alpha=1}(r), & \xi_{\text{left}}^{n} \le r \le \xi_{\text{right}}^{n}, \\ l_n^{\max}, & \xi_{\text{right}}^{n} < r \le r_{max}, \end{cases}, & \text{if } v_n > v_s. \end{cases} \tag{5}$$

When $\alpha \neq 1$ *:*

The first-order partial derivative of the utility function is:

$$\frac{\partial U_n}{\partial l_n} = r\frac{\tau_n a\delta_n}{(T - t(l_n))^\alpha} - \tau_n a\beta v_n. \tag{6}$$

The second-order partial derivative of the utility function is:

$$\frac{\partial U^2}{\partial^2 l_n} = -r\alpha\frac{(\tau_n a\delta_n)^2}{(T - t(l_n))^{\alpha+1}} < 0. \tag{7}$$

Since the second-order derivative is always negative, the utility function is strictly concave with respect to l_n, so it has a unique maximum.

- When $v_n \le v_s$: $\delta_n \le 0$. The first-order derivative is always less than zero, so the best response is $l_n^*(r) = l_n^{\min}$.
- When $v_n > v_s$: Set the first-order derivative to zero and solve for the optimal cut layer:

$$l_n^{0\le\alpha<1}(r) = \frac{\delta_n^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}} \tau_n} r^{\frac{1}{\alpha}} - \frac{T_n}{a\tau_n\delta_n} + \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} - \frac{b}{a}. \tag{8}$$

Since $l_n^{\min} \le l_n \le l_n^{\max}$, we give the following solution:

$$l_n^*(r) = \begin{cases} l_n^{\min}, & 0 < r < \xi_{\text{left}}^{n,0<\alpha<1}, \\ l_n^{0\le\alpha<1}(r), & \xi_{\text{left}}^{n} \le r \le \xi_{\text{right}}^{n,0<\alpha<1}, \\ l_n^{\max}, & \xi_{\text{right}}^{n,0<\alpha<1} < r \le r_{max}, \end{cases} \tag{9}$$

where $\xi_{\text{left}}^{n,0<\alpha<1} = (\frac{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}} \tau_n}{\delta_n^{\frac{1}{\alpha}-1}})^\alpha (l_n^{\min} + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a})^\alpha$, $\xi_{\text{right}}^{n,0<\alpha<1} = (\frac{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}} \tau_n}{\delta_n^{\frac{1}{\alpha}-1}})^\alpha (l_n^{\max} + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a})^\alpha$.

When $0 < \alpha < 1$, the optimal solution function for client n is:

$$l_n^*(r) = \begin{cases} l_n^{\min}, & \text{if } v_n \le v_s, \\ \begin{cases} l_n^{\min}, & 0 < r < \xi_{\text{left}}^{n,0<\alpha<1}, \\ l_n^{0\le\alpha<1}(r), & \xi_{\text{left}}^{n,0<\alpha<1} \le r \le \xi_{\text{right}}^{n,0<\alpha<1}, \\ l_n^{\max}, & \xi_{\text{right}}^{n,0<\alpha<1} < r \le r_{max}, \end{cases}, & \text{if } v_n > v_s. \end{cases} \tag{10}$$

We integrate the solution forms of two cases and find that when $0 < \alpha \le 1$, $\xi_{\text{left}}^{n}(\alpha) = (\frac{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}} \tau_n}{\delta_n^{\frac{1}{\alpha}-1}})^\alpha (l_n^{\min} + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a})^\alpha$, $\xi_{\text{right}}^{n}(\alpha) = (\frac{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}} \tau_n}{\delta_n^{\frac{1}{\alpha}-1}})^\alpha (l_n^{\max} + \frac{T_n}{a\tau_n\delta_n} - \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} + \frac{b}{a})^\alpha$ and $l_n^{\alpha}(r) = \frac{\delta_n^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}} \tau_n} r^{\frac{1}{\alpha}} - \frac{T_n}{a\tau_n\delta_n} + \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} - \frac{b}{a}$.

In summary, the optimal solution function for client n is:

$$l_n^*(r) = \begin{cases} l_n^{\min}, & \text{if } v_n \le v_s, \\ \begin{cases} l_n^{\min}, & 0 < r < \xi_{\text{left}}^{n}(\alpha), \\ l_n^{\alpha}(r), & \xi_{\text{left}}^{n}(\alpha) \le r \le \xi_{\text{right}}^{n}(\alpha), \\ l_n^{\max}, & \xi_{\text{right}}^{n}(\alpha) < r \le r_{max}, \end{cases}, & \text{if } v_n > v_s. \end{cases} \tag{11}$$

Appendix B
Proof of Theorem 2

Under assumption 1, we have $T_n = t_n^{max}$, so when $v_s < v_n$:

$$
\begin{aligned}
l_n^\alpha(r) &= \frac{{\delta_n}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n} r^{\frac{1}{\alpha}} - \frac{t_n^{max}}{a\tau_n\delta_n} + \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} - \frac{b}{a} \\
&= \frac{{\delta_n}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n} r^{\frac{1}{\alpha}} - \frac{\tau_n(-b\delta_n + S_t + \frac{f(L)}{v_s})}{a\tau_n\delta_n} + \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} - \frac{b}{a} \\
&= \frac{{\delta_n}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n} r^{\frac{1}{\alpha}} \geq 0.
\end{aligned} \tag{12}
$$

Therefore, the Assumption 1 yields a candidate cut-layer that is always nonnegative. Under Assumption 1, where the maximum allowable cut layer approaches positive infinity, any such value for users with r and $v_n > v_s$ automatically lies in the feasible interval $[l_n^{\min}, l_n^{\max}]$.

Therefore, we revised the conclusion in the general case and provided the solution under assumption 1. In summary, the optimal solution function for client n under assumption 1 is:

$$
l_n^*(r) = \begin{cases} l_n^{\min}, & \text{if } v_s \geq v_n, \\ l_n^\alpha(r), & \text{if } v_s < v_n. \end{cases} \tag{13}
$$

Appendix C
Proof of Theorem 3

$v_s < v_{min}$: When $v_s < v_n$ (i.e., the server's computational capability is inferior to that of the clients), and under the setting ($\alpha = 1$), the server's cost function can be derived based on the clients' optimal strategies. For simplicity, we define Z as all terms in the equation that are independent of the variable r. After simplification, the cost function takes the form:

$$
\begin{aligned}
C_s(r) = &\sum_{n\in\mathcal{N}} r \cdot \ln\left(r \cdot \frac{\delta_n}{\beta v_n}\right) - \sum_{n\in\mathcal{N}} a\tau_n v_s \beta \frac{1}{a\beta v_n \tau_n} r \\
&+ \sum_{n\in\mathcal{N}} a\tau_n v_s \beta \left[L + \frac{T_n}{a\tau_n\delta_n} - \frac{\left(S_t + \frac{aL+b}{v_s}\right)}{a\delta_n} + \frac{b}{a}\right] \\
= &Nr\ln r + r\sum_{n\in\mathcal{N}} \left[\ln\left(\frac{\delta_n}{\beta v_n}\right) - \frac{v_s}{v_n}\right] + Z.
\end{aligned} \tag{14}
$$

To determine the optimal reward r^*, we analyze the first and second derivatives of $C_s(r)$:

$$
\frac{\partial C_s}{\partial r} = N(1 + \ln r) + \sum_{n\in\mathcal{N}} (\ln\frac{\delta_n}{\beta v_n} - \frac{v_s}{v_n}), \tag{15}
$$

$$
\frac{\partial^2 C_s}{\partial r^2} = \frac{N}{r} > 0 \quad \text{for} \quad r > 0. \tag{16}
$$

The positive second derivative implies that $C_s(r)$ is strictly convex in r, ensuring that any critical point found is the global minimum. Setting the first derivative to zero yields the unique optimal reward:

$$
r^* = \exp\left\{-\frac{\sum_{n\in\mathcal{N}}\left[\ln\left(\frac{\delta_n}{\beta v_n}\right) - \frac{v_s}{v_n}\right]}{N} - 1\right\}. \tag{17}
$$

$v_{min} \leq v_s \leq v_{max}$: When $v_{min} < v_s < v_{max}$, the server's cost function under the proportional fairness setting ($\alpha = 1$) is as follows:

$$
\begin{aligned}
C_s(r) = &\sum_{n\in\mathcal{N}_1} r \cdot \ln\left(r \cdot \frac{\frac{1}{v_s} - \frac{1}{v_n}}{\beta v_n}\right) + \sum_{n\in\mathcal{N}_2} r \cdot \ln\left(T - t_n(l_{\min})\right) \\
&+ \sum_{n\in\mathcal{N}_2} a\tau_n v_s \beta (L - l_{\min}) \\
&+ \sum_{n\in\mathcal{N}_1} a\tau_n v_s \beta (L - (\frac{r}{a\beta v_n \tau_n} - \frac{T_n}{a\tau_n\delta_n} + \frac{S_t + \frac{aL+b}{v_s}}{a\delta_n} - \frac{b}{a})) \\
= &r\left[\sum_{n\in\mathcal{N}_1}\left(\ln\frac{\delta_n}{\beta v_n} - \frac{v_s}{v_n}\right) + \sum_{n\in\mathcal{N}_2} \ln(T - t_n(l_{\min}))\right] \\
&+ N_1 r\ln r + Z,
\end{aligned} \tag{18}
$$

The first-order and second-order derivatives of $C_s(r)$ with respect to r are:

$$
\begin{aligned}
&\frac{\partial C_s}{\partial r} \\
&= N_1(1 + \ln r) + \sum_{n\in\mathcal{N}_1} (\ln\frac{\delta_n}{\beta v_n} - \frac{v_s}{v_n}) + \sum_{n\in\mathcal{N}_2} \ln(T - t_n(l_{\min})).
\end{aligned} \tag{19}
$$

$$
\frac{\partial^2 C_s}{\partial r^2} = \frac{N_1}{r} \geq 0. \tag{20}
$$

The positive second derivative implies strong convexity, ensuring the existence of a unique global minimum, given by:

$$
r^* = \exp\{-\frac{\sum\limits_{n\in\mathcal{N}_1}(\ln\frac{\delta_n}{\beta v_n} - \frac{v_s}{v_n}) + \sum\limits_{n\in\mathcal{N}_2}\ln(T - t_n(l_{\min}))}{N_1} - 1\}. \tag{21}
$$

Appendix D
Proof of Theorem 4

$v_s < v_{min}$: When $v_s < v_n$ (i.e., the server's computational capability is inferior to that of the clients), and under the

setting ($\alpha \neq 1$), the cost function takes the form:

$$
\begin{aligned}
C_s(r) &= \sum_{n\in\mathcal{N}} r\frac{\left(\frac{r(\frac{1}{v_s}-\frac{1}{v_n})}{\beta v_n}\right)^{\frac{1-\alpha}{\alpha}}}{1-\alpha} - \sum_{n\in\mathcal{N}} a\tau_n v_s\beta\frac{\delta_n{}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n} r^{\frac{1}{\alpha}} \\
&+ \sum_{n\in\mathcal{N}} a\tau_n v_s\beta\left[L+\frac{T_n}{a\tau_n\delta_n}-\frac{S_t+\frac{aL+b}{v_s}}{a\delta_n}+\frac{b}{a}\right] \\
=& r^{\frac{1}{\alpha}}\sum_{n\in\mathcal{N}}\left(\frac{\left(\frac{\delta_n}{\beta v_n}\right)^{\frac{1-\alpha}{\alpha}}}{1-\alpha} - a\tau_n v_s\beta\frac{\delta_n{}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n}\right)+Z \\
&= r^{\frac{1}{\alpha}}\sum_{n\in\mathcal{N}}\left(\frac{\delta_n^{\frac{1}{\alpha}-1}}{\beta_n^{\frac{1}{\alpha}-1} v_n^{\frac{1}{\alpha}}}\right)\left(\frac{v_n}{1-\alpha}-v_s\right)+Z,
\end{aligned}
\tag{22}
$$

where z is still all terms in the equation that are independent of the variable r.

To determine the optimal reward r^*, we analyze the first and second derivatives of $C_s(r)$:

$$
\frac{\partial C_s}{\partial r} = r^{\frac{1}{\alpha}-1}\frac{1}{\alpha}\sum_{n\in\mathcal{N}}\left(\frac{\delta_n^{\frac{1}{\alpha}-1}}{\beta_n^{\frac{1}{\alpha}-1} v_n^{\frac{1}{\alpha}}}\right)\left(\frac{v_n}{1-\alpha}-v_s\right), \tag{23}
$$

At this time, the server's cost function is a power function, and its monotonicity is determined by other constant terms:

- When the overall computing power of the user is high, that is, when $\sum_{n\in\mathcal{N}}(\frac{v_n}{1-\alpha}-v_s)>0$, then $r^*=0$.
- When the overall computing power of the user is low, that is, when $\sum_{n\in\mathcal{N}}(\frac{v_n}{1-\alpha}-v_s)<0$, then $r^*=r_{max}$.

$v_{min} \leq v_s \leq v_{max}$: We make $\gamma_1 = \sum_{n\in\mathcal{N}_1}(\frac{\delta_n^{\frac{1}{\alpha}-1}}{\beta_n^{\frac{1}{\alpha}-1} v_n^{\frac{1}{\alpha}}})(\frac{v_n}{1-\alpha} - v_s)$ and $\gamma_2 = \sum_{n\in\mathcal{N}_2}\frac{(T_n-t_n(l_n^{min}))^{1-\alpha}}{1-\alpha}$. Z represents the constant term aggregates all r-independent expressions. When $v_{min} < v_s < v_{max}$, the server's cost function under the setting ($\alpha = 1$) is as follows:

$$
\begin{aligned}
&C_s(r) = \\
&\sum_{n\in\mathcal{N}_1} r\frac{\left(\frac{r(\frac{1}{v_s}-\frac{1}{v_n})}{\beta v_n}\right)^{\frac{1-\alpha}{\alpha}}}{1-\alpha} - \sum_{n\in\mathcal{N}} a\tau_n v_s\beta\frac{\delta_n{}^{\frac{1}{\alpha}-1}}{a\beta^{\frac{1}{\alpha}} v_n^{\frac{1}{\alpha}}\tau_n} r^{\frac{1}{\alpha}} \\
&+ \sum_{n\in\mathcal{N}_1} a\tau_n v_s\beta\left[L+\frac{T_n}{a\tau_n\delta_n}-\frac{S_t+\frac{aL+b}{v_s}}{a\delta_n}+\frac{b}{a}\right] \\
&+ \sum_{n\in\mathcal{N}_2} r\cdot\ln\left(T-t_n(l_{\min})\right) + \sum_{n\in\mathcal{N}_2} a\tau_n v_s\beta(L-l_{\min}) \\
=&\gamma_1 r^{\frac{1}{\alpha}} + \gamma_2 r + Z
\end{aligned}
\tag{24}
$$

$C_s'(r) = \gamma_1\frac{1}{\alpha}r^{\frac{1}{\alpha}-1}+\gamma_2$ and $C_s^{''}(r) = \gamma_1\frac{1}{\alpha}(\frac{1}{\alpha}-1)r^{\frac{1}{\alpha}-2}$.

- When the overall computing power of the users in $\mathcal{N}_1$ is high, that is, when $\gamma_1 > 0$ and $\gamma_2 > 0$, $C_s^{'}(r) > 0$, then $r^* = 0$.
- When the overall computing power of the users in $\mathcal{N}_1$ is low, that is, when $\gamma_1 < 0$ and $\gamma_2 > 0$.

$C_s'(r) = \gamma_1\frac{1}{\alpha}r^{\frac{1}{\alpha}-1}+\gamma_2$ and $C_s^{''}(r) = \gamma_1\frac{1}{\alpha}(\frac{1}{\alpha}-1)r^{\frac{1}{\alpha}-2} < 0$. Therefore, this cost function has no minimum value, and the extreme value is obtained at the boundary. $r^* = argmin_r\{C_s(r=0), C_s(r=r_{max})\}$.