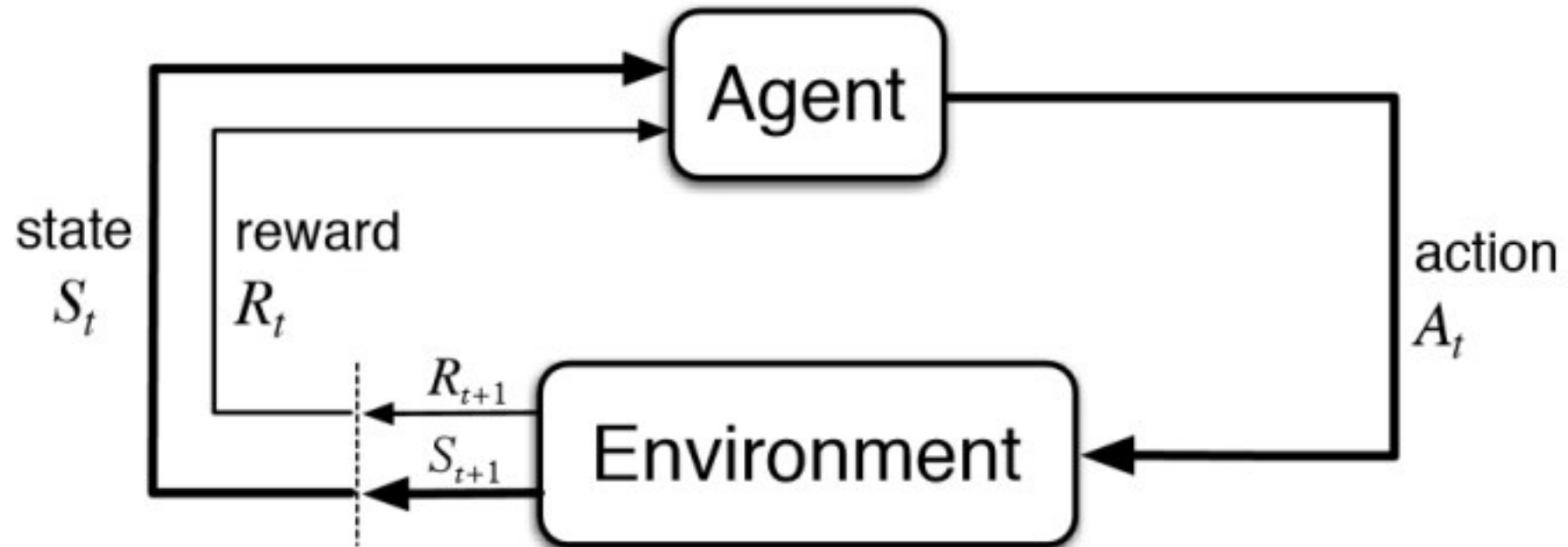


# Tree Search

Week 3

# Recap on Reinforcement Learning



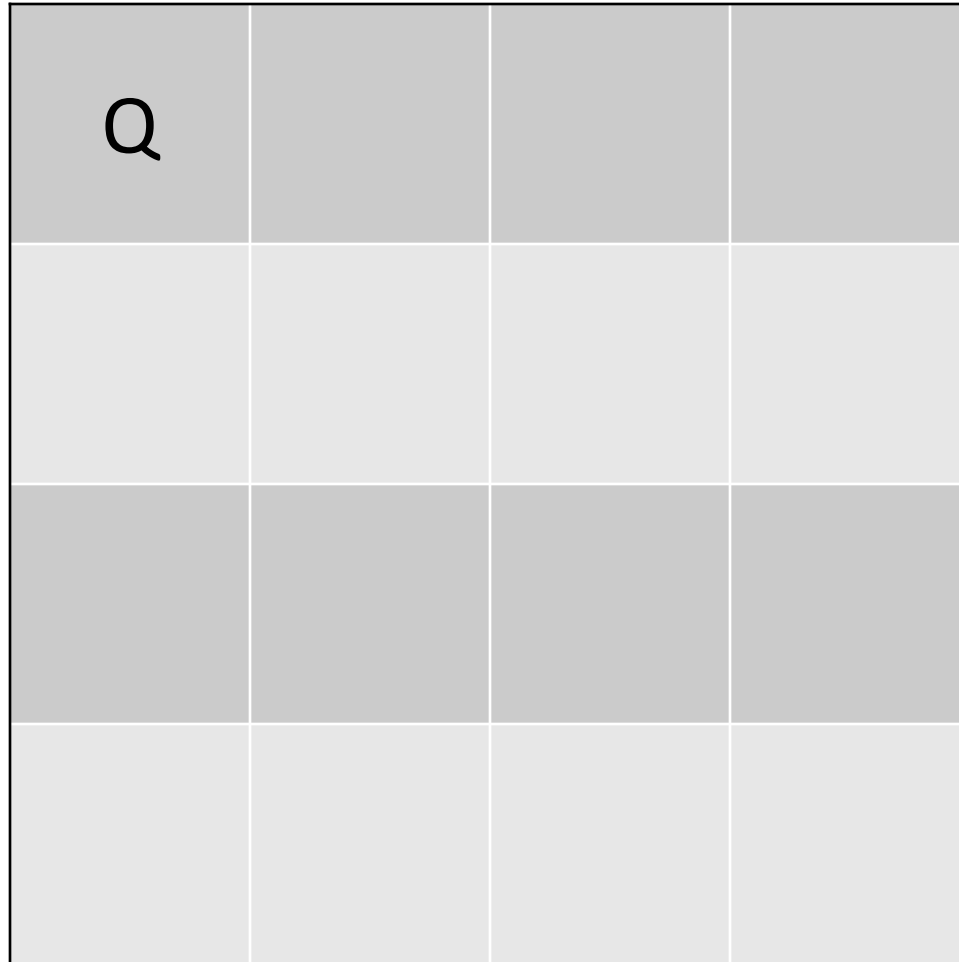
# Formulating Problems as RL

- State Space
  - Start State
  - Goal State
- Actions Space

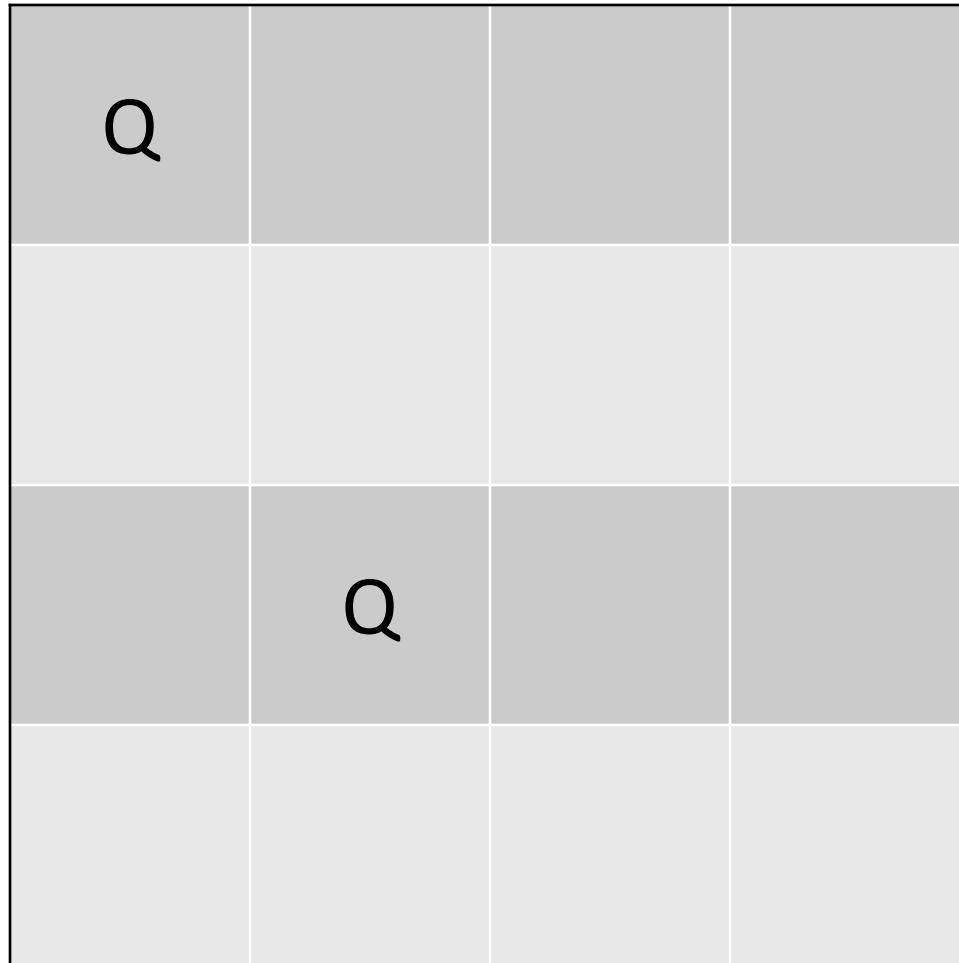
# Four Queens Problem

- Place four queens on a 4 x 4 board such that they do not attack each other.
- Initial State
- Action Space
- State Space
- Goal State

# Four Queens Problem



# Four Queens Problem



# Four Queens Problem

Q			
			Q
	Q		



# Four Queens: Limiting Action Space

- State Space
  - $16 \times 15 \times 14 \times 13 = 43,680$
- Action Space
  - ~14 tiles
- Reformulate
  - One Queen per Column
- State Space
  - $4 \times 4 \times 4 \times 4 = 16$
- Action Space
  - 4 tiles



# N-Queens Problem

- Eight Queens
  - Original, Published 1848
  - State Space  $\sim 10^{14}$  to 2,057
- Hundred Queens
  - State Space  $\sim 10^{400}$  to  $10^{52}$
- Million Queens
  - Literally Impossible (jk)
  - Solved in a few seconds with min-conflicts heuristics.

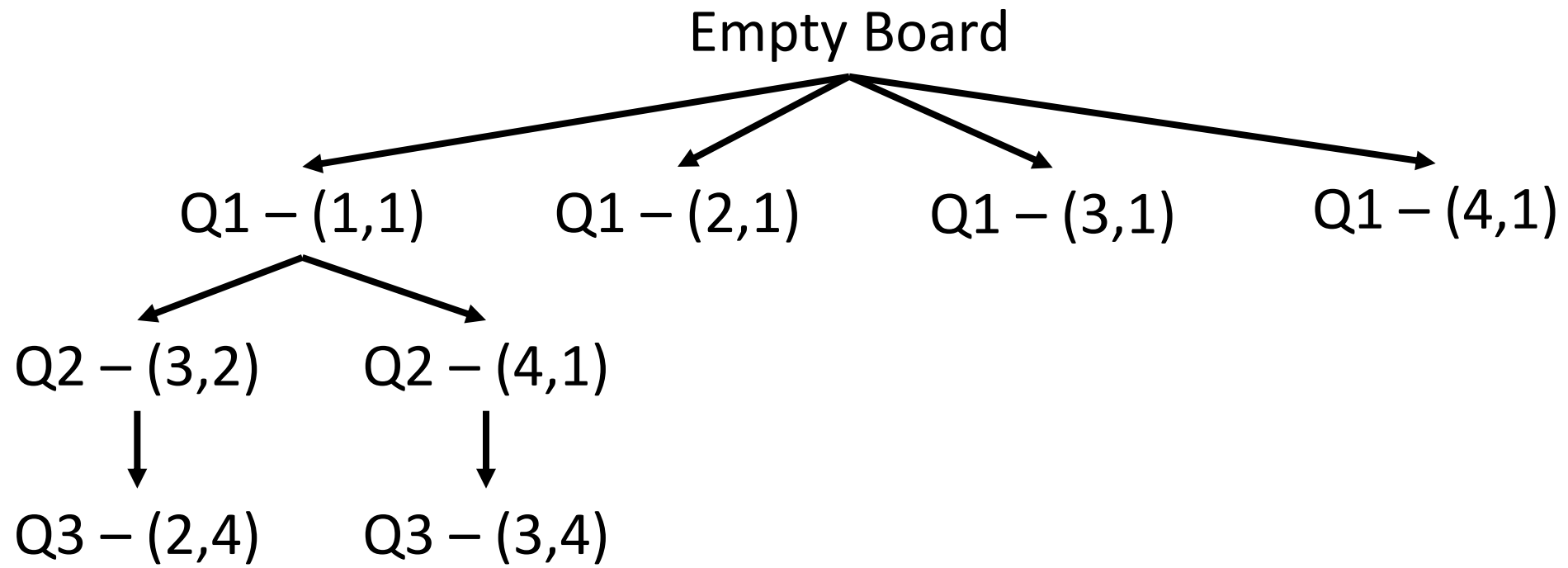
# Four Queens Solution

		Q	
Q			
			Q
	Q		

	Q		
			Q
Q			
		Q	

# Search Tree

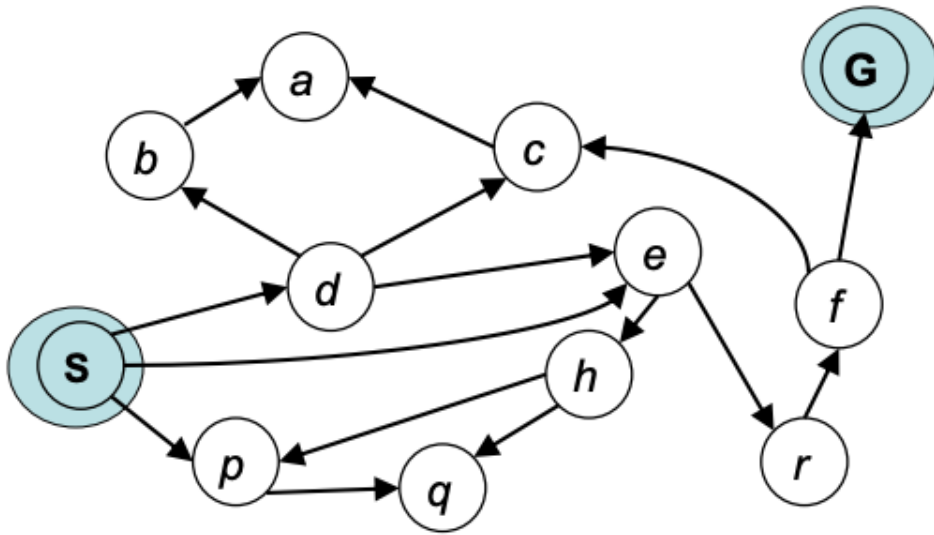
- General strategy to search through possible plans.



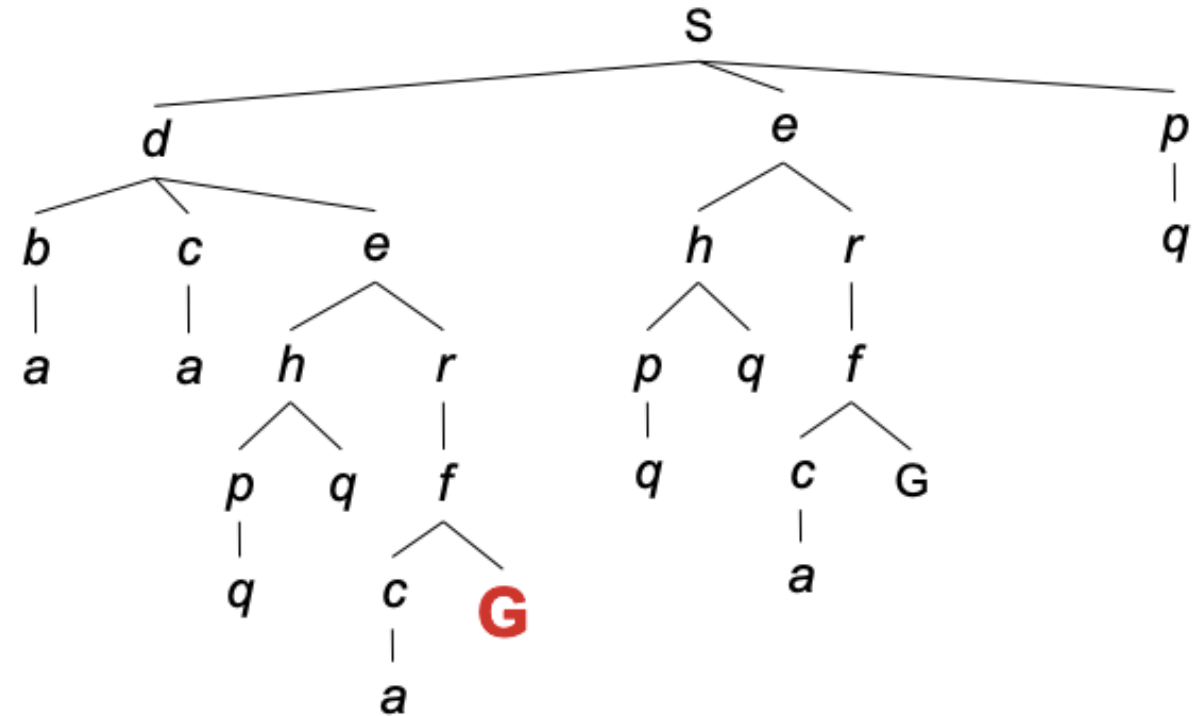
# Tree Search

- Generalized Method of Searching State Space
  - Useful for Most Action Based Problems
  - Useful for NP-Hard Problems
- Breadth First Search
- Depth First Search
- Advanced
  - Uniform Cost Search
  - A\* Search

# Search Problem



## State Graph

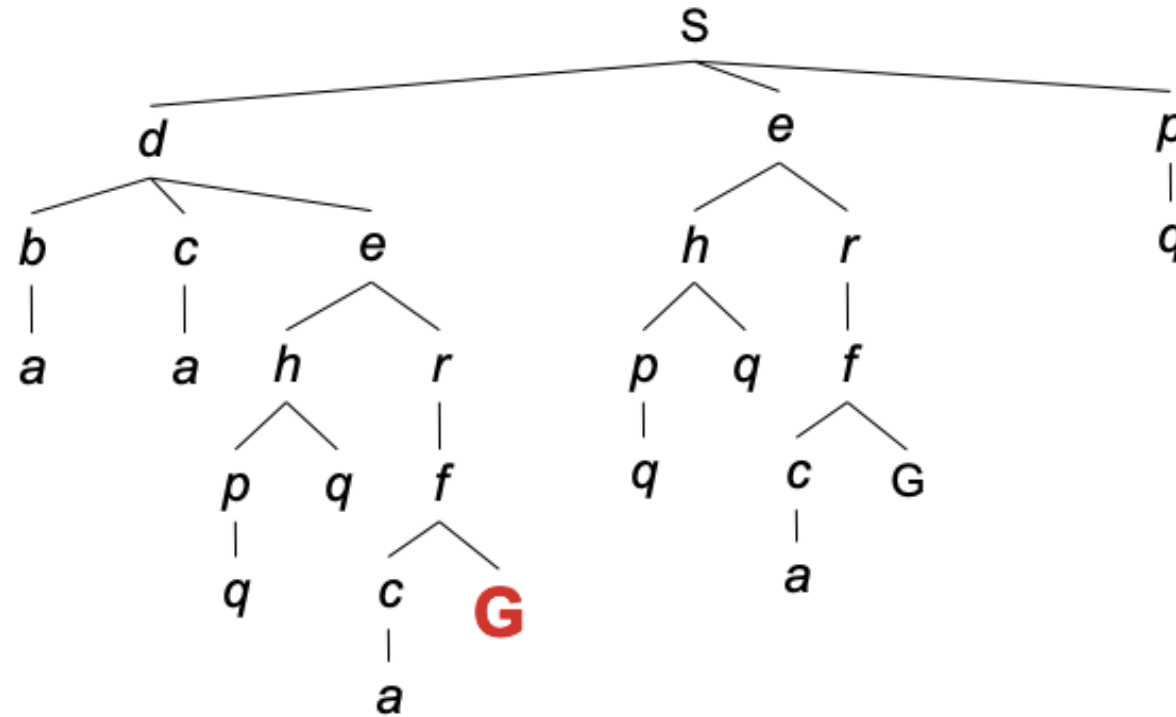


## Search Tree

# Depth First Search

- Strategy
  - Search the first unexplored path.
- Implementation
  - Stack – Last in First Out
  - Recursive

# Depth First Search



- Order – S,p,q,e,r,f,G

# Depth First Search

push first node to stack

while the stack is not empty:

    pop the stack

    if goal:

        end

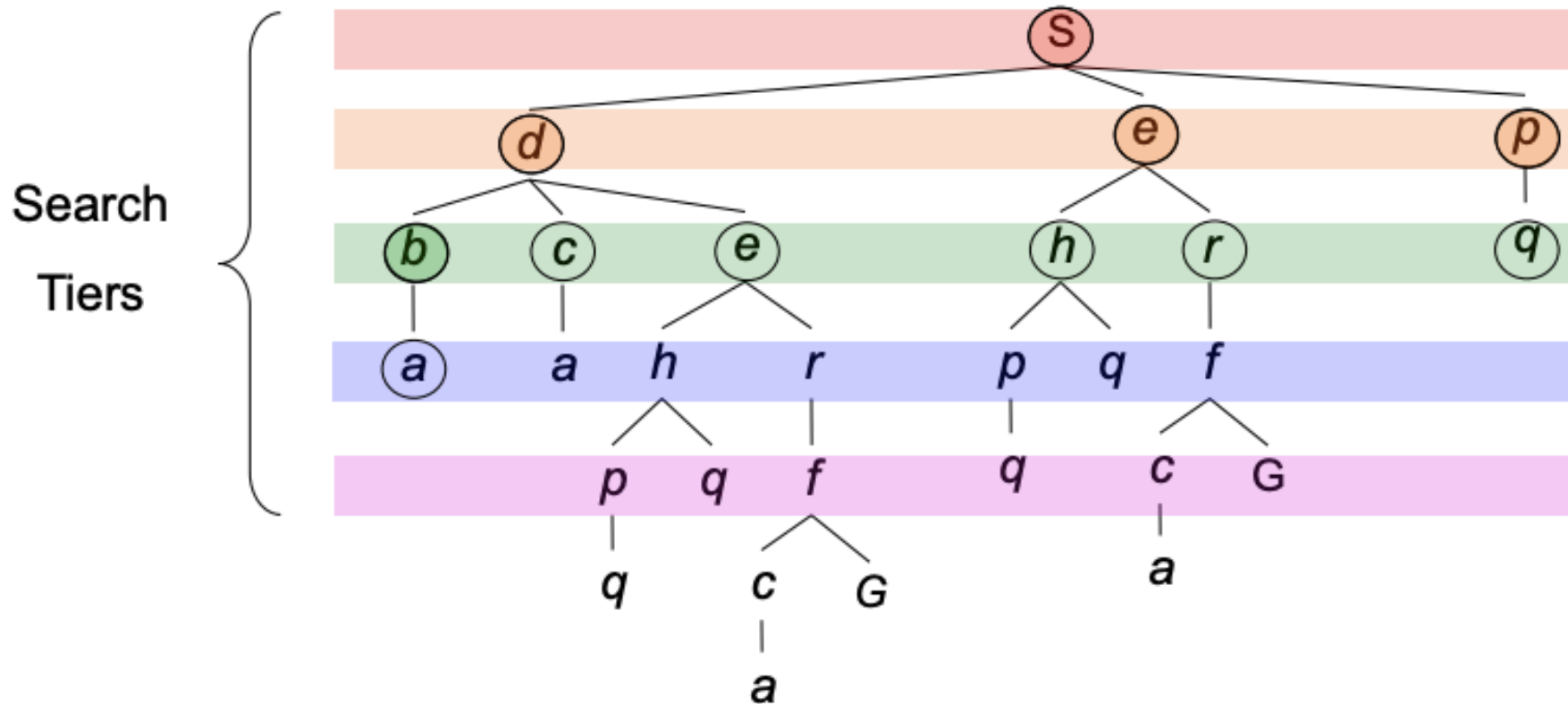
    push children to stack



# Breadth First Search

- Strategy
  - Search each layer at a time.
- Implementation
  - Queue – First in First Out

# Breadth First Search



- Order – S,d,e,p,b,c,e,h,r,q,a,a,h,r,p,q,f,p,q,f,q,c,G

# Breadth First Search

push first node to queue

while the queue is not empty:

    pop the queue

    if goal:

        end

    push children to queue

# Search Algorithm Properties

- $n$  – number of states
- $b$  – max branching factor
- $d$  – min depth of solution
- $m$  – max depth of search tree

# Search Algorithm Properties - DFS

- Complete
  - Yes
- Optimal
  - No
- Time Complexity
  - $O(b^m)$
- Space Complexity
  - $O(bm)$

$n$  – number of states

$b$  – max branching factor

$d$  – min depth of solution

$m$  – max depth of search tree

# Search Algorithm Properties - BFS

- Complete
  - Yes
- Optimal
  - Yes
- Time Complexity
  - $O(b^d)$
- Space Complexity
  - $O(b^d)$

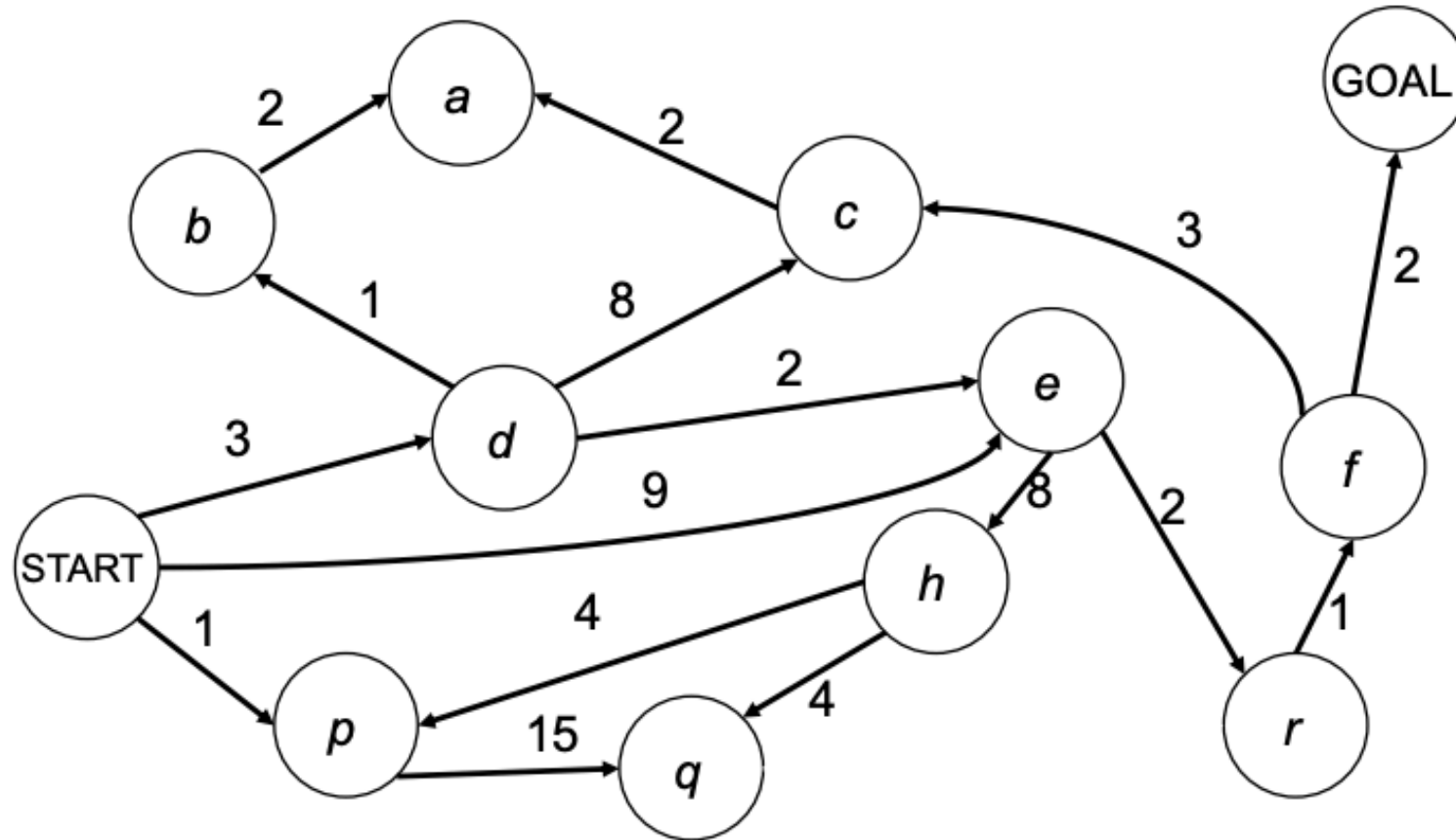
$n$  – number of states

$b$  – max branching factor

$d$  – min depth of solution

$m$  – max depth of search tree

# Actions can have Cost

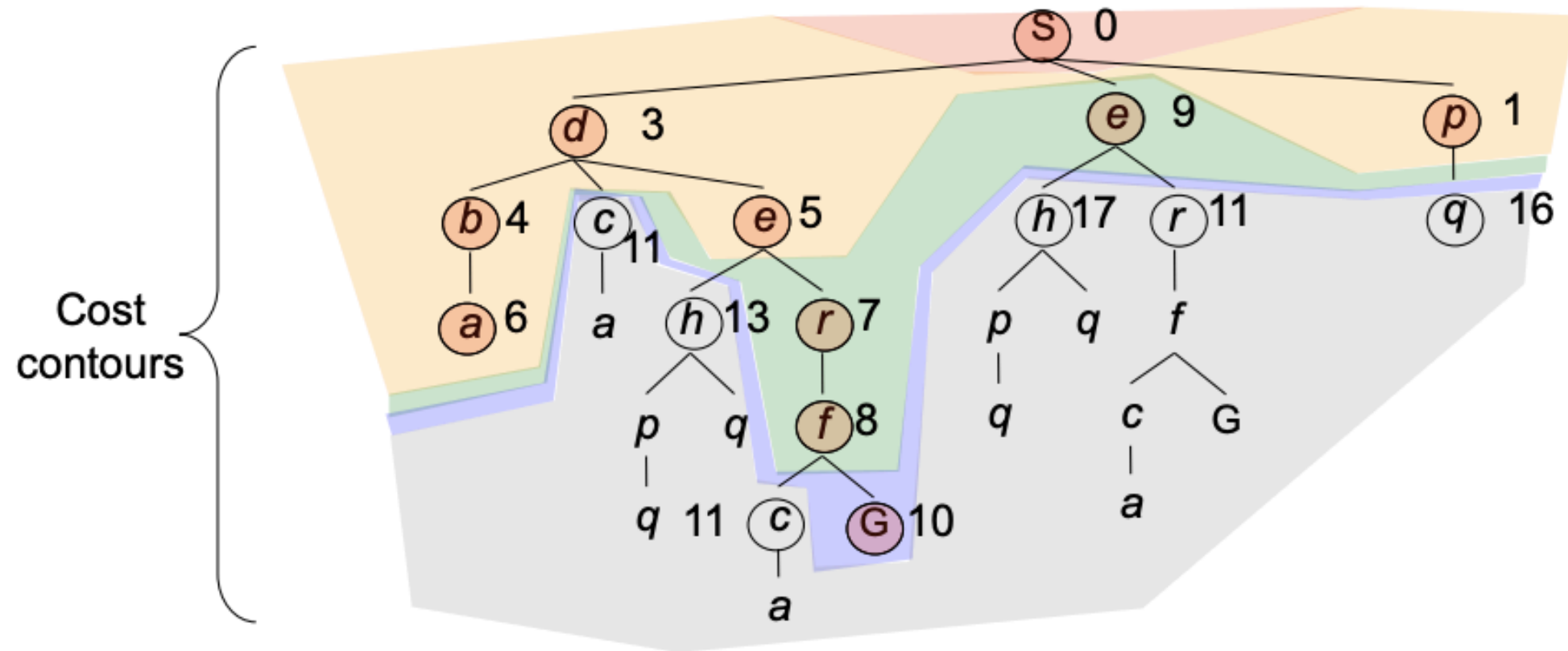


# Uniform Cost Search

- Generalization of Breadth First Search
- Strategy:
  - Search the next cheapest state.
- Implementation:
  - Priority Queue
  - Each state has a cost function  $f(n)$



# Uniform Cost Search



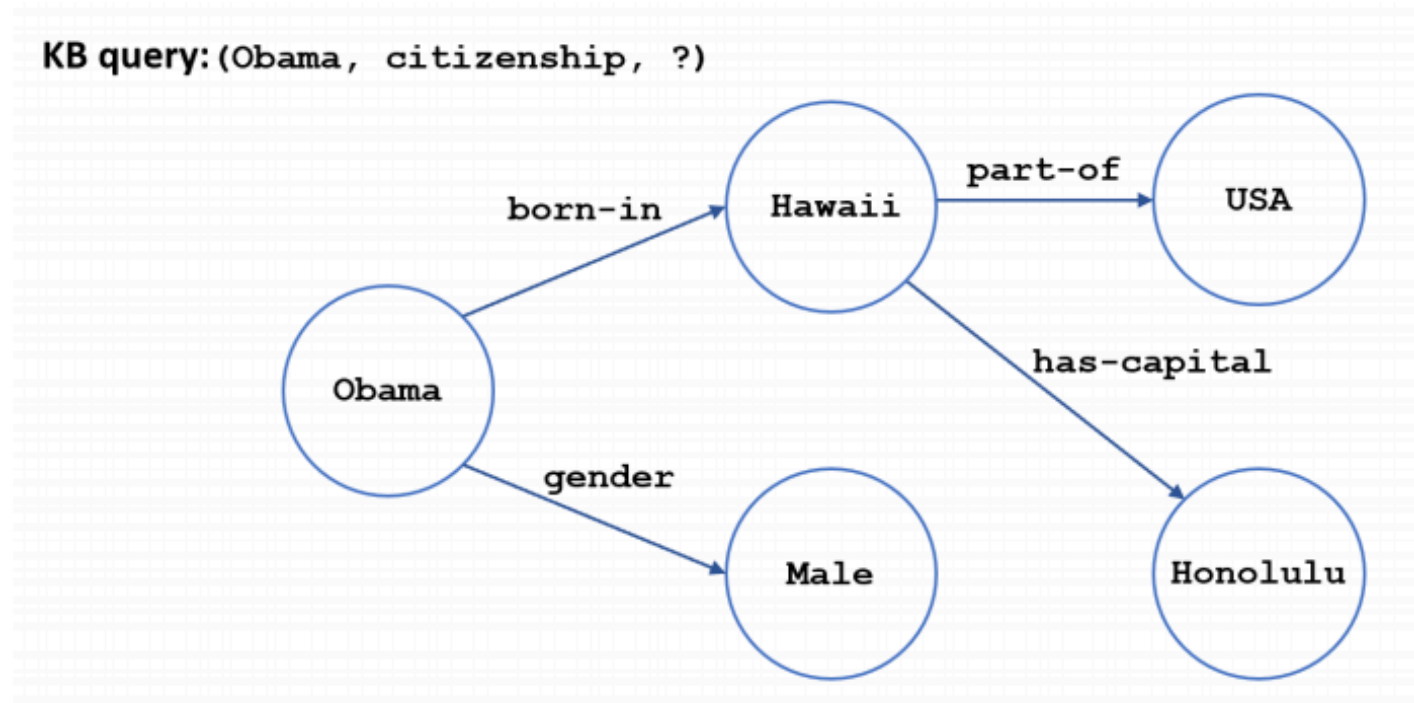
- Order – S,p,d,b,e,a,r,f,e,G

# Trees and Conversational Question Answer

- How to query knowledge bases with natural language?
- Single-Step Reasoning
  - Semantic Parsing
  - Embedding Based
- Multi-Step Reasoning
  - Symbolic Methods
  - Embedding Based
  - Reinforcement Learning

# Multi-Step Reasoning with RL

- What is the citizenship of Obama?



# Multi-Step Reasoning with RL

- DeepPath (Xiong et al., 2017)
- MINERVA (Das et al., 2017)
- M-Walk (Shen et al., 2018)