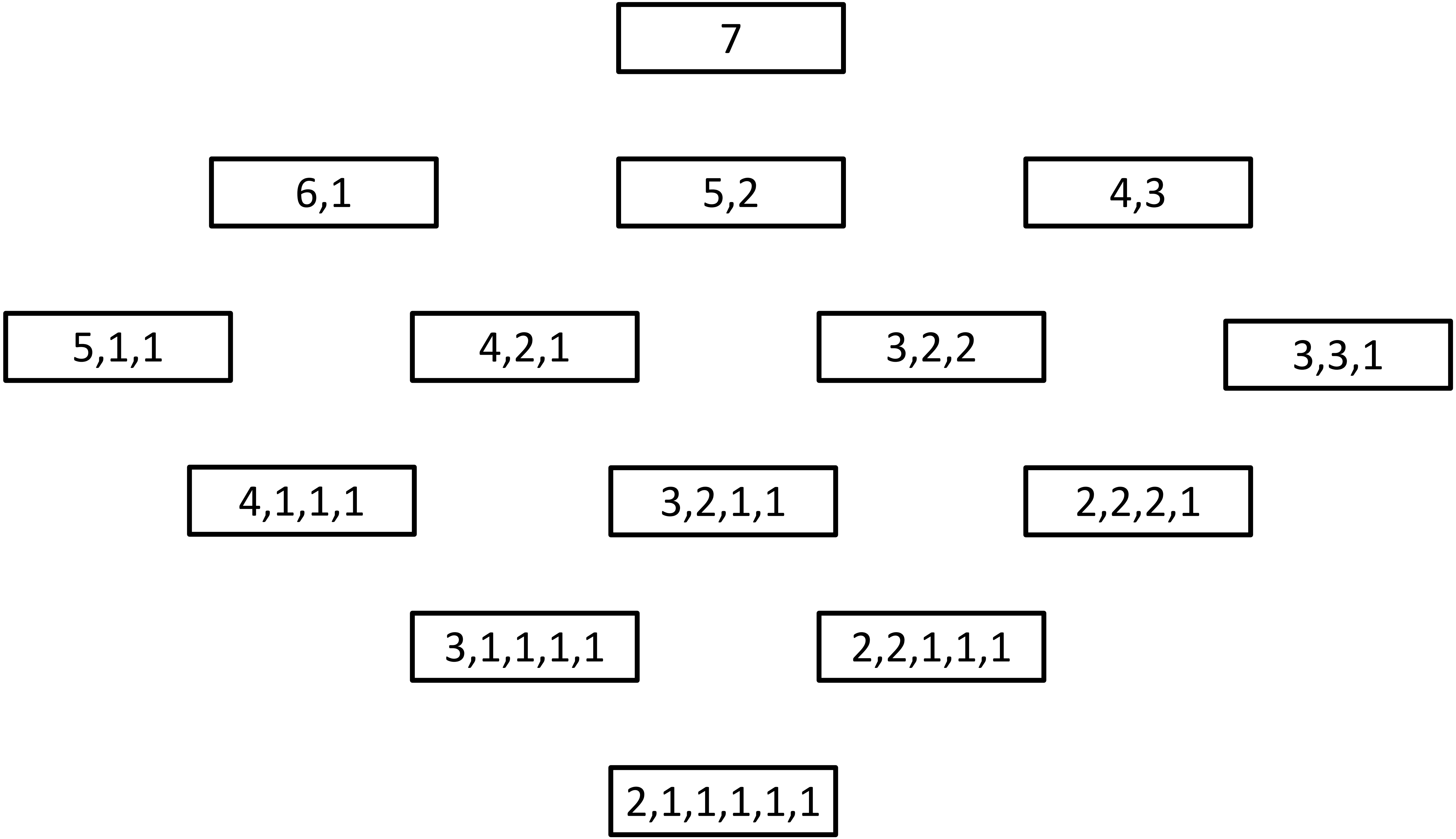
Grundy’s Game of Nim

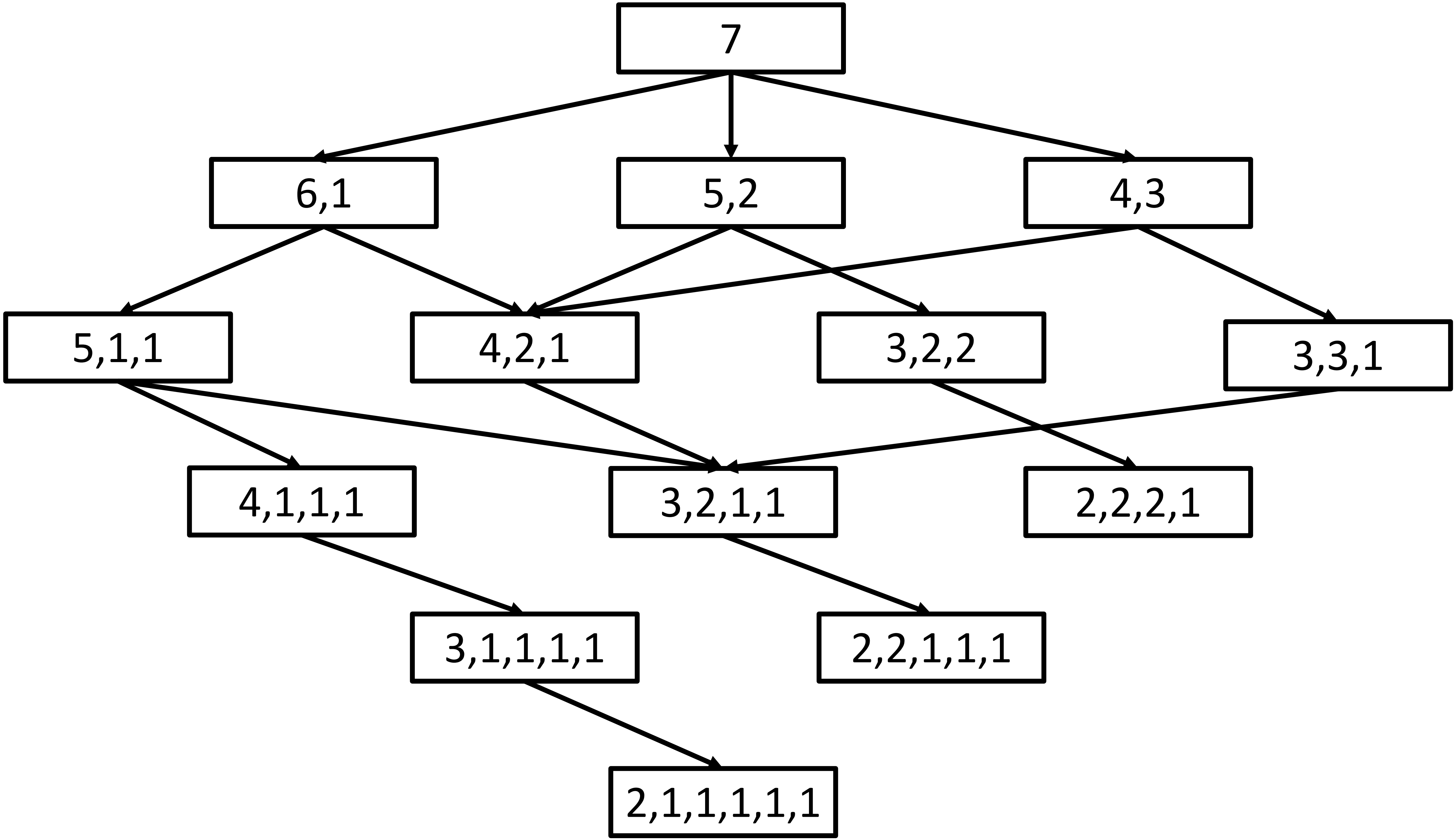
There are two players. The game starts with a single stack of 7 tokens. At each move a player selects one stack and divides it into two non-empty, non-equal stacks. A player who is unable to move loses the game.

Draw the search tree for nim.

Determine the optimal strategy.

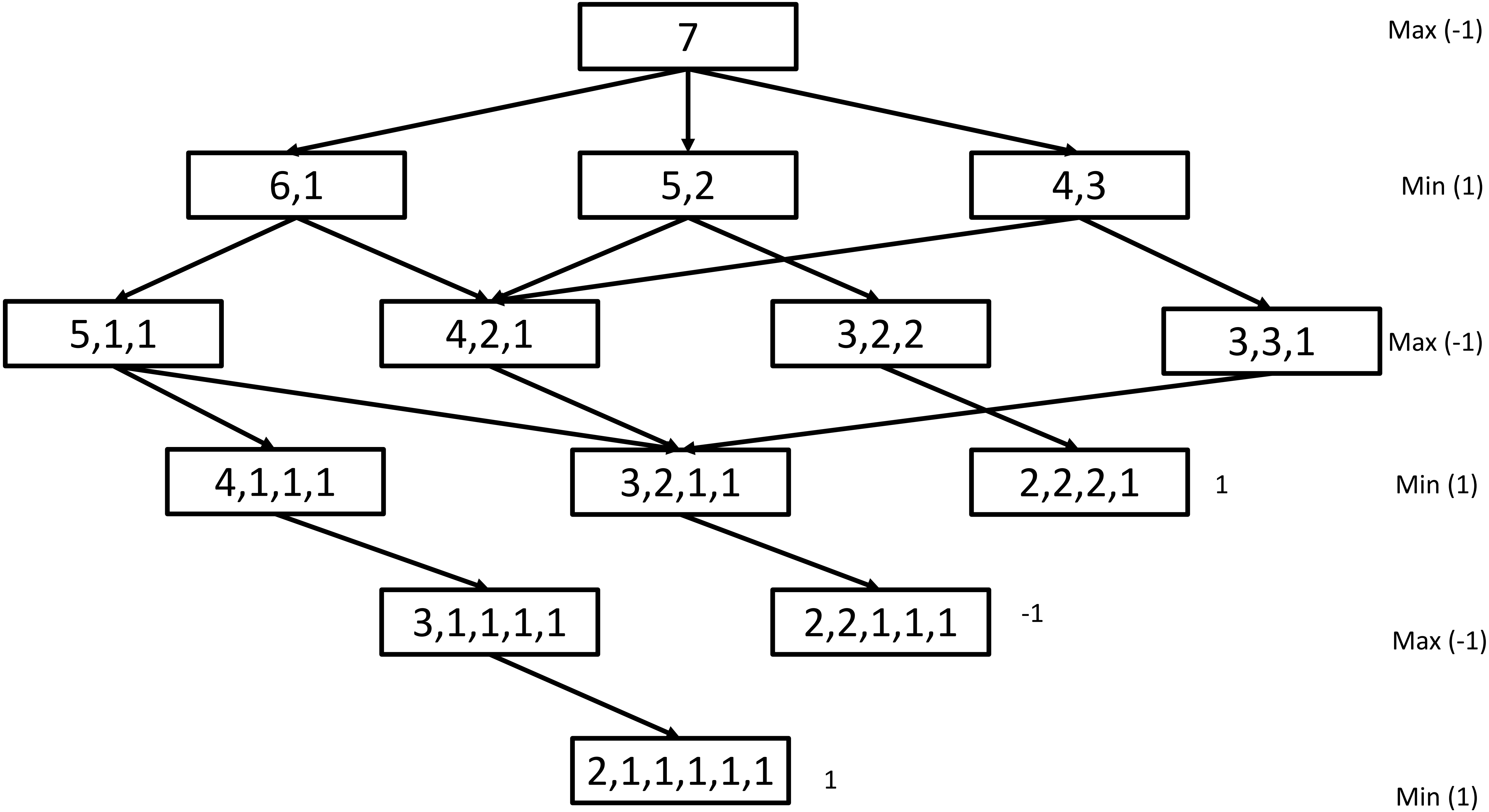
First, draw the game states.


7
6,1
5,2
4,3
5,1,1
4,2,1
3,2,2
3,3,1
4,1,1,1
3,2,1,1
2,2,2,1
3,1,1,1,1
2,2,1,1,1
2,1,1,1,1,1

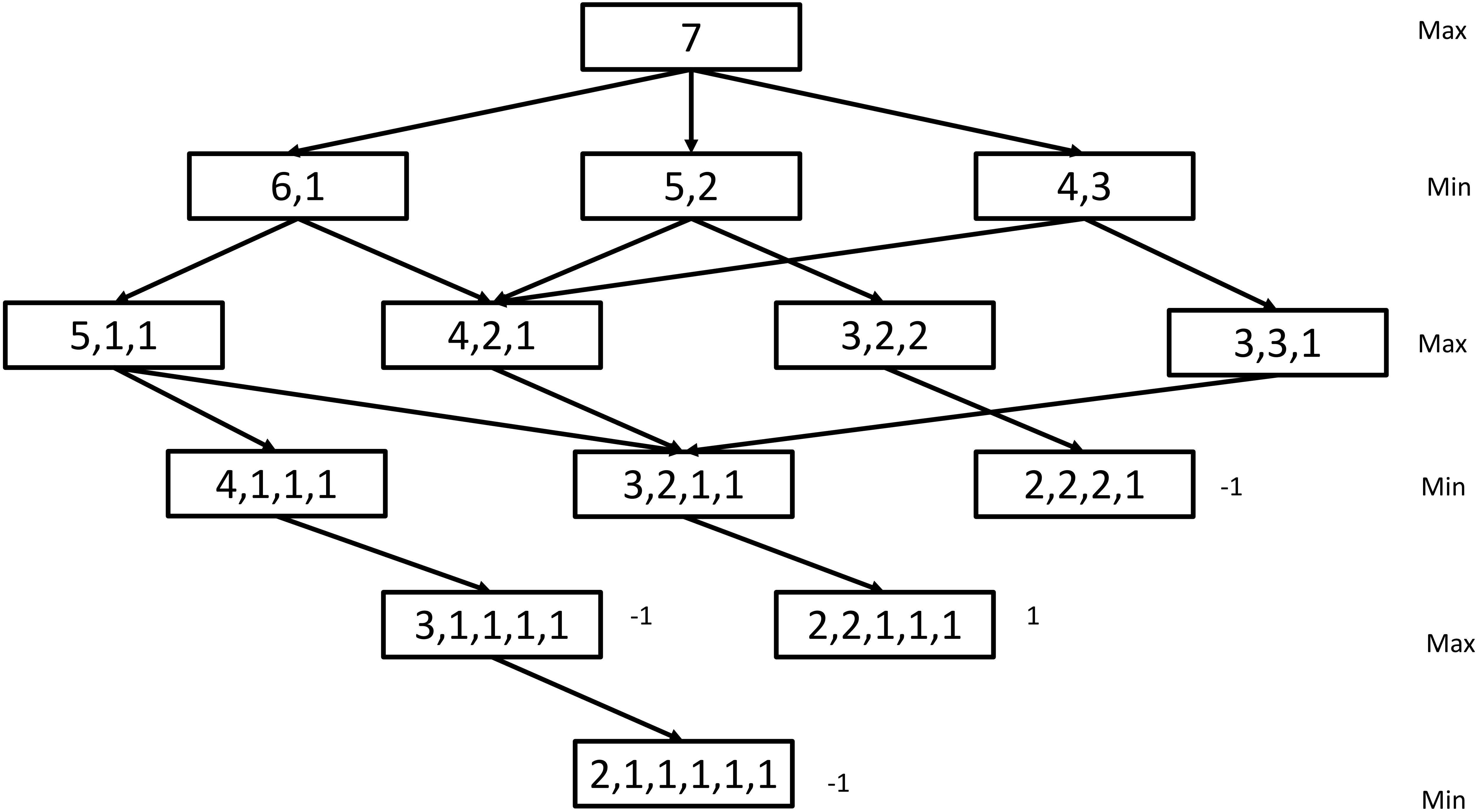

7
6,1
5,2
4,3
5,1,1
4,2,1
3,2,2
3,3,1
4,1,1,1
3,2,1,1
2,2,2,1
3,1,1,1,1
2,2,1,1,1
2,1,1,1,1,1

Add the min max layers

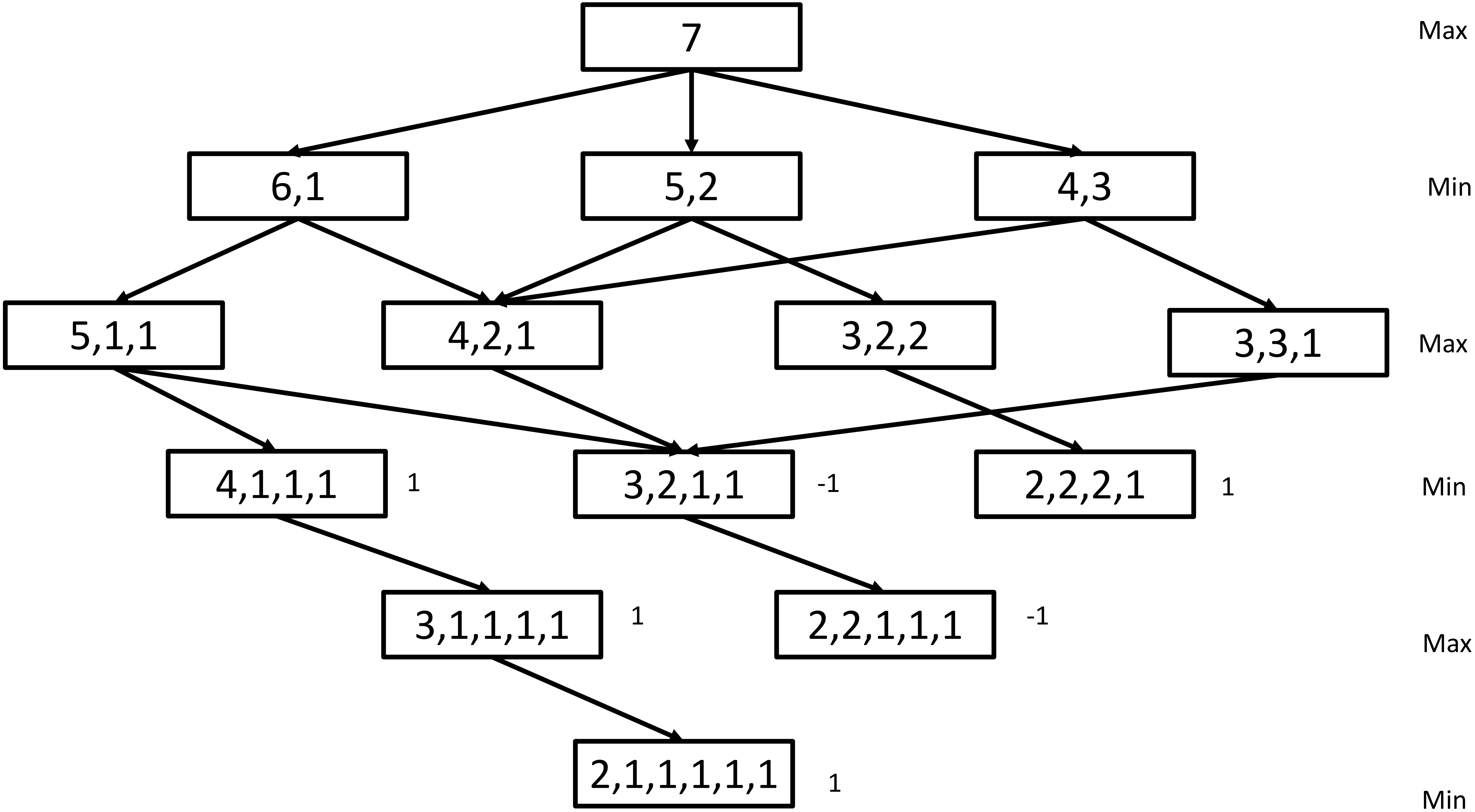
We start with the max layer, and alternate. We assign scores to terminal leaves. To assist, you can determine the score (+1 or -1) that each level would have received if the game ended at that level.


7
Max (-1)
6,1
5,2
4,3
Min (1)
5,1,1
4,2,1
3,2,2
3,3,1
Max (-1)
4,1,1,1
3,2,1,1
2,2,2,1
1
Min (1)
3,1,1,1,1
2,2,1,1,1
-1
Max (-1)
2,1,1,1,1,1
1
Min (1)

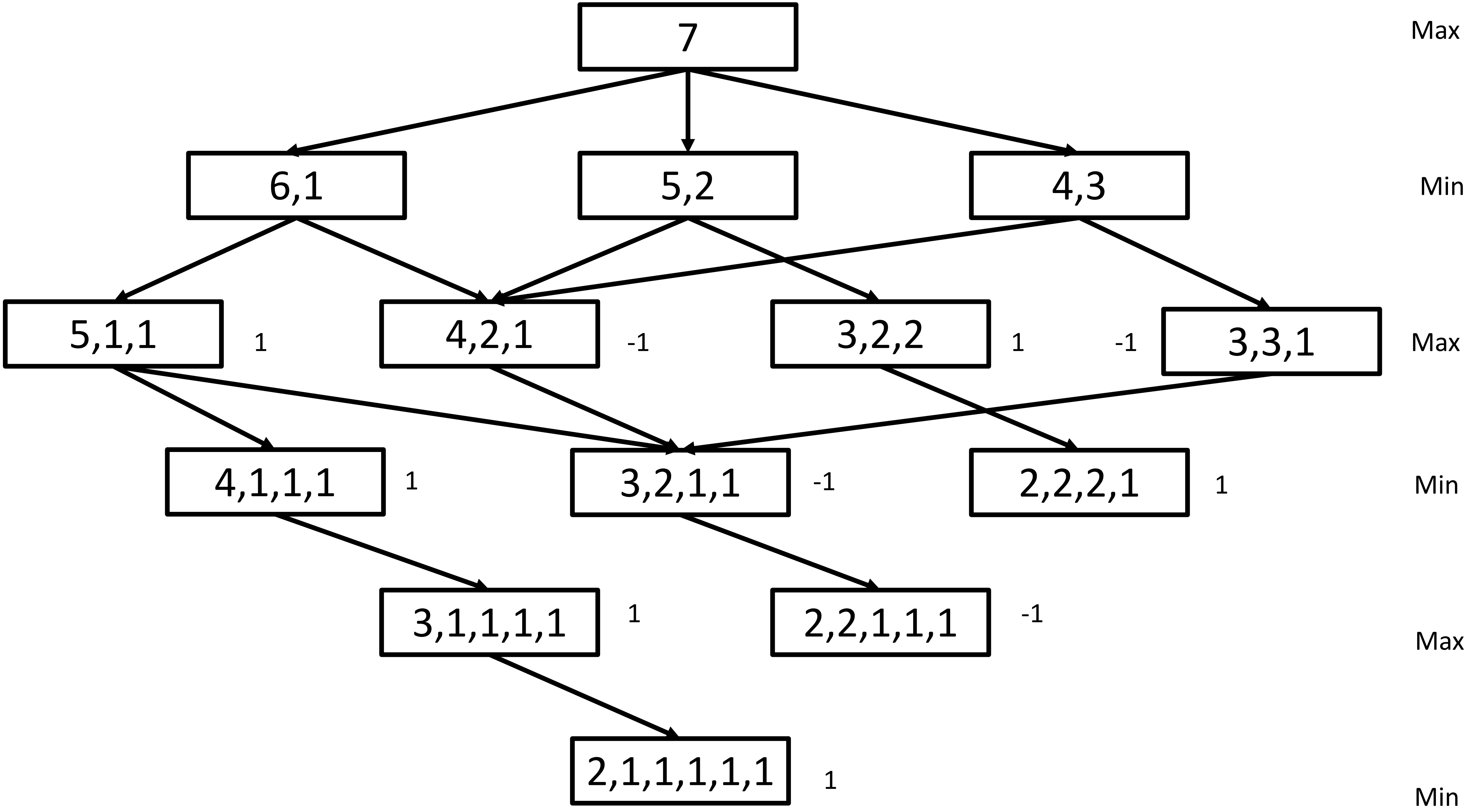
Propagate Scores by Level

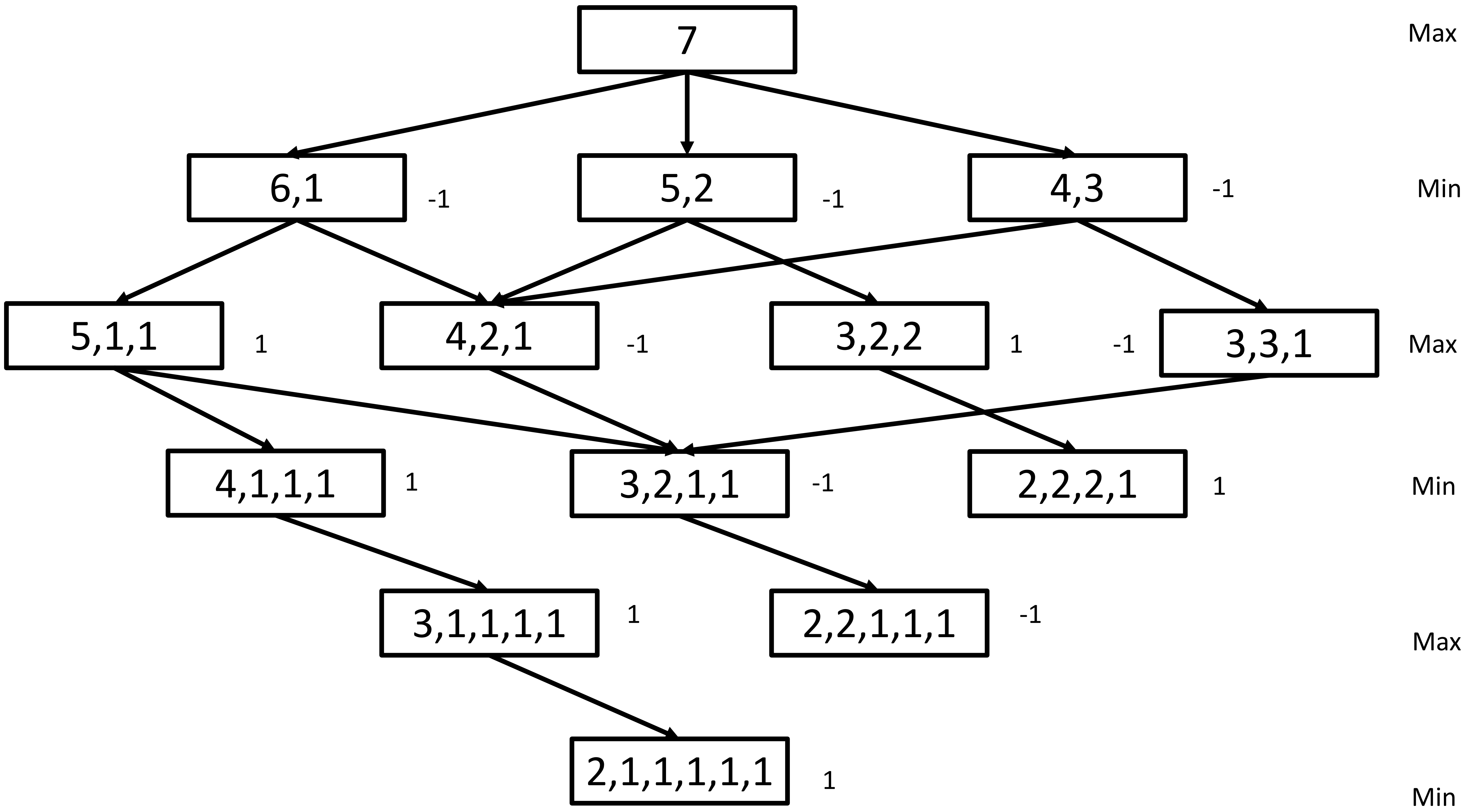


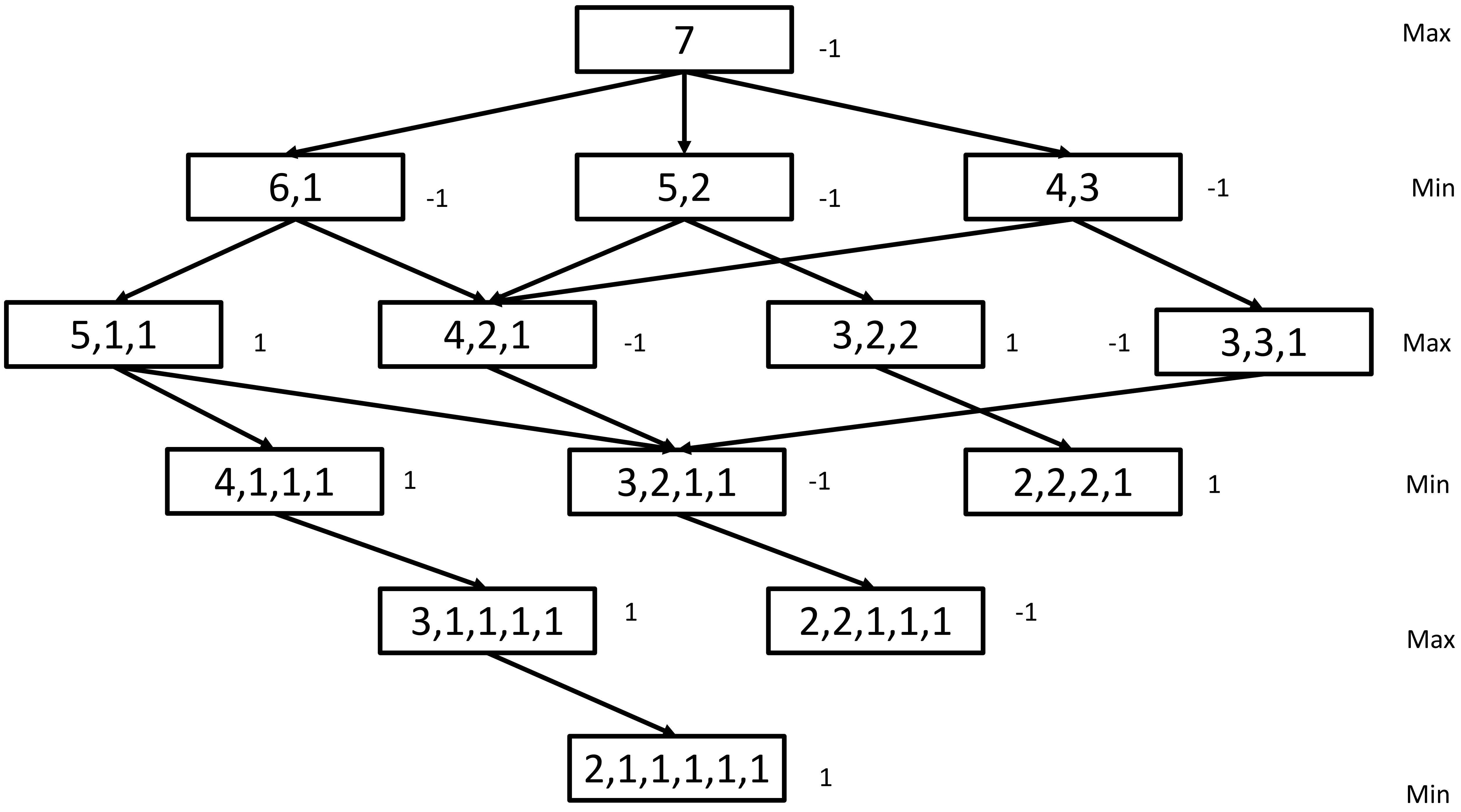
7
6,1
5,2
4,3
5,1,1
4,2,1
3,2,2
3,3,1
4,1,1,1
3,2,1,1
2,2,2,1
-1
3,1,1,1,1
-1
2,2,1,1,1
1
2,1,1,1,1,1
-1
Max
Min
Max
Min
Max
Min



7
Max
6,1
5,2
4,3
Min
5,1,1
4,2,1
3,2,2
3,3,1
Max
4,1,1,1
1
3,2,1,1
-1
2,2,2,1
1
Min
3,1,1,1,1
1
2,2,1,1,1
-1
Max
2,1,1,1,1,1
1
Min


7
Max
6,1
5,2
4,3
Min
5,1,1
1
4,2,1
-1
3,2,2
1
-1
3,3,1
Max
4,1,1,1
1
3,2,1,1
-1
2,2,2,1
1
Min
3,1,1,1,1
1
2,2,1,1,1
-1
Max
2,1,1,1,1,1
1
Min


Max
7
Min
6,1
-1
5,2
-1
4,3
-1
Max
5,1,1
1
4,2,1
-1
3,2,2
1
-1
3,3,1
Min
4,1,1,1
1
3,2,1,1
-1
2,2,2,1
1
Max
3,1,1,1,1
1
2,2,1,1,1
-1
Min
2,1,1,1,1,1
1


7
-1
Max
6,1
-1
5,2
-1
4,3
-1
Min
5,1,1
1
4,2,1
-1
3,2,2
1
-1
3,3,1
Max
4,1,1,1
1
3,2,1,1
-1
2,2,2,1
1
Min
3,1,1,1,1
1
2,2,1,1,1
-1
Max
2,1,1,1,1,1
1
Min

Conclusions

Player 1 will always lose the game of Nim.

For Player 2, an optimal strategy would be to force the game into the 4,2,1 configuration every time on their first turn, since winning is guaranteed from that position.

Bonus Question: Does the outcome of Nim change if you change the number of tokens? Hint: you probably want to code this out.