Lab Report 5 – DETERMINATION OF g BY PENDULUM

XUAN MAI TRAN

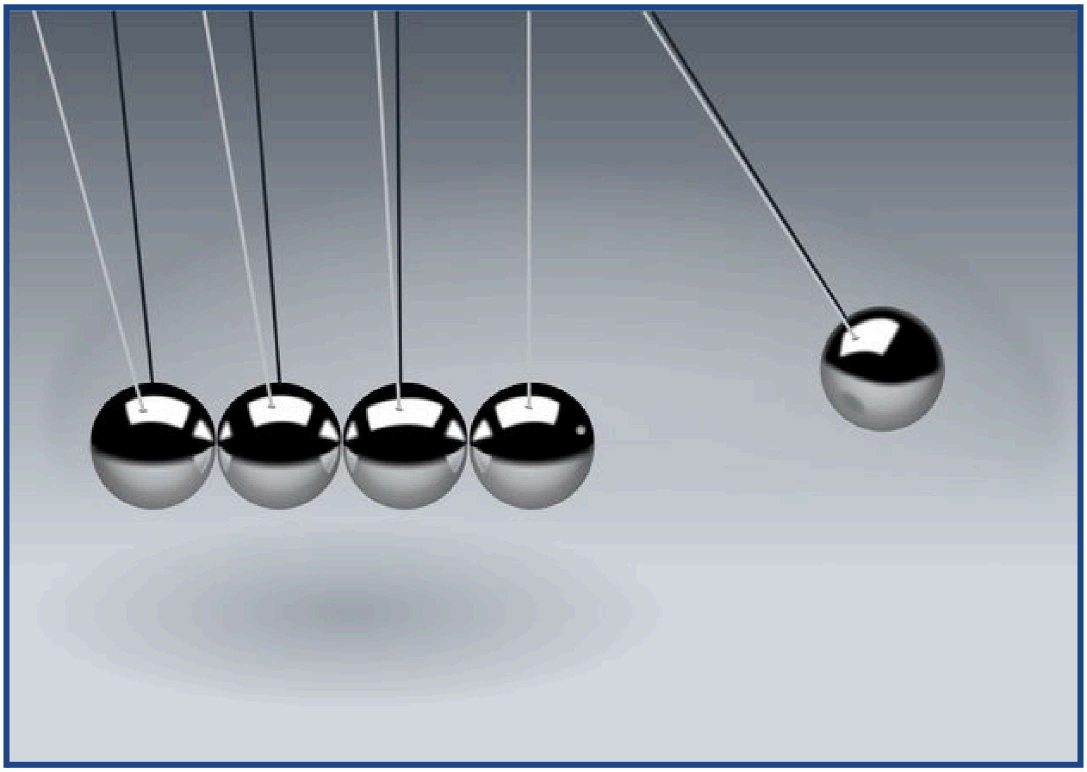
02/13/2025

-

PHYS 2125

-

DR. JOSEPH BARCHAS



Objectives

To measure the period of a simple pendulum and use it to predict local acceleration due to gravity.

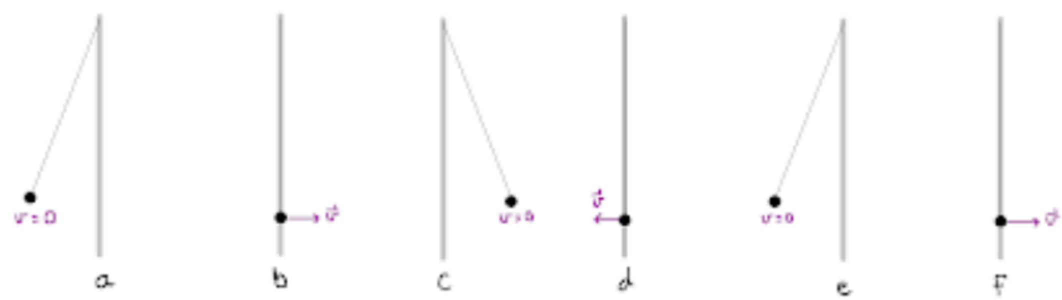
Equipments

The list of equipments used in this experiment is:

- Lab stand
- Pendulum clamp
- String
- Spherical mass
- Meter stick
- Stopwatch

Theory and Equations

1. Pendulum's Oscillation



At the equilibrium state, the mass is at rest and the string hangs vertically as there is a balance between the gravity force and the string force which holds the mass.

When we pull the mass, then release it, so that it can be displaced from the equilibrium. The mass will oscillate back and forth.

T is the time interval between 2 states that share the same position and velocity (for instance, the time interval between state a and e).

If the oscillation is small (<= 10 degrees), we can predict the value of T in theory that depends only on the length of the pendulum and local gravity acceleration as below:

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \text{(Eq. 1)}$$

With L is the distance from the fixed attachment point of the string to the center of the spherical mass.

2. Percentage Difference

After the experiments, we will get 2 different values of g, then we can calculate the average value of g:

$$\text{Percentage Difference} = \left|\frac{g_1 - g_2}{\frac{1}{2}(g_1 + g_2)}\right| * 100\% \qquad \text{(Eq. 2)}$$

3. Percentage Error

The percentage error is calculated based on the experimentally determined measure (exp) and the accepted value (acc). And there are 2 types of accepted values which are theoretical value which is predicted from theory and empirical value which is collected from a reliable source.

$$\text{Percentage Error} = \left|\frac{(exp) - (acc)}{(acc)}\right| * 100\% \qquad \text{(Eq. 3)}$$

Summary of Procedures

First, we set up a stable surface with some measuring tools. We place the foot of the Lab Stand aligned with the outer edge of the table to keep the string's direction in a clear space, so that we can freely adjust the length of the string. Next, we attach the mass to about 2 meters of string, and attach the other end of the string to the pendulum clamp. We use a meter stick to confirm the length (L) of the string (the distance is from the attachment point to the center of the mass).

We gently pull the mass to the side and release it. Then, we use a stopwatch to measure the time interval for 10 periods. We repeat the same process by changing the length of the pendulum with a shorter string of about 5 cm until the value of L is close to 0.500m. This will take about 30 trials.

Data and Observations

L is string length (units: meter) and T is Time (10 periods, units: seconds)

Id	String Length (m)	Time (10 Periods) (s)
1	1.950	28.30
2	1.900	28.30
3	1.850	27.37
4	1.800	26.68
5	1.750	26.36
6	1.700	26.30
7	1.650	25.87
8	1.600	25.26
9	1.550	25.11
10	1.500	24.65
11	1.450	26.95
12	1.400	24.14
13	1.350	23.73
14	1.300	23.08
15	1.250	22.43
16	1.200	22.40
17	1.150	22.20
18	1.100	21.56
19	1.050	20.30
20	1.000	20.57
21	0.950	19.60
22	0.900	19.18
23	0.850	18.75
24	0.800	18.18
25	0.750	17.65
26	0.700	15.45
27	0.650	16.32
28	0.600	15.93
29	0.550	15.14
30	0.500	14.56

Data Analysis

Method 1: Using the Eq. 1 of period formula, we have an example calculation for the id = 1:

$$T/10 = \frac{28.30}{10} = 2.830$$

$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(1.950m)}{(2.830s)^2} = 9.612\, m/s^2$$

Id	String Length (m)	Time (10 Periods) (s)	T/10 (s)	g (m/s^2)
1	1.950	28.30	2.830	9.612
2	1.900	28.30	2.830	9.366
3	1.850	27.37	2.737	9.749
4	1.800	26.68	2.668	9.983
5	1.750	26.36	2.636	9.943
6	1.700	26.30	2.630	9.703
7	1.650	25.87	2.587	9.733
8	1.600	25.26	2.526	9.899
9	1.550	25.11	2.511	9.705
10	1.500	24.65	2.465	9.746
11	1.450	26.95	2.695	7.882
12	1.400	24.14	2.414	9.484
13	1.350	23.73	2.373	9.465
14	1.300	23.08	2.308	9.635
15	1.250	22.43	2.243	9.809
16	1.200	22.40	2.240	9.442
17	1.150	22.20	2.220	9.212
18	1.100	21.56	2.156	9.342
19	1.050	20.30	2.030	10.06
20	1.000	20.57	2.057	9.330
21	0.950	19.60	1.960	9.763
22	0.900	19.18	1.918	9.658
23	0.850	18.75	1.875	9.545
24	0.800	18.18	1.818	9.556
25	0.750	17.65	1.765	9.505
26	0.700	15.45	1.545	11.58
27	0.650	16.32	1.632	9.635
28	0.600	15.93	1.593	9.334
29	0.550	15.14	1.514	9.473
30	0.500	14.56	1.456	9.311

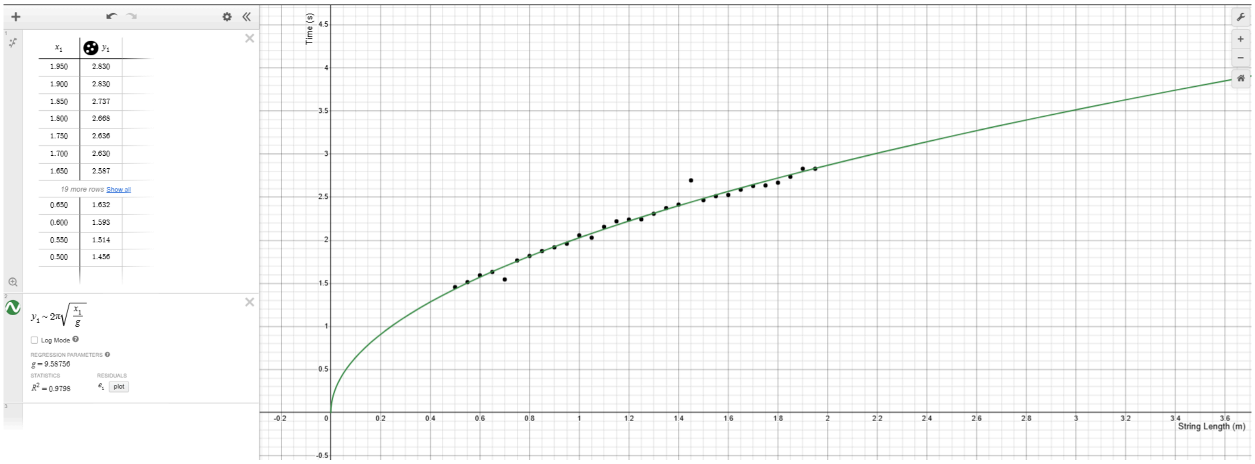
$$g_{average} = \frac{\sum_{1}^{30} g_i}{30} = 9.615$$

Method 2: Using a graph of T versus L

String Length (m)	T/10 (s)
1.950	2.830
1.900	2.830
1.850	2.737
1.800	2.668
1.750	2.636
1.700	2.630
1.650	2.587
1.600	2.526
1.550	2.511
1.500	2.465
1.450	2.695
1.400	2.414
1.350	2.373
1.300	2.308
1.250	2.243
1.200	2.240
1.150	2.220
1.100	2.156
1.050	2.030
1.000	2.057
0.950	1.960
0.900	1.918
0.850	1.875
0.800	1.818
0.750	1.765
0.700	1.545
0.650	1.632
0.600	1.593
0.550	1.514
0.500	1.456

Following to the Eq. 1, we have

$$T \sim 2\pi\sqrt{\frac{L}{g}}$$

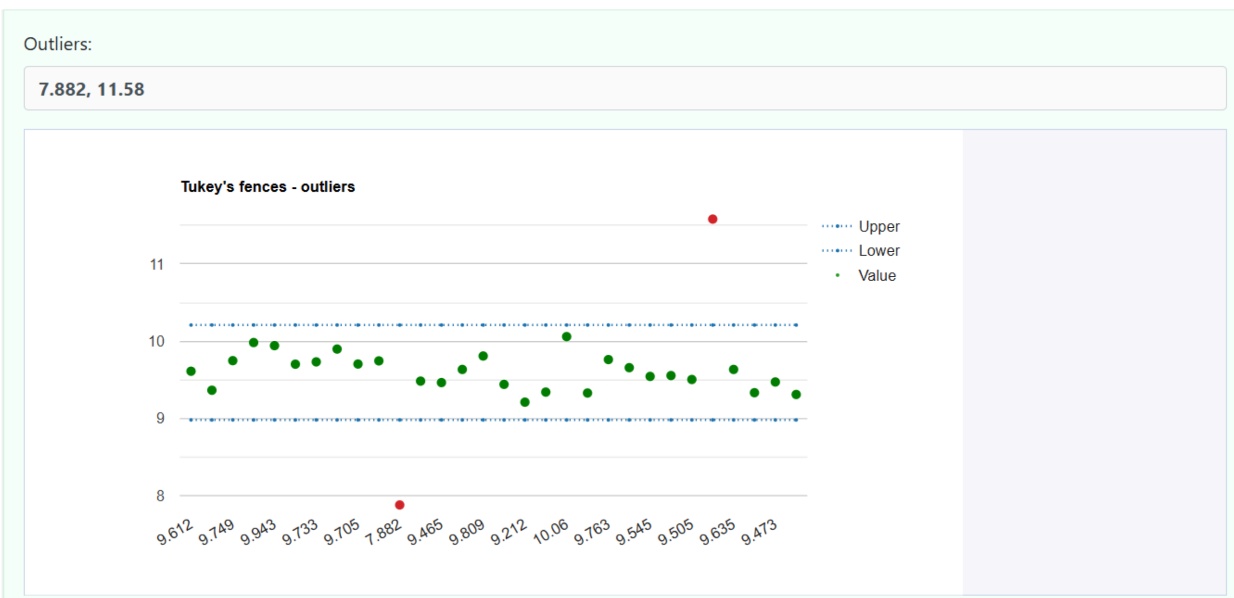


Graph of T versus L

g = 9.58756 m/s^2

R^2= 0.9798

Outlier Analysis



Step-by-step calculation

Method: Tukey's fences

How to calculate Q1 and Q3? go to Q1 and Q3 step by step calculation.

Q1 = **9.442**.

Q3 = **9.749**.

IRQ = Q3 - Q1 = 9.749 - 9.442 = **0.307**.

Lower = Q1 - k * IRQ = 9.442 - 1.5*0.307 = **8.9815**.

Upper = Q3 + k * IRQ = 9.749 + 1.5*0.307 = **10.2095**.

Sample size (n) = **30**.

Outlier count: **2**.

Outliers: 7.882, 11.58.

Source: https://www.statskingdom.com/outlier-calculator.html

Results

Percentage Error

Based on the Eq. 3, we can calculate the difference between the experimental value and the accepted value of g in both methods above.

Accepted value of g (m/s^2)	9.800

- **Method 1**

  Percentage Error $= \left|\frac{(exp)-(acc)}{(acc)}\right| * 100\% = \quad \left|\frac{9.615-9.800}{9.800}\right| * 100\% \ = 1.886\ \%$

- **Method 2**

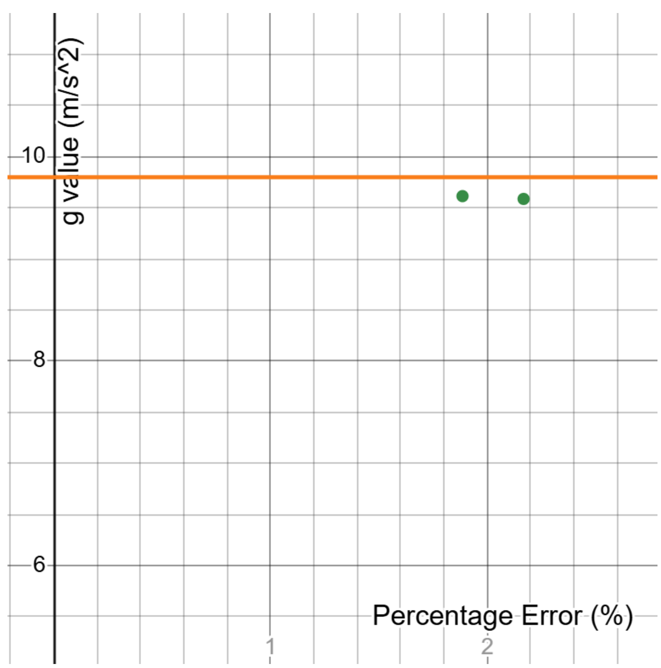
  Percentage Error $= \left|\frac{(exp)-(acc)}{(acc)}\right| * 100\% = \quad \left|\frac{9.58756-9.800}{9.800}\right| * 100\% \ = 2.168\ \%$

Percentage Difference

Based on the Eq. 2, we now want to see the difference between those two (g) values we got from the 2 methods above.

$$\text{Percentage Difference} = \left|\frac{9.615-9.58756}{\frac{1}{2}(9.615+9.58756)}\right| * 100\% \ = 0.2858\ \%$$

	Percentage Error (%)	g (m/s^2)
Method 1	1.886	9.615
Method 2	2.168	9.58756



Comparison between g values of 2 methods

Discussion / Conclusion

Based on the percentage error results, we see that the above two methods used to measure the value (g) are quite accurate with an error of less than 3%. Looking at the data and the graph of "Comparison between g values of 2 methods," we can see obviously that the (g) value in these methods are very close to the accepted value of g (9.800 m/s^2).

Although these 2 (g) values are close to each other with a small percentage difference of 0.2858%, method 1 seems to have a slightly more accurate value (closer to the accepted value of g - the orange horizontal line on the graph) than the (g) value of method 2 (method 1 has (g) value of 9.615 compared to method 2's (g) value of 9.58756).

However, we cannot conclude that method 1 is more accurate than method 2. To explain this confusion, we need to observe the "Graph of T versus L" and the Outlier Analysis section. There are 2 significant outlier (g) values of 7.882 m/s^2 and 11.58 m/s^2. These outliers which might come from human error when using the stopwatch significantly affect the value of (g) average. While method 2's model has a high value of R^2 of 0.9798, it means nearly 98% of data fit to the model or explained well by the model. Hence, we can say that method 2 which uses a function to estimate the (g) value is more accurate than method 1.

In summary, the mean formula affected by outliers will be less accurate in estimating the value of (g), especially when the experiment involves manual measurement methods, while the function method is more objective and accurate in estimating the value of (g). To improve the measurement method in this case, there are many solutions, such as using automatic devices. However, if there is no automatic device, we can use some tools, such as a string or a stick to mark the position of the oscillation state, so that we can better measure the time without relying on eye estimation. Then, we place a mini sponge on the phone screen, right at the "stop" button of the stopwatch. At the last count of the time cycle (the 10th oscillation), we align the phone with the marked string/stick (used to mark the angle of the last oscillation state), the ball will touch the "stop" button on the phone for us. This will reduce the chance of appearing outliers.

Post-Lab Question

$1^{o} = \frac{2\pi}{360} = \frac{\pi}{180} = 0.0175 \text{ rad}$

=> $10^{o} = \frac{2\pi}{360} * 10 = \frac{\pi}{180} * 10 = 0.175 \text{ rad}$

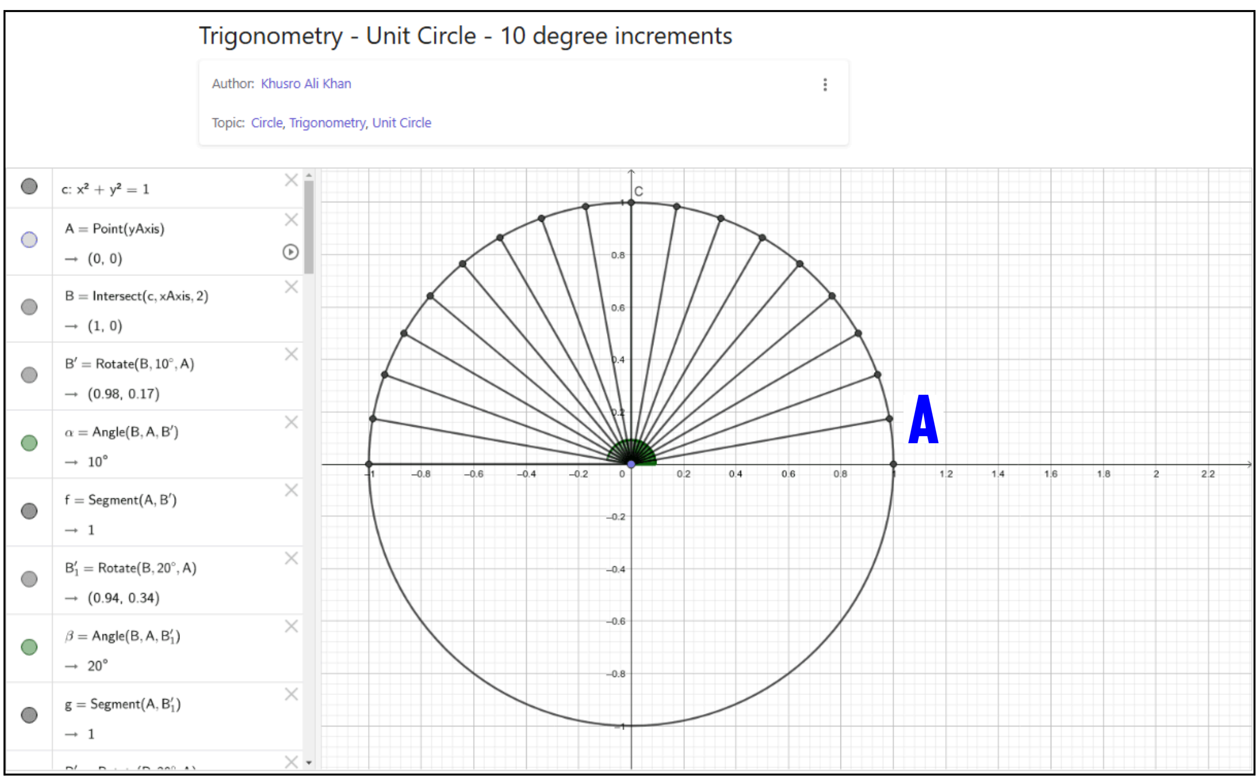
$\sin(10^{o}) = \sin(0.175) = 0.174 \text{ rad}$

To compare the value of 10^{o} and $\sin(10^{o})$, we use the Eq. 2 as below

$$\text{Percentage Difference} = \left| \frac{0.175 - 0.174}{\frac{1}{2}(0.175 + 0.174)} \right| * 100\% = 0.57\ \%$$

We can clearly see that the value of 10^{o} and $\sin(10^{o})$ are approximate to each other with a very small percentage difference of less than 1%.

To explain this, we can use the Trigonometry circle as below.



Trigonometry - Unit Circle - 10 degree increments (Source: geogebra.org)

In the case of the angle of 10^{o}, it lies on the 1st Quadrant, so its positive value is reasonable.

We mark the angle of 10^{o} at point A on the graph above and have:

$$\sin(10^{o}) = \frac{opposite\ length}{hypotenuse\ length} = \frac{y_A}{radius} = y_A \qquad \text{Eq. 4}$$

As the angle of 10^{o} is too small, so we can see that the arc (angle) is approximate to the chord (y_A).

Hence, we have an approximation of Eq. 4 as below:

$$\sin(10^{o}) = y_A = \text{chord} \sim angle \sim 10^{o} \sim 0.175\ rad$$

References

Khan, K. A. (n.d.). *Trigonometry - Unit Circle - 10 degree increments*. GeoGebra. https://www.geogebra.org/m/pw7cqjwn

StatsKingdom. (n.d.). Outlier calculator. StatsKingdom. https://www.statskingdom.com/outlier-calculator.html