Lab Report 4 – PROJECTILE MOTION

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PHYS 2125

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Objectives

To confirm the validity of projectile trajectories.

Equipments

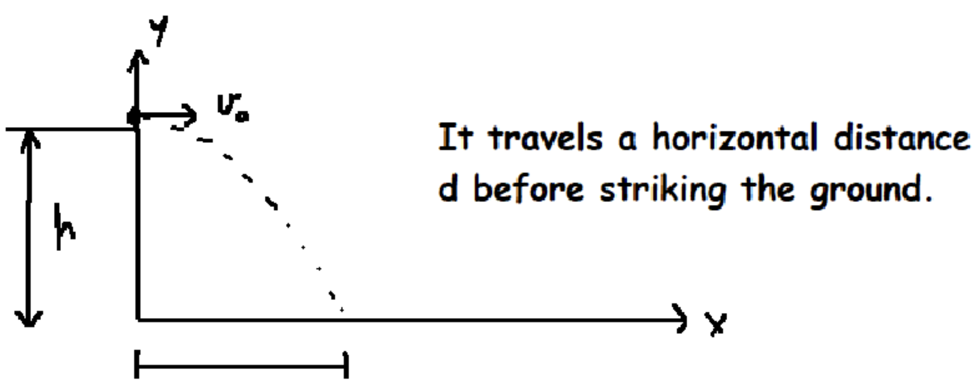
The list of equipments used in this experiment is:

- Projectile Launcher
- Stand
- Computer Interface
- Photogate
- Carbon Paper
- Time-of-Flight Pad
- Meter Stick
- Angle Indicator
- Lab Jack
- Projectile

Theory and Equations

There are 2 situations of projectile motion that we need to concern.

1. Object Launching Horizontally



d

We have the height formula:

$$y = h - \frac{1}{2}gt^2$$

$$0 = h - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}}$$

Also, the distance formula is:

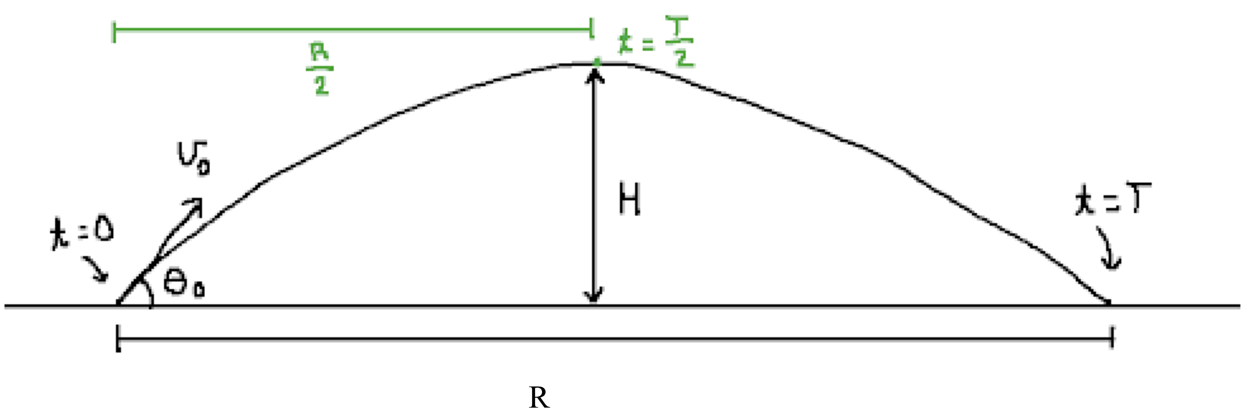
$$x = d = v_0 t$$

$$v_0 = \frac{d}{t}$$

Finally, we have:

$$v_0 = d\sqrt{\frac{g}{2h}} \qquad \text{(Eq. 1)}$$

2. Object Launching and Landing at the Same Vertical Height



We have:

$$R = \frac{v_0^2}{g} sin(2\theta_0) \qquad \text{(Eq. 2)}$$

$$H = \frac{v_0^2}{2g}[sin(\theta_0)]^2$$

$$T = \frac{2v_0}{g} sin(\theta_0) \qquad \text{(Eq. 3)}$$

3. Percentage Error

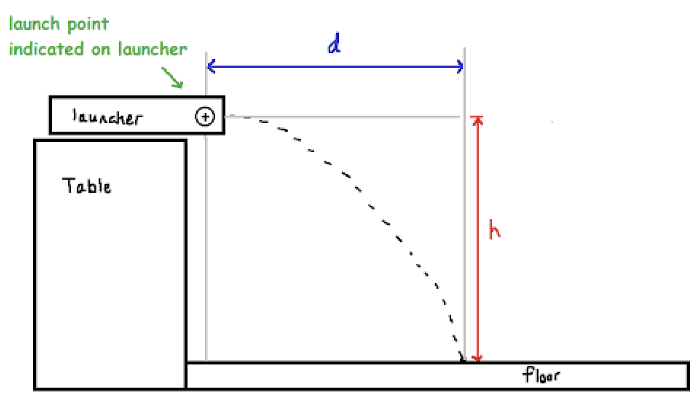
After the experiments, we will get 3 different values of v_0, then we can calculate the average value of v_0:

$$Average_{v_0} = \frac{(Value\ 1) + (Value\ 2) + (Value\ 3)}{3}$$

$$\text{Percentage Difference} = \left| \frac{(Individual\ Value) - (\ Average_{v_0}\ )}{Average_{v_0}} \right| * 100\%$$

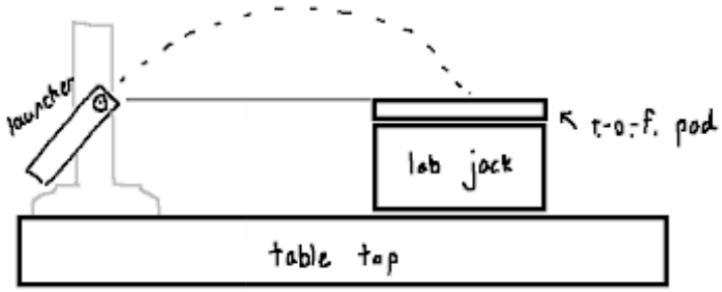
Summary of Procedures

First, we set up a stable surface with some measuring tools. We place the Projectile Launcher with its edge aligned with the outer edge of the table and keep its direction in a clear space, so that the ball can be launched without hitting any obstacles before reaching the ground as shown below.



We equip the launcher with a ball and fire it. Then we note where the bullet falls, where we will put a sheet of paper on the ground and place a sheet of carbon paper on top with the black side down. Without changing the position of the launcher, we repeat the firing 3 more times. The ball will hit the paper, leaving a carbon mark on the paper. We measure h with a meter ruler. To measure d, we remove the carbon and circle the marks with a circle of the smallest possible radius. Then we measure the distance (d) from the center of the circle to the point on the ground directly below the launch point.

For the 2nd part of the experiment, we placed the time-of-flight pad on top of the lab jack, then increased the height of the lab jack to the height of the projectile launcher as shown below.



We tried to shoot the ball to estimate the landing position of the ball and adjusted the position of the lab jack. Then we taped a sheet of paper to the time-of-flight pad and placed a sheet of carbon paper on top with the black side facing down. We could replace the paper as needed during the experiment. Next, we adjusted the launcher so that the angle indicator read 80^o and placed the photogate right at the muzzle of the launcher. We needed to make sure the computer interface was getting data by manual testing. We equipped the launcher with a ball and launched the projectile and repeated this process 2 more times. Then we circled 3 points on the paper and measured the distance (R) from the center of the circle to the launch point.

We repeated the 2nd experiment by changing the launcher according to the list of angles: 80^o, 75^o, 70^o, 65^o, 60^o, 55^o, 50^o, 45^o, 40^o, 35^o, 30^o.

Data and Observations

Part 1

We have height (h) is the distance from the projectile launcher on the table to the floor, and the distance (d) is from the center of the circle to the point on the ground directly below the launch point.

h and d are converted to meter for convenient calculation as shown in the table below.

Height (m)	Distance (m)
1.099	1.505

Part 2

We have 11 different values of the launch angle of the projectile as shown in the table below. At each launch angle level, we shoot the ball 3 times to collect 3 different time data on the computer software representing Time 1, Time 2 and Time 3. Finally, the distance R is from the center of the circle to the launch point.

Degree (θ_o)	Time 1 (s)	Time 2 (s)	Time 3 (s)	Distance (R) (m)
80	0.613	0.613	0.612	0.301
75	0.602	0.602	0.602	0.455
70	0.583	0.583	0.582	0.607
65	0.564	0.563	0.566	0.726
60	0.542	0.539	0.542	0.819
55	0.511	0.513	0.514	0.899
50	0.483	0.485	0.485	0.927
45	0.448	0.450	0.452	0.949
40	0.409	0.409	0.408	0.947
35	0.368	0.368	0.369	0.910
30	0.319	0.318	0.325	0.844

Data Analysis

Part 1

Accepted value of g is 9.800 m/s^2.

$$v_0 = d\sqrt{\frac{g}{2h}} = 1.505\sqrt{\frac{9.800}{2*1.099}} = 3.179\ (m/s)$$

Part 2

Average the 3 time-of-flight: $$T_{average} = \frac{(Time\ 1) + (Time\ 2) + (Time\ 3)}{3}$$

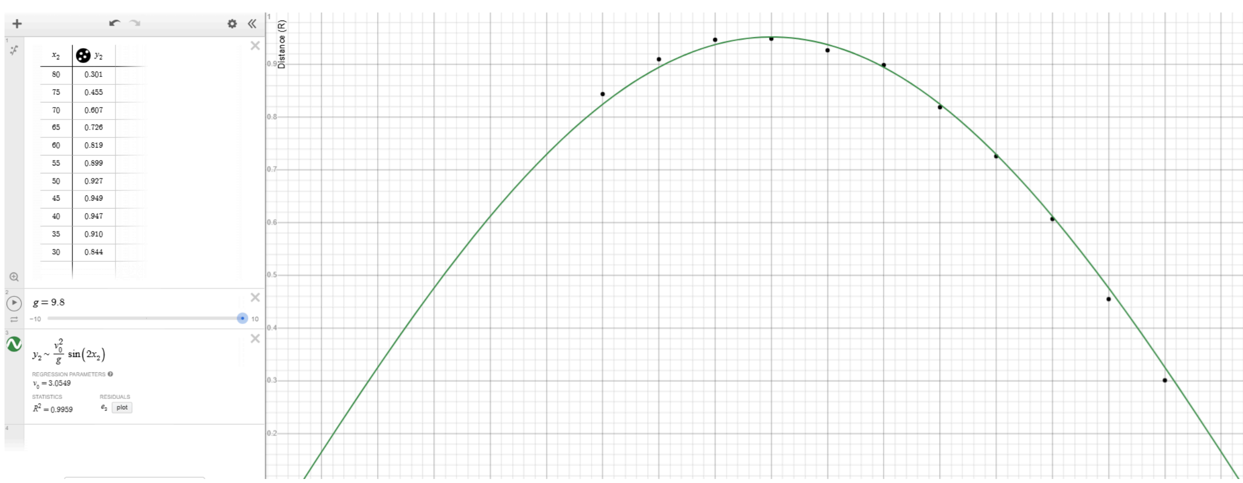
Sample Calculation for Angle 80^o:

$$T_{average} = \frac{0.613 + 0.613 + 0.612}{3} = 0.613$$

Thus, we have the average time-of-flight calculated for each measurement as shown below.

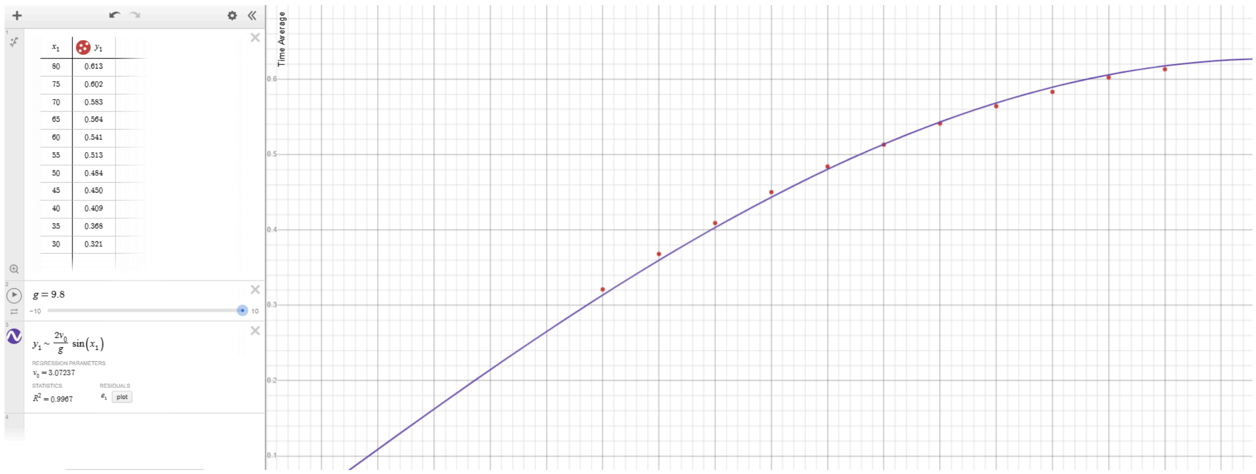
Degree (θ_o)	Time 1 (s)	Time 2 (s)	Time 3 (s)	Time Average	Distance (R) (m)
80	0.613	0.613	0.612	0.613	0.301
75	0.602	0.602	0.602	0.602	0.455
70	0.583	0.583	0.582	0.583	0.607
65	0.564	0.563	0.566	0.564	0.726
60	0.542	0.539	0.542	0.541	0.819
55	0.511	0.513	0.514	0.513	0.899
50	0.483	0.485	0.485	0.484	0.927
45	0.448	0.450	0.452	0.450	0.949
40	0.409	0.409	0.408	0.409	0.947
35	0.368	0.368	0.369	0.368	0.910
30	0.319	0.318	0.325	0.321	0.844

Based on the Eq. 2 above, we produce a graph of R vs. θ_o to find a fit function and estimate the value of v_0.



From the graph above, the value of v_0 **is 3.0549 m/s** with a high $R^2 = 0.9959$.

Then, we use the Eq. 3 to estimate the value of v_0 based on the graph of $T_{average}$ vs. θ_o.



From the graph above, the value of v_0 is **3.07237 m/s** with a high $R^2 = 0.9967$.

Results

Average Value of v_0:

After the calculation in Part 1 experiment and the graphs in Part 2 experiment, we have 3 values of v_0. Now we can calculate their average value.

$$v_{0_{AVERAGE}} = \frac{3.179 + 3.0549 + 3.07237}{3} = 3.102\ m/s$$

Comparison of each individual v_0:

$$\text{Percentage Error 1} = \left|\frac{(v_0\ of\ Part\ 1) - v_{0_{AVERAGE}}}{v_{0_{AVERAGE}}}\right| * 100\% = \left|\frac{(3.179) - 3.102}{3.102}\right| * 100\% = 2.479$$

	v0 - Part 1 (m/s)	v0 - Part 2 - Graph 1 (m/s)	v0 - Part 2 - Graph 2 (m/s)	Average v0 (m/s)
	3.179	3.0549	3.07237	3.102
Percentage Error (%)	2.479	1.521	0.958	

Discussion / Conclusion

Based on the 2 experiments above, we can determine the initial speed of an object at launching time v_0 in different methods. In general, the 3 different values of v_0 are quite close to each other with less than 3% of percentage error. We can also see that v_0 collected from Part 1 has the highest value of 3.179 m/s with the highest percentage error of 2.479 % compared to the Part 2 experiment. The reason for that might come from the longer travel distance of the ball in Part 1 experiment, as according to Eq. 1, a bigger value of (d) will increase the value of v_0. Besides, the Part 2 experiment uses more automatic and sensitive devices and software, such as the time-of-flight pad, photogate, and software interface, to measure more accurate data.
Moreover, as the value of v_0 is quite similar to each other via different experiments and calculation methods, it confirms that the value of R in corresponding to the value of v_0 is a quadratic function shown as a parabola in the graph 1. Particularly, the value of R starts from 0.301 m and increases gradually to reach maximum at 0.949 m when the angle decreases from 80^o to 45^o. Then, the value of R suddenly decreases gradually when the angle continually decreases to 30^o. This change is totally fit to the parabola model and Eq. 2. A high value of $R^2 of 0.9959$ again confirms for the well explanation of the quadratic model with most of the points lying on the graph line.
Furthermore, based on the column R of the data table,we can see that the projectile reaches its maximum value at the 45^o angle of the launcher. It means 45^o is the optimal angle to shoot the projectile the farthest. Following to the Eq. 2, we can also see that the distance R is maximized when $\theta = 45^o$ as:

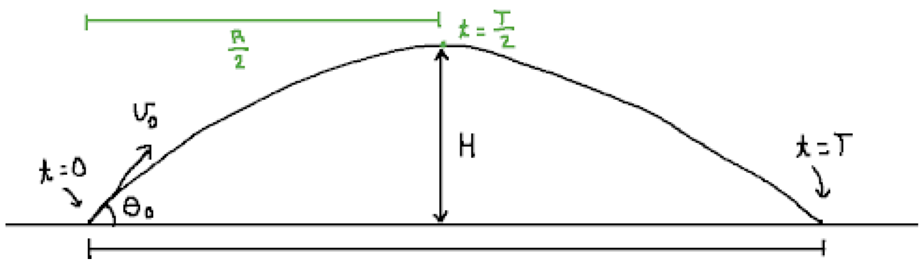
$$R = \frac{v_0^2}{g} sin(2\theta_0)$$

And,

$$\sin(2*45^o) = \sin(90^o) = 1$$

To re-examine the impact of (h) and (d) in Part 1 experiment on the value of v_0, we can experiment with different values of those pairs in a future experiment. Alternatively, we can also use automated measurement tools in Part 2 experiment to test the impact of (h) and (d) to increase the precision of Part 1 experiment.

Post-Lab Question



Given kinematic equations:

$$X = X_0 + (V_0 \cos\theta_0)t$$

$$Y = Y_0 + (V_0 \sin\theta_0)t - \frac{1}{2}gt^2$$

$$V_x = V_0 \cos\theta_0$$

$$V_y = V_0 \sin\theta_0 - gt$$

When the ball reaches the floor, we have Y = Y_0 = 0. Then,

$$Y = (V_0 \sin\theta_0)t - \frac{1}{2}gt^2 = 0$$

$$t(V_0 \sin\theta_0 - \frac{1}{2}gt) = 0$$

There are 2 cases of t:

$$t = 0 \quad \text{or} \quad t = \frac{2V_0 \sin\theta_0}{g}$$

As t > 0 to be meaningful, we get the following formula of Eq. 3 as below.

$$T = \frac{2V_0 \sin\theta_0}{g}$$

When the ball reaches the floor, its distance is the value of R. At that point, we have:

$$R = X \text{ at } t = T$$

From the kinematic equation, we have:

$$X = V_0 \cos\theta_0 \cdot t$$

From the Eq. 3 derived above, we have:

$$t = T = \frac{2V_0 \sin\theta_0}{g}$$

Thus:

$$R = X = V_0 \cos\theta_0 \cdot t$$

$$R = V_0 \cos\theta_0 \cdot \frac{2V_0 \sin\theta_0}{g}$$

$$R = \frac{V_0^2}{g} \cdot (2 \sin\theta_0 \cos\theta_0)$$

As we have:

$$2 \sin\theta_0 \cos\theta_0 = \sin(2\theta_0)$$

Thus we rewrite R and get the Eq. 2 as below:

$$R = \frac{V_0^2}{g} \sin(2\theta_0)$$