Es Riposso - * 2

1) Calcola n° anagrammi delle parole:

- RICORSO: $\frac{7!}{2! \cdot 2!}$
- CARPACCIO: $\frac{9!}{2! \cdot 3!}$
- BARATTO: $\frac{7!}{2! \cdot 2!}$
- ISTANZA: $\frac{7!}{2!}$

2)

X\Y	3	4
1	1/4	1/5
2	1/2	0
3	0	1/20

$p_Y \, (3) = \frac{3}{4} \quad (4) = \frac{5}{20} = \frac{1}{4}$

$p_X \, (1) = \frac{9}{20}$, $(2) = \frac{1}{2}$, $(3) = \frac{1}{20}$

a) p marginali: vedi margini

$\Sigma = \frac{10}{20} + \frac{1}{2} = \frac{20}{20} = 1$ ✓

$\Sigma = \frac{3}{4} + \frac{1}{4} = 1$ ✓

b) $E[X]$? $E[Y]$?

$E[X] = 1 \cdot \frac{9}{20} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{20} = \frac{32}{20} = \frac{8}{5} \simeq 1{,}6$

$E[Y] = 3 \cdot \frac{3}{4} + 4 \cdot \frac{1}{4} = \frac{13}{4} \simeq 3{,}25 \quad (3 < \cdot < 4$ ✓$)$

c) $E[XY^2]$? $\quad g(X,Y) = X \cdot Y^2$

$E[g(X,Y)] = \sum_i \sum_j g(x_i, y_j) \cdot p(x_i, y_j)$

$= 1 \cdot 3^2 \cdot \frac{1}{4} + 1 \cdot 4^2 \cdot \frac{1}{5} + 2 \cdot 9 \cdot \frac{1}{2} + 3 \cdot 16 \cdot \frac{1}{20} = 16{,}85$

d) $Cov(X,Y) = E[XY] - E[X]E[Y]$

$E[XY] = 1 \cdot 3 \cdot \frac{1}{4} + 1 \cdot 4 \cdot \frac{1}{5} + 2 \cdot 3 \cdot \frac{1}{2} + 3 \cdot 4 \cdot \frac{1}{20} = 5,15$

$Cov(X,Y) = 5,15 - (1,6 \cdot 3,25) = 0,05$ senza approssimazioni

e) Poiché $Cov(X,Y) \neq 0$, X e Y sono dipendenti

3) Dalle 18.00 t ha distr exp $\lambda = 6,9$

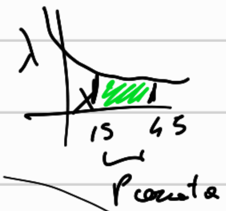
a) $P(18.15 - 18.30) = \int_{1/4}^{1/2} 6,9\, e^{-6,9x} = \left. \frac{\cancel{6,9}\, e^{-6,9x}}{-\cancel{6,9}} \right|_{1/4}^{1/2}$

$= -e^{-6,9 \cdot 0,5} + e^{-6,9 \cdot 0,25} = 0,146 = 14,6\%$

b) $P(<19.00) = -e^{-6,9x}\Big|_0^1 = -e^{-6,9} + 1 = 99,9\%$

c) $P(<18.45)$ sapendo che non è arrivato < 18.15 $\approx P(\leq \frac{3}{4} \mid > \frac{1}{4})$

$= P(<18.45) - \underbrace{P(<18.15)}_{\text{Tolgo l'area}}$



$= -e^{-6,9x}\Big|_0^{3/4} + e^{-6,9x}\Big|_0^{1/4}$

$e^{-6,9x}\Big|_0^{1/2} = -e^{-6,9 \cdot \frac{1}{2}} + 1 \approx 0,80$

$= -e^{-6,9 \cdot \frac{3}{4}} + \cancel{1} + e^{-6,9 \cdot \frac{1}{4}} - \cancel{1} \approx 0,173 = 17,3\%$

(oppure Bayes oppure:)
equivalente calcolare $P(x < \frac{3}{4} \mid x > \frac{1}{4}) = 1 - P(x > \frac{3}{4} \mid x > \frac{1}{4})$

$= 1 - P(x > \frac{1}{4} + \frac{1}{2} \mid x > \frac{1}{4}) = 1 - P(x > \frac{1}{2}) = P(x \leq \frac{1}{2})$ numericamente (non considera $<\frac{1}{4}$ come richiesto)

sicuramente $> 1/4$

4) 85 lanci, 15 teste Calcola stima max veros. di p testa

$X = \{T, C\}$ $p = p(\text{testa})$ $1-p = p(\text{croce})$

$L(T, X) = \prod f_X(X)$ 15 teste e 70 croce
$= p^{15}(1-p)^{70}$

$\ln L = 15 \ln p + 70 \ln(1-p)$

pongo a zero

$\frac{dL}{dx} = \frac{15}{p} \underset{\substack{\text{der} \\ 1-p}}{-} \frac{70}{1-p} = \frac{15 - 15p - 70p}{p(1-p)} = 0$

$\rightarrow 85p = 15 \rightarrow p = \frac{15}{85}$

5) 20 7d dadi a) $P(1 \sigma 2)$?

$d \begin{cases} P(1\ 2\ 3) = 1/9 \\ P(4\ 5\ 6) = 2/9 \end{cases}$

$P(1) = P(1|\sigma) \cdot P(\sigma) + P(1|d) \cdot P(d)$
$= \frac{1}{6} \cdot \frac{2}{9} + \frac{1}{9} \cdot \frac{7}{9}$
$= \frac{2}{36} + \frac{7}{81} = 0{,}14$

$P(2) = P(1) = 0{,}14$ $P(1 \cup 2) = P(1) + P(2) = 0{,}28$

b) Supposto di aver fatto $1 \sigma 2$, P onesto?

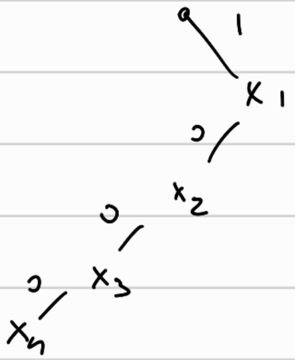
$$P(\sigma | 1\sigma 2) = \frac{P(1 \cup 2 | \sigma) \cdot P(\sigma)}{P(1 \cup 2 | \sigma) \cdot P(\sigma) + P(1 \cup 2 | d) \cdot P(d)}$$

$$= \frac{(P(1|\sigma) + P(2|\sigma)) \cdot P(\sigma)}{(P(1|\sigma) + P(2|\sigma))P(\sigma) + (P(1|d) + P(2|d))P(d)}$$

$$= \frac{\left(\frac{1}{6}\cdot\frac{1}{6}\right)\frac{2}{9}}{\left(\frac{1}{6}\cdot\frac{1}{6}\right)\frac{2}{9} + \left(\frac{1}{9}\cdot\frac{1}{9}\right)\frac{7}{9}} = 0{,}3$$

6) $X = \{x_1, x_2\ x_3\ x_4\}$

a) es. di cod bin univ. ma no ist



provando le stringhe e' u.d.

non e' istantanea perche' ho simb. come nodi intermedi

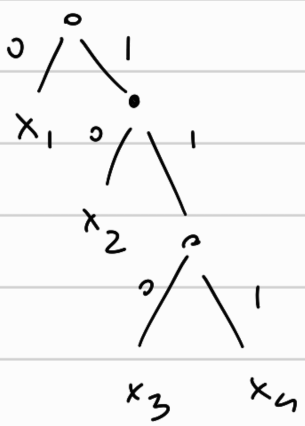
x_1	1
x_2	10
x_3	100
x_4	1000

$\sum 2^{-l_i} = 1\,?$ $\quad 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4}$

$= 0{,}9375$ soddisfa Kraft - McMillan

(condizione non sufficiente però)

b) ist e u.d.



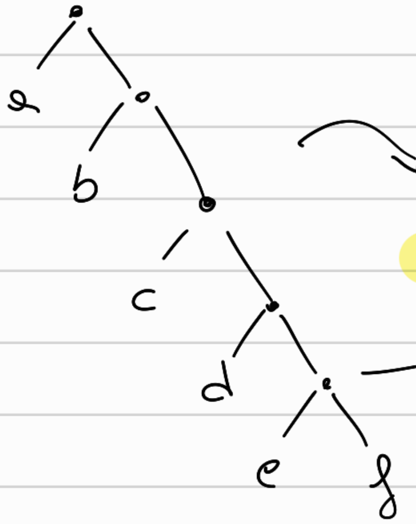
ist ✓ $\longrightarrow$ u.d.

cod. di Huffman

7) Calcola cod Huffman per $X = \{a, b, c, d, e, f\}$
$p(a) = \frac{1}{2}$, $p(b) = \frac{3}{16}$, $p(c) = p(d) = p(e) = \frac{1}{12}$, $p(f) = \frac{1}{16}$

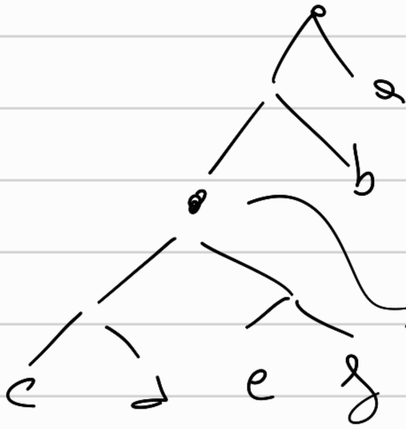
già parte in ord. decr.

dove i nodi fittizi hanno $p = \sum p$ figli



non va bene: !

$\rightarrow p > p(c)!$



$\rightarrow \sum p > p(b)$

non è un problema, li pongo allo stesso livello

posso osservare se $H \approx L_{media}$

8) μ noto, σ campioni indip. $D = \{x_1, \dots, x_n\}$
Det stimatore di σ di max verosim.

$$f_x(X) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$L(\sigma | X) = \prod_{i=1}^{n} f_x = \frac{1}{(\sqrt{2\pi}\,\sigma)^n} \cdot e^{-\frac{\sum (x-\mu)^2}{2\sigma^2}} \rightarrow \text{somma esp.}$$

$$\ln L(\sigma | X) = n \ln\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right) + \ln e^{-\frac{\sum (x-\mu)^2}{2\sigma^2}}$$

$$= n \ln\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right) - \frac{\sum (x-\mu)^2}{2\sigma^2}$$

$$= n \ln 1 - n \ln(\sqrt{2\pi}\,\sigma) - \text{''}$$

$$= 0 - \text{''} - \text{''}$$

$$\frac{dL}{d\delta} = \frac{\cancel{\delta^3} - n\cancel{\sqrt{2\pi}}}{\cancel{\sqrt{2\pi}}\,\delta} + \frac{\cancel{\delta^3}\,\cancel{2}\sum (x-\mu)^2}{\cancel{2}\,\cancel{\delta^3}}$$ → pensandomi la sommatoria come un numero noto (ho x_i e μ noti)

$$= 0 \cdot \cancel{\delta^2}$$

$$\rightarrow \quad \hat{\delta}^2 = \frac{1}{n}\left(\sum_i^n (x_i - \mu)\right)^2$$

è uno stimatore corretto, perchè δ^2 nella Gaussiana corrisponde alla varianza

Combinatoria, E[Y], E[spaziale]
P, P cond, Bayes,
C congiunte
H
codifiche
inf. freq. (max veros.)
inf. Bayesiana (ho max a posteriori, ma vuoi: priori = posteriori)
montecarlo

Teoria veza (cod ocitm)