Correzione esercizi 2B e 5 del Tutorato 2

$$\begin{cases} \dfrac{x+4}{3} - \dfrac{x^2-2}{2} \geq x-3 \\ (x+\sqrt{5})^2 - 2(x-\sqrt{5}) \leq 2(x^2+5) \end{cases}$$

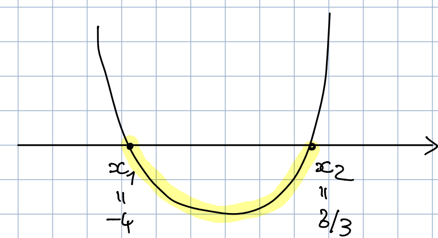
1. $2(x+4) - 3(x^2-2) \geq 6(x-3)$

$\iff 2x+8-3x^2+6-6x+18 \geq 0$

$\iff -3x^2-4x+32 \geq 0$

$\iff 3x^2+4x-32 \leq 0$

$$x_{1,2} = \frac{-2 \pm \sqrt{4+96}}{3} = \begin{cases} -4 = x_1 \\ \dfrac{8}{3} = x_2 \end{cases}$$



$\iff x \in [-4, 8/3]$

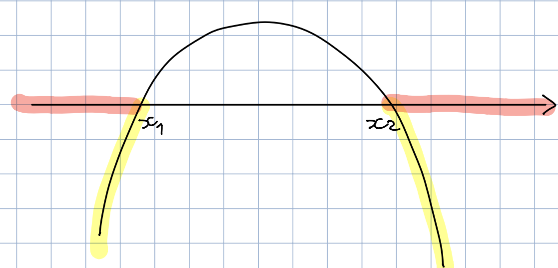
2. $(x+\sqrt{5})^2 - 2(x-\sqrt{5}) \leq 2(x^2+5)$

$\iff x^2+2\sqrt{5}x+5-2x+2\sqrt{5} \leq 2x^2+10$

$\iff -x^2+(2\sqrt{5}-2)x+2\sqrt{5}-5 \leq 0 \quad \iff \ldots$

$$x_{1,2} = \frac{(1-\sqrt{5}) \pm \sqrt{(1-\sqrt{5})^2+(2\sqrt{5}-5)}}{-1} = \frac{(1-\sqrt{5}) \pm \sqrt{1-2\sqrt{5}+5+2\sqrt{5}-5}}{-1}$$

$$= \frac{1-\sqrt{5} \pm 1}{-1} = \sqrt{5}-1 \pm 1 = \begin{cases} \sqrt{5}-2 = x_1 \\ \sqrt{5} = x_2 \end{cases}$$

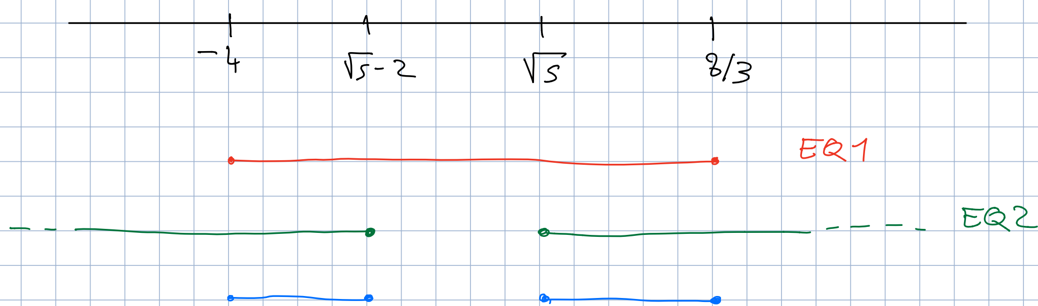


$\ldots \iff x \in (-\infty, \sqrt{5}-2] \cup [\sqrt{5}, +\infty)$

Quindi

$\{ \dots \quad \Longleftrightarrow \quad x \in \left[-4, \frac{8}{3}\right] \cap \left((-\infty, \sqrt{5}-2] \cup [\sqrt{5}, +\infty) \right)$

$\Longleftrightarrow \quad x \in [-4, \sqrt{5}-2] \cup \left[\sqrt{5}, \frac{8}{3}\right]$



$\frac{8}{3} \overset{?}{<} \sqrt{5} \quad \Longleftrightarrow \quad \frac{64}{9} < 5 \quad \Longleftrightarrow \quad 64 < 45 \quad$ no

5. Consideriamo la funzione $f(x) = \sqrt{x^2 - 4} - 1$.

(a) Determinare il dominio A e l'immagine B di f.

(b) Stabilire se è iniettiva, surgettiva e/o bigettiva.

(c) Determinare un intervallo $I \subseteq A$ tale che la funzione f ristretta ad I, che denotiamo con $f|_I$, sia invertibile, e calcolare $(f|_I)^{-1}$.

a) • dom f

$f(x) = \sqrt{x^2 - 4} - 1$

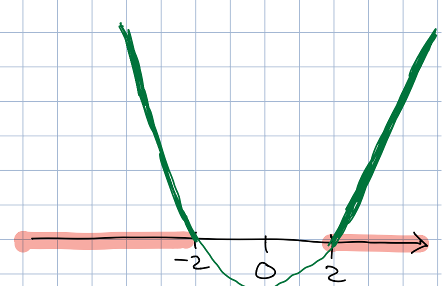
$x \in \operatorname{dom} f \iff x^2 - 4 \geq 0 \iff x^2 \geq 4 \iff x \in (-\infty, -2] \cup [2, +\infty)$

Quindi $\operatorname{dom} f = (-\infty, -2] \cup [2, +\infty)$

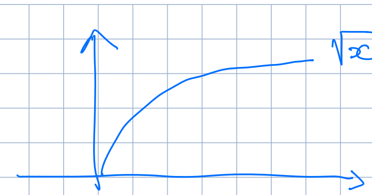
• Im f

$f(x) = \sqrt{x^2 - 4} - 1 = \sqrt{x^2 - 4} - 1$

$h(x) = x^2 - 4, \quad h : \operatorname{dom} f \to \mathbb{R}$



$\text{Im}\, h = [0, +\infty)$



$g(x) = \sqrt{x^2 - 4} = \sqrt{h(x)}$

$\text{Im}\, g = [0, +\infty)$

$f(x) = g(x) - 1 \implies \text{Im} f = [-1, +\infty)$

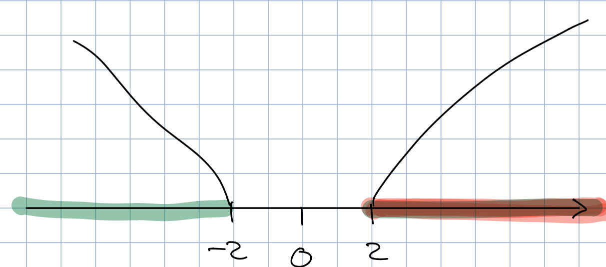
b) iniettiva: NO : $f(-2) = f(2) \quad (= -1)$

surgettiva: NO : $-2 \notin \text{Im} f$

bigettiva: NO

c) $f(x) = \sqrt{x^2-4} - 1$

$f(x) = f(-x)$



Prendiamo $I = [2, +\infty)$

$B = [-1, +\infty)$

cerco $f^{-1} : B \longrightarrow \mathbb{R}$ prendo $y \in B$ $(y \geq -1)$

cerco $x \in I$ t.c. $f(x) = y$ (porrò $f^{-1}(y) = x$)

$f(x) = y \iff \sqrt{x^2-4} - 1 = y$

$\iff 0 \leq \sqrt{x^2-4} = y + 1 \geq 0$

$\iff x^2 - 4 = (y+1)^2$

$\iff x^2 = (y+1)^2 + 4$

$\iff x = \begin{cases} \cancel{-\sqrt{(y+1)^2 + 4}} \\ +\sqrt{(y+1)^2+4} \end{cases}$

$$f^{-1}(y) = \sqrt{(y+1)^2+4} \quad \rightsquigarrow \quad f^{-1}(x) = \sqrt{(x+1)^2 + 4}$$