$$f(x) = \frac{e^{1/x}}{x-2}$$

$$f'(x) = \frac{e^{1/x}\left(-\frac{1}{x^2}\right)(x-2) - e^{1/x}}{(x-2)^2}$$

$$= \frac{e^{1/x}}{(x-2)^2}\left(\frac{2-x}{x^2} - 1\right)$$

$$= \underbrace{\left(\frac{e^{1/x}}{x-2}\right)}_{f(x)} \underbrace{\frac{\overbrace{2-x-x^2}^{n(x)}}{\underbrace{(x-2)x^2}_{d(x)}}}_{\boxed{g(x) = \frac{n(x)}{d(x)}}}$$

$$f''(x) = f'(x)\, g(x) + f(x)\, g'(x)$$

$$= f(x)\, g(x)\, g(x) + f(x)\, g'(x)$$

$$= f(x)\left[g^2(x) + g'(x)\right]$$

$$g^2(x) + g'(x) = \frac{n^2(x)}{d^2(x)} + \frac{n'(x)\, d(x) - d'(x)\, n(x)}{d^2(x)}$$

$$g^2(x) + g'(x) = \frac{1}{d^2(x)} \underline{\left(n^2(x) + n'(x)\, d(x) - d'(x)\, n(x) \right)}$$

$$n(x) = 2 - x - x^2 \qquad n'(x) = -2x - 1$$

$$d(x) = (x-2)\, x^2 \qquad d'(x) = 3x^2 - 4x$$

$$n^2(x) = x^4 + x^2 + 4 - 4x - 4x^2 + 2x^3 = \underline{x^4 + 2x^3 - 3x^2 - 4x + 4}$$

$$n'(x)\, d(x) = (-2x-1)(x-2)\, x^2$$

$$= -2x^4 + 4x^3 - x^3 + 2x^2 = \underline{-2x^4 + 3x^3 + 2x^2}$$

$$d'(x)\, n(x) = (3x^2 - 4x)(2 - x - x^2)$$

$$= 6x^2 - 8x - 3x^3 + 4x^2 - 3x^4 + 4x^3$$

$$= \underline{-3x^4 + x^3 + 10x^2 - 8x}$$

$$g^2(x) + g'(x) = \Big(x^4 + 2x^3 - 3x^2 - 4x + 4$$

$$- 2x^4 + 3x^3 + 2x^2$$

$$+ 3x^4 - x^3 - 10x^2 + 8x \Big) \Big/ d^2(x)$$

$$= \underline{\left(2x^4 + 4x^3 - 11x^2 + 4x + 4 \right)} \Big/ d^2(x)$$