Es foglio 3 pre-esame

1) X pdf: $f_X(x)$ $\quad f_Y(Y)?$ dove $Y = |X|$

PDF

$$f_Y = f_X(f^{-1}(y)) \cdot \frac{d f^{-1}(y)}{dy} = f_X\left(\frac{1}{|x|}\right) \cdot \frac{|x|}{x}$$

2)

$X \backslash Y$	1	3	4
2	$1/6$	$1/3$	0
3	0	$1/4$	$1/4$

a) $p_Y(1)$ $\quad p_Y(3) \quad p_Y(4) \quad p_X(2) \quad p_X(3)$

$1/6 \quad 1/3 + 1/4 \quad 1/4 \quad 1/6 + 1/3 \quad 1/2$

b) $E(X) = 2\left(\frac{1}{6} + \frac{1}{3}\right) + \frac{3}{2} = \frac{5}{2}$

$E(Y) = \frac{1}{6} + \frac{21}{12} + 1 = \frac{35}{12}$

c) $E[X \cdot Y] = \sum_i \sum_j (x_i \cdot y_j) \cdot p(x_i, y_j)$

$= \cancel{2} \cdot 1 \cdot \frac{1}{\cancel{6}_3} + 2 \cdot \cancel{3} \cdot \frac{1}{\cancel{3}} + 3 \cdot 3 \cdot \frac{1}{4} + 3 \cdot \cancel{4} \cdot \frac{1}{\cancel{4}}$

$= \frac{1}{3} + 5 + \frac{9}{4} = \frac{91}{12}$

d) $Cov(X, Y) = E[XY] - E[X]E[Y]$

$= \frac{91}{12} - \frac{5}{2} \cdot \frac{35}{12} = \frac{7}{24}$

e) X e Y dipendenti

f) $P\{X \leq 3, Y \leq 3\} = F(3,3)$

$$= \sum_i^3 \sum_j^3 p(x_i, y_j) = p(2,1) + p(2,3) + p(3,3)$$
$$= \frac{1}{6} + \frac{1}{3} + \frac{1}{4} = \frac{9}{12} = \frac{3}{4}$$

3) Se X e Y indip. $\rightarrow$ $E[X \cdot Y] = E[X]E[Y]$

a) Poiché $Cov(X,Y) = E[XY] - E[X]E[Y]$

esso è $= 0$ x var indip.

b) [non richiesto]

4) $E[X_i] = 0$ $\quad Var(X_i) = 1$ $\quad X_i$ indip.

$Y_1 = X_1 + X_2$ $\quad Y_2 = X_2 + X_3$ $\quad Y_3 = X_3 + X_4$

Calcolo le correlazione (Y_1, Y_2) e (Y_1, Y_3)

$$p(Y_1, Y_2) = \frac{Cov(Y_1, Y_2)}{Var(Y_1) Var(Y_2)}$$

$$Var(Y_1) = \underbrace{Var(X_1) + Var(X_2)}_{\text{poiché indip } Cov = 0} = 2 = Var(Y_2) = Var(Y_3)$$

$$Cov(Y_1, Y_2) = E[(X_1 + X_2)(X_2 + X_3)] - \underbrace{E[X_1 + X_2]}_{\substack{EX_1 + EX_2 \\ 0 + 0}} \underbrace{E[X_2 + X_3]}_{\substack{EX_2 + EX_3 \\ 0 + 0}}$$

$$= E[X_1X_2 + X_1X_3 + \overbrace{X_2X_2}^{\text{non sono indip}} + X_2X_3] - 0$$

$= \sum E$ dove ora X_i sono indip

$$E[X_1 \cdot X_2] = EX_1 \cdot EX_2 = 0$$

$Cov(Y_1, Y_2) = 0 = Cov(Y_1, Y_3)$ con i medesimi calcoli

Dunque $\rho(Y_1, Y_2) = \rho(Y_1, Y_3) = 0$

5) Da Chebyshev

$$P(|\bar{X} - \mu| \geq \epsilon) \leq \frac{\delta^2}{n\epsilon^2}$$

$$\rightarrow P(|\bar{X} - \mu| \geq 0{,}5) \leq \frac{1}{n \cdot \frac{1}{4}}$$

Confidenza = Probabilità

Ricerco i valori "buoni", ovvero che mantengono la precisione di 0,5 cm:

$$\underset{\substack{\uparrow \\ \text{cambio} \\ \text{di probabilità} \\ \text{(non più <)}}}{1 - 0{,}90 \geq} \frac{4}{n} \rightarrow \underset{\substack{\uparrow \\ \text{cerco un } n \text{ "necessario",} \\ \text{almeno } n \text{ ne servono}}}{0{,}10 =} \frac{4}{n} \rightarrow n = 40$$

6) No lim centrale