TUSTINS (BILINEAR)

$$H(S) = \frac{Y(S)}{X(S)} = \frac{\omega_c}{S+\omega_c}$$

$$\left[S = \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}} \right]$$

$$H(z) = \frac{\omega_c}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} + \omega_c}$$

$$\frac{Y(z)}{X(z)} = \frac{\omega_c}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} + \omega_c}$$

$$Y(z)\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} + \omega_c\right) = X(z)\,\omega_c$$

$$Y(z)\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} + Y(z)\omega_c = X(z)\omega_c$$

$$Y(z)\frac{2}{T}(1-z^{-1}) + Y(z)\omega_c(1+z^{-1}) = X(z)\omega_c(1+z^{-1})$$

$$Y(z)\frac{2}{T} - Y(z)\frac{2}{T}z^{-1} + Y(z)\omega_c + Y(z)z^{-1}\omega_c =$$

$$X(z)\omega_c + \omega_c X(z) z^{-1}$$

$$Y(z)\left(\frac{2}{T} + \omega_c\right) + Y(z)z^{-1}\left(\omega_c - \frac{2}{T}\right) = X(z)\omega_c + \omega X(z)z^{-1}$$

$$\left(\frac{2}{T} + \omega_c\right) y[k] + y[k-1]\left(-\frac{2}{T}\right) = \omega_c \cdot x(k) + x[k-1]$$

$$Y(z) = \frac{X(z)\,\omega_c}{\left(\frac{2}{T}+\omega_c\right)} + \frac{X(z)z^{-1}}{\left(\frac{2}{T}+\omega_c\right)} - \frac{Y(z)z^{-1}\left(\omega_c - \frac{2}{T}\right)}{\left(\frac{2}{T}+\omega_c\right)}$$

$$y[k] = x[k]\underbrace{\left(\frac{\omega_c}{\frac{2}{T}+\omega_c}\right)}_{\alpha} + x[k-1]\underbrace{\left(\frac{\omega_c}{\frac{2}{T}+\omega_c}\right)}_{\beta} + y[k-1]\cdot\underbrace{\left(\frac{\omega_c - \frac{2}{T}}{\frac{2}{T}+\omega_c}\right)}_{\gamma}$$

$$y[k] = x[k]\left(\frac{\omega_c}{\left(\frac{2+T\omega_c}{T}\right)}\right) + x[k-1]\left(\frac{\omega_c}{\left(\frac{2+T\omega_c}{T}\right)}\right) + y[k-1]\cdot\left(\frac{\left(\frac{\omega_c T - 2}{T}\right)}{\frac{2+T\omega_c}{T}}\right)$$

$$= x[k]\left(\frac{T\omega_c}{2+T\omega_c}\right) + x[k-1]\left(\frac{T\omega_c}{2+T\omega_c}\right) + y[k-1]\left(\frac{\omega_c T - 2}{T\omega_c + 2}\right)$$