The Data Science of Particle Physics
Section I : Statistical data analysis in particle physics

Lecture 1: intro & basic concepts I

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The Data Science of Particle Physics

- Week 1 - 5 sessions with me (JK)
  - All course content for week 1 is available in this git repo GitHub link
  - Introduction to particle physics - no physics experience necessary!
  - The statistics of discoveries
  - 2-3 Open-ended data analysis assignments using Real Data from the ATLAS experiment at CERN!
- Weeks 2 and 3 with Dr. Julia Gonski (Stanford)

Who am I?

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DEPARTMENT OF PHYSICS
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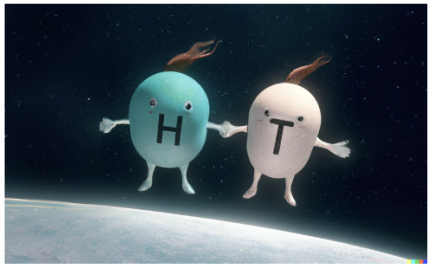
Particle physics with ATLAS
- seeking new physics with **Higgs bosons** and **Top quarks**
- **Real-Time AI – Anomaly detection in the Trigger**

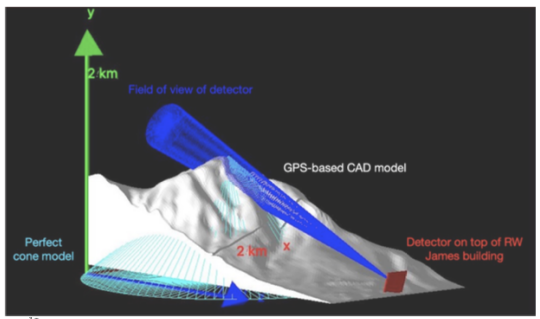
Low-cost Medical PET imaging
- Quantum Tech meets AI!

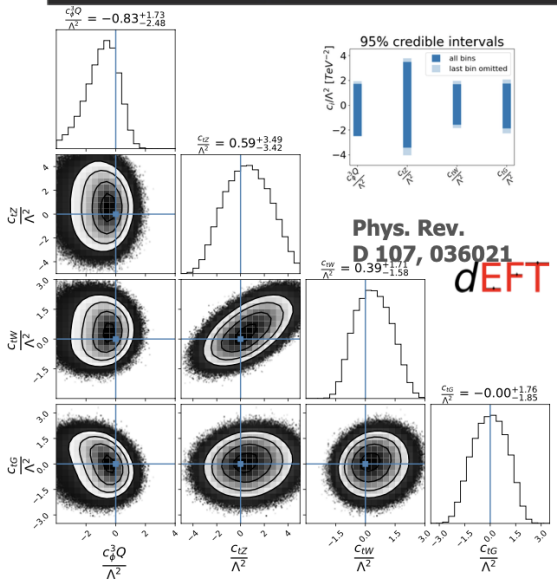
Detector development
- **ATLAS** inner tracker
- **µCT** - muon tomography

Particle phenomenology
- constraining new physics via Effective Field Theory

- **Outreach**
  - ATLAS Open Data for education







ATLAS EXPERIMENT

OPPENHEIMER MEMORIAL TRUST

dEFT

µCT

UCT-CERN

Read more about my research in the media... If you like :)

- OMT Article
- Engineering News Article
- UCT News Article

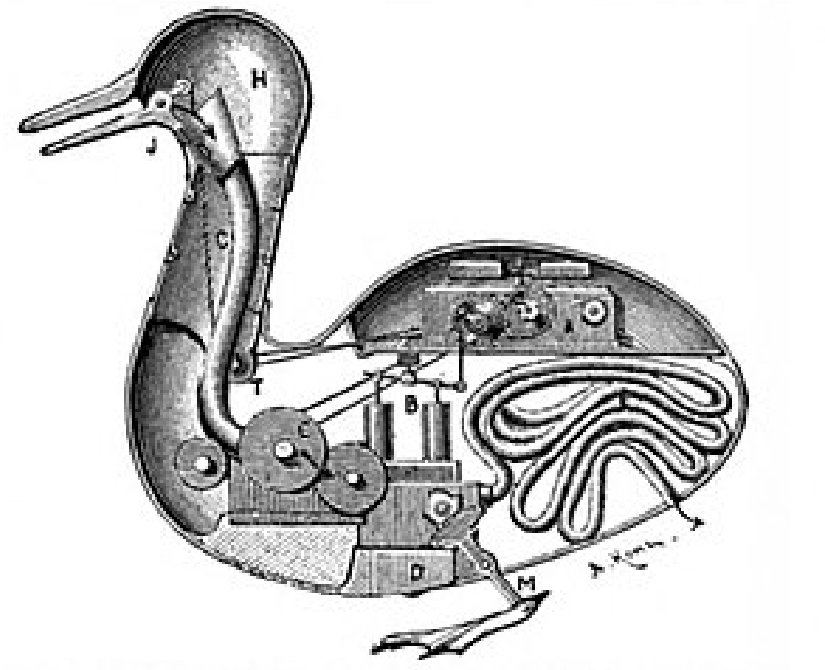
Ice-breakers (No wrong answers)

- What is particle physics?
- What is data science?

Ice-breakers 2 (No wrong answers)

- What do you hope to get out of this module?

Reductionism



INTERIOR OF VAUCANSON'S AUTOMATIC DUCK.

A, clockwork; *B*, pump; *C*, mill for grinding grain; *F*, intestinal tube; *J*, bill; *H*, head; *M*, feet.

Figure

Particle Physics as *Reductionism*

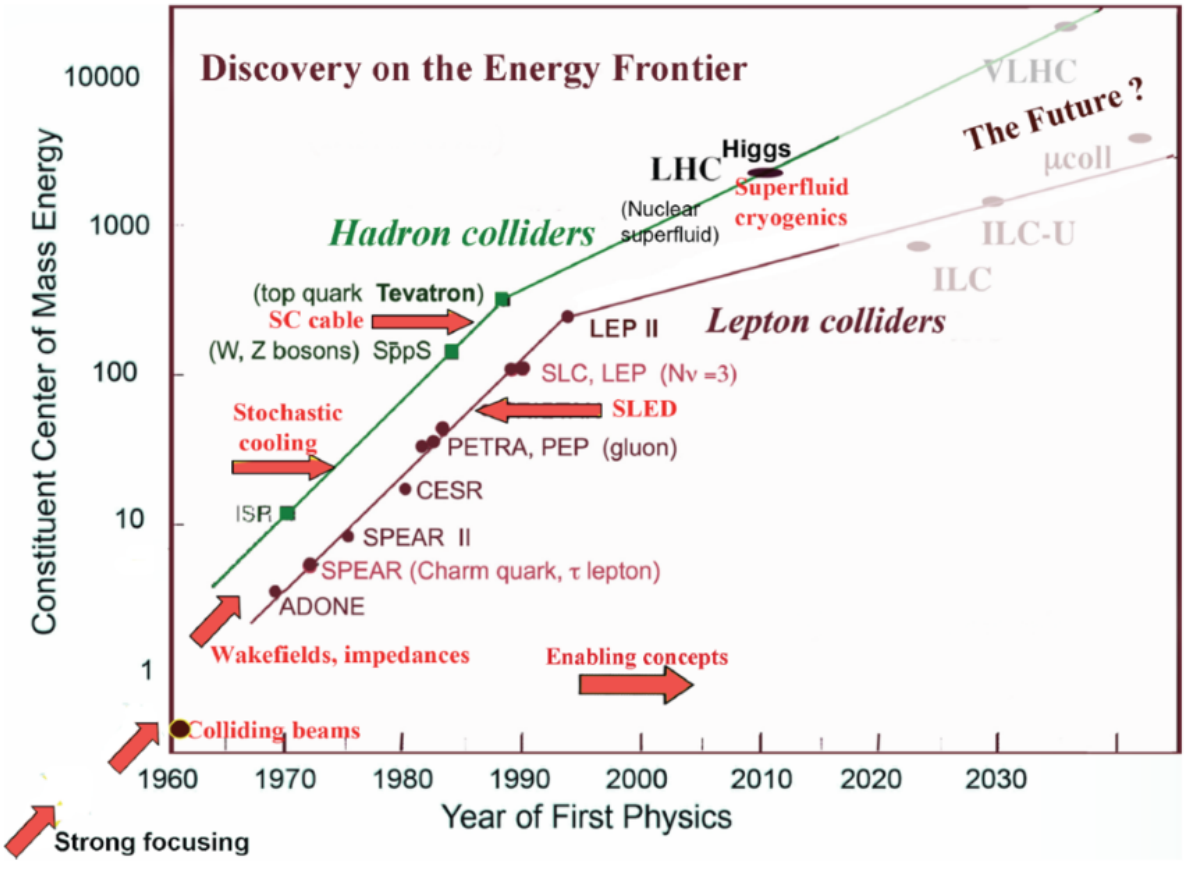
- **Methodological reductionism**: the scientific attempt to provide explanation in terms of ever smaller entities.

- **Particle physics** represents (**for now**) the culmination of the reductionist approach to understanding the universe

Physics at the end of the (experimental) road

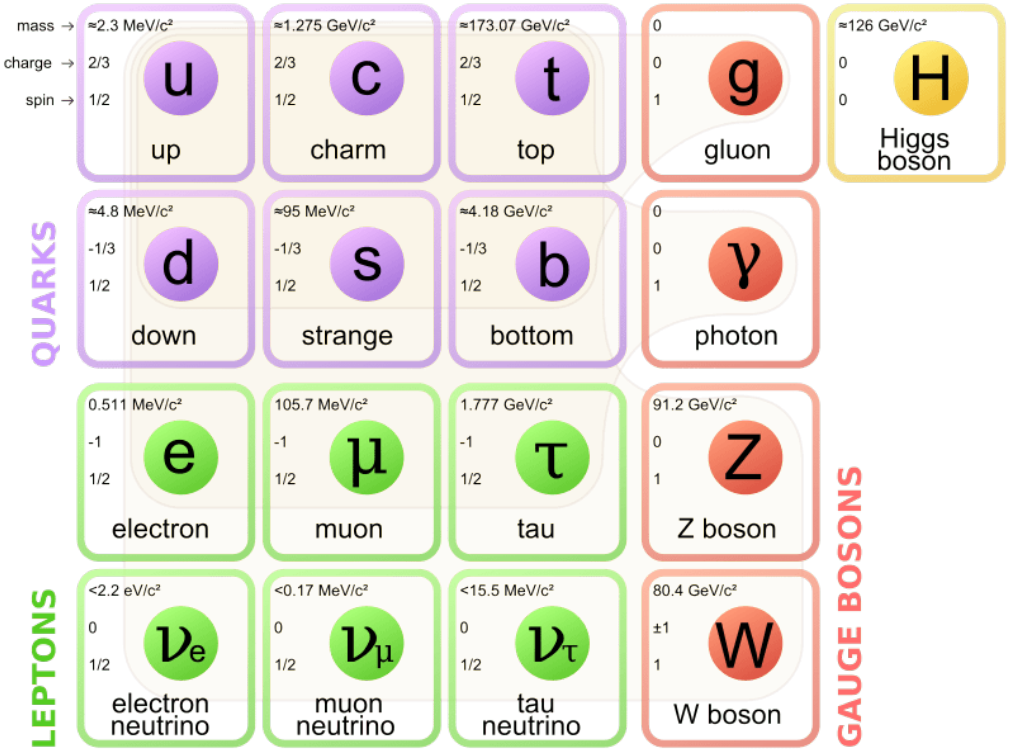
- Following the reductionist approach means experimentally probing smaller and smaller distance scales ℓ
- But from Quantum Mechanics we know $\ell = \hbar c / E$
- Smaller distance scales means larger energy scales! $\ell \downarrow \equiv E \uparrow$
- **Colliding particles at the highest-ever energies allows us to probe nature at the smallest-ever distance scales**

Colliders as microscopes



- LHC has been colliding protons at 13 TeV since 2015

So what do we know?



- *u*, *d* and *e* form all *normal* matter
- 3 fundamental forces: Electromagnetic, Weak, and Strong
  - each mediated by the exchange of elementary bosons

What do we know: fundamental fermions

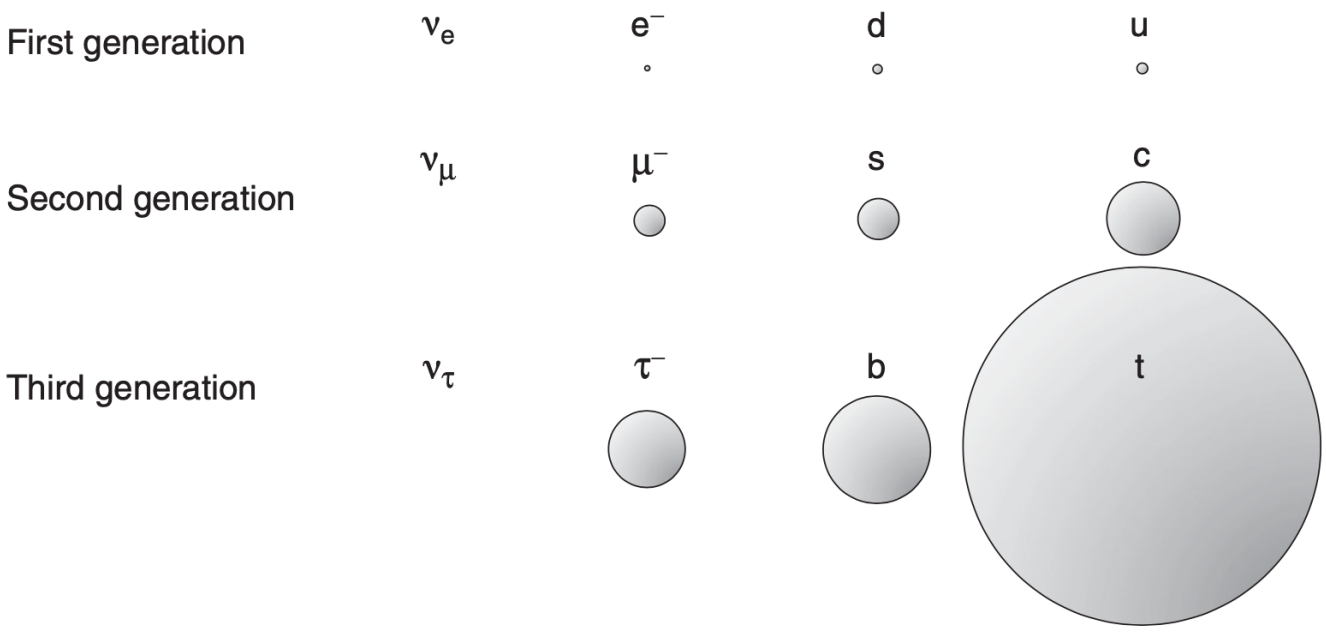


Figure: fundamental particles drawn as spheres with $V \propto m$

- Why are there three *generations*?
- Why the random masses if these are fundamental?
- $m_{top} \approx m_{Au\ atom}$!! Γ Γ

What do we know: forces through boson *exchange*

- fundamental particles *interact*: scatter, decay, annihilate...

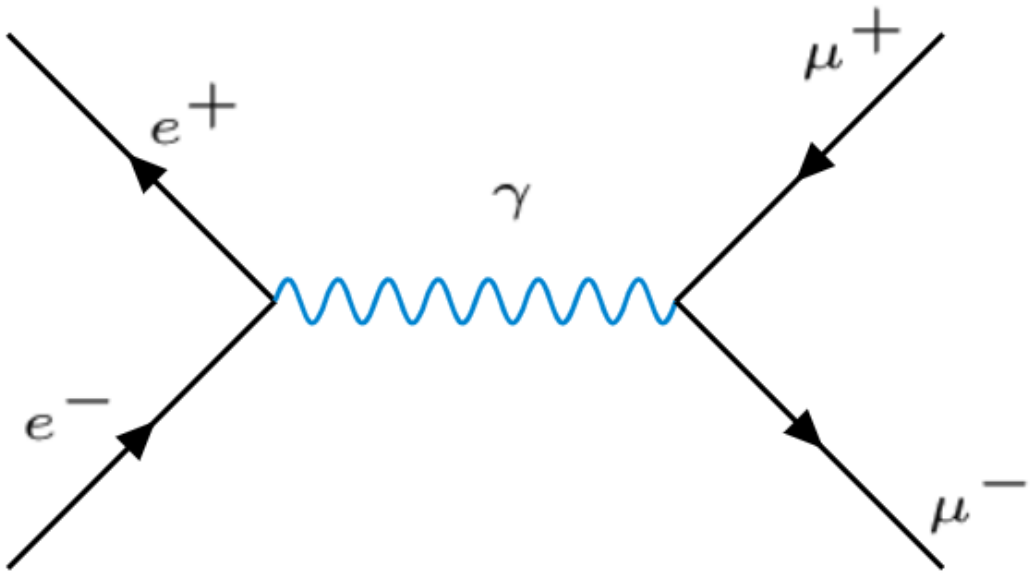


Figure: Feynman diagram for Bhabha scattering ($e^+e^- \to \mu^+\mu^-$)

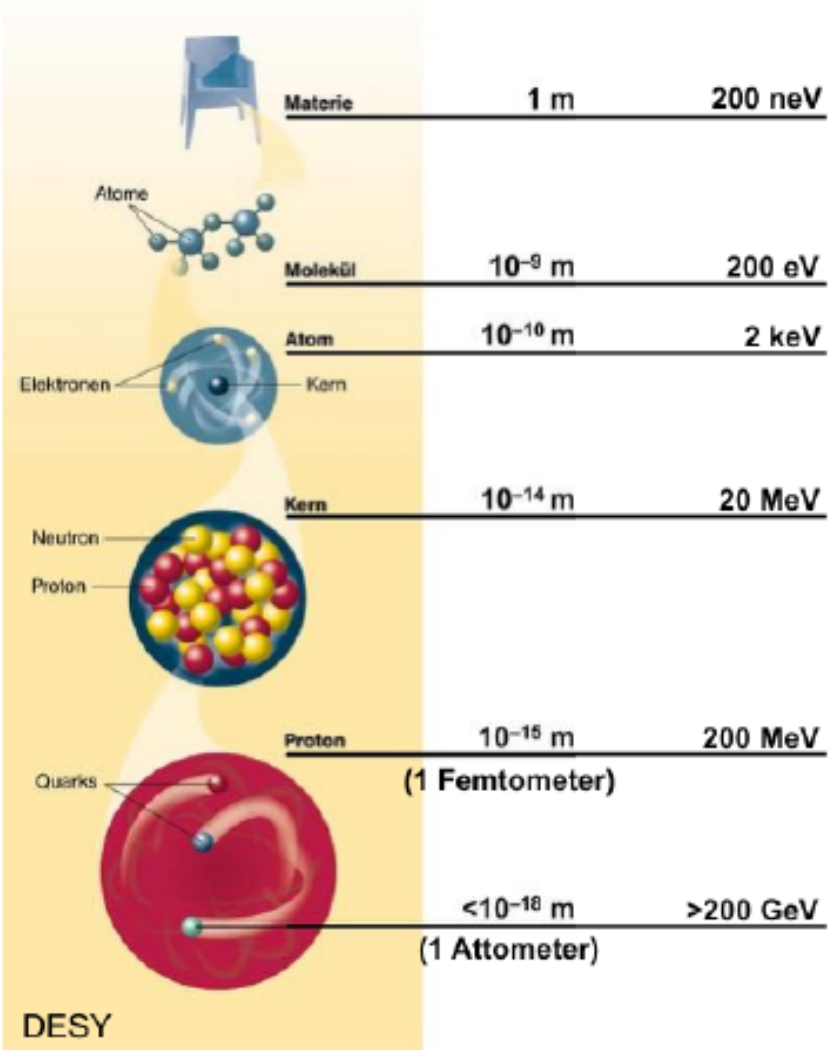
- basic interactions (EM, weak, strong) understood as due to boson exchange (γ, $W^\pm$ or Z, g)
- Nobel-prize winning discovery of the Higgs boson in 2012 *completes* the Standard Model, but...

What **don't** we know?

- we have no idea how to include gravity in the Standard Model
- we cannot reconcile our understanding of gravity with the structure and evolution of the universe without assuming $\approx 95\%$ of the universe is of unknown nature (Dark Matter/Energy), not accounted for in Standard Model
- No explanation for the masses of the Fundamental particles, especially the Higgs boson
- No sign yet of a discovery that would answer these questions...

The Large Hadron Collider

Why particles accelerators?



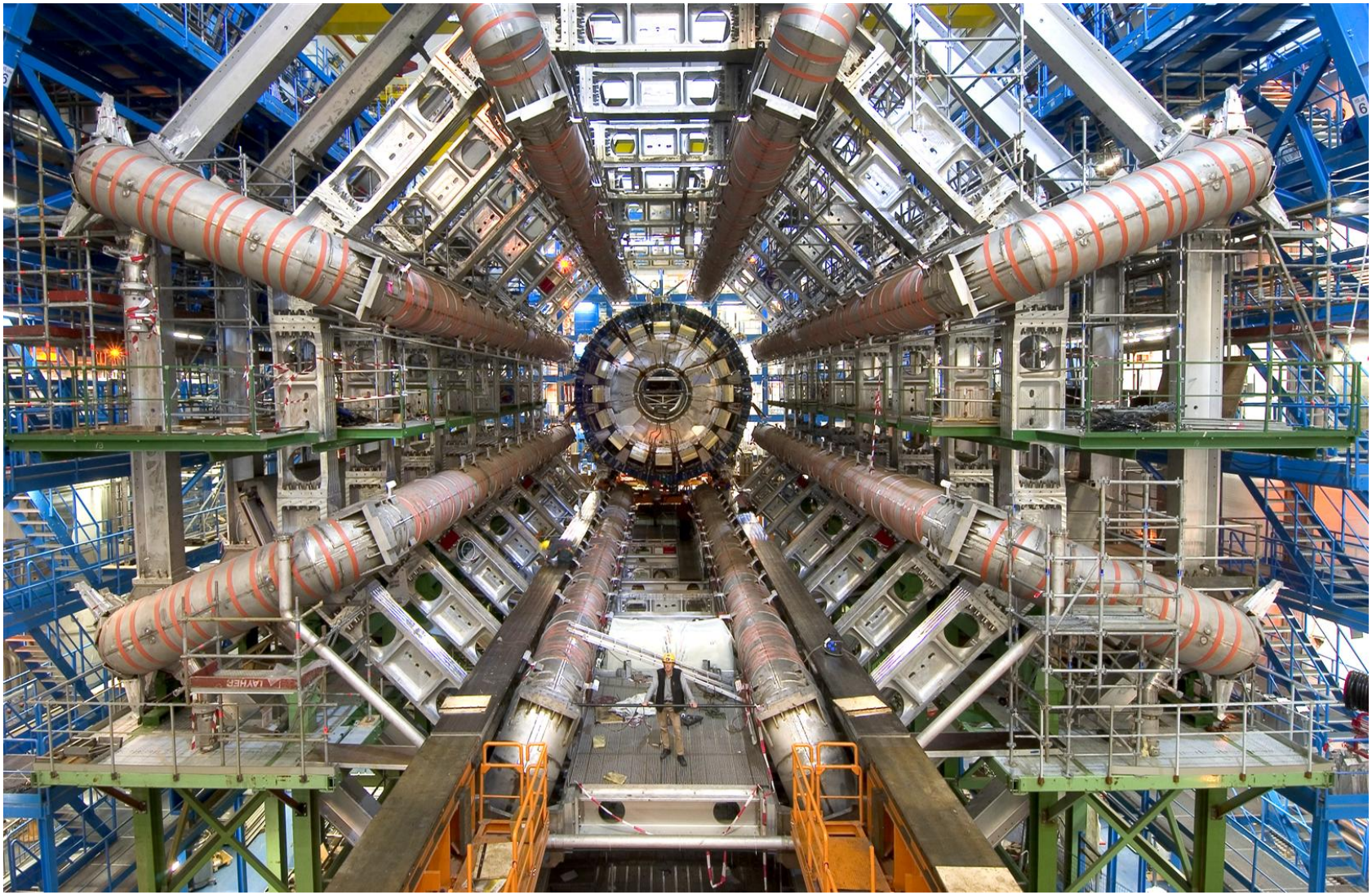
- Production of ***new heavy particles*** ($E = mc^2$)
  - Maximum mass of produced particle = centre-of-mass energy of elementary collision, e. g. parton-parton collision
- Resolution of ***small structures***: accelerator as very powerful ***microscope***
  - ***De-Broglie wave length*** of particle beam

$$\lambda = \frac{h}{p} = \frac{2\pi\hbar c}{pc} \rightarrow \lambda\,[\text{fm}] \approx \frac{1.24}{p[\text{GeV}]}$$

  - For example

$$p = 1\,\text{GeV} \rightarrow \lambda = 1.24 \cdot 10^{-15}\,\text{m}$$
$$p = 1\,\text{TeV} \rightarrow \lambda = 1.24 \cdot 10^{-18}\,\text{m}$$

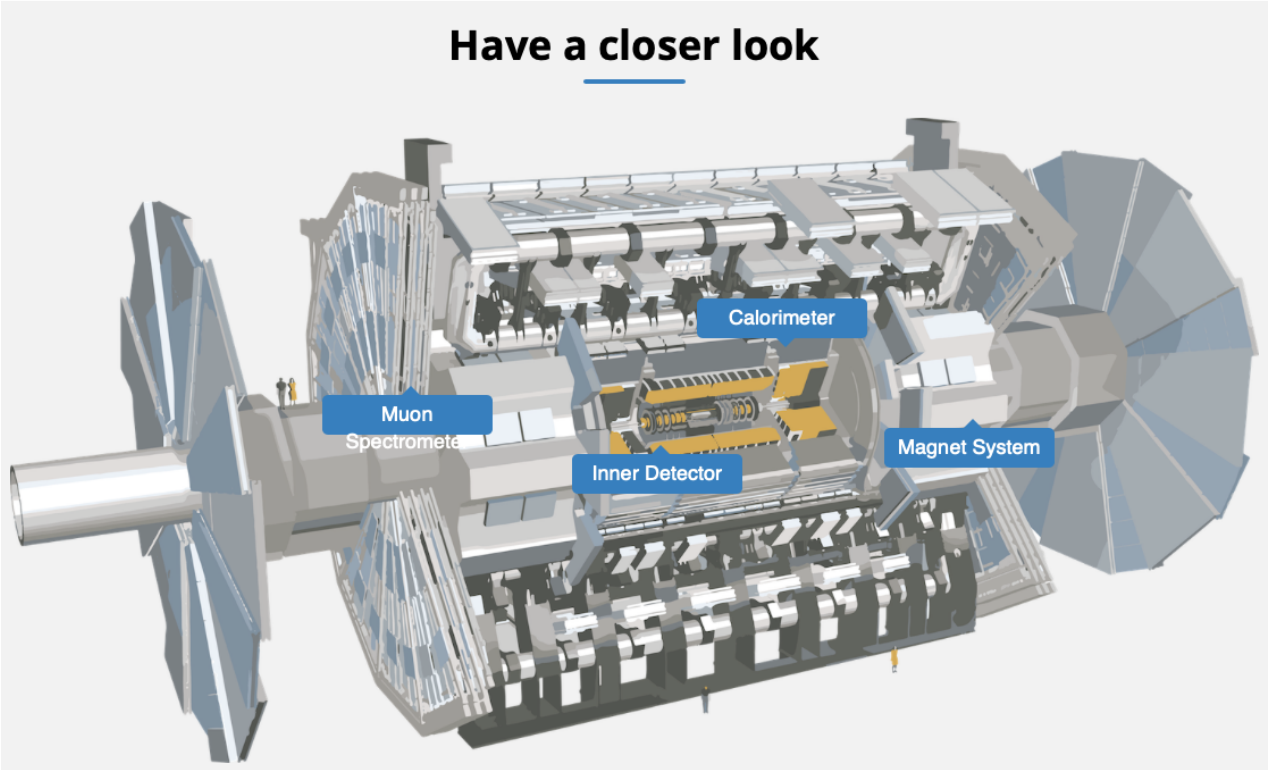
The ATLAS Detector



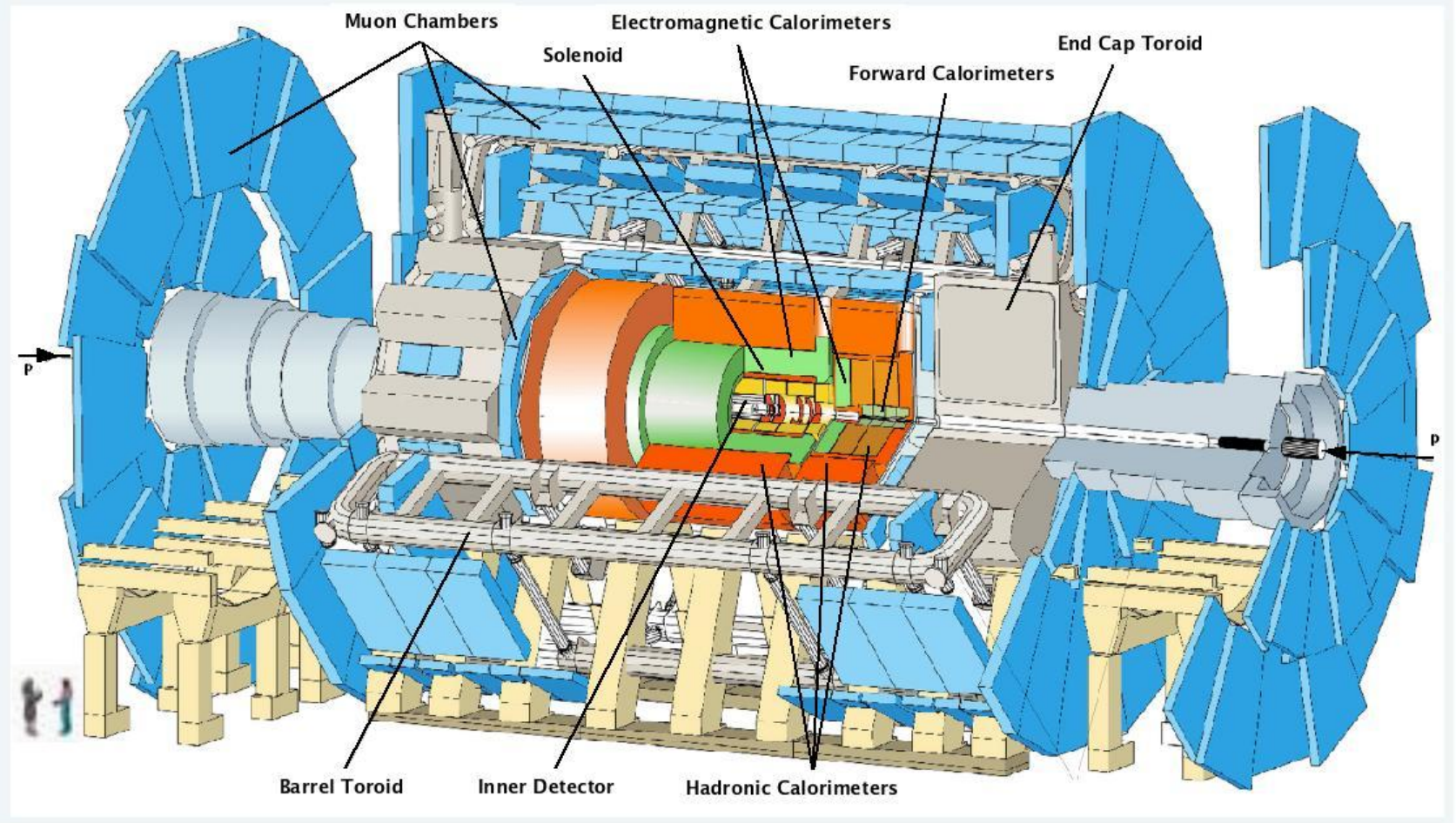
The ATLAS Detector

The ATLAS Detector

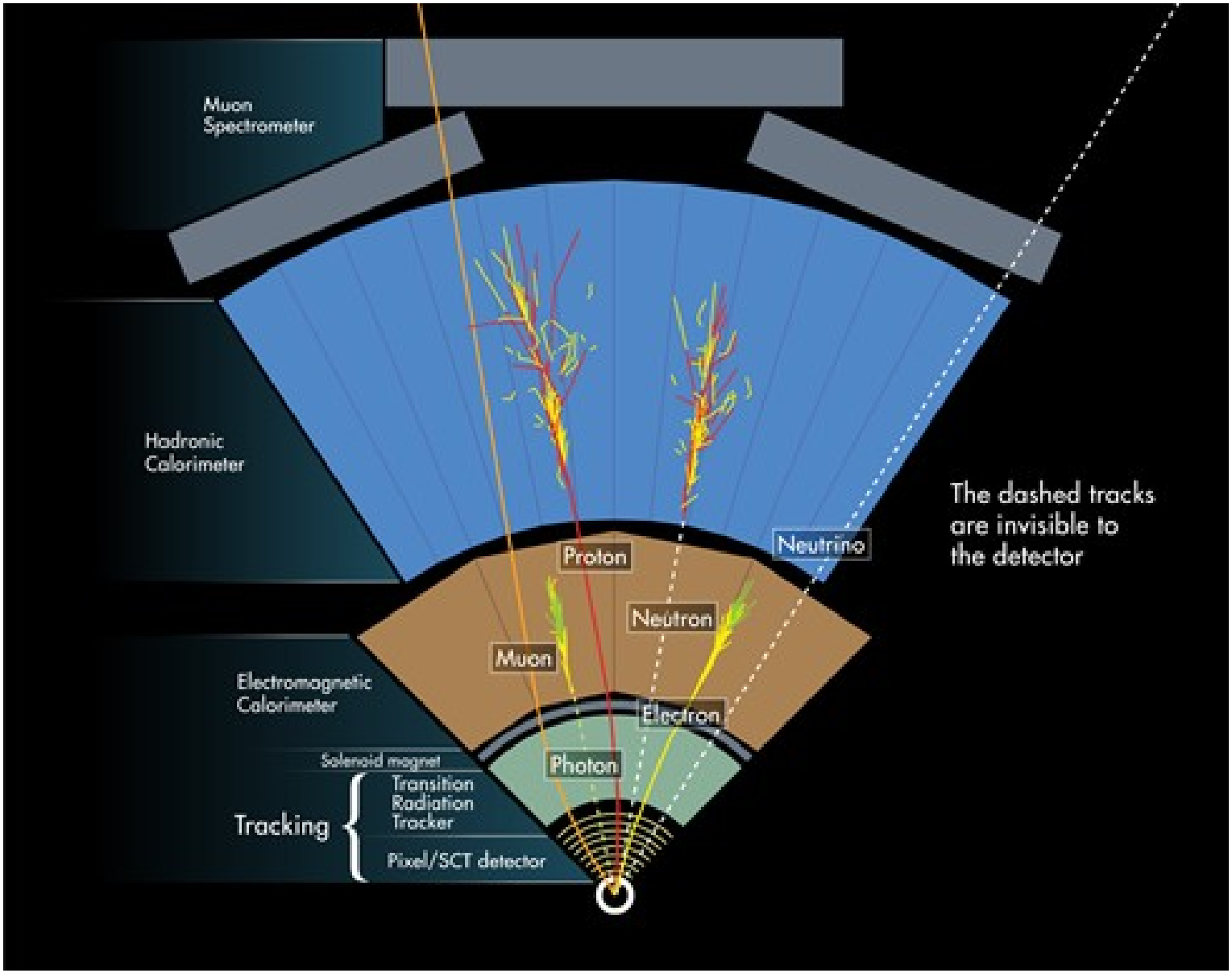
- Take a virtual tour of the ATLAS detector here!



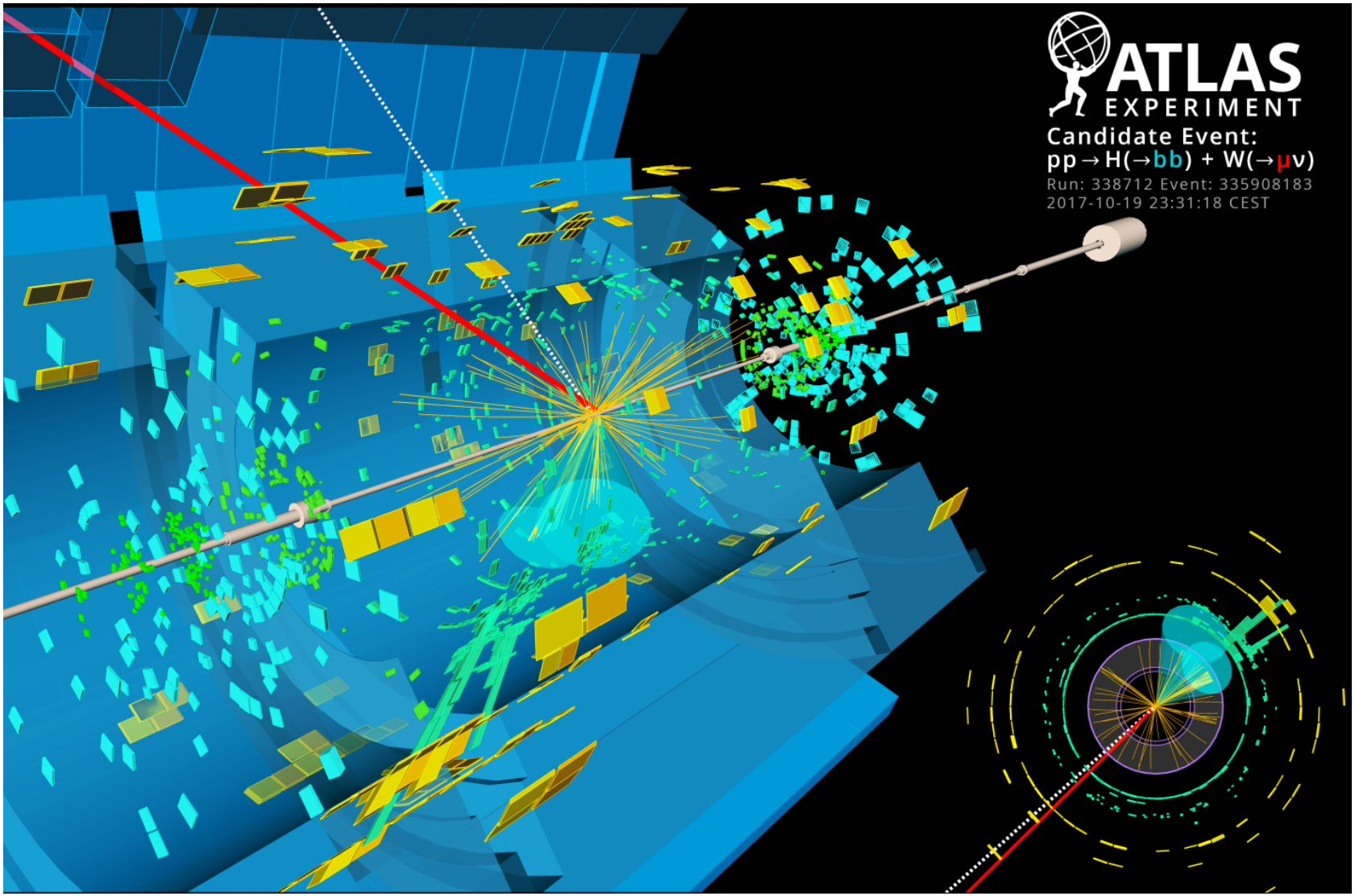
The ATLAS Detector



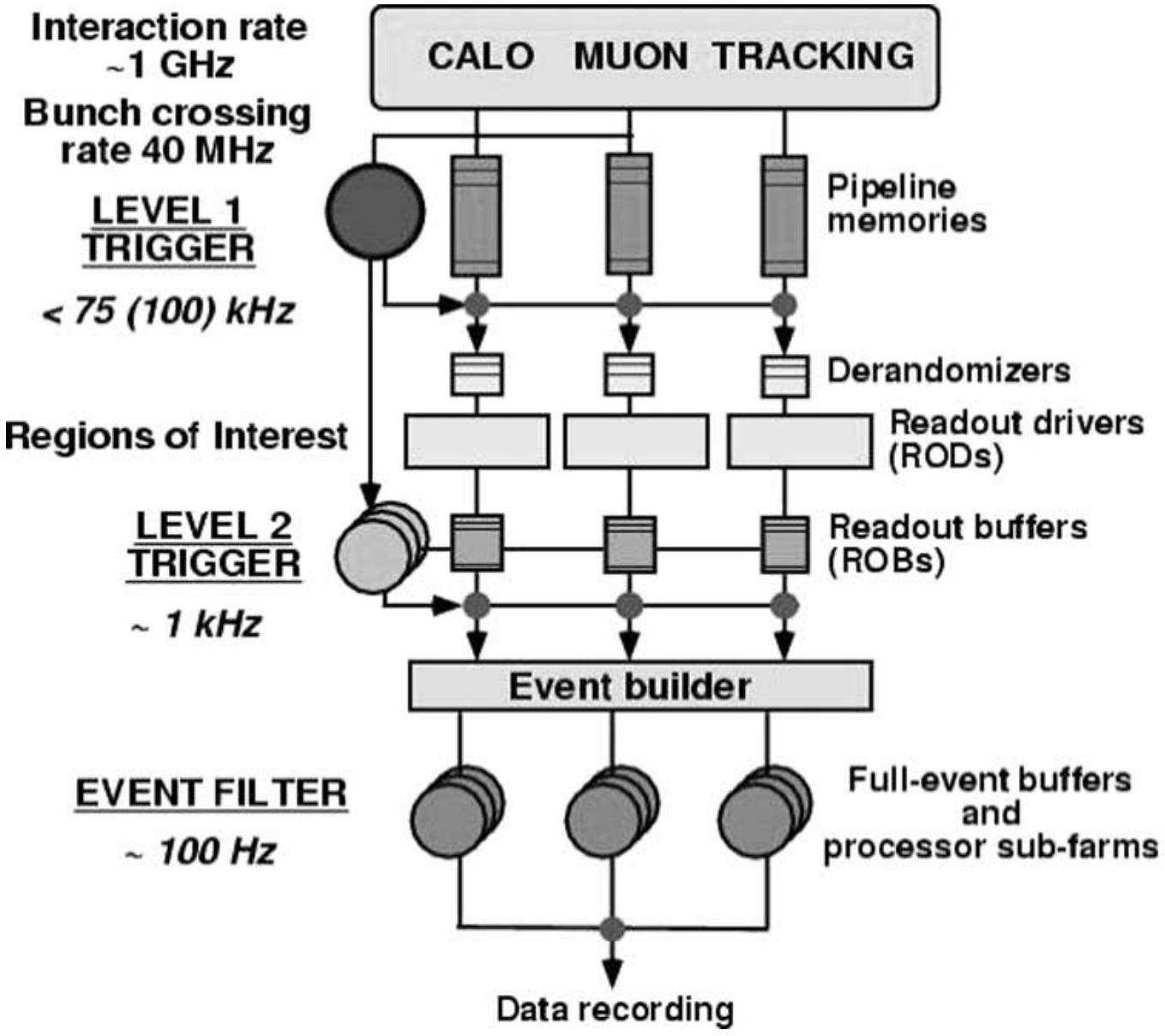
The ATLAS Detector



What happens when you collide protons?



The ATLAS Trigger



So what does ATLAS data look like?

- surprisingly simple!

```
df
✓ 0.1s                                                                                                                    Python
```

	muon_E	muon_pt	muon_phi	muon_eta	muon_charge	muon_etcone20	muon_ptcone30
0	[59232.8828125 55261.05078125]	[49085.51953125 39126.15625]	[0.14771345257759094 -2.867905855178833]	[0.6324113607406616 0.8795363306999207]	[1 -1]	[-991.5115356445312 603.2068481445312]	[0.0 0.0]
1	[127074.1875 33773.015625]	[65642.484375 33766.76953125]	[-1.462382197380066 -2.7814879417419434]	[1.2791037559509277 -0.018980734050273895]	[-1 1]	[-788.6085205078125 -1062.7930908203125]	[1063.2587890625 0.0]
2	[201868.75 54080.953125]	[38357.2578125 32836.46484375]	[-2.7715253829956055 0.05025695636868477]	[-2.344670295715332 -1.0837055444717407]	[-1 1]	[-320.12200927734375 670.6602172851562]	[0.0 0.0]
3	[79745.3828125 28305.017578125]	[69548.1328125 27166.60546875]	[1.5498226881027222 -0.946570873260498]	[-0.5351100564002991 0.2884726822376251]	[-1 1]	[39.91961669921875 -149.05303955078125]	[0.0 0.0]
4	[28867.724609375 98152.65625]	[27808.724609375 25207.083984375]	[-1.008341670036316 1.500999927520752]	[0.2750834822654724 -2.0356335639953613]	[-1 1]	[121.45053100585938 1119.2376708984375]	[0.0 0.0]
...	...	...	...	...	...	...	...
549252	[37704.515625 53827.45703125]	[33141.77734375 31681.8828125]	[2.1597113609313965 -1.0514856576919556]	[-0.5188854932785034 1.1224994659423828]	[-1 1]	[-380.786376953125 2450.8642578125]	[0.0 1780.0516357421875]
549253	[38264.09765625 99136.9921875]	[36691.76953125 30524.439453125]	[2.4906346797943115 -0.590939462184906]	[0.2917047142982483 1.846530795097351]	[-1 1]	[188.41143798828125 -137.44154357910156]	[0.0 0.0]
549254	[39036.56640625 114896.96875]	[36656.05859375 28371.234375]	[0.8913777470588684 -2.3450729846954346]	[0.35846027731895447 2.076202630996704]	[-1 1]	[-491.8928527832031 -164.48634338378906]	[0.0 0.0]
549255	[37635.5546875 63254.0546875]	[36309.703125 34960.17578125]	[-0.868657648563385 2.3575997352600098]	[0.269410103559494 -1.1991113424301147]	[1 -1]	[-358.3528747558594 -358.3528747558594]	[0.0 0.0]
549256	[61195.4609375 35183.0625]	[47037.4921875 33874.4453125]	[-2.48432993888855 1.0605766773223877]	[-0.7576252222061157 0.2770580649375915 5]	[1 -1]	[2067.195556640625 889.0148315429688]	[0.0 0.0]

549257 rows × 7 columns

Lorentz Four-Vectors

- Four-vectors are mathematical objects that combine space and time components
- In special relativity, a four-vector is represented as $(\vec{x}, t)$ or $(ct, \vec{x})$
- Four-vectors are used to describe physical quantities in a way that is consistent with the principles of special relativity
- Examples of four-vectors:
  - Four-position: $x^\mu = (ct, \vec{x})$
  - Four-momentum: $p^\mu = (E/c, \vec{p})$
  - Four-velocity: $u^\mu = \gamma(c, \vec{v})$

Lorentz Transformations and Invariance

- Lorentz transformations are a set of linear transformations that relate the coordinates of one inertial frame to another
- Four-vectors transform under Lorentz transformations in a way that preserves their inner product, known as Lorentz invariance
- Lorentz invariance ensures that the laws of physics are the same in all inertial reference frames
- The inner product of two four-vectors A^μ and B^μ is defined as:
$$A^\mu B_\mu = A^0 B_0 - \vec{A} \cdot \vec{B}$$
- Explicitly, for two four-vectors $A^\mu = (A^0, \vec{A})$ and $B^\mu = (B^0, \vec{B})$:
$$A^\mu B_\mu = A^0 B^0 - A^1 B^1 - A^2 B^2 - A^3 B^3$$
- Lorentz-invariant quantities, such as the rest mass and proper time, play a crucial role in particle physics

Four-vectors in Particle Physics

- Lorentz invariance is a fundamental principle in the Standard Model of particle physics
- Four-vectors and Lorentz-invariant quantities are used to construct kinematic variables and selection criteria in particle physics analyses
- Lorentz invariance ensures that the results of particle physics experiments are independent of the reference frame, allowing for consistent comparisons and interpretations

The Statistics of Discoveries

- Statistics is
  - peculiar, counter-intuitive, often seems easier than it is
  - elusive: (you think you understand it, you realise you don't)^N
  - **fundamental to modern experimental particle physics**
- Incorrect statistical analysis can mean the difference between a discovery and not a discovery

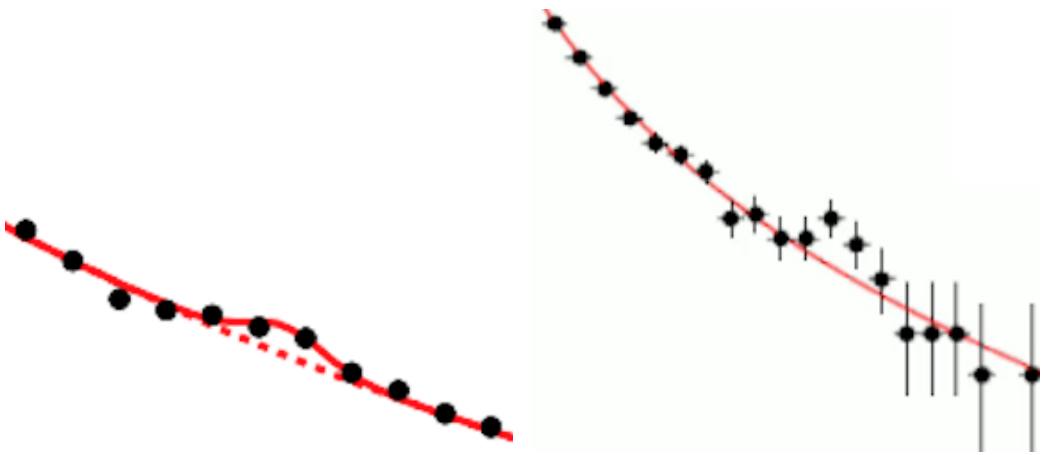


Figure: One of these *bumps* is a real discovery, the other is not...

The Statistics of Discoveries

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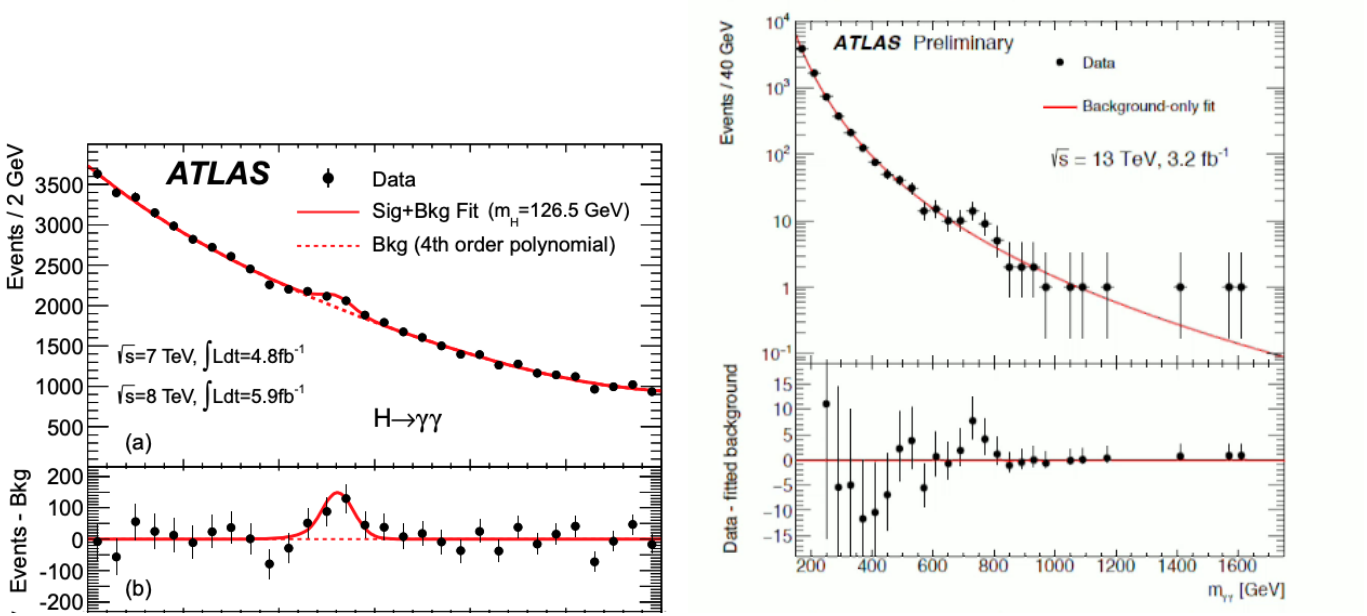
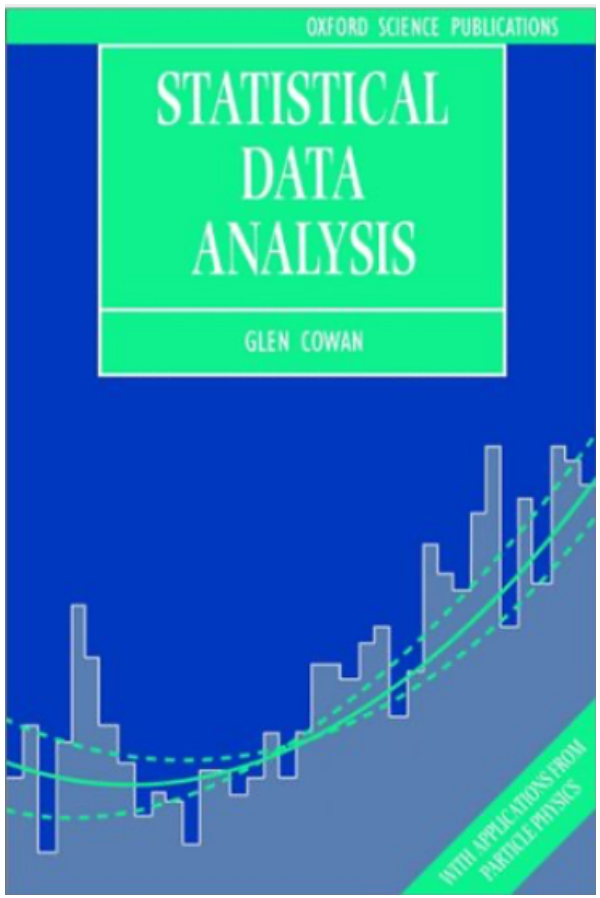


Figure: Discovery (left), Not a Discovery (right)

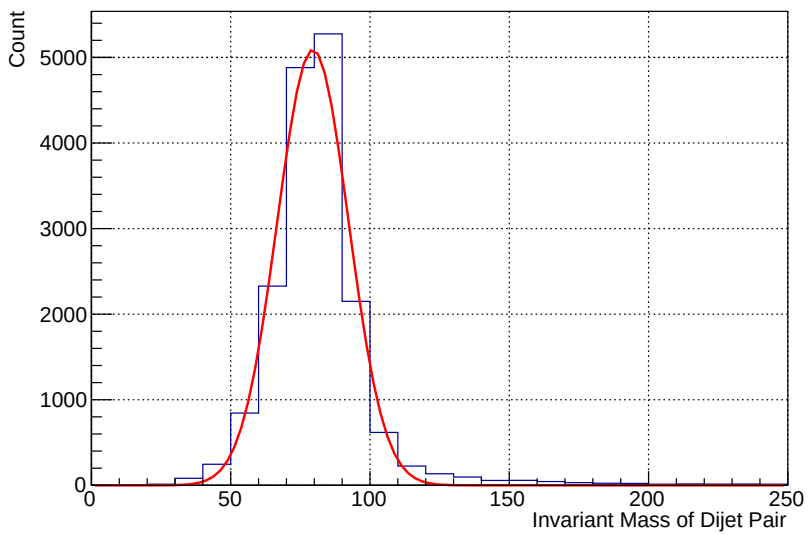
The Statistics of Discoveries

- Statistics is
  - elusive: (you think you understand it, you realise you don't)^N
- often need to refer back to textbooks...



Basic concepts: random variables and probability

- Results of repeated "identical", experiments may vary.
  - Instability in apparatus/environment/experimenter
  - Fundamental QM unpredictability of the system
- A variable is **random** when it cannot be predicted with absolute certainty



Basic concepts: probability

- Statistics and Probability: two schools of thought
  - **Bayesian**: Given some data/evidence, we assign probability to some *hypothesis*, e.g. given this LHC data, how sure are we the Higgs boson exists?
  - **Frequentist**: Given some *hypothesis*, how likely is the data we observe, e.g. assuming the Higgs boson exists, how likely is the data that we observe?
- Frequentist approaches are more popular in particle physics
- I will mainly discuss frequentist ideas

Basic concepts: random variables and probability

- Frequentist Probability
  - interpreted as a **limiting frequency**
- Imagine a *repeatable* experiment repeated n times, with S the set of all possible results
- A is a subset of possible results

$$P(A) = \lim_{n \to +\infty} \frac{N_{\text{result in A}}}{n}$$

- This definition satisfies the **3 axioms of probability**:
  1. $P(A) > 0$ for all A - probabilities can't be negative
  2. $\int_S P(A) = 1$ - *something* must happen
  3. For two mutually exclusive sets A and B, $(A \cap B = 0)$, $P(A \cup B) = P(A) + P(B)$.

Ice-breaker 1

- What does the *mean* mean?

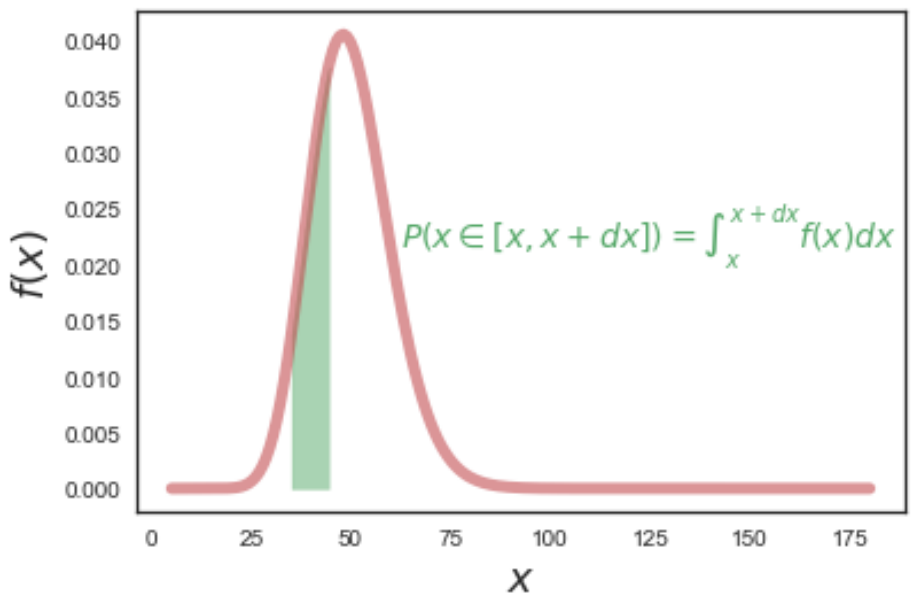
Python code / notebooks

- Code used to make the following plots (unless stated otherwise) available at GitHub link

Basic concepts: probability density functions (pdf)

- Imagine an experiment with all possible results characterised by a single continuous variable x
- S corresponds to the $(1D)$ space of all possible results
- What is the probability of observing a result in the interval $[x, x + dx]$?
  - given by $f(x)$ (pdf)

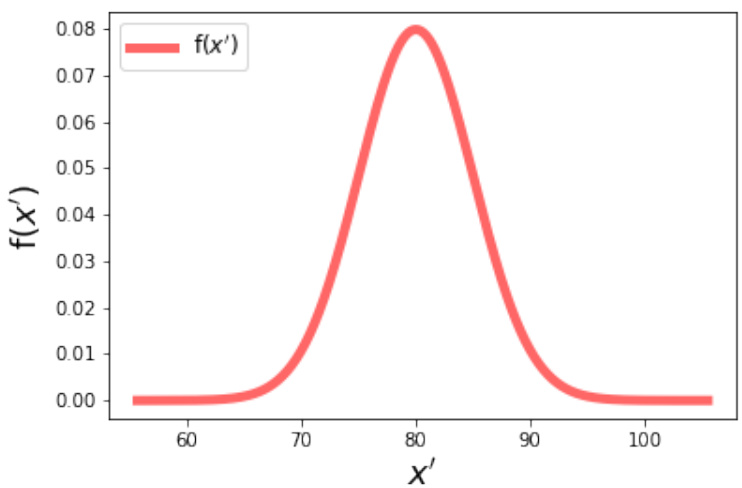
$$P(x \in [x, x + dx]) = \int_{x}^{x+dx} f(x)dx$$

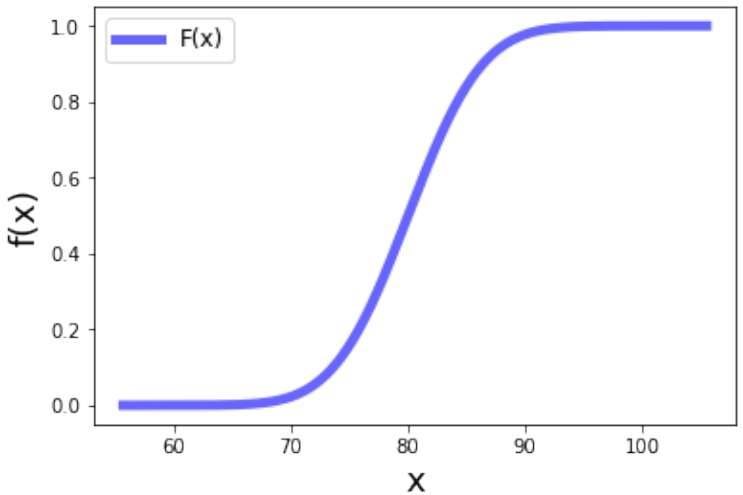


Basic concepts: cumulative density functions (cdf)

- cdf: $F(x)$
  - probability for x' to have a value $\leq x$

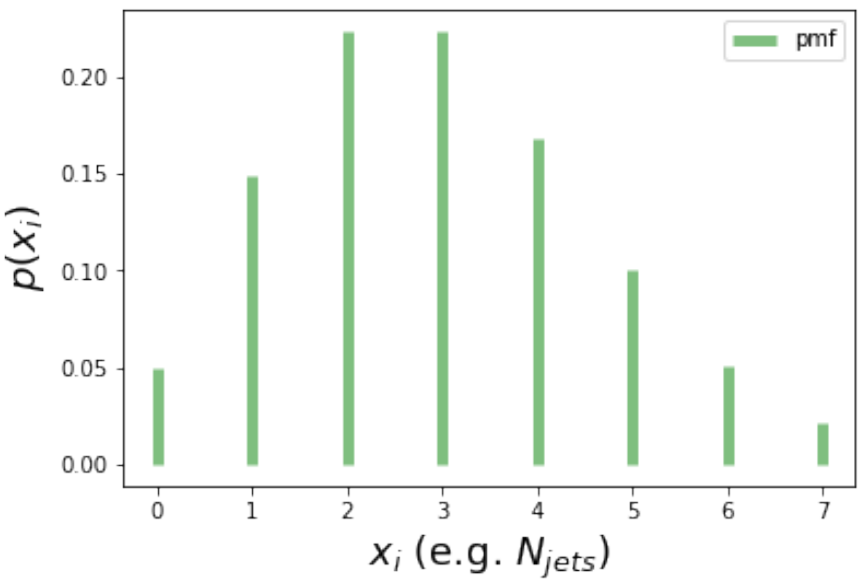
$$F(x) = \int_{-\infty}^{x} f(x)dx'$$





Basic concepts: probability mass function (pmf)

- If x can only assume discrete values (x_i), we use a *pmf* to describe its distribution
- pmf: $p(x_i) = P(x = x_i)$ where P is a probability.



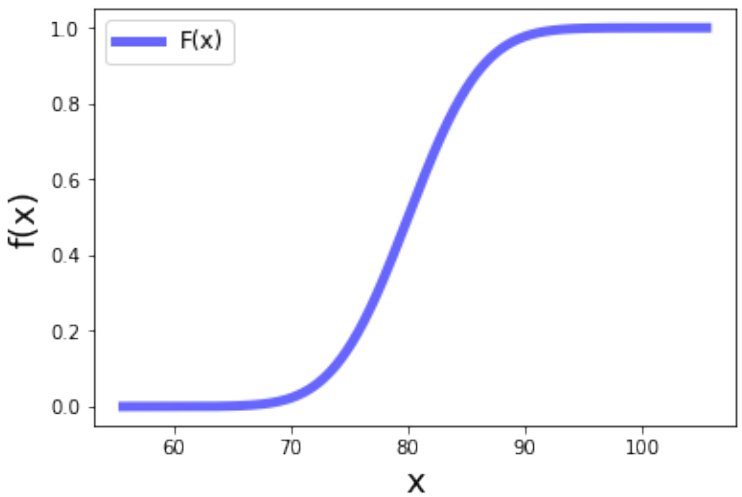
$$\sum_{x_i} p(x_i) = 1$$

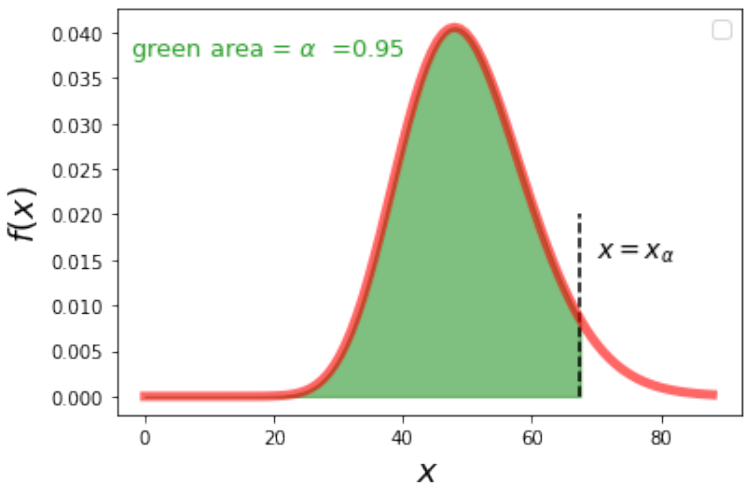
- Many examples of discrete observables in particle physics!

Basic concepts: quantiles

- also known as α points
- the quantile x_α is the value of x such that $F(x_\alpha) = \alpha$
- simply the inverse of the cdf

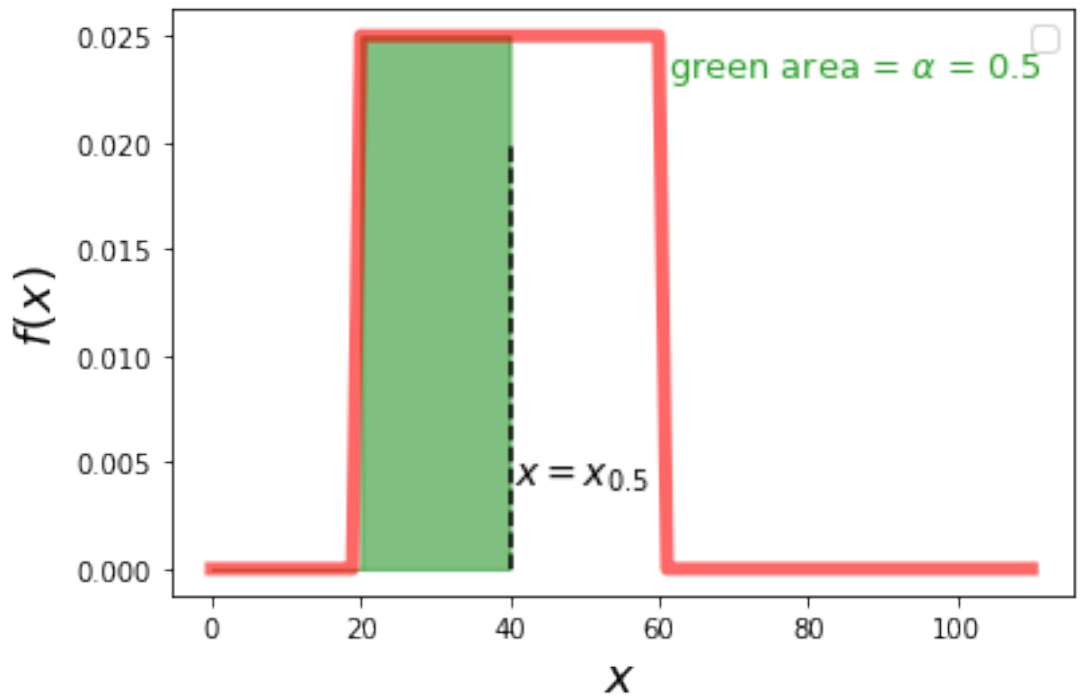
$$x_\alpha = F^{-1}(\alpha)$$





Basic concepts: median

- $x_{0.5}$ is a special case known as the **median**
- median often interpreted as the *typical location of x*
- when can this interpretation break down?



Basic concepts: median

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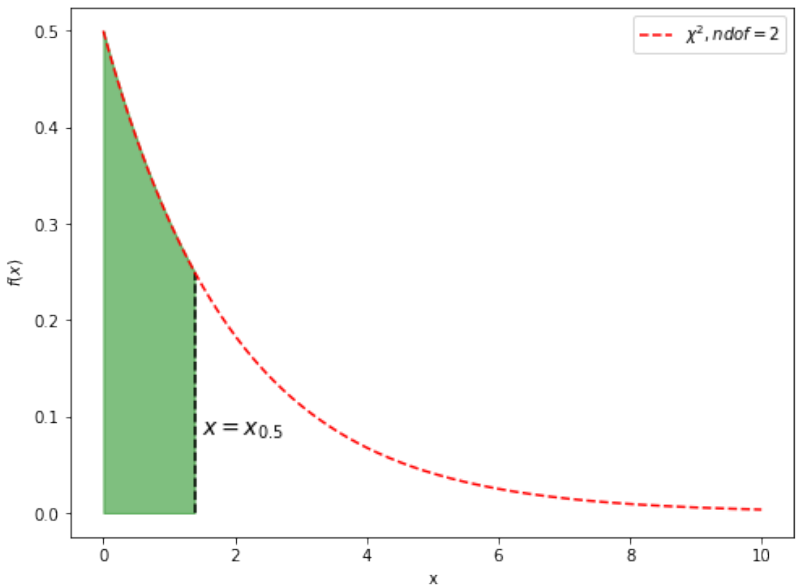
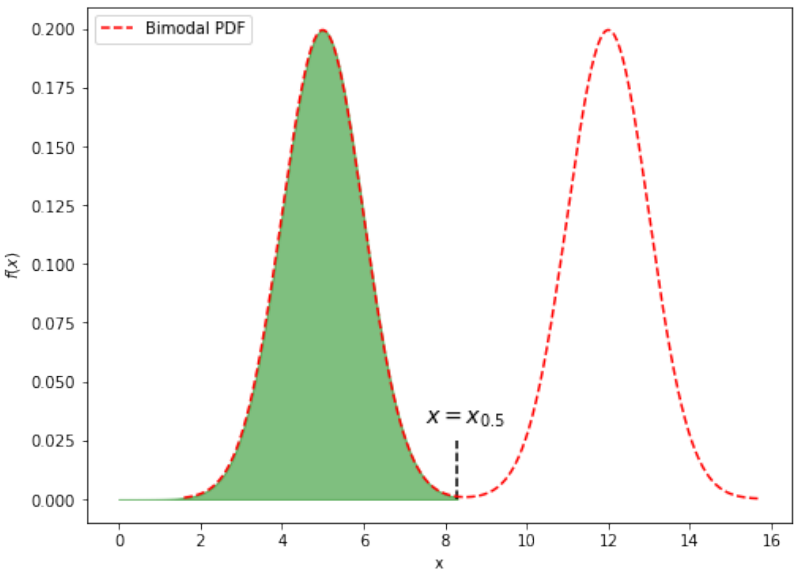
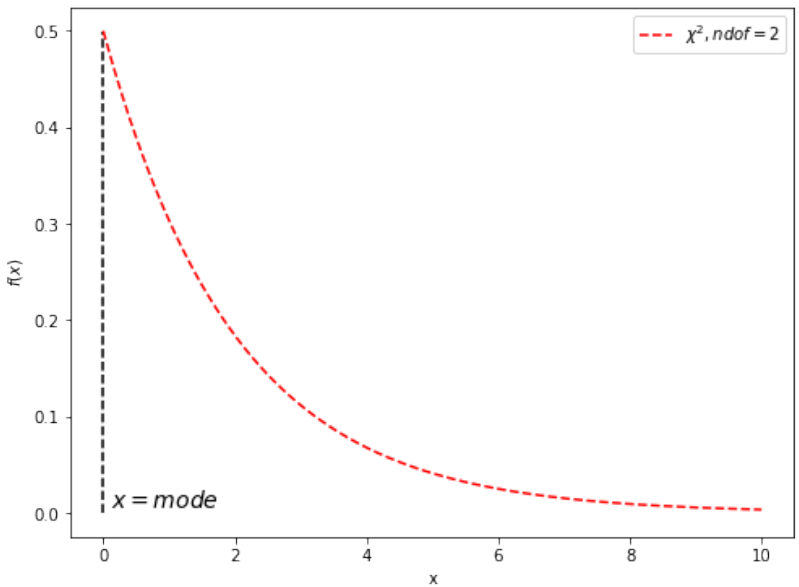
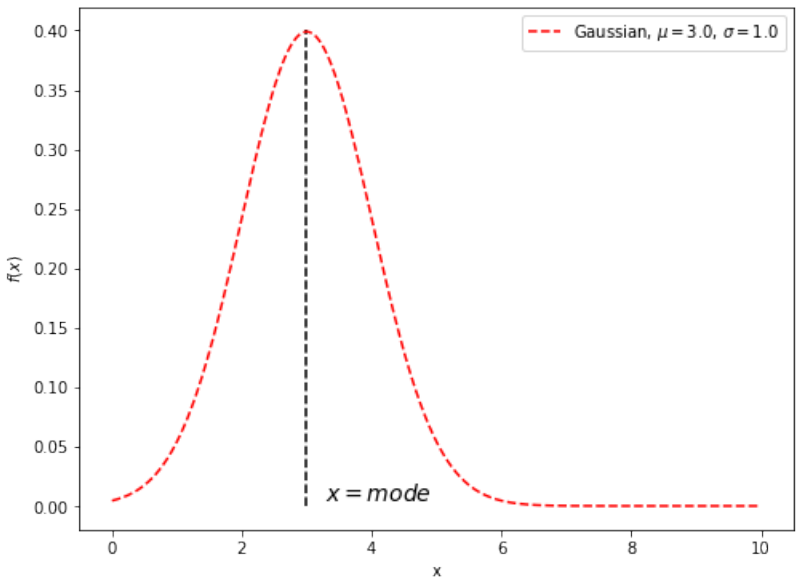




Basic concepts: mode

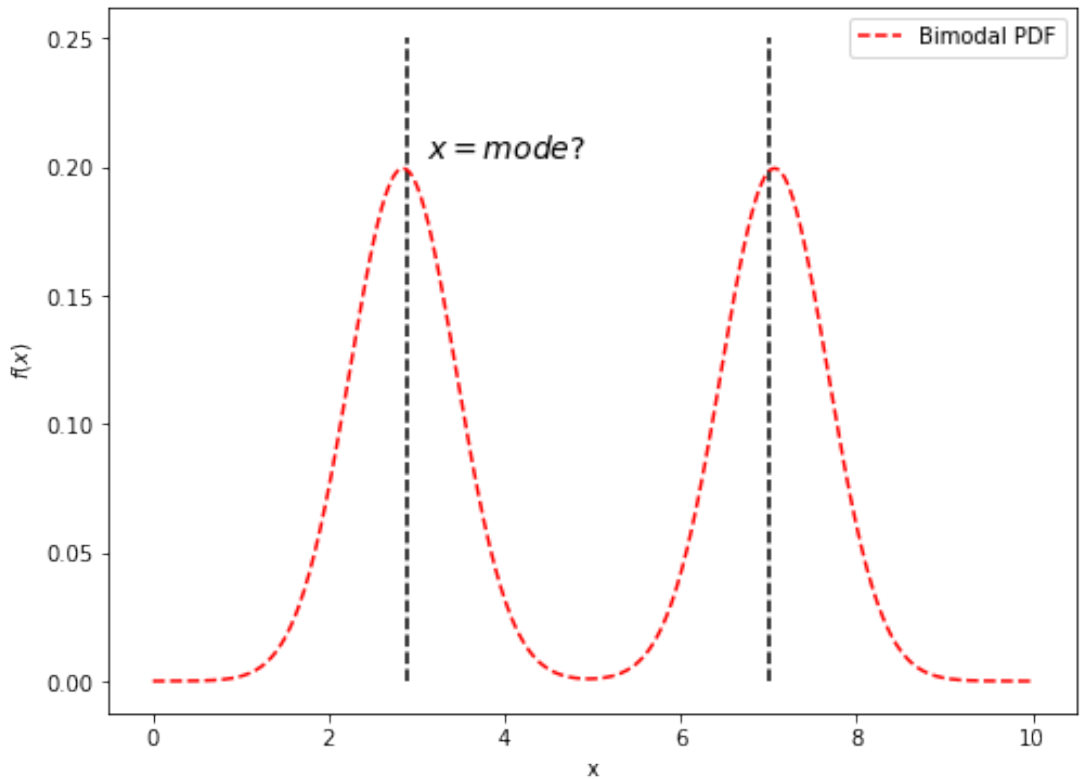
- The **mode** is the value of x for which $pdf\,(x)$ is maximal
  - The *typical location of the variable* is often better captured by the mode





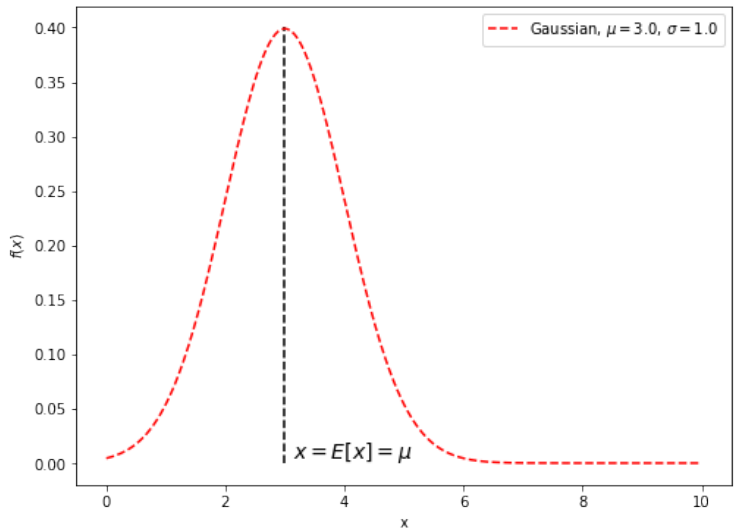
Basic concepts: mode

- mode is the value of x for which $pdf(x)$ is maximal
- when can this breakdown?



Basic concepts: expectation value

- The **expectation value** $E[x]$ of a variable x distributed according to $f(x)$ is often referred to as the **mean** μ.
- $E[x]$ is **not** a function of x, rather depends on form of $f(x)$.

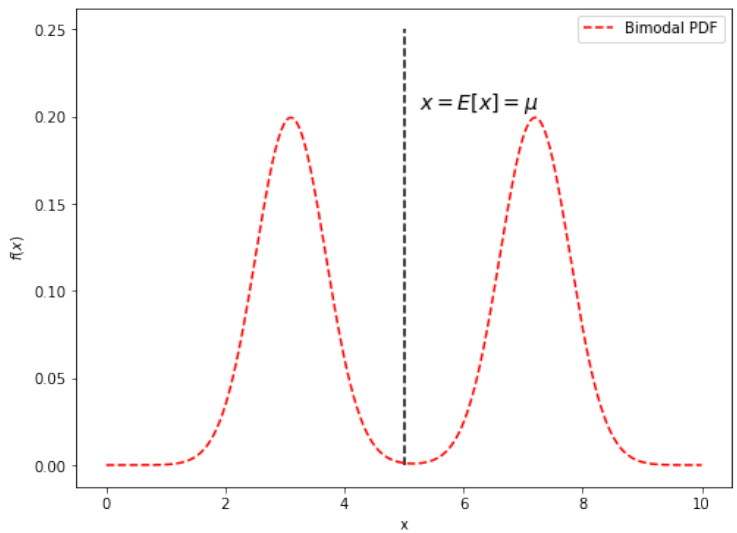


$$E[x] = \int_{-\infty}^{\infty} x.f(x)dx = \mu$$

- If the $f(x)$ is *concentrated in one region*, $E[x]$ represents a measure of where values of x are likely to be observed.
- **When can this interpretation break down?**

Basic concepts: expectation value

- What if $f(x)$ is *multimodal*?, e.g, two gaussian peaks



- x is never equal to μ!

$$E[x] = \int_{-\infty}^{\infty} x.f(x)dx = \mu$$

Basic concepts: variance

- Functions of x also have expectation values
  - e.g. the expectation value of the squared difference between x and μ.
- $E[(x-\mu)^2]$ is called the **variance** V
  - V measures how *spread out* $f(x)$ is
  - Note $E[(x-\mu)^2] = E[x^2] - \mu^2$
- usually use the **standard deviation** σ instead
  - $\sigma = \sqrt{V}$

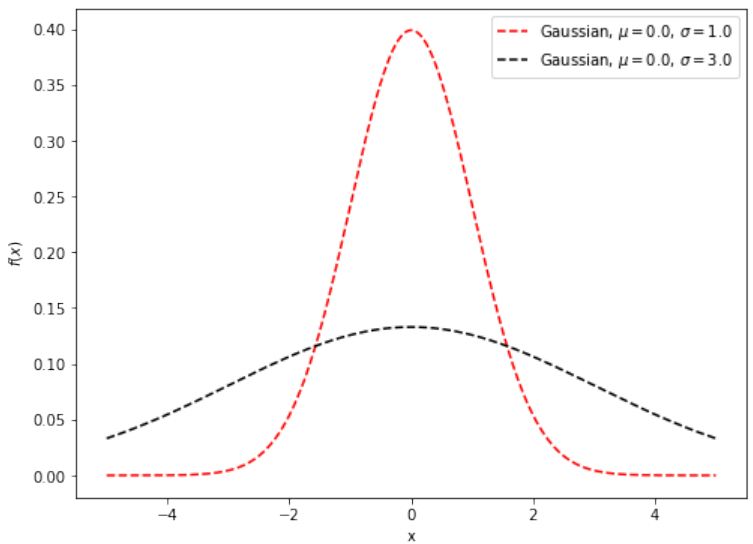


Figure: The two pdfs have the same μ but different σ

Basic concepts: variance

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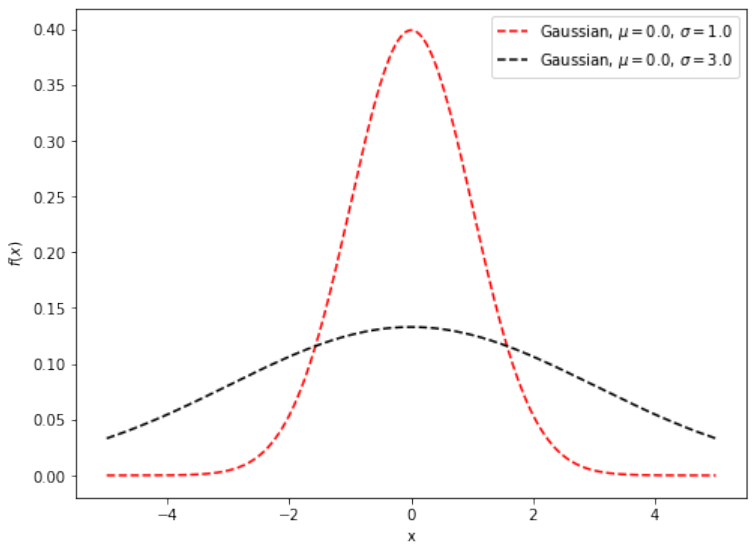
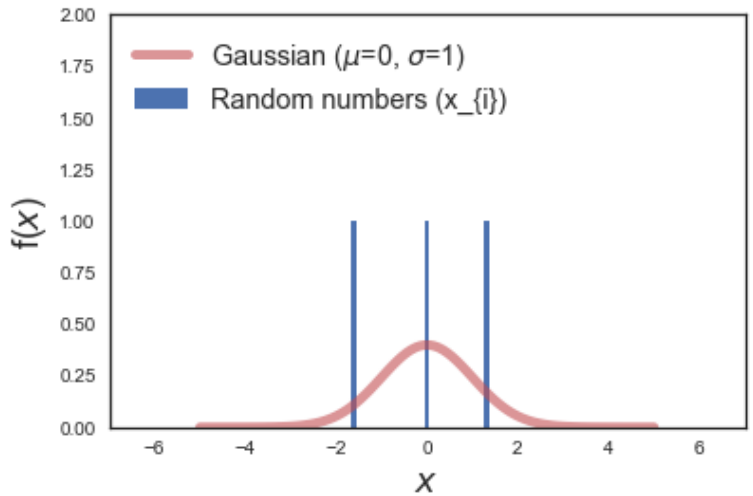


Figure: The two pdfs have the same μ but different σ

Basic concepts: *random* numbers

- We have been talking about abstract notions of probability
  - but what about real data?
  - imagine some data x_i: n observations of some quantity x
  - what then is the μ and σ of x_i?

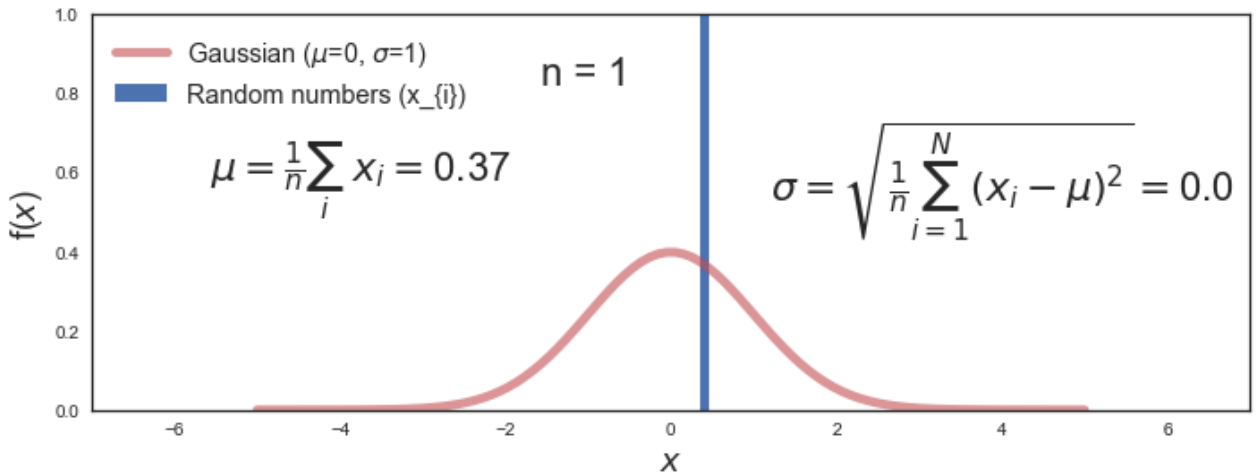


$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i, \qquad \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{N}(x_i - \mu)^2}$$

- Let's think about how these definitions correspond to the defns. for pdfs

Basic concepts: *random* numbers

- *Random* numbers are useful in simulating data that is governed by a pdf
- Software tools can generate random numbers that are governed by any pdf...



Basic concepts: *random* numbers

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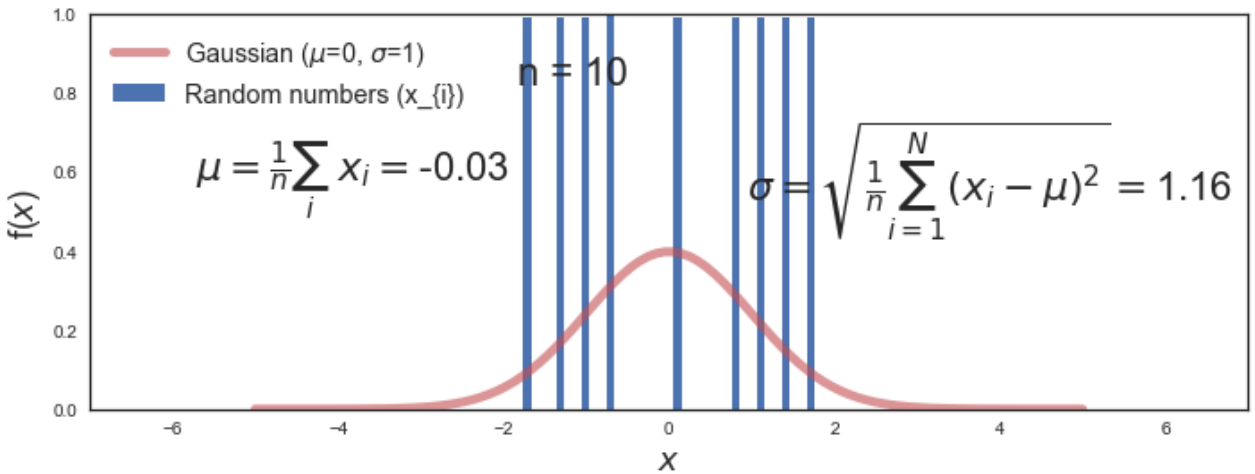

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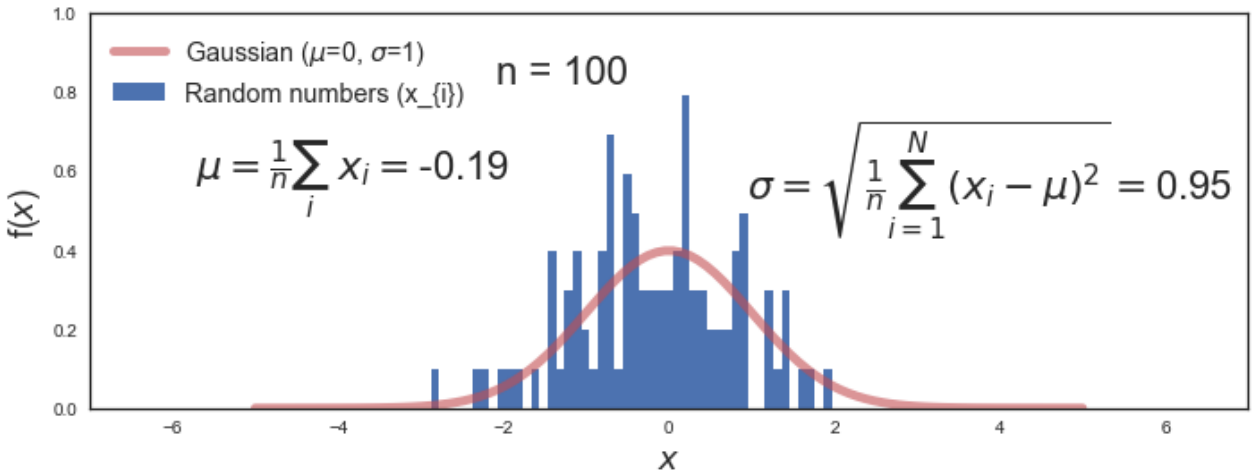

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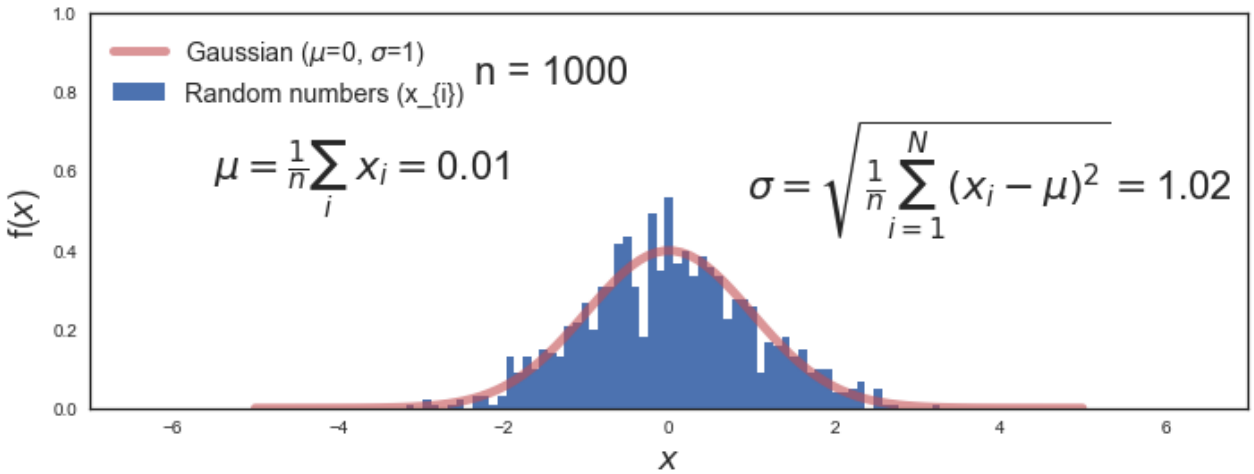

Basic concepts: *random* numbers

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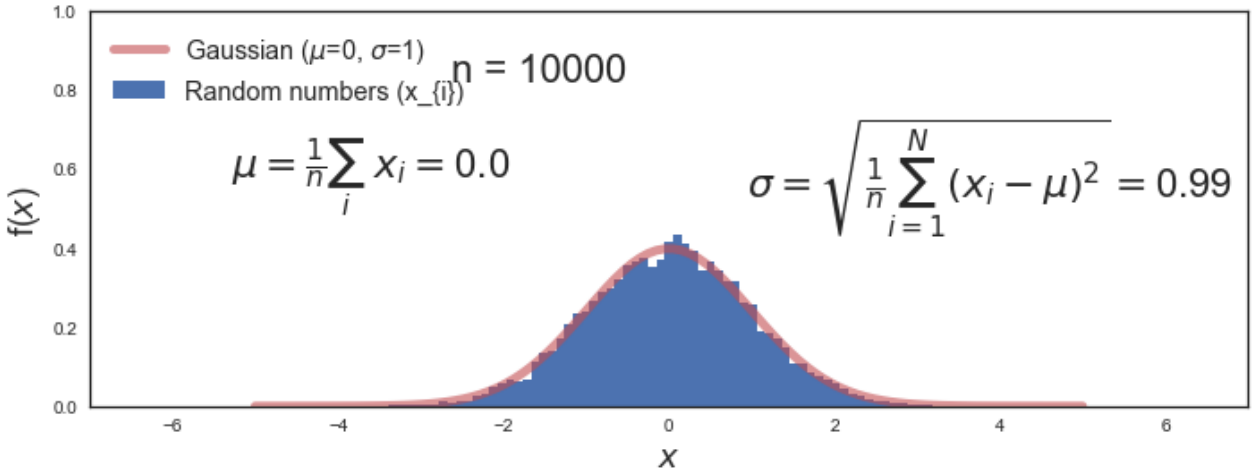
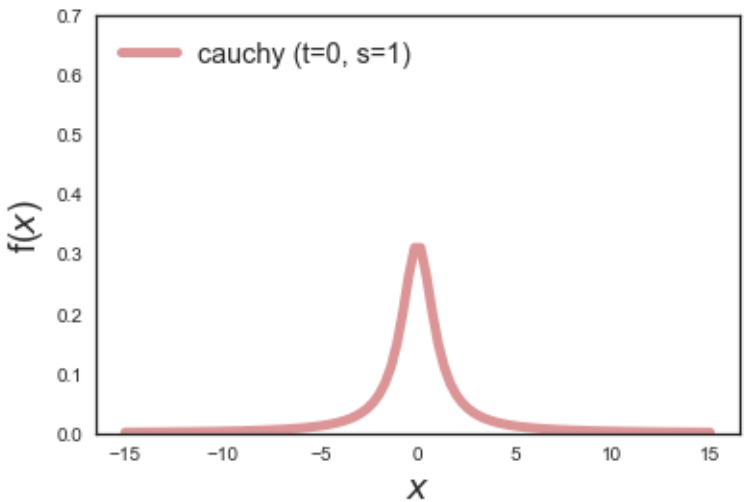


Basic concepts: mean & standard deviation limitations

- When the pdf has *fat tails*, μ and σ stop being useful
  - e.g. the Cauchy pdf

$$f(x; x_0, \gamma) = \frac{1}{\pi\gamma}\left[\frac{\gamma^2}{(x - x_0)^2 + \gamma^2}\right],$$

- This pdf comes up a lot in physics
- $E[x]$ is undefined!
- $E[(x - \mu)^2]$ is undefined!



Basic concepts: mean & standard deviation limitations

- $E[x]$ is undefined!
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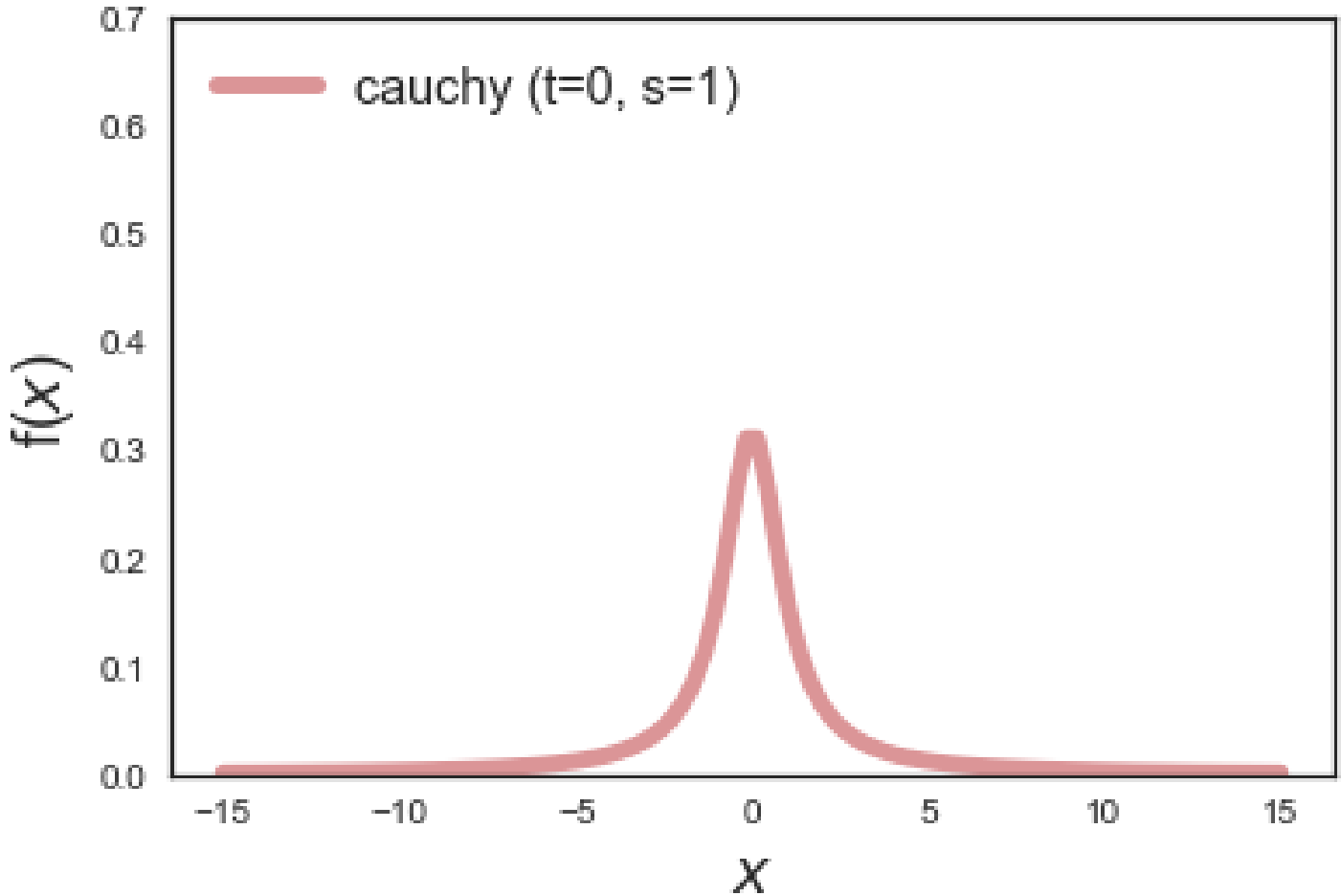
- Taking the μ and σ of random numbers distributed according to a Cauchy does not work



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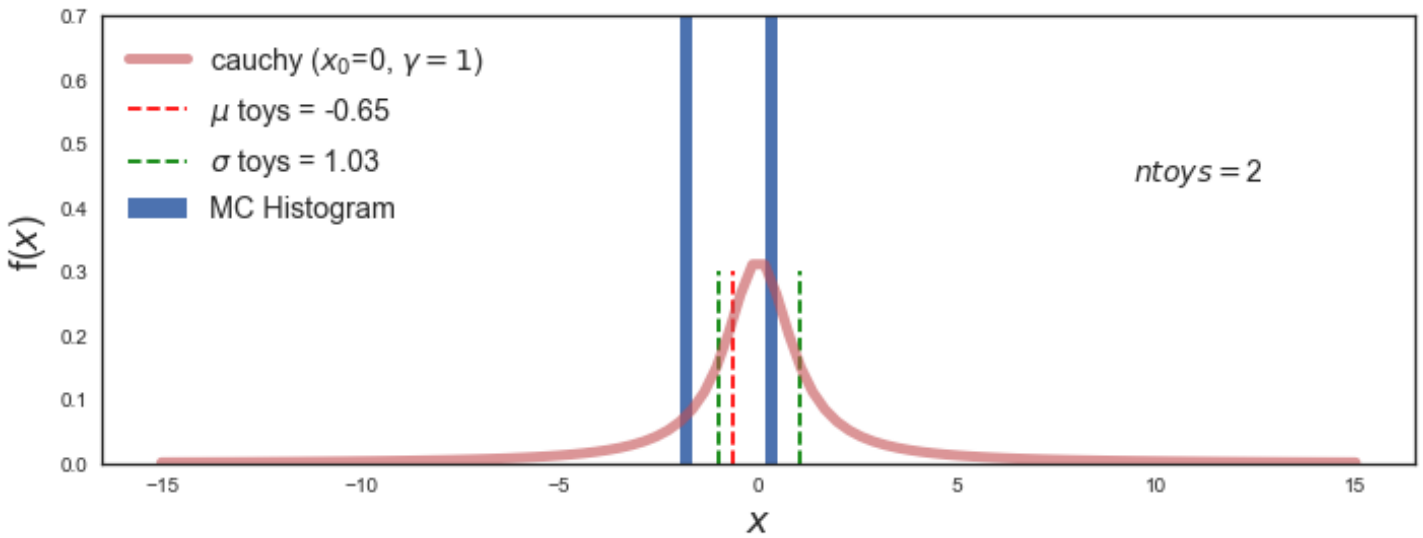

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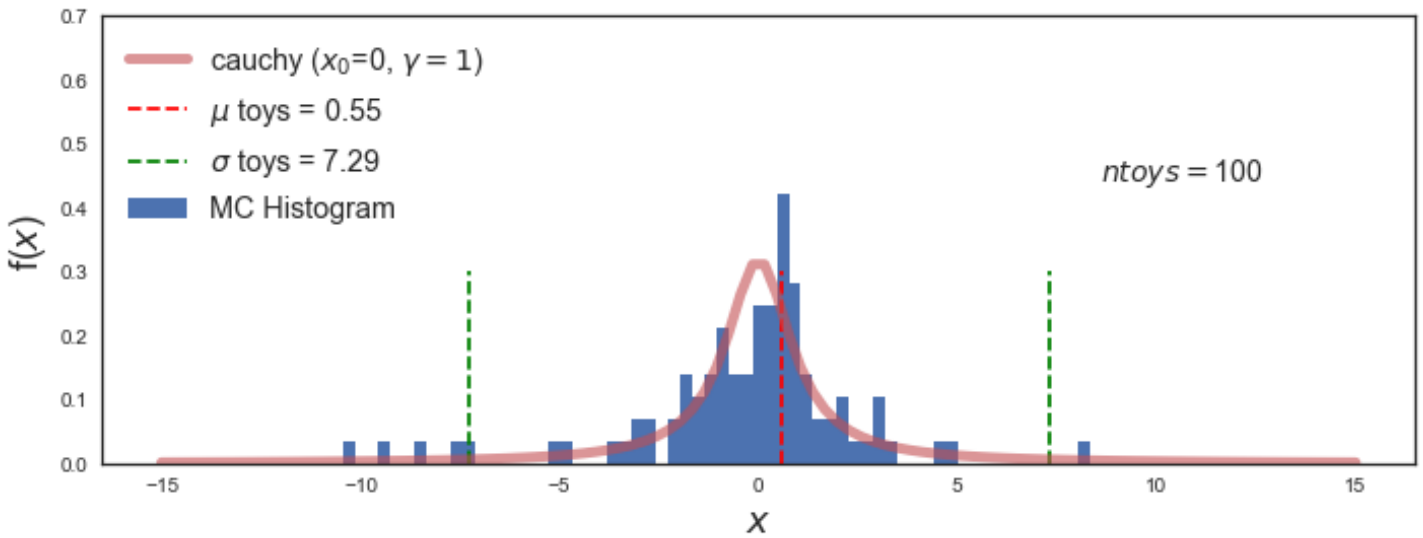

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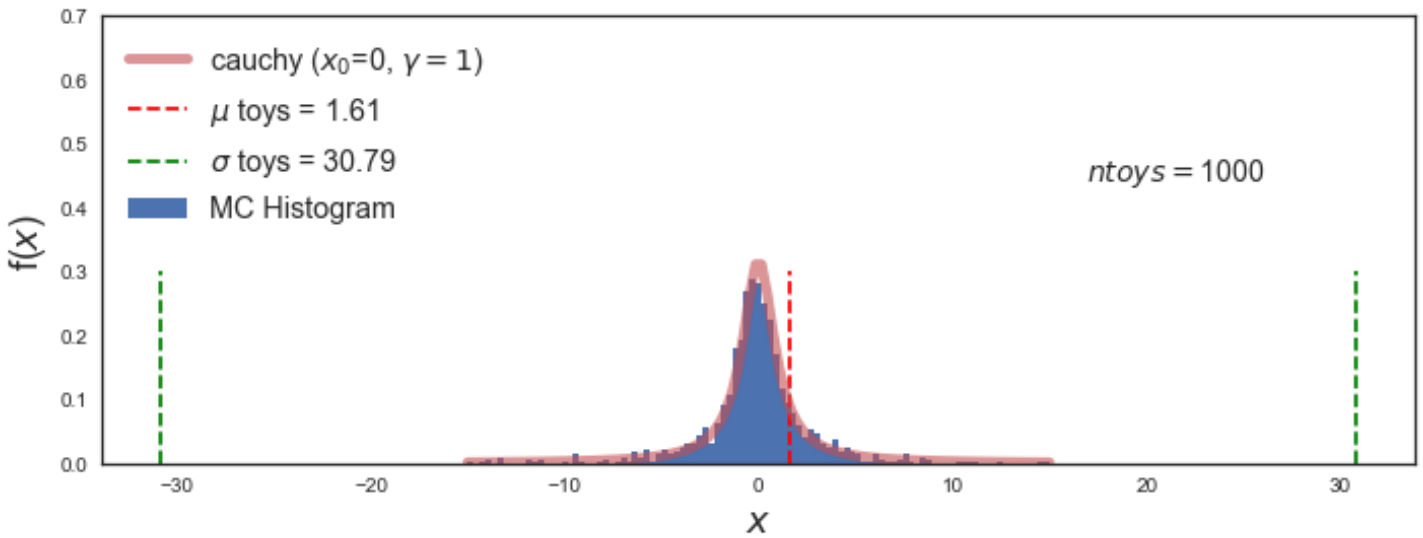

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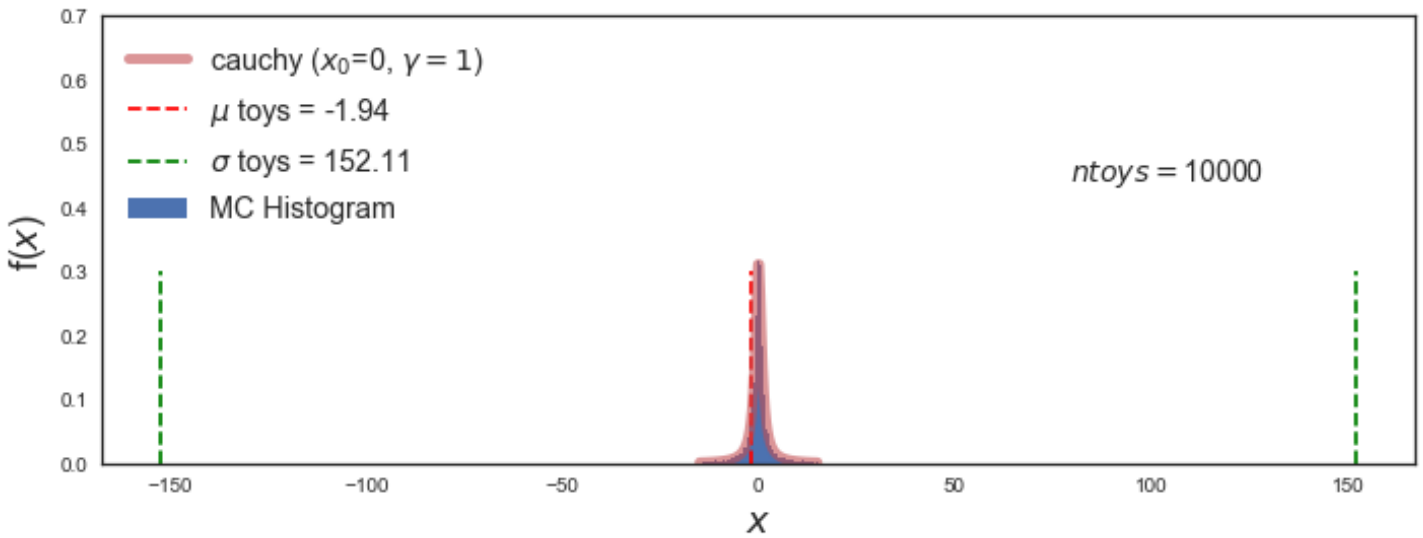
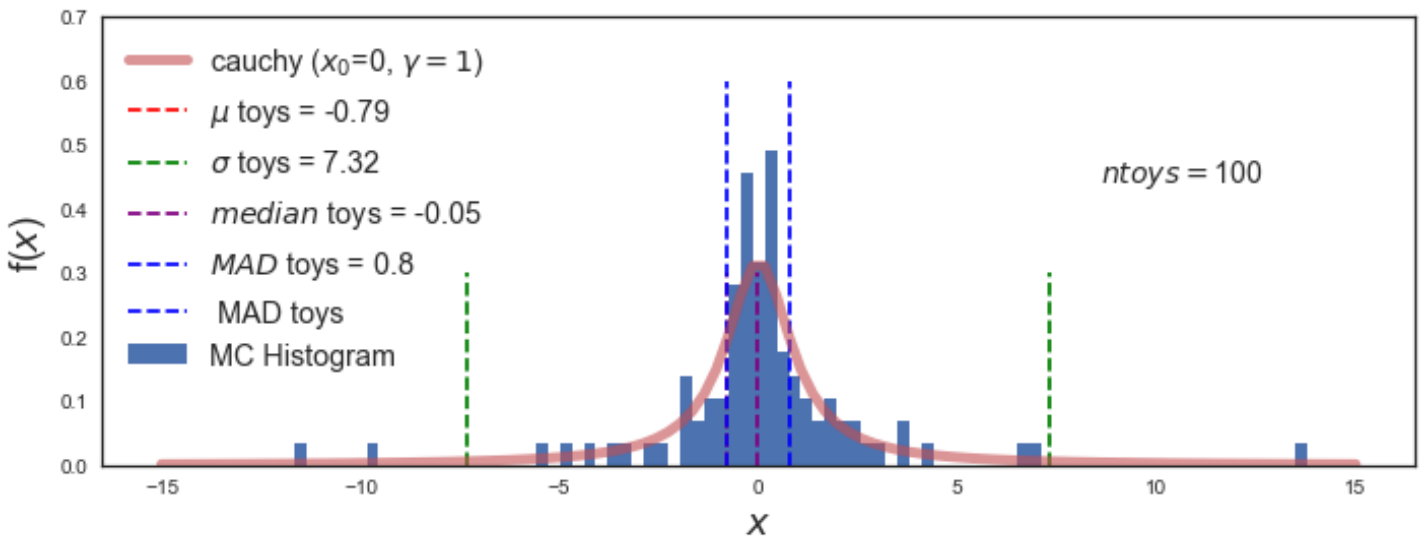


Basic concepts: alternatives: median and MAD

- If you suspect your data has fat tails, it's can be better to avoid the μ and σ
- Instead of μ how about the median?
- Instead of σ how about something MAD? (Mean Absolute Deviation)

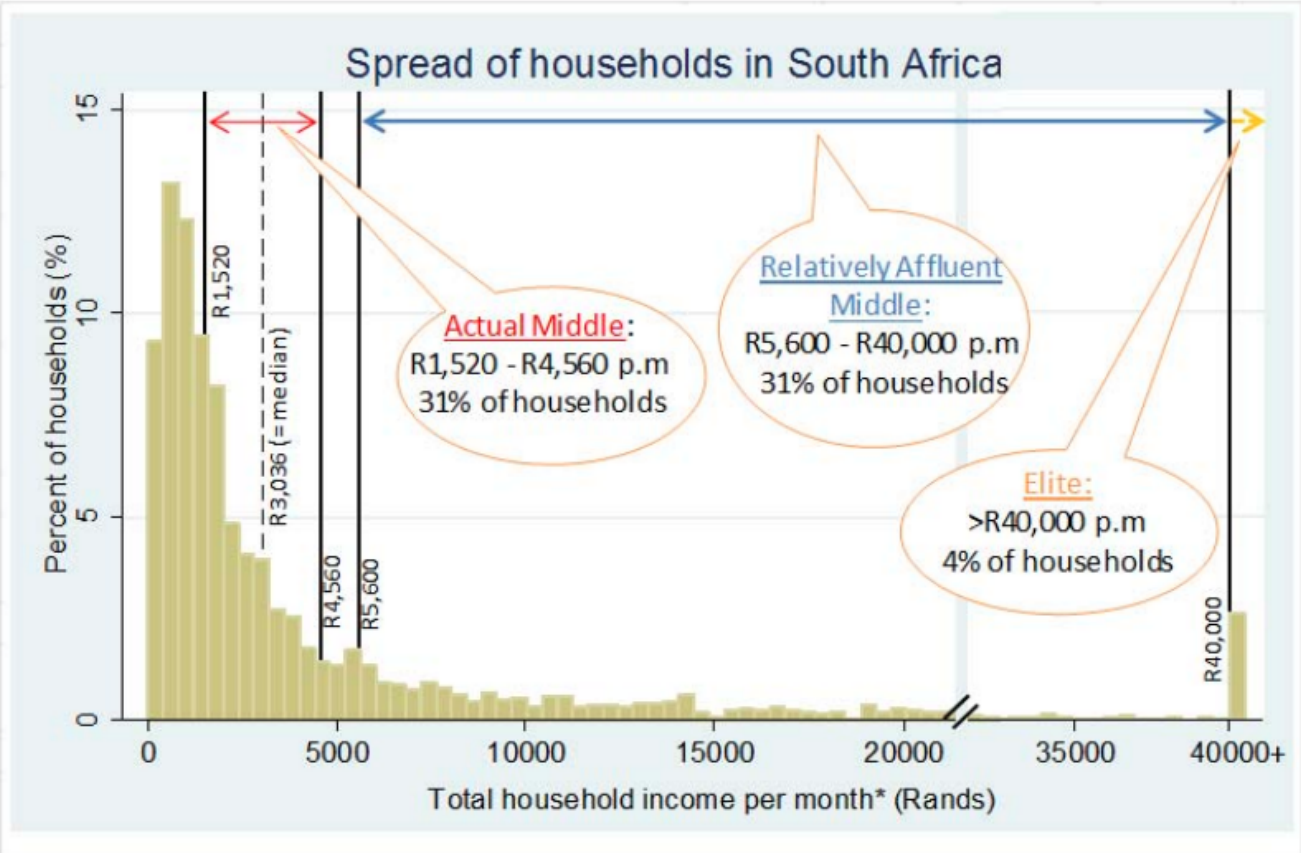
$$MAD = \frac{1}{n}\sum_{i=1}^{n} |x_i - \mu(x)|$$



Basic concepts: alternatives: mode

- When does the median fail?

Figure 1: The spread of households within the income distribution in South Africa, 2008



Source: NIDS 2008, own estimates

Figure: Source: *Who are the middle class in South Africa? Does it matter for policy? Visagie 2013*