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Quantum Walks are Computationally Universal

Quantum Circuit as Quantum Walk-propagation

Quantum Circuit as Quantum Walk-propagation

Evolution of amplitude using 4d Grover coin

Quantum Circuit as Quantum Walk- example gate

FORMAL LANGUAGES

Quick summary:

- A formal language is a set of words from some alphabet Σ
- Example $\mathcal{L}_{eq} = \{a^m b^m | m \in \mathbb{N}\} = \{ab, aabb, aaabbb, \ldots\}$
- Want to find machines which can distinguish words in a given language
- Some, such as $\mathcal{L}_{eq}$ can be recognised by models of computation which are not Turing universal

Motivation for work

Quantum circuits have not been implemented beyond a few qubits

The best quantum computation devices currently are based on liquid state NMR

Latvian quantum finite automata have been specifically developed to model these devices- these are characterised in terms of language acceptance

Spatially distributed input- setup

Designate encoding for input:
Graph structure then directs amplitude encoding words in the language accepted by the graph to an accepting node.

Modulus squared of the amplitude at the accepting node after a designated number of steps = probability of acceptance.

WALK RECOGNISING $\mathcal{L}_{eq}$

Advantages and drawbacks

Well suited to recognising languages with at most one word of each length.
Spatially distributed input allows arbitrarily long strings to be processed in a fixed, small number of steps …

…but increase in input length requires an increase in the number of nodes.

Number of nodes required to accept a given language can be held constant regardless of input length if each input symbol is fed into the structure in turn

Sequentially distributed input- setup

The input can be treated sequentially if we start with it along a chain with two links between each node. The two symbols are represented:

$$a = \begin{pmatrix} \alpha \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ \alpha \\ 0 \\ 0 \end{pmatrix} \tag{1}$$

Coin on this part of the graph is $\sigma_x \otimes \mathbb{I}_2$.
The shape of the graph and the coins at each node determine which words will be accepted by directing amplitude to ‘accepting’ or ‘rejecting’ paths/nodes.

Walks accepting particular strings

Walk accepting $\mathcal{L}_{ab}$

Walk accepting $\mathcal{L}_{ab}$

Quantum inputs

We have so far examined classical inputs. Our setup allows for **quantum inputs**: superpositions of words.

- Each symbol can be in superposition of a or b, $xa + yb$ such that $|x|^2 + |y|^2 = \alpha^2$
- Where symbols match, amplitude is allocated to that symbol as for the classical encoding
- Otherwise amplitude is distributed between the a and b states accordingly

Using quantum inputs, language acceptance becomes *quantum state discrimination*.

Example state discrimination scheme

$$
\begin{aligned}
|\psi_1^4\rangle &= \frac{1}{\sqrt{3}}\left(-|h\rangle + \sqrt{2}\mathrm{e}^{-2\pi i/3}|v\rangle\right) \\
|\psi_2^4\rangle &= \frac{1}{\sqrt{3}}\left(-|h\rangle + \sqrt{2}\mathrm{e}^{+2\pi i/3}|v\rangle\right) \\
|\psi_3^4\rangle &= \frac{1}{\sqrt{3}}\left(-|h\rangle + \sqrt{2}|v\rangle\right) \\
|\psi_4^4\rangle &= |h\rangle
\end{aligned}
$$

[Clarke/VK/Chefles/Barnett/Riis/Sasaki PRA 64 012303 2001]

SUMMARY

We have:

- Introduced new way to implement computation using the discrete time quantum walk
- Shown two ways to do this and discussed examples of each
- Developed the concept of a quantum input...
- ... and applied the results to the problem of quantum state discrimination

FUTURE WORK

- Find more examples of quantum walks accepting formal languages
- Which examples we concentrate on depends on direction we want to take the work
- Further develop concept of quantum input, extend to quantum languages?
- and **lots more!**

Thank you very much for listening!