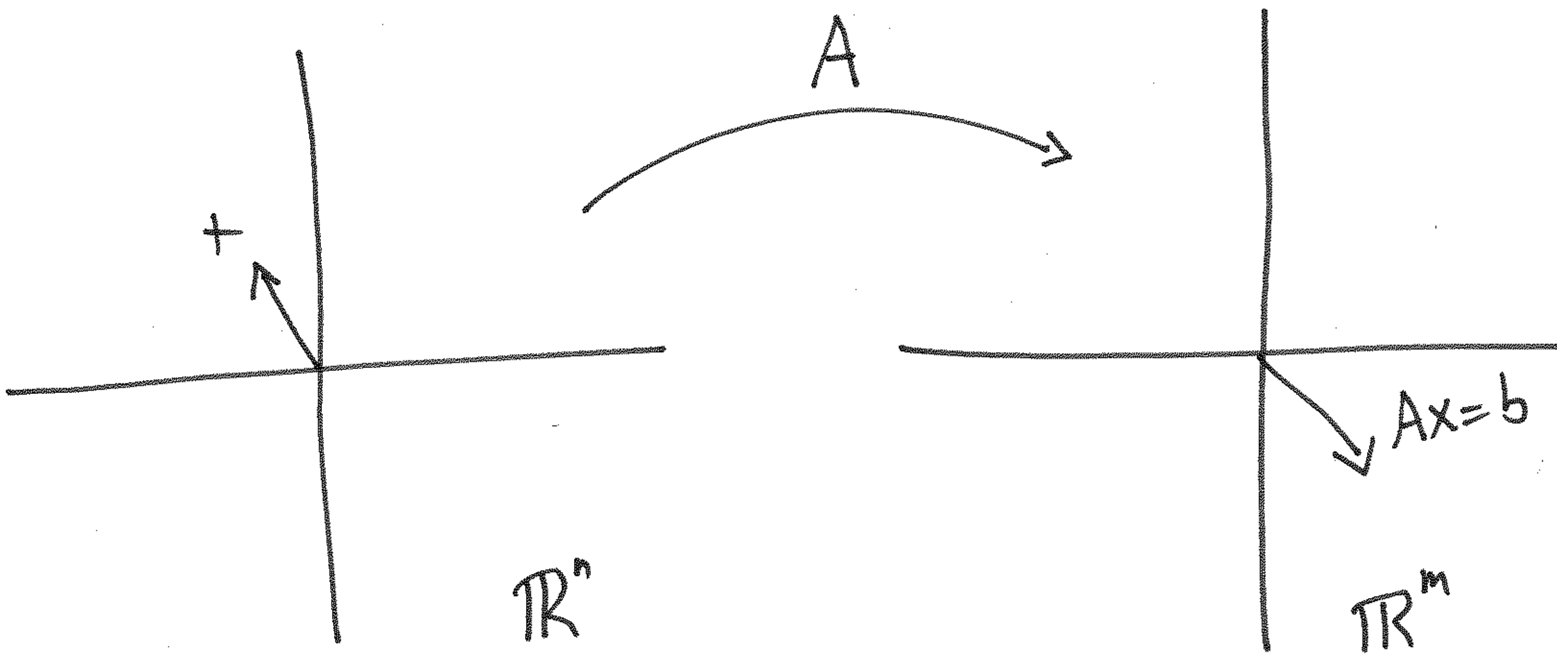
Linear Algebra Picture # 1

the transformational view of a matrix
(version one)

Suppose A is an $m \times n$ matrix.
Then A makes a kind of function.



domain
(set of inputs)

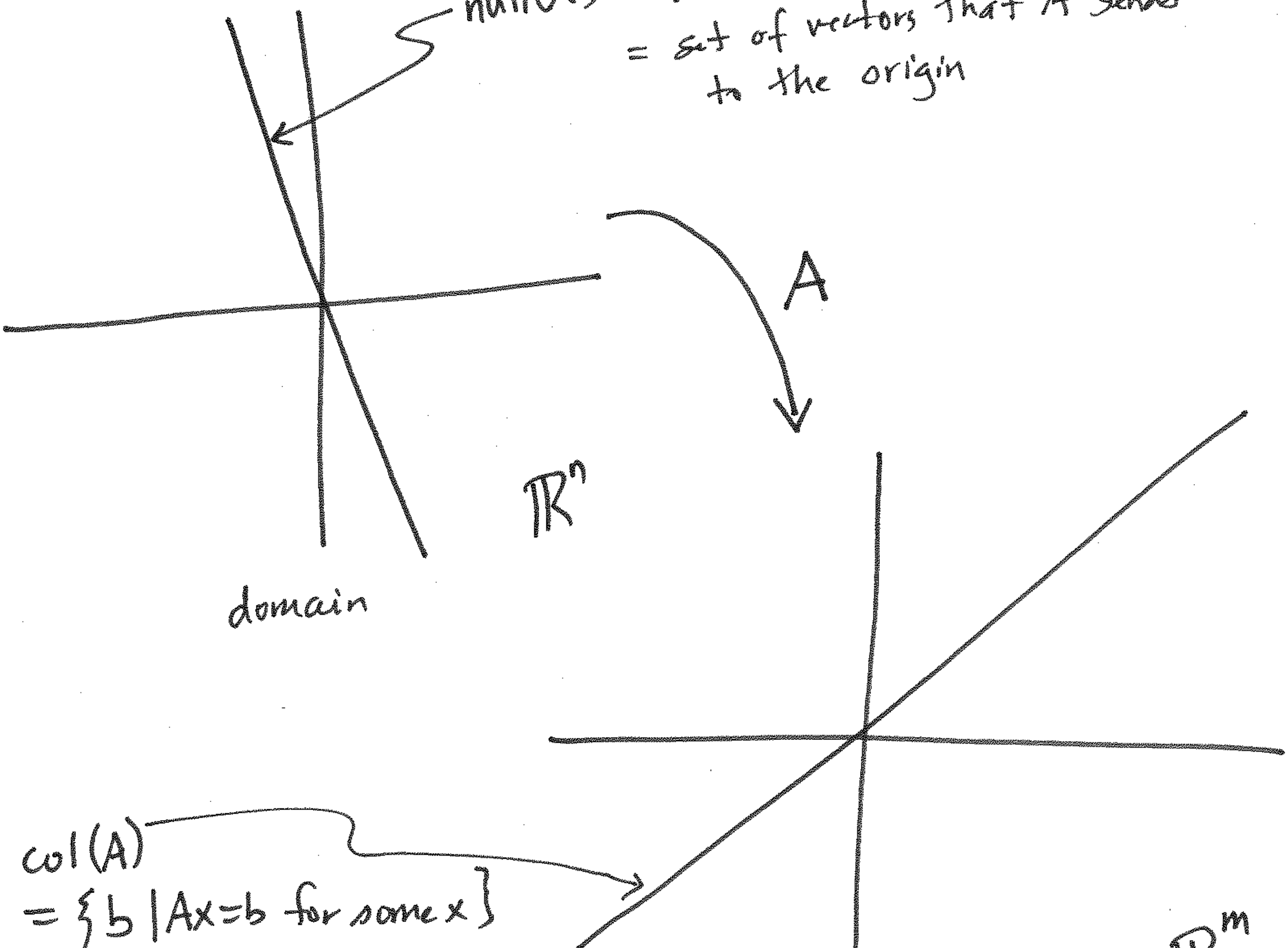
target
(things which could be an output)

Linear Algebra Picture #2

the transformational view of a matrix (version two)

Suppose A is an $m \times n$ matrix. Then A makes a kind of function

$\text{null}(A) = \{x \mid Ax = 0\}$ = the nullspace
= set of vectors that A sends to the origin

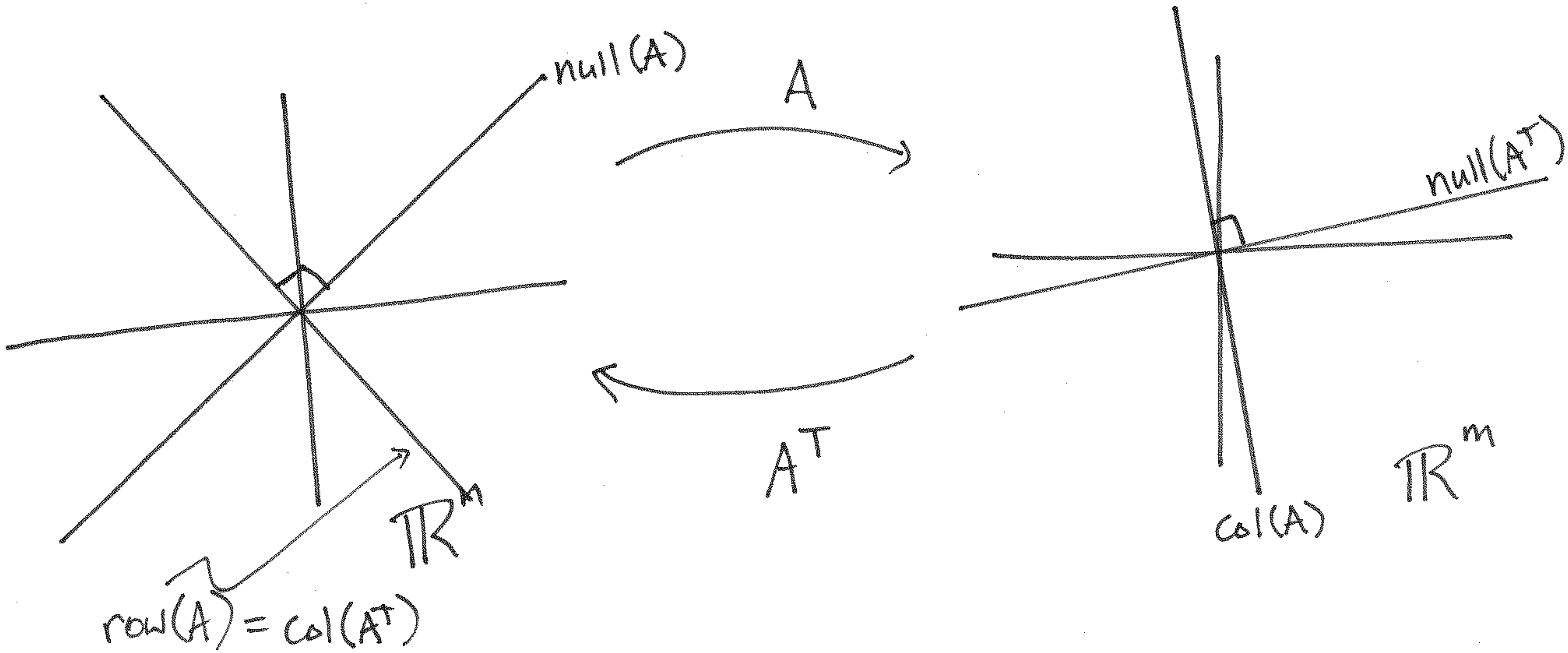


$\text{col}(A)$
$= \{b \mid Ax = b \text{ for some } x\}$
= column space
= the image of $\mathbb{R}^n$ under A
= the collection of all realized outputs.

Linear Algebra Picture #3

the transformational view of a matrix (version three)

Suppose A is an $m \times n$ matrix. Then A makes a function $\mathbb{R}^n \to \mathbb{R}^m$, and A^T makes a function $\mathbb{R}^m \to \mathbb{R}^n$



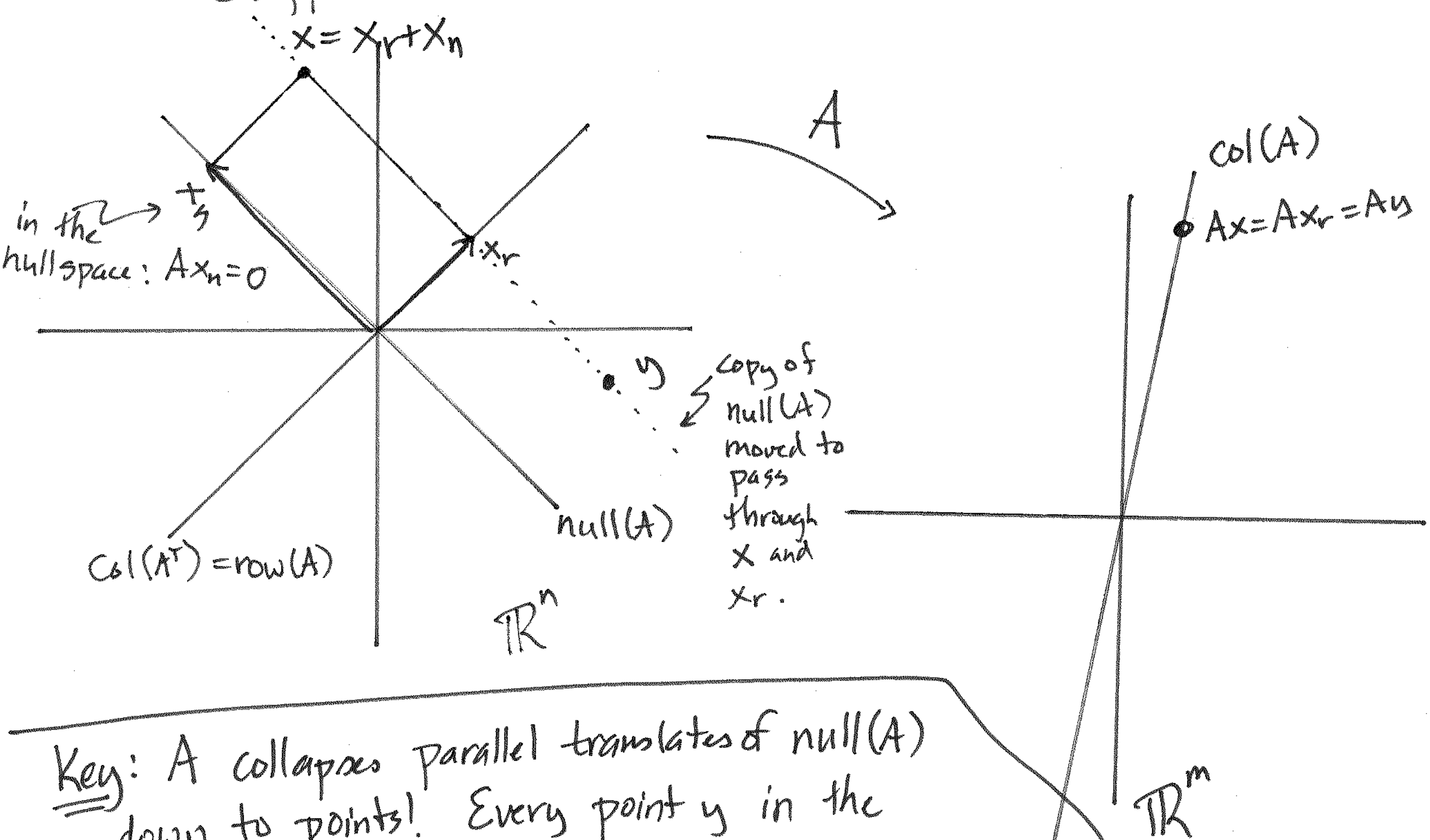
<u>Fundamental Theorem of Linear Algebra</u>: Each of these pairs forms a pair of orthogonal complementary subspaces

- the null space & the row space
- the column space and the left nullspace

Linear Algebra Picture #4

The transformational view of a matrix (final version)

Suppose A is an $m \times n$ matrix.



Key: A collapses parallel translates of null(A) down to points! Every point y in the set $y + \mathrm{null}(A)$ has the same image $Ay = Ax = Ax_r$