Name: __Key_____

Exercise 1. *What is the domain of the function*

$$f(x) = \sqrt{4x - 3}?$$

Domain is where $4x - 3 \geq 0$.

$$4x - 3 \geq 0$$
$$4x \geq 3$$
$$x \geq \frac{3}{4}$$

Domain: $\left[\frac{3}{4}, \infty\right)$

Exercise 2. *Determine if the following functions are even, odd, or neither:*

(a) $f(x) = 2x^5 + 6x^2 + 2$

(b) $g(x) = x^2 \cos(x)$

(c) $h(x) = 3x^3 - x.$

a) $f(-x) = 2(-x)^5 + 6(-x)^2 + 2$
$$= -2x^5 + 6x^2 + 2$$

Since $f(x) \neq f(-x) \neq -f(x)$, f is neither.

b) $g(-x) = (-x)^2 \cos(-x) = x^2 \cos(x) = g(x)$. Even

c) $h(-x) = 3(-x)^3 - (-x) = -3x^3 + x = -(3x^3 - x) = -h(x)$

Odd

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Exercise 3. *Calculate the limit*

$$\lim_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3}.$$

$$\lim_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3} = \lim_{x \to -3} \frac{(x-3)(x+3)}{(x-1)(x+3)} = \lim_{x \to -3} \frac{x-3}{x-1}$$

$$= \frac{-3-3}{-3-1}$$

$$= \frac{-6}{-4} = \boxed{\frac{3}{2}}$$

Exercise 4. *Show the the equation*

$$3x^3 - 4x^2 + x - 1 = 0$$

has a solution between 1 and 2.

Let $f(x) = 3x^3 - 4x^2 + x - 1$. Since f is a polynomial, f is continuous everywhere. Then we see that

$$f(1) = 3(1)^3 - 4(1)^2 + (1) - 1 = -1 < 0$$

$$f(2) = 3(2)^3 - 4(2)^2 + (2) - 1 = 24 - 16 + 2 - 1 = 9 > 0.$$

Thus, the IVT says there is a c in $(1,2)$ such that $f(c) = 0$.

Exercise 5. *Find*

$$\lim_{x \to 4^+} \frac{2x}{4x - 16} \quad and \quad \lim_{x \to 4^-} \frac{2x}{4x - 16}.$$

$$\lim_{x \to 4^\pm} \frac{2x}{4x - 16} = \lim_{x \to 4^\pm} \left[(2x) \left(\frac{1}{4x - 16} \right) \right]. \quad \text{Since} \quad \lim_{x \to 4^\pm} 2x = 8,$$

$$\lim_{x \to 4^+} \frac{1}{4x - 16} = \infty, \quad \text{and} \quad \lim_{x \to 4^-} \frac{1}{4x - 16} = -\infty, \quad \text{we have}$$

$$\lim_{x \to 4^+} \frac{2x}{4x - 16} = \infty \quad \& \quad \lim_{x \to 4^-} \frac{2x}{4x - 16} = -\infty.$$

Exercise 6. *You are walking across the Green checking out the latest on Instragram when your annoying friend Mike (You can't stand Mike!) comes up behind you and scares you. As you flail about in terror, you throw your phone into the air with an initial velocity of 12 ft/s. The position function for your phone is given by*

$$s(t) = -16t^2 + 12t + 4.$$

You watch in horror as your phone careens toward the pavement. After one second, your phone slams into the unforgiving concrete. What was the velocity of your phone when it hits the ground shattering not only your screen, but any hope that you would have enough money for that spring break trip you were planning?

The velocity function $v(t)$ is found by taking the first derivative of $s(t)$. Thus,

$$v(t) = s'(t) = \frac{d}{dt}\left[-16t^2 + 12t + 4 \right]$$

$$= -16(2t) + 12(1) + 0$$

$$= -32t + 12$$

Then,

$$v(1) = -32(1) + 12 = -20 \ ft/s.$$

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Exercise 7. *Using the **definition of the derivative**, show that the derivative of $f(x) = x^2 + x$ is $f'(x) = 2x + 1$.*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h + 1)}{h}$$

$$= \lim_{h \to 0} 2x + h + 1$$

$$= 2x + 1$$

Exercise 8. *Differentiate the following functions:*

(a) $f(x) = 3x^3 - 4x^2 + 5x - 1$

(b) $g(x) = x^2(1 - 2x)$

(c) $h(x) = 2^{40}$

(d) $r(x) = \frac{3}{x^3} - 10\sin(x)$

a) $f'(x) = 3(3x^2) - 4(2x) + 5(1) + 0$

$\qquad = 9x^2 - 8x + 5$

b) $g(x) = x^2(1-2x) = x^2 - 2x^3$

$\quad g'(x) = 2x - 2(3x^2) = 2x - 6x^2$

c) Since 2^{40} is a constant,

$\qquad h'(x) = 0.$

d) $r(x) = 3x^{-3} - 10\sin(x)$

$\quad r'(x) = 3(-3x^{-4}) - 10(\cos(x))$

$\qquad = -9x^{-4} - 10\cos(x)$

(e) $u(x) = x^3 \tan(x)$

(f) $m(x) = \frac{\cos(x)}{1-\sin(x)}$

(g) $k(x) = (x^2 + 1)\cos(x)$

(h) $v(x) = \frac{3x^2+1}{x^2-4}$

e) $u'(x) = x^3 \frac{d}{dx}\left(\tan(x)\right) + \tan(x)\frac{d}{dx}\left(x^3\right)$

$= x^3 \sec^2 x + 3x^2 \tan(x)$

f) $m'(x) = \dfrac{(1-\sin(x))\frac{d}{dx}\left(\cos(x)\right) - \cos(x)\frac{d}{dx}\left(1-\sin(x)\right)}{\left(1-\sin(x)\right)^2}$

$= \dfrac{(1-\sin(x))(-\sin(x)) - \cos(x)\left(-\cos(x)\right)}{\left(1-\sin(x)\right)^2}$

$= \dfrac{-\sin(x) + \sin^2(x) + \cos^2(x)}{\left(1-\sin(x)\right)^2} = \dfrac{1-\sin(x)}{\left(1-\sin(x)\right)^2} = \dfrac{1}{1-\sin(x)}$

g) $k'(x) = (x^2+1)\frac{d}{dx}\left(\cos(x)\right) + \cos(x)\frac{d}{dx}\left(x^2+1\right)$

$= (x^2+1)(-\sin(x)) + 2x\cos(x)$

h) $v'(x) = \dfrac{(x^2-4)\frac{d}{dx}\left(3x^2+1\right) - (3x^2+1)\frac{d}{dx}\left(x^2-4\right)}{\left(x^2-4\right)^2}$

$= \dfrac{(x^2-4)(6x) - (3x^2+1)(2x)}{\left(x^2-4\right)^2}$

$= \dfrac{6x^3 - 24x - 6x^3 - 2x}{\left(x^2-4\right)^2} = \dfrac{-26x}{\left(x^2-4\right)^2}$