MA125-8B Final Exam Practice

Name: _____

Exercise 1. *Find the domain of the function*

$$f(x) = \frac{3}{\sqrt{5x-2}}.$$

Exercise 2. *Determine if the following functions are even, odd, or neither:*

(a) $f(x) = 2x^5 + 6x^2 + 2$

(b) $g(x) = \cos(x) + \sin^2(x)$

(c) $h(x) = x^3 - 5x$.

Exercise 3. *Calculate the limit*

$$\lim_{x \to -3} \frac{x^2 - 9}{2x^2 + 7x + 3}.$$

Exercise 4. *Determine if the following limits exists. If it does, find the limit. If it does not, explain why.*

(a) $\lim_{x \to 3} (2x + |x - 3|)$

(b) $\lim_{x \to -6} \frac{2x + 12}{|x + 6|}$

Exercise 5. *Find the limit*

$$\lim_{x \to 0} \frac{\sin(3x)\sin(5x)}{x^2}.$$

Exercise 6. *Show the the equation*

$$3x^3 - 4x^2 + x - 1 = 0$$

has a solution between 1 and 2.

Exercise 7. *Find*

$$\lim_{x\to 2^+} \frac{x}{2x-4} \quad \textit{and} \quad \lim_{x\to 2^-} \frac{x}{2x-4}.$$

Exercise 8. *Evaluate the limit*

$$\lim_{x\to\infty} \frac{2x^2 - x + 3}{x^2 - 6x + 13}.$$

Exercise 9. *If the position of an intergalactic spaceship is given by*

$$s(t) = \frac{e^x}{100} + x,$$

find the instantaneous velocity of the ship at time $t = 15$ seconds.

Exercise 10. *Differentiate the following functions:*

(a) $f(x) = 3x^3 - 4x^2 + 5x - 1$

(b) $f(x) = x^2(1 - 2x)$

(c) $f(x) = 2^{40}$

(d) $f(x) = \frac{3}{x^3} - 10\sin(x)$

(e) $f(x) = x^3 \tan(x)$

(f) $f(x) = \frac{\cos(x)}{1-\sin(x)}$

(g) $f(x) = \sin(x\cos(x))$

(h) $f(x) = (4x - 15x^3)^{48}$

(i) $f(x) = \ln\left(\frac{x^2-3}{x^3+2x+1}\right)$

(j) $f(x) = (x^2+1)\ln|\sin(x^2)|$

(k) $f(x) = xe^x + xe^{x^2}$

(l) $f(x) = \ln|e^3|$

Exercise 11. *Use a linear approximation to estimate* $(8.05)^{\frac{2}{3}}$.

Exercise 12. *Show that*

$$2x - 1 - \sin x = 0$$

has exactly one real root.

Exercise 13. *Given the function*

$$f(x) = \frac{x}{x^2 - 9}$$

determine the domain, intercepts, symmetry, and asymptotes of the function. Additionally, find where the function is increasing/decreasing as well as any local min/max values. Finally, sketch the curve.

Exercise 14. *A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) A minimum?*

Exercise 15. *The function* $f(x) = x^5 - 3x^3 - 2$ *has one positive real root. Determine between which two consecutive positive integers does this root exist. Using the larger of these two numbers as your* x_0*, use Newton's method to find* x_1*.*

Exercise 16. *Using the right endpoint rule, use a Riemann sum with $n = 3$ terms to approximate the integral*

$$\int_1^4 \frac{x^2 - 1}{x}\, dx.$$

Is it possible to determine if this is an over or under approximation?

Exercise 17. *Evaluate the following integrals.*

(a) $\displaystyle\int_0^3 x^2 \cos(x^3)\, dx$

(b) $\displaystyle\int_0^{\frac{\pi}{8}} \sec(2\theta)\tan(2\theta)\, d\theta$

(c) $\displaystyle\int_0^1 \frac{x^2}{3x^3+1}\, dx$

(d) $\displaystyle\int_1^2 e^x \cos(2e^x)\, dx$

(e) $\int 2x(x^2+1)\,dx$

(f) $\int \cos(x)\sin(x)\,dx$

(g) $\int \frac{(\ln x)^2}{x}\,dx$

(h) $\int xe^{2x^2+1}\,dx$

Exercise 18. *Determine the inverse function* $f^{-1}(x)$ *for*

$$f(x) = 1 + \sqrt{2 + 3x}.$$

Exercise 19. *Find the average value of the function* $f(x) = \sqrt[3]{x}$ *on the interval* $[1, 8]$.

Exercise 20. *Determine the derivative of the function*

$$f(x) = \int_x^3 \frac{1}{t^2 + 1}\, dt$$