Homework 4

§§2.6, 2.7, 2.9

1. (2 points) Which of the following must be true for any set of seven vectors in $\mathbb{R}^5$? Answer "YES" (always true), "NO" (always impossible), or "MAYBE" (if it is sometimes possible and sometimes not). You do not have to give reasons or show your work. No partial credit in this question.

   (a) The vectors span $\mathbb{R}^5$.

   (b) If we put the seven vectors as the columns of a matrix A, then the matrix equation $AX = 0$ must have infinitely many solutions.

   (c) Suppose we put the seven vectors as the columns of a matrix A. Then for each b in $\mathbb{R}^5$, the matrix equation $Ax = b$ must be consistent.

   (d) If every vector b in $\mathbb{R}^5$ can be written as a linear combination of our seven vectors, then in fact every b in $\mathbb{R}^5$ can be written in infinitely many different ways as a linear combination of our seven vectors.

2. (3 points) Circle **TRUE** if the statement is always true, otherwise circle **FALSE**. No partial credit in this question.

   (a) If A is a 3×100 matrix of rank 2, then $\dim(\text{Nul } A) = 97$.

   A. **TRUE** B. **FALSE**

   (b) There exists a 3×5 matrix with rank 4.

   A. **TRUE** B. **FALSE**

   (c) If A is an 9×4 matrix with a pivot in each column, then $\text{Nul } A = \{0\}$.

   A. **TRUE** B. **FALSE**

3. (3 points) Consider the following matrix A and its row reduced echelon form:

$$\begin{pmatrix} 2 & 4 & 7 & -16 \\ 3 & 6 & -1 & -1 \\ 5 & 10 & 6 & -17 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

(a) (1 point) Find Null space of the matrix A.

(b) (2 points) For each of the following vectors v_1 and v_2, decide if they are in $\mathrm{Nul}(A)$, and if so, write them as a linear combination of your basis from the previous part.

$$v_1 = \begin{pmatrix} 7 \\ 3 \\ 1 \\ 2 \end{pmatrix}, v_2 = \begin{pmatrix} -5 \\ 2 \\ -2 \\ -1 \end{pmatrix}$$

4. (4 points) Find bases for the column space and null space of

$$A = \begin{pmatrix} 0 & 1 & -3 & 1 & 0 \\ 1 & -1 & 8 & -7 & 1 \\ -1 & -2 & 1 & 4 & -1 \end{pmatrix}$$

5. (4 points) Find a basis for the subspace V of $\mathbb{R}^4$ given by:

$$V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; x + 2y - 3z + w = 0 \right\}.$$

6. (4 points) Find the rank of the following matrix and specify which columns are independent and which columns are dependent. If they are dependent, specify how.

$$A = \begin{pmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{pmatrix}$$