Homework 3

§§2.3–2.5

1. (8 points) The professor in your widgets and gizmos class is trying to decide between three different grading schemes for computing your final course grade. The schemes are based on homework (HW), quiz grades (Q), midterms (M), and a final exam (F). The three schemes can be described by the following matrix A:

$$\begin{array}{l} \\ \textbf{Scheme 1} \\ \textbf{Scheme 2} \\ \textbf{Scheme 3} \end{array} \begin{array}{c} \begin{array}{cccc} \textbf{HW} & \textbf{Q} & \textbf{M} & \textbf{F} \end{array} \\ \begin{pmatrix} 0.1 & 0.1 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0.4 & 0.4 \\ 0.1 & 0.1 & 0.6 & 0.2 \end{pmatrix} \end{array}$$

Suppose that you have a score of x_1 on homework, x_2 on quizzes, x_3 on midterms, and x_4 on the final, with potential final course grades of b_1, b_2, b_3.

(a) (1 point) Write a matrix equation $Ax = b$ to relate your final grades to your scores.

(b) (2 points) Row reduce the corresponding augmented matrix until you reach reduced row echelon form.

(c) (1 point) Looking at the final matrix in (b), what equation in terms of b_1, b_2, b_3 must be satisfied in order for $Ax = b$ to have a solution?

(d) (1 point) The answer to (c) also defines the span of the columns of A. Describe the span geometrically.

(e) (3 points) Solve the equation in (c) for b_1. Looking at this equation, is it possible for b_1 to be the largest of b_1, b_2, b_3? In other words, is it ever possible for the grade under Scheme 1 to be the highest of the three final course grades? Why or why not? Which scheme would you argue for?

2. (2 points) Suppose the solution set of a certain system of linear equations is given by

$$x_1 = 9 + 8x_4, x_2 = -9 - 14x_4, x_3 = 1 + 2x_4, x_4 = x_4$$

(x_4 is free). Write the solution set in parametric vector form.

3. (2 points) Which option describes the span of the columns $\begin{pmatrix} 0 & 2 & 0 \\ 1 & 3 & 1 \\ 1 & 3 & 1 \end{pmatrix}$? Justify your answer with calculations or explanation.

   A. It is a plane through the origin.

   B. It is three lines through the origin.

   C. It is all of $\mathbb{R}^3$.

4. (4 points) Consider the following consistent system of linear equations.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= -2 \\ 3x_1 + 4x_2 + 5x_3 + 6x_4 &= -2 \\ 5x_1 + 6x_2 + 7x_3 + 8x_4 &= -2 \end{aligned}$$

   (a) (2 points) Find the parametric vector form for the general solution.

(b) (2 points) Find the parametric vector form of the corresponding homogeneous equations.

5. (4 points) Determine whether the following set of vectors is linearly independent or dependent. Show your work.

   1. $$\left\{\begin{pmatrix}3\\3\\4\end{pmatrix}, \begin{pmatrix}0\\10\\20\end{pmatrix}, \begin{pmatrix}0\\5\\7\end{pmatrix}\right\}$$

   2. $$\left\{\begin{pmatrix}3\\3\\4\end{pmatrix}, \begin{pmatrix}0\\10\\20\end{pmatrix}, \begin{pmatrix}0\\5\\7\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$$