HB Model

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Methods

Statistical Analysis

Leaf level

In order to model leaf size variation within plants, we model each leaf k form plant i and species j as a random sample from a lognormal distribution centered at a_{ij} (mean leaf size for plant i), as follows:

$$\log_e A_{kij} \sim \mathcal{N}(a_{ij}, \sigma_A^2)$$

where σ_A^2 is the leaf size variation within plant i. The estimate a_{ij} was later used as a predictor of the total leaf surface area for each of the sampled plants.

Individual level

In order to model intraspecific variation in the total number of leaves per plant (N_{ij}), and thus total leaf surface area per plant ($a_{ij} * N_{ij}$), we assumed N_{ij} to be a random sample of a lognormal distribution as follows:

$$\log_e N_{ij} \sim \mathcal{N}(\beta_j + \delta_j * D_{ij} + \phi_j * L_{ij} + \mu_j * M_{ij}, \sigma_N^2)$$

where σ_N^2 is the intra-specific residual variation of N and β, δ, ϕ, and μ are species-specific parameters that estimate, respectively: the number of leaves of an average-sized sapling growing in the shade ($L = 1$); the effect of sapling size (D; log-transformed) on N; the effect of light on N; and the effect of leaf mean dry mass (M; log-transformed) on N.

Now, to model the relationship between mean leaf mass (M) and mean leaf area (a) per plant we modeled a_{ij} as a function of m_{ij} as $a_{ij} \sim \mathcal{N}(SLA_j + M_{ij}, \sigma_a^2)$, where SLA_j is the natural logarithm of the specific leaf area for species j and σ_a^2 is the intraspecific variance in mean leaf area. Additionally, to estimate species average leaf mass we modeled M_{ij} as $M_{ij} \sim \mathcal{N}(m_j, \sigma_M^2)$, where m_j is the species-specific mean leaf mass and σ_M^2 is the intraspecific variance in mean leaf mass. Of course, to derive species-specific mean leaf area we can simply multiply SLA_j by m_j.

Species level

Finally, to model the interspecific variation in N we modeled all species-specific parameters (β_j, δ_j, ϕ_j, and μ_j, or simply Γ_j) as samples from a multivariate normal distribution as follows:

$$\Gamma_j \sim \mathcal{N}_4(\gamma, \Sigma_\Gamma)$$

where γ is the vector with the means of the hyperparameter distributions for each species-specific parameter and Σ_Γ is the variance-covariance matrix containing the interspecific variations and correlations among these parameters.

We also modeled the across species distribution of leaf sizes (m_j) and specific leaf area (SLA_j) respectively as $m_j \sim \mathcal{N}(\bar{m}, \sigma_m^2)$ and $SLA_j \sim \mathcal{N}(sla, \sigma_{SLA}^2)$.

Posterior predictions

Finally, to compare the growth strategies of large- and small-leafed saplings, we estimated total leaf surface area per plant (A_T) for standardized plants (medium-sized, grown in the shade) of different leaf sizes and specific leaf areas (SLA) as follows:

$$A_T =$$