# HB Model

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2020-12-17 20:19:39

## Methods

### Statistical Analysis

#### Leaf level

In order to model leaf size variation within plants, we model each leaf $k$ form plant $i$ and species $j$ as a random sample from a lognormal distribution centered at $a_{ij}$ (mean leaf size for plant $i$), as follows:

$$\log_e A_{kij} \sim \mathcal{N}(a_{ij}, \sigma_A^2)$$

where $\sigma_A^2$ is the leaf size variation within plant $i$. The estimate $a_{ij}$ was later used as a predictor of the total leaf surface area for each of the sampled plants.

#### Individual level

In order to model intraspecific variation in the total number of leaves per plant ($N_{ij}$; log-transformed), we assumed $N_{ij}$ to be a random sample of a normal distribution as follows:

$$N_{ij} \sim \mathcal{N}(\beta_j + \delta_j * D_{ij} + \phi_j * L_{ij} + \mu_j * M_{ij} + \theta_j * SLA_{ij}, \sigma_N^2)$$

where $\sigma_N^2$ is the intra-specific residual variation of $N$ and $\beta$, $\delta$, $\phi$, and $\mu$ are species-specific parameters that estimate, respectively: the number of leaves of an average-sized sapling growing in the shade ($L = 1$); the effect of sapling size ($D$; centered and log-transformed) on $N$; the effect of light on $N$; the effect of leaf mean dry mass ($M$; log-transformed) on $N$; and finally the effect of $SLA$ on $N$.

Now, to model the relationship between mean leaf mass ($M$) and mean leaf area ($a$) per plant we modeled $a_{ij}$ as a function of $m_{ij}$ as $a_{ij} \sim \mathcal{N}(SLA_{ij} + M_{ij}, \sigma_a^2)$, where $SLA_{ij}$ is the log-transformed specific leaf area for species $j$ and $\sigma_a^2$ is the intraspecific variance in mean leaf area. Additionally, to estimate species average leaf mass we modeled $M_{ij}$ as $M_{ij} \sim \mathcal{N}(m_j, \sigma_M^2)$, where $m_j$ is the species-specific mean leaf mass and $\sigma_M^2$ is the intraspecific variance in mean leaf mass. Finally, to estimate species-specific $SLA$, we modeled $SLA_{ij}$ as $SLA_{ij} \sim \mathcal{N}(sla_j, \sigma_{SLA}^2)$. Of course, to estimate species-specific mean leaf area we can just sum $SLA_j$ and $m_j$.

#### Species level

Finally, to model the interspecific variation in $N$ we modeled all species-specific parameters ($\beta_j$, $\delta_j$, $\phi_j$, $\mu_j$, and $\theta_j$, or simply $\Gamma_j$) as samples from a multivariate normal distribution as follows:

$$\Gamma_j \sim \mathcal{N}_5(\gamma, \Sigma_\Gamma)$$

where $\gamma$ is the vector with the means of the hyperparameter distributions for each species-specific parameter and $\Sigma_\Gamma$ is the variance-covariance matrix containing the interspecific variations and correlations among these parameters.

We also modeled the across-species distribution of leaf sizes ($m_j$) and specific leaf area ($sla_j$) respectively as $m_j \sim \mathcal{N}(\bar{m}, \sigma_m^2)$ and $sla_j \sim \mathcal{N}(s\bar{l}a, \sigma_{sla}^2)$.

**Posterior predictions**

Finally, to compare the growth strategies of large- and small-leafed saplings, we estimated total leaf surface area per plant ($A_T$; log-transformed) for standardized plants (i.e., medium-sized [$D = 0$], grown in the shade [$L = 0$]) of species mean leaf mass ($m_j$) and specific leaf area ($sla_j$) as follows:

$$A_{T_j} = \hat{N}_j + \hat{a}_j = (\beta_j + \mu_j * m_j + \theta_j * sla_j) + (sla_j + m_j)$$

thus $A_{T_j} = \beta_j + (1 + \mu_j) * m_j + (1 + \theta_j) * sla_j$.