Parameter Identification and Digital Control of Speed of a Permanent Magnet DC Motors

Sanel Galijašević, Šemsudin Mašić, Senad Smaka, Abdulah Akšamović, and Dinko Balić
Department for Electrical Power Engineering
Faculty of Electrical Engineering Sarajevo
Sarajevo, Bosnia and Herzegovina
{sg14950, smasic, ssmaka, aaksamovic, db15045}@etf.unsa.ba

Abstract— **The digital control of speed of a permanent magnet DC motor based on a digital computer is analyzed in this paper. Datasheet information about the DC motor was initially unknown. Therefore, the parameter identification and parameter estimation of the motor was done. A type of PID controller, which parameters were optimized (PI), was used in process of controlling the speed of the motor. DAQ acquisition card was used for the purposes of motor's parameter identification and realization of computer control of motor speed in closed feedback loop. Manipulations with the same DAQ card were done using Matlab/Simulink software package. Experimental results were compared with the results obtained by simulation.**

Keywords - Permanent magnet DC motor; DAQ; parameter identification; PID control.

I. INTRODUCTION

DC motors are type of collector motors attached to DC voltage. They are the oldest types of electrical machines and were invented after the invention of the first sources of DC current, galvanic elements. Thanks to their advantage over other AC machines in controlling the rotating speed, they found applications in many areas of industry. They are used for driving many working mechanisms, where the rotating speed must be changeable in a wide range. They find their place in many products like consumer goods and are used for industrial, medical and military purposes [1].

The most frequent requirement is the possibility to be able to control both the rotation speed and the direction [2]. Apart from these, it is also possible to control the position of a rotor and electromagnetic torque. However, the rotation speed of rotor is the most important control variable, and the variable which was being controlled in this paper [3] – [6].

In spite of the problems related with the complexity of collector – brush system (commutator) which needs constant maintenance, high production costs and problems related with the commutation as the power and speed limiter [7], the advantages of DC motors in controlling the rotation speed in wide range make it concurrent to cheaper AC machines.

The DC motor being tested drives no load, except for two gear wheels, which means it does not operate in regime of ideal idling. The value of armature current is less than 20 mA, so it is possible to drive this motor with voltage signal coming directly from the analog output of the acquisition card (the maximum value of the current through the acquisition card is then limited to 30 mA). By using this approach, a better understanding of proposed principles of automatic control of analyzed DC motor is achieved, because the transfer function of power amplifier, which would separate the power part from the signal part, is not taken into consideration [8].

Although all the specifications of the controlled object – DC motor were unknown, using corresponding electrical sensors integrated on to the analyzed DC motor (tachogenerator and encoder), relevant information about the motor being controlled in closed loop are acquired. After that, by knowing the mathematical model of a permanent magnet DC motor [9], the process of motor's parameter estimation can begin [10], [11]. The process of identifying motor parameters using Siemens PLC S7 200 CPU is the subject of paper given in [12].

After the parameter identification process of the DC motors, the type of controller is being selected (PI controller was chosen) [13], [14], and its parameters are optimized [15]. Design of the fuzzy logic control for a permanent magnet DC motor is given in [16].

The paper is organized as follows. The mathematical model and parameter identification procedure are described in the Section II. The design of PI controller and optimization of PI controllers parameters are presented in Section III. The Section IV shows the simulation model and results. Comparison of simulation and experimental results are shown in section V. The conclusions are written in Section VI.

II. PARAMETER IDENTIFICATION OF DC MOTOR

A. The transfer function of a DC motor

The mathematical model of a permanent magnet DC motor is described by the following set of equations:

$$u_a = R_a i_a + L_a \frac{di_a}{dt} + c\omega_{mech}, \tag{1}$$

$$ci_a = T_l + J\frac{d\omega_{mech}}{dt}, \tag{2}$$

where u_a and i_a are current values of armature voltage (V) and current (A), c is the construction constant of the motor (V·s/rad), ω_{mech} is the mechanical angular speed of the motor's shaft (rad/s), T_l is the current value of the load torque (N·m),

978-1-4577-0746-9/11/$26.00 ©2011 IEEE

R_a and L_a are the armature resistance (Ω) and inductance (H), J is the moment of inertia (kg·m^2).

By neglecting the electrical transient (di_a/dt=0), and applying the *Laplace* transform to (1) and (2), with zero initial conditions of every time dependent variable, we get:

$$U_a(s) = R_a I_a(s) + c\Omega_{mech}(s), \tag{3}$$

$$cI_a(s) = T_l(s) + J \cdot s\Omega_{mech}(s). \tag{4}$$

By neglecting the load torque (T_l=0), eliminating $I_a(s)$ from (4) and including it in (3), we get the transfer function of the motor as:

$$G_m(s) = \frac{\Omega_{mech}(s)}{U_a(s)} = \frac{K_m}{1+T_m \cdot s}, \tag{5}$$

where K_m=1/c and T_m=R_aJ/c^2 are constants which should be determined in parameter identification process.

Therefore, a permanent magnet DC motor in idling mode can, with satisfactory accuracy, be defined with first order transfer function.

B. Parameter identification procedure

It is necessary to perform the acquisition of corresponding data in order to determine the parameters of the motor. MiniLab1008 acquisition card was used for that purpose, and later for the purpose of controlling the analyzed motor as well. The procedure of identification is done using Matlab/Simulink software package.

It is necessary to have acquired data about the motor response to the given voltage sequence (*random* sequence as input signal – voltage is usually used, which is send from Matlab/Simulink to the analog output of the acquisition card, and data about tachogenerator voltage, which corresponds to the speed of motor, is acquired from the analog input). Fig. 1 shows the basic block diagram for the parameter identification.

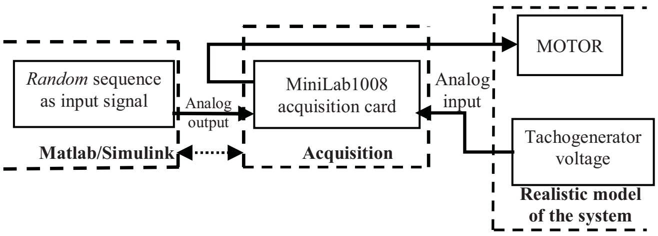

Figure 1. Basic block diagram for the parameter identification

In order to correctly perform parameter identification, it is necessary to known the dependence between the mechanical speed of motor and measured voltage of tachogenerator.

The mentioned dependence is determined experimentally, in the way that the value of tachogenerator voltage is measured for each value of the armature voltage. According to the output signal of the encoder specifying the position of the slowest gear wheel, angular speed is determined. In respect to determined ratio between the shaft and that gear wheel, the value of mechanical angular speed of DC motor's shaft is determined. After the linearization of obtained results using the *least squares* method had been performed, the following relation between the mechanical angular speed of the DC motor's shaft ω_{mech} (rad/s) and tachogenerator voltage U_{tg} (V) is established:

$$\omega_{mech} = 10.61 \cdot U_{tg} + 0.04. \tag{6}$$

Now, in accordance with Figure 1, it is possible to create the Simulink model in order to determine unknown motor parameters from the transfer function (5). Fig. 2 shows the corresponding Simulink model for the parameter identification of the analyzed DC motor.

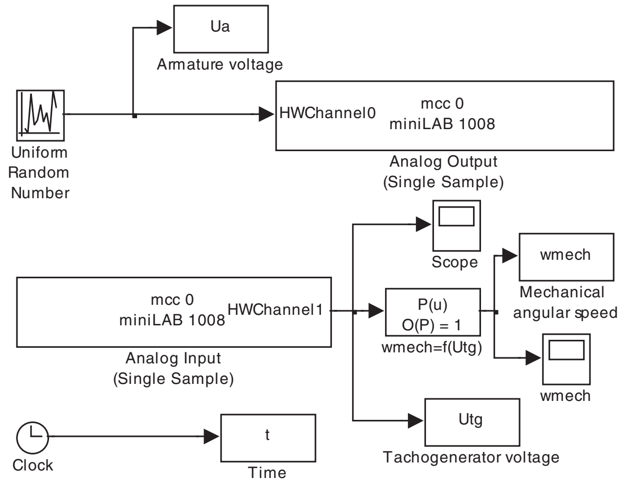

Figure 2. Simulink model for the parameter identification

After the output data (random voltage sequence) and input data (tachogenerator voltage – spinning speed) had been gathered, *System Identification Tool* was opened by entering the *ident* command in Matlab's workspace, where data about variables (armature voltage and mechanical angular speed) was being imported from Matlab's workspace. Under the *Process Models* dialog box, the type of transfer function can be chosen (the first order transfer function is chosen because of (5)). Then, the process of motor estimation as the first order transfer function begins. Fig. 3 shows this process.

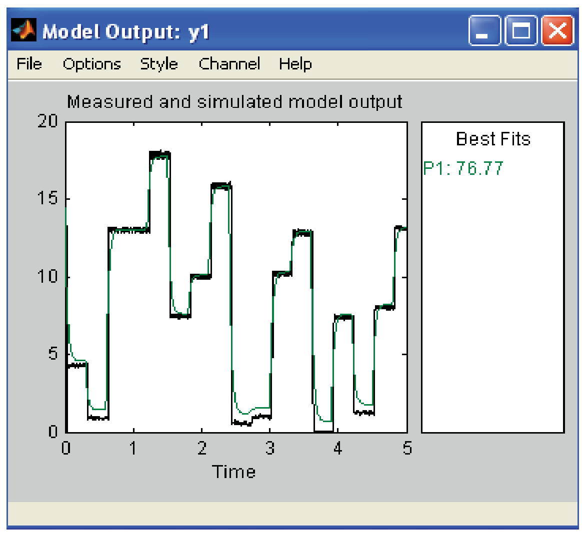

Figure 3. The process of motor estimation using the first order transfer function

Taking into account the results of the estimation process, the analyzed motor in idling mode can be defined with the following transfer function:

$$G_m(s) = \frac{3.7854}{1 + 0.032631 \cdot s}, \tag{7}$$

with the 76.77% accuracy (Fig. 3).

Since the measured value of resistance is R_a=0.43 Ω, we can calculate the basic parameters of the motor:

$$c = \frac{1}{K_m} = 0.2642 \text{ V} \cdot \text{s/rad},$$

$$J = \frac{T_m c^2}{R_a} = 0.0053 \text{ kg} \cdot \text{m}^2 .$$

III. THE SYNTHESIS OF A CONTROLLER

A. Block diagram of a PI controller

The implemented controller in the system for automatic control of speed of the motor is a PI controller. PI controller is mostly used when the referent value is constant.

Block diagram of a PI controller is shown in Fig. 4.

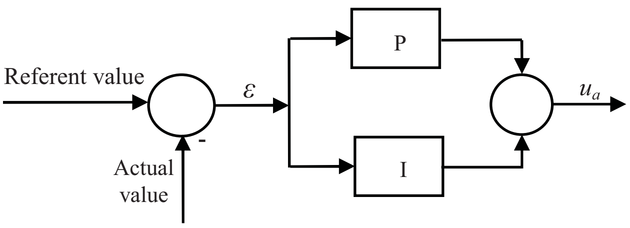

Figure 4. Block diagram of a PI controller

The output of a PI controller, in accordance to Fig. 4, is defined by the following relation:

$$u_a(t) = K_P \varepsilon(t) + \frac{K_P}{T_I} \int_0^t \varepsilon(\tau) d\tau, \tag{8}$$

where K_p is proportional component (amplification) and T_I is time constant of integrator.

Applying *Laplace* transform to (8), we get:

$$U_a(s) = K_P \mathrm{E}(s) + \frac{K_P}{T_I \cdot s} \mathrm{E}(s) = K_P \frac{1 + T_I \cdot s}{T_I \cdot s} \mathrm{E}(s) . \tag{9}$$

From (9) the transfer function of the controller can be derived as:

$$G_{PI}(s) = \frac{U_a(s)}{\mathrm{E}(s)} = K_P \frac{1 + T_I \cdot s}{T_I \cdot s} . \tag{10}$$

B. Optimization of a PI controller's parameters

Since the transfer function of the used PI controller is already derived in digital form in Matlab/Simulink, it is necessary to derive the optimal values of parameters from (10). Therefore, we analyze the system closed with negative feedback loop in order to observe the system's response to the given excitation. The corresponding block diagram is shown in Fig. 5.

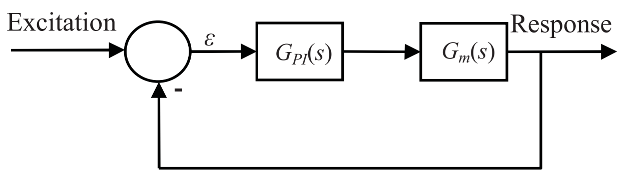

Figure 5. Block diagram of the system for determining the parameters of PI controller

The transfer function of the whole system shown in Fig. 5 is defined with the following relation:

$$G(s) = \frac{G_{PI}(s) \cdot G_m(s)}{1 + G_{PI}(s) \cdot G_m(s)} . \tag{11}$$

Inserting (7) and (10) into (11) we get:

$$G(s) = \frac{3.7854 K_P \left(1 + T_I \cdot s\right)}{T_I \cdot s \left(1 + 0.032631 \cdot s\right) + 3.7854 K_P \left(1 + T_I \cdot s\right)} . \tag{12}$$

If the time constant of integrator equals the time constant of the motor, it is not difficult to realize that zero of the transfer function (12) is cancelled with one of its poles (which comes as a result of controller's effect to the system). Therefore, according to (12), we calculate the value of one coefficient of our PI controller T_I = 0.032631 s, so its transfer function, according to (10), is now defined as:

$$G_{PI}(s) = K_P \frac{1 + 0.032631 \cdot s}{0.032631 \cdot s}, \tag{13}$$

what is left to find is the optimal value for parameter K_p.

In order to determine K_p, iterative optimization method is used by choosing the *Signal Constraint* block found in *Simulink Design Optimization* library (or in *Simulink Response Optimization* library in older versions of Matlab). Fig. 6 shows the corresponding Simulink model for optimization of parameter K_P.

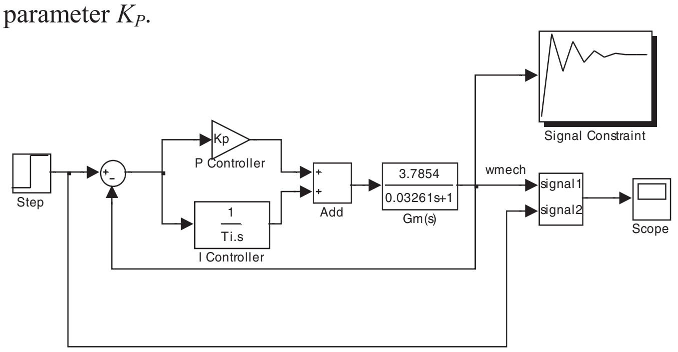

Figure 6. Simulink model for determining K_P

Time constant of integrator is assumed known (whose value is T_I=0.032631 s). The excitation of system is represented with *step* function whose initial value equals zero, while the final value (at t = 0) equals 5 (this represents the value of armature voltage that can be generated at the acquisition card to drive the motor).

After running the *Signal Constraint* block, *Block Parameters: Signal Constraint* dialog box opens. From the *Goals* drop menu we choose *Desired Response,* after which a dialog box of the same name appears, where the option for specifying step response parameters can be activated.

After the process of optimization had finished, the optimal value of parameter K_P is calculated as 1.0 (tacking into account the basic requests that were put on the system response in transient and stationary state). Finally, the transfer function of implemented controller is defined by the following relation:

$$G_{PI}(s) = \frac{1 + 0.032631 \cdot s}{0.032631 \cdot s}. \tag{14}$$

C. Determining the sampling rate

Controlling the speed of the DC motor is a continuous process. At the other side, gathered data (tachogenerator voltage) about the control variable are brought to analog inputs of acquisition card, after which A/D conversion of the gathered data takes place. For the purpose of deriving the reliable information about the control variable from its discretized form, the sampling rate was chosen according to Nyquist criterion:

$$T_s \leq \frac{\pi}{\omega_c}, \tag{15}$$

where T_s is the sampling period (s), and ω_c is critical angular frequency (rad/s).

For the purpose of determining the critical angular frequency ω_c or value of sampling period T_s, the amplitude – frequency characteristics of the motor was determined. Starting from the transfer function of the motor (7) and allowing $\sigma \to 0$ (where $s = \sigma + j\omega$ is the complex variable), we get:

$$G_m(j\omega) = G_m(s)\Big|_{s=j\omega} = \frac{3.7854}{1 + 0.032631 \cdot j\omega}. \tag{16}$$

From (16) we obtain:

$$\left|G_m(j\omega)\right| = \frac{\sqrt{14.329 + 0.015\omega^2}}{1 + 0.001\omega^2}. \tag{17}$$

Fig. 7 shows amplitude – frequency characteristics defined by (17) for positive angular frequencies.

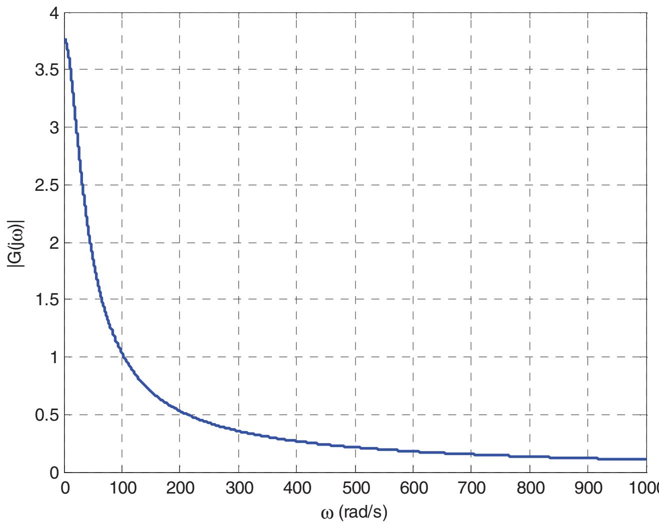

Figure 7. Amplitude – frequency characteristics of the motor

Fig. 7 shows that the amplitude spectrum of the motor is bounded function and approaches zero when angular velocity approaches infinity. It can be seen that the value of amplitude is small when compared to initial value starting from angular velocities greater than 200 rad/s. Therefore, critical angular frequency can be chosen as ω_c=200 rad/s, which means that frequency spectrum outside of [0 rad/s, 200 rad/s] is neglected. According to Nyquist crtiterion given by (15) we have:

$$T_s \leq \frac{\pi}{200} \approx 15.71 \text{ ms}. \tag{18}$$

The value of sampling period is chosen as T_s=10 ms, which satisfies the condition given by (18).

D. Checking the stability of the system

After determining the transfer function of the motor in idling state, choosing controller's parameters and sampling rate, it is necessary to check the system's stability.

System's stability check is carried out based on the position of zeros and poles in the complex plane. Fig. 8 shows the position of zeros and poles of the system closed with negative feedback loop, which is drawn by using the *pzmap* command in the Matlab workspace.

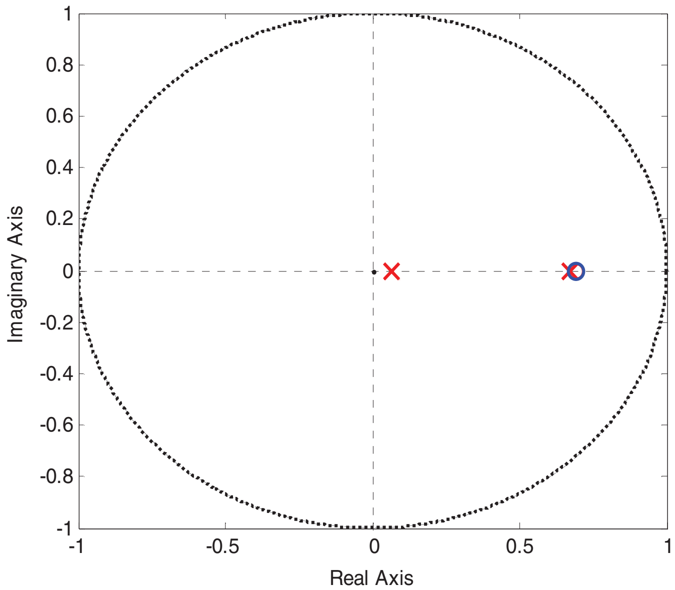

Figure 8. Pole – zero map

Fig. 8 shows that both poles of the system are positioned inside the unity circle, thus the system is stable. That, from the stability point of view, is a confirmation of well designed parameters for our PI controller and well chosen value for sampling rate.

IV. Simulation Model and Results

A. Making the simulation model

After completing the parameter identification process of the motor and choosing the optimal parameters for the controller, the corresponding simulation model of the system can be created. Fig. 9 shows the block diagram of the system.

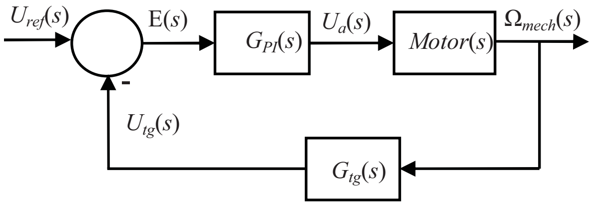

Figure 9. Block diagram of system for automatic control of speed of a DC motor

$U_{ref}(s)$, $U_a(s)$, $U_{tg}(s)$, $\Omega_{mech}(s)$ and E(s) represent the referent (given) voltage (which represents the given speed), armature voltage, tachogenerator voltage, mechanical angular speed of the motor and error, respectively, expressed in *Laplace* domain space, and $G_{PI}(s)$ and $G_{tg}(s)$ represent transfer functions of a PI controller and tachogenerator, respectively. Block "Motor(s)" equals the following block diagram in *Laplace* domain space (Fig. 10), according to (3) and (4):

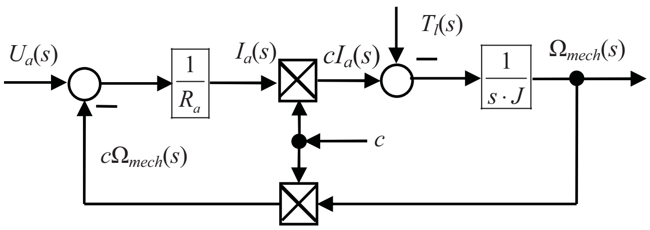

Figure 10. The content of „Motor(s)" block

Fig. 11 shows the corresponding Simulink model which directly follows the block diagram from Fig. 9.

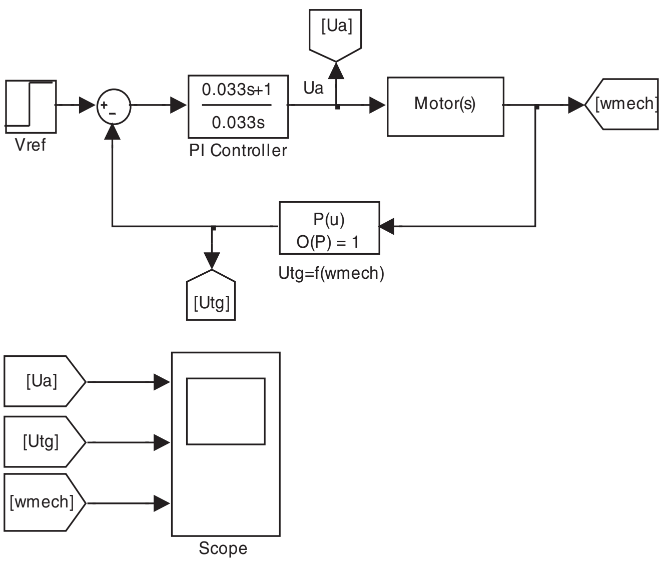

Figure 11. Simulink model for the simulation of automatic control of the analyzed DC motor's speed

The value of referent voltage (V_{ref}) from Fig. 11 needs to be calculated separately for the desired value of mechanical angular speed or armature voltage.

The content of "Motor(s)" block from Fig. 11 is shown in Fig. 12.

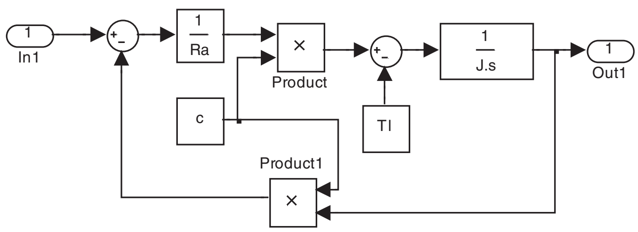

Figure 12. The structure of "Motor(s)" subsystem

B. Calculating the value of referent voltage

According to Fig. 9, we have:

$$G_{PI}(s) = \frac{U_a(s)}{\mathrm{E}(s)} = \frac{U_a(s)}{U_{ref}(s) - U_{tg}(s)}. \tag{19}$$

The referent voltage in *Laplace* domain space $U_{ref}(s)$ can be derived from (19) as:

$$U_{ref}(s) = U_{tg}(s) + \frac{U_a(s)}{G_{PI}(s)}. \tag{20}$$

Let's assume that desired mechanical angular speed is constant:

$$\omega_{meh} = C_{\omega} = const. \tag{21}$$

According to (6), the particular tachogenerator voltage $U_{tg}=C_{tg}=const.$ corresponds to mechanical angular speed from (21). For the obtained value of tachogenerator voltage, the value of armature voltage is set to $U_a=C_a$=const. Therefore, *Laplace* transforms of these values are given as:

$$U_a(s) = \frac{C_a}{s}, \quad \mathrm{Re}\{s\} > 0, \tag{22}$$

$$U_{tg}(s) = \frac{C_{tg}}{s}, \quad \mathrm{Re}\{s\} > 0. \tag{23}$$

By inserting (14), (22) and (23) into (20), we get:

$$U_{ref}(s) = \frac{C_{tg}}{s} + \frac{C_a}{s + 30.3}, \quad \mathrm{Re}\{s\} > 0 . \tag{24}$$

By inverse *Laplace* transform, we obtain:

$$u_{ref}(t) = C_{tg} + C_a e^{-30.3t}, \ (t \geq 0). \tag{25}$$

The second term in (25) is negligible when compared to the first one, which means that the value of referent signal, from Fig. 11, should be set to:

$$V_{ref} = C_{tg}. \tag{26}$$

C. Simulation Results

Fig. 13 shows the comparison of the simulation results with and without the PI controller for the value of referent voltage set to 1.6 V, when motor is in idling mode. Simulation results with and without the PI controller for the motor in idling mode match, which once again confirms that the controller was well chosen. However, when the load is applied, mechanical angular speed decreases without the use of the PI controller. Fig. 14 shows that the speed achieves the desired one when the load of 1 N·m is applied and the PI controller is used. However, without the controller, and with the same load applied, the speed reaches only 65% of the desired one.

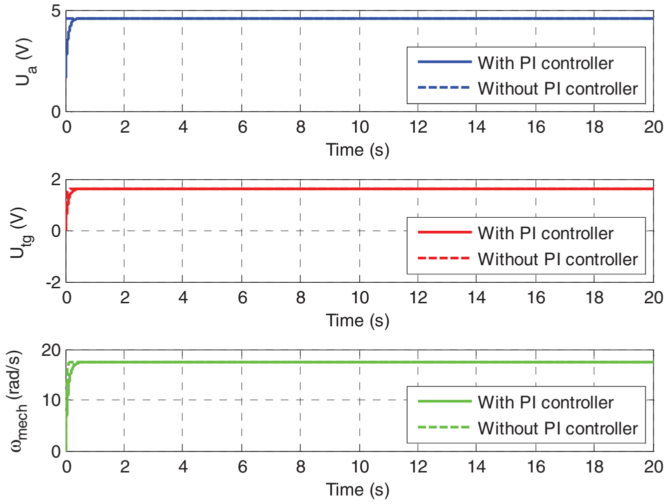

Figure 13. Results of the simulation model (idling mode)

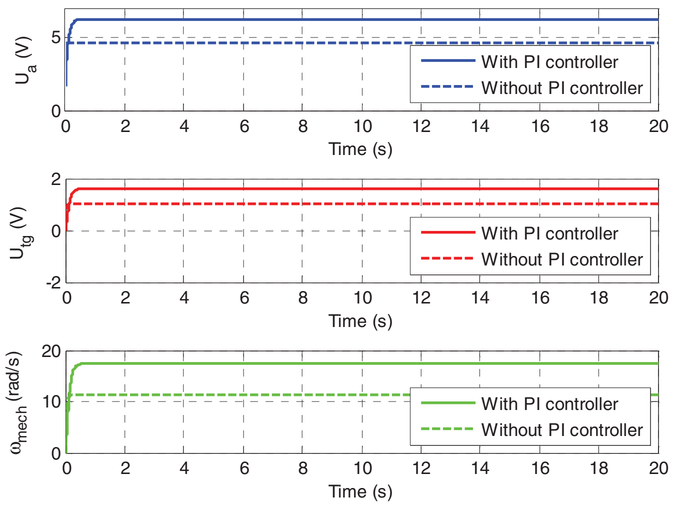

Figure 14. Results of the simulation model (with the load of 1 N·m)

The simulation results indicate that the chosen PI controller works well and that its parameters were well optimized. The correctness of motor's parameter identification process, as well as the whole implemented procedure for the purposes of controlling the motor, is confirmed by tests carried out on the real model, and comparing those results with the simulation results.

V. Experimental Verification

Controlling the speed of the DC motor is based on computer using Matlab/Simulink software package, through the acquisition card used. Fig. 15 shows the created Simulink model for the purposes of digital control of analyzed DC motor. "DC Motor" subsystem contains analog output of the acquisition card on which armature voltage is being generated, and analog input of the same card from which the information about tachogenerator voltage is being gathered. Fig. 16 shows the content of "DC motor" subsystem.

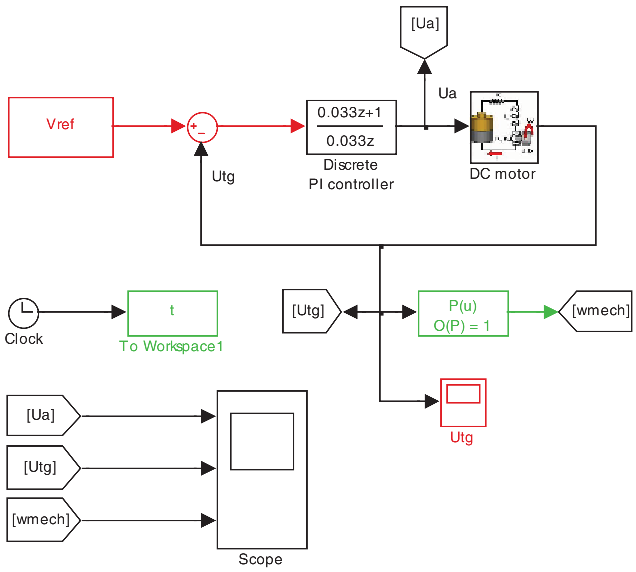

Figure 15. Simulink model for the digital control of the DC motor

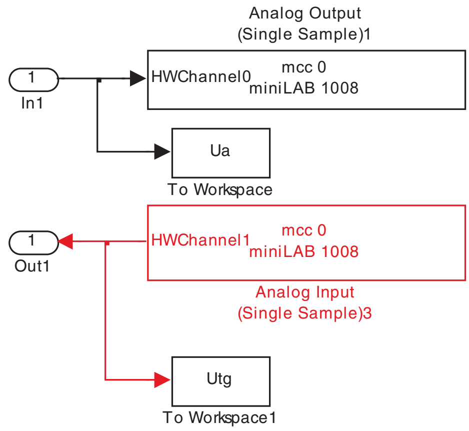

Figure 16. The content of the "DC Motor" subsystem from Figure 15

The photography of analyzed DC motor in system for automatic control is shown in Fig. 17.

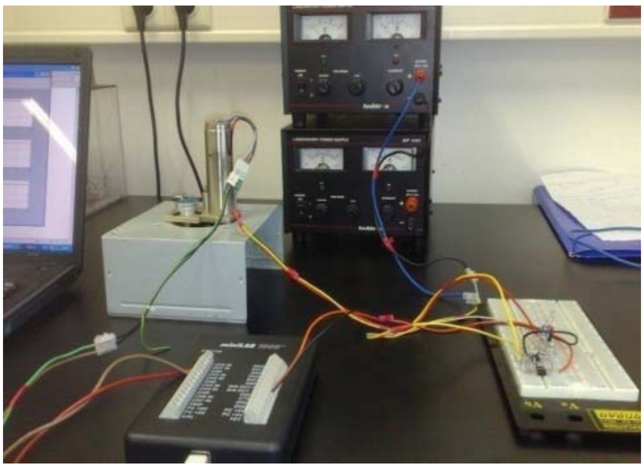

Figure 17. Experimental setup

Fig. 18 shows the results of testing the physical model in the system for automatic control based on the PC, using the Simulink model shown in Fig. 15.

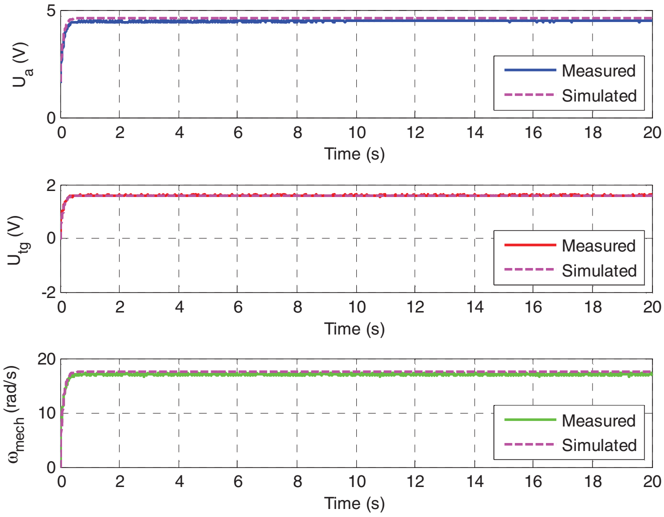

Figure 18. The comparison of experimental results with the simulation results.

VI. Conclusions

The digital control of speed of a permanent magnet DC motor is achieved using a PC and Matlab/Simulink software package. It was necessary to identify parameters of the motor since its datasheet information was unknown.

The implemented controller is of PID type of controllers, directly used from the corresponding Simulink model of the system. The parameters of the controller were optimized, and the system's stability was checked for those parameters' values and the sampling rate value. It was found that the system was stable.

Experimental results carried out on the physical model agreed with the results obtained by the corresponding simulation, which verifies the quality of control achieved by the proposed solution, which consists of clever identification of motor parameters and optimization of controller parameters, accompanied with the properly chosen sampling rate according to Nyquist criterion.

References

[1] K. Hameyer, R. J. M. Belmans, "Permanent Magnet Excited Brushed DC Motors", *IEEE Tranactions on Industrial Electronics*, vol.43, no.2, April 1996, pp. 247-255.

[2] G. Moleykutty, "Speed Control of Separately Excited DC Motor", *American Journal of Applied Sciences*, 2008, pp. 227-233.

[3] B. Allaoua, B. Gasbaoui, B. Mebarki "Setting Up PID DC Motor Speed Control Alteration Parameters Using Particle Swarm Optimization Strategy", *Leonardo Electronic Journal of Practices and Technologies*, January-June 2009, pp. 19-32.

[4] S.-M. Baek, T.-Y. Kuc, "An adaptive PID learning control of DC motors", *IEEE International Conference on Systems, Man and Cybernetics, 1997, Computational Cybernetics and Simulation, 1997*, Orlando, Florida, USA, October 1997, pp. 2877-2882.

[5] M. N. Kamarudin, S. M. Rozali, "Simulink implementation of digital cascade control DC motor model - a didactic approach", *IEEE 2nd International Power and Energy Conference PECon 2008*, Johor Bahru, Malaysia, December 2008, pp. 1043-1048.

[6] M. Koksal, F. Yenici, A. N. Asya, "Position Control of a Permanent Magnet DC Motor by Model Reference Adaptive Control", *IEEE International Symposium on Industrial Electronics ISIE 2007*, Vigo, Spain, June 2007, pp. 112-117.

[7] D. Ishak, A. H. A. Hassan, "Analytical modeling of permanent magnet excited brushed DC motor for low-cost applications", *5th International Symposium on Mechatronics and Its Applications ISMA 2008*, Amman, Jordan, May 2008, pp. 1-5.

[8] C.R. Dorf, R.H. Bishop, "Modern Control Systems", *Prentice Hall 10th Edition, 2005.*

[9] Z. Glowacz, A. Zdrojewski, "Mathematical modelling of commutator DC motor in failure conditions", *5th IEEE International Symposium on Diagnostics for Electric Machines, Power Electronics and Drives SDEMPED 2005.* , Vienna, Austria, September 2005, pp. 1-5.

[10] C. Rongmin, H. Zhongsheng, H. Jian, "The Model-free Learning Adaptive Control for DC Motor Rotate Speed Systems", *Control Conference Chinese CCC 2007*, Hunan, China, June/July 2007, pp. 738-742.

[11] C. Schneider, D. Filbert, "Parameter estimation and residual analysis a comparison", *Proc. IFAC Fault Detection, Supervision and Safety for Technical Processes*, Espoo, Finland, 1994, pp. 701–705.

[12] J. Velagić, K. Obarčanin, E. Kapetanović, S. Huseinbegović, N. Osmić, "Design of PLC-based PI controller for the permanent magnet DC motor under real constraints and disturbances", *22nd International Symposium on Information, Communication and Automation Technologies ICAT 2009,* Sarajevo, Bosnia and Herzegovina, October 2009, pp. 1-5.

[13] K. Ang, G. Chong, Y. Li, "PID control system analysis, design and technology", *IEEE Transactions on Control System Technology*, vol. 13, no. 4, July 2005, pp. 559 – 576.

[14] A. S. O. Al-Mashakbeh, "Proportional Integral and Derivative Control of Brushless DC Motor", *European Journal of Scientific Research*, vol. 35, no.2, 2009, pp. 198-203.

[15] El-Gammal, El-Samahy, "Adaptive Tuning of a PID Speed Controller for DC Motor Drives Using Multi-objective Particle Swarm Optimization", *11th International Conference on Computer Modelling and Simulation UKSIM 2009,* Cambridge, Great Britain, March 2009, pp. 398-404.

[16] J. Velagić, A. Galijašević, "Design of Fuzzy Logic Control of Permanent Magnet DC Motor under Real Constraints and Disturbances", *IEEE Multi-Conference on Systems and Control* (MSC2009), *International Symposium on Intelligent Control (ISIC'09),* Saint Petersburg, Russia, July 2009, pp. 461-466.

[17] N. Boules, "Design Optimization of Permanent Magnet DC Motors", *IEEE Transaction on Industry Applications*, vol.26, no.4, July/August 1990, pp. 786-792.

[18] A. Dumitrescu, D. Fodor, T. Jokinen, M. Rosu, S. Bucurencio, "Modeling and Simulation of electric drive systems using Matlab/Simulink environment", *International Conference on Electric Machines and Drives IEMD* 1999, Seattle, Washington, USA, May 1999, pp. 451-453.

[19] T. Wada, M. Ishikawa, R. Kitayoshi, I. Maruta, T. Sugie, "Practical modeling and system identification of R/C servo motors", *Control Applications, (CCA) & Intelligent Control, (ISIC), 2009 IEEE*, Saint Petersburg, Russia , July 2009, pp. 1378-1383.

[20] H. Garnier, M. Gilson, O. Cervellin, "Latest developments for the Matlab CONTSID toolbox", *14th IFAC Symposium on System Identification*, 2006, pp. 714-719.

[21] The MathWorks Inc., Matlab & Simulink R2010a, *Control System Toolbox* , Product help