

Assignment 1

AI1110: Probability and Random Variables

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Question : A fair coin is tossed four times, and a person win Rs 1 for each head and lose Rs 1.5 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution.

Let X be the random variable that represents the amount of money won or lost after four coin tosses we can define X as:

$X = 1 * (\text{number of heads}) - 1.5 * (\text{number of tails})$

- (i) $X = 1 * (4) - 1.5 * (0) = 4$ when four heads appear
- (ii) $X = 1 * (3) - 1.5 * (1) = 2.5$ when three heads and one tail appear
- (iii) $X = 1 * (2) - 1.5 * (2) = -0.5$ when two heads and two tails appear
- (iv) $X = 1 * (1) - 1.5 * (3) = -3.5$ when one head and three tails appear
- (v) $X = 1 * (0) - 1.5 * (4) = -6$ when four tails appear

Now, let's calculate the probability of each value of X using the pmf. The pmf gives the probability of each possible value of the random variable. In this case, the pmf is given by:

- (i) Probability of having Rs 4 is

$$\Pr(X = 4) = {}^4C_4 \times \left(\frac{1}{2}\right)^4 = \frac{1}{16} \quad (1)$$

- (ii) Probability of having Rs 1.5 is

$$\Pr(X = 1.5) = {}^4C_3 \times \left(\frac{1}{2}\right)^4 = \frac{1}{4} \quad (2)$$

- (iii) Probability of having Rs -1 is

$$\Pr(X = -1) = {}^4C_2 \times \left(\frac{1}{2}\right)^4 = \frac{3}{8} \quad (3)$$

- (iv) Probability of having Rs -3.5 is

$$\Pr(X = -3.5) = {}^4C_1 \times \left(\frac{1}{2}\right)^4 = \frac{1}{4} \quad (4)$$

- (v) Probability of having Rs -6 is

$$\Pr(X = -6) = {}^4C_0 \times \left(\frac{1}{2}\right)^4 = \frac{1}{16} \quad (5)$$

Therefore, the possible amounts of money that can be won or lost after four coin tosses are 4, 2.5, -0.5, -3.5, and -6, and the probability of each amount is 1/16, 1/4, 3/8, 1/4, and 1/16, respectively.