

# Optimisation

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## I. PROBLEM STATEMENT

A wire of length 28 m is cut into two pieces. One of the pieces to be made into a square and the other into a circle. What should be the length of each piece so that the combined area of the two is minimum

### To Find:

The value of the length of each piece so that the combined area of the two is minimum from the two figures that are square and circle.

### Given:

Length of the wire is 28m

## II. SOLUTION

length of the square is  $x$  m. (1)

Then length of the other piece for the shape of the circle is

$$(28 - x)m \quad (2)$$

Perimeter of the square with side  $a$  is given by:

$$\text{Perimeter of the square} = 4a \quad (3)$$

Now, we know from the above condition that the total length of all four sides of the square is  $x$ .

Then, by using equation 3 we get:

$$x = \frac{a}{4} \quad (4)$$

Similarly, we know the formula for the circumference of the circle with radius  $r$  is given by:

$$\text{Circumference of a circle} = 2\pi r \quad (5)$$

So, the total length is

$$4x + 2\pi r = 28 \quad (6)$$

Now, we know from the above condition that the total length of all circles is  $(28-x)$ . Then, by using equation 5 we get:

$$r = \frac{28 - 4x}{2\pi} \quad (7)$$

The standard equation of the line in conics is given as :

$$n^T \mathbf{x} = c \quad (8)$$

$$(4 \quad 2\pi) \mathbf{x} = 28 \quad (9)$$

$$\mathbf{x} = \begin{pmatrix} x \\ r \end{pmatrix}, \quad (10)$$

Now by using the formula for the area of the circle and square is:

$$\text{Area of square} = a^2 \quad (11)$$

$$\text{Area of the circle} = \pi r^2 \quad (12)$$

Now, the combined area(A) after substituting the value of  $a$  as  $x^4$  and  $r$  is

$$A = a^2 + \pi r^2 \quad (13)$$

$$A = \frac{x^2}{16} + \frac{(28 - 4x)^2}{4\pi} \quad (14)$$

The standard equation of the conics is given as :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (15)$$

$$\mathbf{V} = \begin{bmatrix} 4\pi + 16 & 0 \\ 0 & 0 \end{bmatrix} \quad (16)$$

$$\mathbf{u}^T = [800 \quad 0] \quad (17)$$

$$f = 16(28^2) \quad (18)$$

The minimum value is calculated by using gradient descent method.

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \quad (19)$$

$$\Rightarrow x_{n+1} = x_n - \alpha \left( \frac{x}{8} - \frac{28 - x}{0.636} \right) \quad (20)$$

where

- 1)  $\alpha = 0.001$
- 2)  $x_{n+1}$  is current value
- 3)  $x_n$  is previous value
- 4) precession = 0.00000001
- 5) maximum iterations = 100000000

The minimum values obtained from the python code

The given function has minimum value at 6.863 i.e

$$\frac{21}{4 + \pi} \quad (21)$$

Then, the length of the circle wire is  $50-x$  and substitute the  $x$  value in it, we get:

$$\frac{150\pi}{4 + \pi} \quad (22)$$

Hence, the length of square is  $\frac{21}{4+\pi}$  m and circle wire is  $\frac{150}{4+\pi}m$  to get the minimum area.