

CIRCLE ASSIGNMENT

Somisetty Kedaeswari
mail2kedari@gmail.com

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Assignment

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1 Problem

Let C be the circle with centre $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and radius 3 units. Find the equation of the locus of the mid-points of the chords which subtend an angle of $\frac{2\pi}{3}$ at its center.

2 Construction

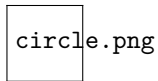


Figure of construction

3 Solution

Circle equation : $x^2 + y^2 = 9$

The standard equation of the conics is given as :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1)$$

The given circle can be expressed as conics with parameters

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = -\begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -9 \quad (2)$$

Radius and Centre are

$$r = \sqrt{\mathbf{u}^T \mathbf{u} - f}, \mathbf{O} = -\mathbf{u} \quad (3)$$

$$r = 3 \quad (4)$$

$$\text{Angle between A and B is } \cos\theta = \frac{(\mathbf{A})^T \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|}$$

$$\cos 120^\circ = \frac{(\mathbf{A})^T \mathbf{B}}{9}$$

$$(\mathbf{A})^T \mathbf{B} = \frac{-9}{2}$$

Let \mathbf{R} is the rotation matrix of given circle

$$\mathbf{R} = \begin{pmatrix} -0.5 & -0.866 \\ 0.866 & 0.5 \end{pmatrix} \quad (6)$$

Let \mathbf{B} be the another end point of chord

$$\mathbf{B} = \mathbf{R}\mathbf{A} \quad (7)$$

Let \mathbf{M} be the mid point of chord of the circle

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (8)$$

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{R}\mathbf{A}}{2} \quad (9)$$

$$\mathbf{M} = \frac{\mathbf{A}(\mathbf{I} + \mathbf{R})}{2} \quad (10)$$

$$\mathbf{A} = 2\mathbf{M}[\mathbf{I} + \mathbf{R}]^{-1} \quad (11)$$

STEPS TO FIND THE LOCUS OF THE MIDPOINT OF CHORD OF THE CIRCLE:

By substituting A value in quadratic form of the circle we get

$$[2\mathbf{M}(\mathbf{I} + \mathbf{R})^{-1}]^T [2\mathbf{M}(\mathbf{I} + \mathbf{R})^{-1}] + 2[2\mathbf{M}(\mathbf{I} + \mathbf{R})^{-1}] \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 9 = 0 \quad (12)$$

where

$$(\mathbf{I} + \mathbf{R})^{-1} = \begin{pmatrix} 0.5 & 0.86 \\ -0.86 & 0.5 \end{pmatrix} \quad (13)$$

Let \mathbf{p} be the midpoint of chord of the circle

$$\mathbf{M} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (14)$$

Therefore

$$[2\mathbf{M}(\mathbf{I} + \mathbf{R})^{-1}]^T = \begin{pmatrix} x + 1.72y \\ -1.72x + y \end{pmatrix} \quad (15)$$

And

$$[2\mathbf{M}(\mathbf{I} + \mathbf{R})^{-1}] = \begin{pmatrix} x + 1.72y & -1.72x + y \end{pmatrix} \quad (16)$$

Now the quadratic equation of given circle becomes

$$\begin{pmatrix} x + 1.72y \\ -1.72x + y \end{pmatrix}^T \begin{pmatrix} x + 1.72y \\ -1.72x + y \end{pmatrix} + 2 \begin{pmatrix} x + 1.72y \\ -1.72x + y \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}^T - 9 = 0 \quad (17)$$

$$(x + 1.72y)^2 + (-1.72x + y)^2 = 9 \quad (18)$$

$$x^2 + 3y^2 - 2xy + 3x^2 + y^2 + 2xy = 9 \quad (19)$$

$$4x^2 + 4y^2 = 9 \quad (20)$$

FINALLY LOCUS OF MIDPOINT OF CHORD OF THE GIVEN CIRCLE:

$$\mathbf{x}^2 + \mathbf{y}^2 = \frac{9}{4} \quad (21)$$

The quadratic form of locus of the given circle

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (22)$$

The given circle can be expressed as conics with parameters

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = -\begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -9 \quad (23)$$

Radius and Centre are

$$r = \sqrt{\mathbf{u}^\top \mathbf{u} - f}, \mathbf{O} = -\mathbf{u} \quad (24)$$

$$r = \sqrt{3} \quad (25)$$

termux commands :

```
bash mat2.sh.....using shell command
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Below python code realizes the above construction :

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https://github.com/kedareswari200/fwc-module1/blob/Matri\_Circle/cir.py
```