

Sep 2022

MATRIX ASSIGNMENT

0.1 Problem:

Construct a triangle ABC in which $BC=8\text{cm}$, $\angle B = 45^\circ$ and $AB - AC = 3.5 \text{ cm}$.

0.2 Solution

The input parameters for this construction are

Symbol	Value	Description
BC	a	where a is 8cm
$\angle B$	45°	$\triangle ABC$
k	3.5	constant value

Calculating Other Coordinate:

The coordinates of B and C are X_2, Y_2 respectively.

$$\text{Let } \mathbf{A} = c \times \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$

Using the Cosine formula in $\triangle ABC$,

$$b^2 = a^2 + c^2 - 2accosB$$

$$(b+c)(b-c) = a^2 - 2accosB$$

Given

$$c - b = k$$

Upon Simplification we get:-

$$(b+c)(-k) = a^2 - 2accosB$$

$$-kc - kb + 2accosB = a^2$$

$$-kb - c(-k + 2acosB) = a^2$$

From the above, we obtain the matrix equation:-

$$\begin{pmatrix} -k & k + 2acosB \\ -1 & 1 \end{pmatrix} \begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} k \\ a^2 \end{pmatrix}$$

$$\begin{pmatrix} -3.5 & 3.5 + 2(8)cos45^\circ \\ -1 & 1 \end{pmatrix} \begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} 3.5 \\ 64 \end{pmatrix}$$

$$\text{Augmented Matrix} \Rightarrow \begin{pmatrix} -3.5 & 3.5 + 2(8)cos45^\circ & 3.5 \\ -1 & 1 & 64 \end{pmatrix}$$

Reducing to echelon form:-

$$\left(\begin{pmatrix} 1 & -1 & \frac{7}{2} \\ -\frac{7}{2} & \frac{78154172560113}{1000000000000} & 64 \end{pmatrix} \xrightarrow{-R_1 \leftarrow R_1} \right)$$

$$\left(\begin{pmatrix} 1 & -1 & \frac{7}{2} \\ 0 & 1 & \frac{517500000000000}{43154172560113} \end{pmatrix} \xrightarrow{\frac{1000000000000R_2 \leftarrow R_2}{43154172560113}} \right)$$

$$\left(\begin{pmatrix} 1 & 0 & \frac{732920792079209}{86308345120226} \\ 0 & 1 & \frac{517500000000000}{43154172560113} \end{pmatrix} \xrightarrow{R_1 + R_2 \leftarrow R_2} \right)$$

$$\text{Reduced Echelon Form: } \left(\begin{pmatrix} 1 & 0 & 8.491887905604763 \\ 0 & 1 & 11.991887905604763 \end{pmatrix} \right)$$

$$\begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} 11.99 \\ 8.49 \end{pmatrix}$$

The vertices of $\triangle ABC$ are

$$\mathbf{A} = 11.99 \begin{pmatrix} \cos 45^\circ \\ \sin 45^\circ \end{pmatrix} = \begin{pmatrix} 8.4 \\ 8.4 \end{pmatrix}$$

$$(1) \quad \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$(2) \quad \mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

$$(3) \quad \mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

Below python code realizes the above construction :

$$(4) \quad \text{https://github.com/kedareswari200/fwc-moudle1/blob/Matri_lines/triangle.py}$$

$$(5)$$

$$(6)$$

0.3 Construction

