Optimization

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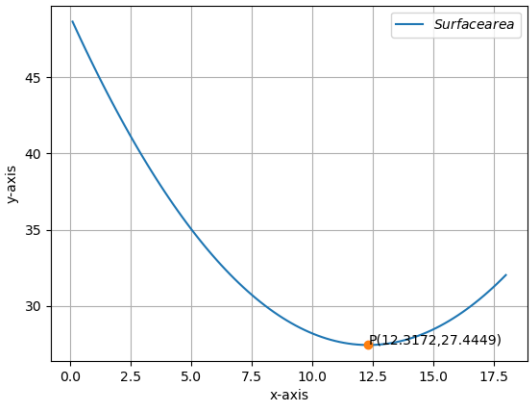
I. Problem Statement

A wire of length 28 m is cut into two pieces. One of the pieces to be made in to a square and the other into a circle. What should be the length of each piece so that the combined area of the two is minimum

To Find:
The value of the length of each piece so that the combined area of the two is minimum from the two figures that are square and circle.
Given:
Length of the wire is 28m

II. Construction



III. solution

$$\text{length of the circle is} \quad \mathbf{x} \quad \text{m.} \tag{1}$$

Then length of the other piece for the shape of the square is

$$(28-x)\text{m} \tag{2}$$

Perimeter of the square with side a is given by:

$$\text{Perimeter of the square = } 4x \tag{3}$$

$$\text{Circumference of a circle= } 2\pi r r = \frac{2\pi}{x} \tag{4}$$

$$r = \frac{x}{2\pi} \tag{5}$$

So, the total length is

$$4x + 2\pi r = 28 \tag{6}$$

The standard equation of the line in conics is given as :

$$n^\top \mathbf{x} = c \tag{7}$$

$$\begin{pmatrix} 4 & 2\pi \end{pmatrix} \mathbf{x} = 28 \tag{8}$$

$$\mathbf{x} = \begin{pmatrix} x \\ r \end{pmatrix} \tag{9}$$

Now by using the formula for the area of the circle and square is:

$$\text{Area of square= } a^2 \tag{10}$$

$$\text{Area of the circle= } \pi r^2 \tag{11}$$

Now, the combined area(A)

$$A = a^2 + \pi r^2 \tag{12}$$

$$A = \frac{x^2}{4\pi} + \frac{(28-x)^2}{4} \tag{13}$$

The area of two figures is grepresented as :

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{14}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & \pi \end{pmatrix} \tag{15}$$

$$u^\top = \begin{pmatrix} 0 & 0 \end{pmatrix} \tag{16}$$

$$f = 0 \tag{17}$$

The minimum area is

$$\min_x \mathbf{x}^\top \mathbf{V}\mathbf{x} \tag{18}$$

IV. Calculation of Minima using Gradient Descent algorithm

minima using Gradient Descent method

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \tag{19}$$

$$\implies x_{n+1} = x_n - \alpha \left(\frac{x}{8} - \frac{28 - x}{0.636} \right) \tag{20}$$

Taking $x_0 = 0.5, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

Minima length of circle = 12.31

Minimum length of square = 15.7

V. Calculation of Minima using CVXPY algorithm

minima using CVXPY method

Constraint is,

$$\begin{pmatrix} 4 & 2\pi \end{pmatrix} \mathbf{x} - 28 == 0 \tag{21}$$

Solving using cvxpy, we get

$$\min_x \mathbf{x}^\top \mathbf{V} \mathbf{x} = 27.45 \quad m^2 \tag{22}$$

The length of each piece is
Circle = 12.3 m
Square = 15.7 m

VI. Calculation of Minima using Calculus

minima using conventional method

$$\frac{dA}{dx} = \frac{x^2}{4\pi} + [\frac{28 - x}{4}]^2 \tag{23}$$

To find minima

$$\frac{dA}{dx} = 0 \tag{24}$$

$$\frac{x}{2\pi} + \frac{28 - x}{8} = 0 \tag{25}$$

$$4x = 28\pi - x\pi \tag{26}$$

$$x[4 + \pi] = 28\pi \tag{27}$$

$$x = \frac{28\pi}{4 + \pi} \tag{28}$$

The length of each piece is
Circle length =12.3 m
Square length =15.7 m