

Sep 2022

MATRIX ASSIGNMENT

0.1 Problem:

Construct a triangle ABC in which $BC=8\text{cm}$, $\angle B = 45^\circ$ and $AB - AC = 3.5 \text{ cm}$.

0.2 Solution

The input parameters for this construction are

Symbol	Value	Description
BC	a	where a is 8cm
AB	b	AB distance is denoted as c
AC	c	AC distance is denoted as b
$\angle BC$	45°	$\triangle ABC$
C	$\begin{pmatrix} a \\ 0 \end{pmatrix}$	BC length is equal to a
A	$\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$	using the cosine formula in $\triangle ABC$
k	3.5	constant value

Calculating Other Coordinate:

The coordinates of B and C are X_2, Y_2 respectively.

$$\text{Let } \mathbf{A} = c \times \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

Using the Cosine formula in $\triangle ABC$,

$$b^2 = a^2 + c^2 - 2accos\mathbf{B} \quad (1)$$

$$(b+c)(b-c) = a^2 - 2accos\mathbf{B} \quad (2)$$

Given

$$c - b = k \quad (3)$$

Upon Simplification we get:-

$$(b+c)(-k) = a^2 - 2accos\mathbf{B} \quad (4)$$

$$-kc - kb + 2accos\mathbf{B} = a^2 \quad (5)$$

$$-kb - c(-k + 2accos\mathbf{B}) = a^2 \quad (6)$$

From the above, we obtain the matrix equation:-

$$\begin{pmatrix} -k & k + 2accos\mathbf{B} \\ -1 & 1 \end{pmatrix} \begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} k \\ a^2 \end{pmatrix}$$

$$\begin{pmatrix} -3.5 & 3.5 + 2(8)\cos45^\circ \\ -1 & 1 \end{pmatrix} \begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} 3.5 \\ 64 \end{pmatrix}$$

$$\begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} 11.99 \\ 8.49 \end{pmatrix}$$

The vertices of $\triangle ABC$ are

$$\mathbf{A} = 11.99 \begin{pmatrix} \cos45^\circ \\ \sin45^\circ \end{pmatrix} = \begin{pmatrix} 8.4 \\ 8.4 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

Below python code realizes the above construction :

https://github.com/kedareswari200/fwc-moudle1/blob/Matri_lines/triangle.py

0.3 Construction

