

CIRCLE ASSIGNMENT

Somisetty Kedareshwari
mail2kedari@gmail.com

FWC22062

IITH Future Wireless Communication (FWC)

Assignment

October 19, 2022

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1 Problem

Let C be the circle with centre $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and radius 3 units. Find the equation of the locus of the mid-points of the chords which subtend an angle of $\frac{2\pi}{3}$ at its center.

2 Construction

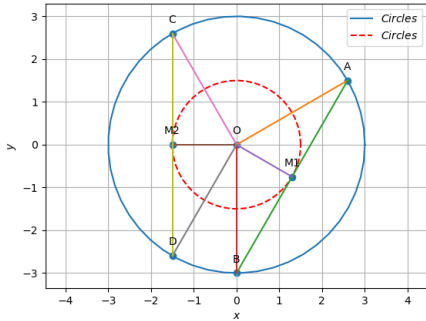


Figure of construction

3 Solution

Circle equation : $x^2 + y^2 = 9$

The standard equation of the conics is given as :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1)$$

The given circle can be expressed as conics with parameters

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = -\begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -9 \quad (2)$$

Radius and Centre are

$$r = \sqrt{\mathbf{u}^T \mathbf{u} - f}, \mathbf{O} = -\mathbf{u} \quad (3)$$

$$r = 3 \quad (4)$$

$$\text{Angle between A and B is } \cos \theta = \frac{(\mathbf{A})^T \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|}$$

$$\cos 120^\circ = \frac{(\mathbf{A})^T \mathbf{B}}{9}$$

$$(\mathbf{A})^T \mathbf{B} = \frac{-9}{2} \quad (5)$$

Let \mathbf{R} is the rotation matrix of given circle

$$\mathbf{R} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (6)$$

Let \mathbf{B} be the another end point of chord

$$\mathbf{B} = \mathbf{R} \mathbf{A} \quad (7)$$

Let \mathbf{M} be the mid point of chord of the circle

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (8)$$

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{R} \mathbf{A}}{2} \quad (9)$$

$$\mathbf{M} = \frac{\mathbf{A}(\mathbf{I} + \mathbf{R})}{2} \quad (10)$$

$$\mathbf{A} = 2[\mathbf{I} + \mathbf{R}]^{-1} \mathbf{M} \quad (11)$$

STEPS TO FIND THE LOCUS OF THE MIDPOINT OF CHORD OF THE CIRCLE:

By substituting A value in quadratic form of the circle we get

$$(2(\mathbf{I} + \mathbf{R})^{-1} \mathbf{M})^T (2(\mathbf{I} + \mathbf{R})^{-1} \mathbf{M}) + 2(2(\mathbf{I} + \mathbf{R})^{-1} \mathbf{M}) \begin{pmatrix} 0 & 0 \end{pmatrix} + f = 0 \quad (12)$$

$$(2(\mathbf{I} + \mathbf{R})^{-1} \mathbf{M})^T (2(\mathbf{I} + \mathbf{R})^{-1} \mathbf{M}) + f = 0 \quad (13)$$

$$\left\| (2(\mathbf{I} + \mathbf{R})^{-1} \mathbf{M}) \right\|^2 + f = 0 \quad (14)$$

$$(2(\mathbf{I} + \mathbf{R})^{-1} \mathbf{M})^T (2(\mathbf{I} + \mathbf{R})^{-1} \mathbf{M}) + f = 0 \quad (15)$$

$$(2(\mathbf{I} + \mathbf{R})^{-1})^T (\mathbf{M})^T 2(\mathbf{I} + \mathbf{R})^{-1} \mathbf{M} + f = 0 \quad (16)$$

$$(\mathbf{M})^\top (\mathbf{2}(\mathbf{I} + \mathbf{R})^{-1})^\top \mathbf{2}(\mathbf{I} + \mathbf{R})^{-1} \mathbf{M} + f = 0 \quad (17)$$

Let

$$\mathbf{V} = (\mathbf{2}(\mathbf{I} + \mathbf{R})^{-1})^\top \mathbf{2}(\mathbf{I} + \mathbf{R})^{-1} \quad (18)$$

Where

$$\mathbf{2}(\mathbf{I} + \mathbf{R})^{-1} = \begin{pmatrix} 1 & 1.72 \\ -1.72 & 1 \end{pmatrix} \quad (19)$$

$$(\mathbf{2}(\mathbf{I} + \mathbf{R})^{-1})^\top = \begin{pmatrix} 1 & -1.72 \\ 1.72 & 1 \end{pmatrix} \quad (20)$$

By solving this we get

$$\mathbf{V} = \mathbf{I} \quad (21)$$

FINALLY THE LOCUS OF MIDPOINT OF CHORD OF
THE GIVEN CIRCLE IS:

$$\mathbf{M}^\top \mathbf{V} \mathbf{M} + f = 0 \quad (22)$$

where

$$\mathbf{V} = \mathbf{I}, f = -9 \quad (23)$$

Radius

$$r = \sqrt{-f} = \sqrt{3} \quad (24)$$

termux commands :

```
bash sh2.sh.....using shell command
```

Below python code realizes the above construction :

```
https://github.com/kedareswari200/  
fwc-module1/blob/Matrix1/cir.py
```