

Sep 2022

MATRIX ASSIGNMENT

0.1 Problem:

Let C be the circle with centre (0, 0) and radius 3 units. To find the equation of the locus of the mid points of the chords of the circle C that subtend an angle of $2\pi/3$ at its center

0.2 Solution

The input parameters for this construction are

| Symbol | Value | Description |
|--------------|--|---------------------------------|
| r | 3cm | Radius of a circle given as 8cm |
| $\angle AOB$ | 120° | Angle between two chords |
| O | $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ | Centre Point |

Calculating Locus:

Let $\theta_1 = 30^\circ$ and $\theta_2 = 120^\circ - \theta_1$

$$\text{Chord A} = \begin{pmatrix} r \cos \theta_1 \\ r \sin \theta_1 \end{pmatrix}$$

$$\text{Chord B} = \begin{pmatrix} r \cos \theta_2 \\ -r \sin \theta_2 \end{pmatrix}$$

Upon Simplification we get

$$A = \begin{pmatrix} 2.59 \\ 1.5 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$$

Finding Mid-Point

$$M = (A + B)/2. \quad (1)$$

Upon Simplification we get

$$M = \begin{pmatrix} 1.299 \\ -0.75 \end{pmatrix}$$

Finding Radius of locus(circle)

$$r^2 \implies (O - M) = \sqrt{M \cdot M^T} \quad (2)$$

$$r^2 = \frac{9}{4}$$

Finding Circle Equation

$$(x - x_0)^2 + (y - y_0)^2 = r^2 \quad (3)$$

$$\text{Where, } (x_0, y_0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (4)$$

Upon Simplification we get

$$x^2 + y^2 = r^2$$

Equation of locus is

$$x^2 + y^2 = \frac{9}{4} \quad (5)$$

Below python code realizes the above construction :

https://github.com/kedareswari200/fwc-moudle1/blob/Matrix_Circle/cir.py

0.3 Construction

