Problems for Fun

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T HIS text consists of problems that should be attempted for fun. It is based on what we have learned so far, however, that is just to be used as a guidance. Have fun. Find out what *kind* of problems are challenging and then perhaps work on the fundamentals. Some points to keep in mind:

1. It may be a good idea to print this out on both sides of a paper and attempt the answers with a pen or pencil on a separate paper.

2. There are no points. A quiz may contain errors. If a question is unclear, be sure to get it clarified.

3. It is a quiz of sorts. The time is not really limited, but you should plan on focusing for about an hour each time you take the quiz.

4. Give each problem enough thought and time and then present your solutions.

5. Have fun. Hopefully, you will struggle to get through the problems. Perhaps you will make silly mistakes. Don't worry, it is all part of the game. You will get better only if you have fun doing it. Note that some problems are quite difficult and you may be stuck. Being stuck is okay.

Problems

1. Multiply (see [1]) your phone number (disregard its area code) by 8. Write down the following three numbers:

    1. Your phone number,

    2. 8, and

    3. The product of your phone number and 8.

   Add *all* the digits in those three numbers. If the sum is more than one digit, add again. Continue this way until a single digit is reached.

   What's the digit? What digit do you get with your friend's phone number? Why?

2. If you had ten bananas (see [1]) and a monkey stole all but six, how many bananas would you have left?

3. (a) If $100^{\frac{1}{2}} = x$, what is the value of x?

   (b) What power (exponent) of 8 is 32?

4. In how many different ways (see [2]) can the missing digits in this short multiplication be completed?

$$
\begin{array}{r}
\square\,6 \\
\times\ \square \\
\hline
\square\,2\,8
\end{array}
$$

5. How might you use the Pascal's triangle to find out $(a - b)^4$? When both a and b are same, can you verify the answer you get?

6. In the wonderful state of Sunbathia, license plates of cars are all five characters. The first two characters are the letters of the English alphabet and each of the last three letters is a hexadecimal digit. How many total license plates are possible?

7. Fill the grid so every row, column, and outlined region contains all the numbers 1 through 5 (See [3]). Hint: never guess!
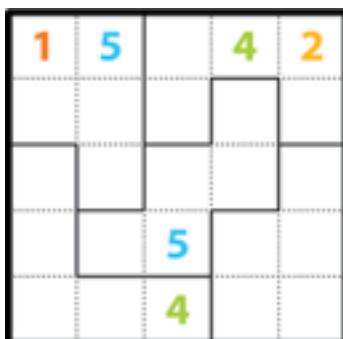

Figure 1: S for Sudoku

8. A number of Puppies and another number of Kittens are in two pens. Two players take turns making one of three possible moves: taking any number of puppies, or any number of kittens, or the *same number* of each. So, for example, if there are 8 puppies and 6 kittens, a player could, in a move, take 4 puppies, or 2 kittens, or 3 puppies and 3 kittens.

One player decides the starting number of puppies and kittens and the other player decides who goes first. The winner is the player who takes the last animal remaining.
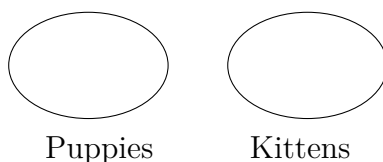
Puppies   Kittens

For any starting number of puppies and kittens, is there an optimal strategy so one player is guaranteed to win? (See [3])

9. Are you convinced that the following statement is true?

    "If the sum s of the digits of a four-digit number N is divisible by 3, then the number N is divisible by 3".

    Make a sound argument using algebra.

References

[1] Gardner, Martin. MENTAL MAGiC Surefire Tricks to Amaze Your Friends. Dover Publications, 2009.

[2] Borovik Alexandre, Gardiner Tony. Essence of Mathematics. Open Book Publishers, 2019.

[3] MT Circular. Fall 2019. Page 13.