Notes and Problems from

Principles of Mathematical Analysis, III edition

by Walter Rudin

Kedar Mhaswade

06 February 2026

Reflection 1. These are the author's highly personal notes based on this great book. They may occasionally sound like author's conversation with the reader (or even himself). Even if they are personal and appear in the public domain, they may help the reader, although the author isn't exactly sure how. He wrote them for 1) LaTeX practice (of writing hopefully readable mathematics, albeit longer than appearing in standard texts), and 2) reliable archival (paper, on which most math is created, tends to get lost) that incurs minimal overhead.

These notes are interspersed with "reflection boxes" like this one. They are meant to express author's 'rough' thinking process, of which frustration is often an inseparable part. The author writes mostly in first person for an authentic conversational tone.

1 The Real and Complex Number Systems

1.1 Introduction

Rudin presents an elegant proof of the irrationality of $\sqrt{2}$. We shall skip that here, but solve the following problem in our own way (he solves it himself) instead.

Problem 1.1. Let $\mathbb{A} = \{p \in \mathbb{Q}^+ \mid p^2 < 2\}$. Show that $\mathbb{A}$ contains no largest number.

Proof. Let $p = \frac{m}{n}$, where $m, n \in \mathbb{N}, n \neq 0, m^2 < 2n^2$.

We want to show that there is a rational number greater than p that is less than $\sqrt{2}$. We can achieve this by adding some positive number to p and showing that the sum is less than $\sqrt{2}$.

Reflection 2. Rudin readily considers a rational, $q = \frac{2p+2}{p+2}$ and shows that if $p < \sqrt{2}$, i.e., $p \in \mathbb{A}$, 1) $q > p$, and 2) $q < \sqrt{2}$, i.e., $q \in \mathbb{A}$. But, how did he think of this new fraction? Where did that insight come from?

Math books do not write about that. They are not a place for writing the long-winding, often frustrating, nature of mathematical discovery. However, these are my notes. I have no restriction like Rudin.

∎