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Self-study Notes, Problems, and Solutions from A. P. French’s Newtonian Mechanics

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Welcome

These notes are from my self-study session based on A. P. French's Newtonian Mechanics [1]. There are two main objectives of these notes. The first is to study from a principal resource of interest this essential part of physics and the second is to be able to teach it to interested and driven students of mechanics. I call this a principal resource because textbooks are notoriously difficult to pick and one has got to settle with something because it is through a combination of *pure reflection*, hard work, reasonable experimentation, and following of good books that a decent understanding of anything in science may be achieved. I scoured the web, consulted the venerable Physics Forums [2] and other resources and settled with [1] *in the hope that French's style will resonate with me*. Of course, from time to time, I have referred to other resources where appropriate. The second objective is due, in part, to Richard Feynman who believed that teaching is a solid way to learn. Fortunately, I have access to at least one driven and talented student eager to discuss these things with me and there is some time before my engagement with him on physics starts. So, since even stalwarts like Donald Knuth require thorough preparation, I must prepare well before discussing anything on this topic with anyone. What follows is the summary of my preparation. Of course, these notes are *not* to be thought of as a license to teach, rather just a prerequisite. I am keeping this in the public domain in the naive hope that someone knowledgeable reviews it at least partially and that it may prove beneficial to someone.

These notes are typeset in LaTeX with the main font set to IBM Plex Serif.

In keeping with Halmos's advice [3]:

> "Audience, level, and treatment—a description of such matters is what prefaces are supposed to be about."

, I believe an interested high-school student with understanding of algebra, some geometry and calculus should be able to follow this material with some struggle. This is not a monograph written by me per se, but rather a collection of ideas and treatises gathered from elsewhere and written in my own words. I believe the treatment is more elementary than advanced. Finally, the focus is on solving the problems thoroughly. In solving problems, I have not referred to anything else, presumably of similar nature (e.g. "Solutions to problems from …"). As an additional help, the book provides brief answers to problems. Thus, these notes are written for a personal benefit (in some abstract sense) and only incidentally for a probable benefit to someone else.

Regarding the format of *notes*, I am trying something new. I am going to enumerate items in each section. This style is reminiscent of that of the books written in the 18^{th} and 19^{th} centuries. It may not always make the text more readable, but since the nature of this text is a review, it might be helpful. An attempt is made to solve problems serially and naturally; liberty is taken to skip some. Figures are used where appropriate.

Prologue

1. The book is divided into three main themes:
   (a) Newton's *approach* to dynamics or motion
   (b) Classical mechanics at work (heart of the contents)
   (c) Some special topics
2. Changes of motion of any object are the result of *forces* acting upon it.
3. Although dealing with it has become our second nature[1], historically, *motion* has been difficult to define and precisely understand. Zeno, an ancient Greek scholar, even argued that *motion couldn't exist*! Florian Cajori wrote a detailed 10-article treatise on Zeno's arguments against motion [4] that is worth referring to. See a summary of these articles at Appendix A.
4. We routinely (or sometimes through hard work) carry out complex mechanical tasks like catching a high fly ball in baseball, driving an outswinger through covers in cricket, doing the seemingly impossible, gravity-defying gymnastic feats on floor and equipment. But it is the task of classical mechanics to *discover and formulate the physical principles so that they can be applied to any situation involving forces and motions of objects big and small interacting with each other over a distance*.
5. The greatest triumph of classical mechanics was Newton's own success at explaining the workings of our solar system. This feat was so impressive that Alexander Pope wrote:

   > Nature and Nature's Laws lay hid in the night,
   > God said, "Let Newton be," and all was light.
   >
   > – *Alexander Pope*

[1] Our subconscious familiarity with using mechanics is only second to our effortless use of a natural language

This was later corrected (as it usually happens in science):

> It did not last; the Devil, howling "Ho,
> Let Einstein be!" restored the status quo.
>
> – *Sir John Squire*

6. We have got to look at the scientific discoveries in a historical context. Nicolaus Copernicus had proposed [5] a heliocentric system in 1543. Tycho Brahe, a danish nobleman[2], painstakingly and comprehensively documented later in the 16^{th} century the movements of celestial bodies across the night sky. His student, Johannes Kepler, after wrestling with a tonne of data, found what looked like an ounce of truth, which he formulated in terms of Kepler's Laws (based on observations and some calculations):

   (a) Some bodies (planets) appear to move in ellipses (and not circles) with sun at the focus.

   (b) The line joining a planet and the sun sweeps out *equal areas in equal periods of time*.

   (c) The ratio of the square of a planet's year (period of time it takes for it to come to the exact same position in the sky) to the cube of its mean distance from sun is the same for all the planets.

   However, it still looked like *description* and not *theory*[3]. Newton went on to formulate the *inverse-square law of gravitation* and demonstrated that Kepler's Laws were an instance of his splendid theory of gravitation.

7. Newton *quantitatively* explained the following:

   (a) The bulging of earth and Jupiter because of their rotation.

   (b) The variation of *acceleration due to gravity* with the latitude[4].

   (c) The generation of tides on earth by the combined action of sun and moon.

   (d) The paths of comets through the solar system.

   (e) The slow but steady change in the direction of earth's axis.

[2] One should listen to James Kaler's lectures on astronomy to better enjoy this historical context.

[3] In science, we tend to use common English words precisely. We use 'theory' to mean 'a well-substantiated explanation of some aspect of the natural world that usually incorporates facts, laws, inferences, calculations, and hypotheses'.

[4] The small circle parallel to the great equatorial circle.

8. An astonishing achievement was the prediction of a planet to explain the discrepancy in the observations on another (nearby) planet! Indeed, the story of the discovery [6] of Neptune is fascinating!

9. Creation of any scientific theory requires an interplay between theory and experimentation, intuition, guesswork, objectivity, and imagination. It has elements of both induction and deduction. Newton's theory is a great achievement of the human intellect, however, physics was far from complete, although a few physicists felt that way toward the end of the 19^{th} century. This was ironic (for these physicists), since the first half of the 20^{th} century saw the greatest upheaval in science since Newton!

10. It is necessary to look at Newtonian Mechanics along with the boundaries where it is applicable and understand (though experiencing those conditions is difficult[5]) its limitations.

11. The currently accepted sizes and speeds of particles weave a matrix of applicable theories as shown in Figure 1.

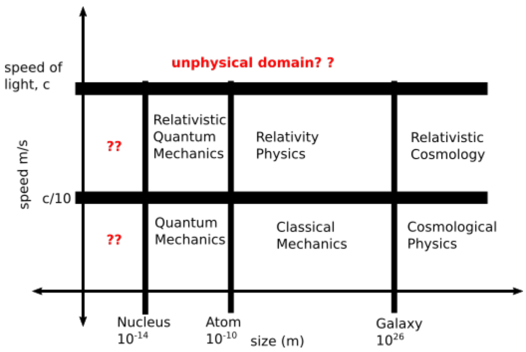

Figure 1: The Size, Speed, and Applicability of Physical Theories (dimensions not to scale)

12. Although it is only too obvious that induction (the method of devising a general governing principle after studying only a few concrete instances) leads to error. Doing classical mechanics is more like applying inductive hypotheses that are gradually strengthened. It is not cut-and-dried like, for example, applied mathematics, where the rules are already known and one has got to apply them rigorously. With all its uncertainties, inductive hypotheses only can lead to new discoveries.

[5] Just like how Galileo Galilei had to postulate in the 17^{th} century that in vacuum, the gravitational acceleration of a body is independent of its mass.

0.1 Exercises – Hors D'ouevres

Hors D'ouevre means "outside work", a teaser, an appetizer. Although this section is named "Exercises" and not "Problems" (perhaps because the questions are based on what should be known beforehand and the answers could be found somewhat mechanically) there are interesting questions here.

The "art of guessing" [7] is emphasized in exercises. I too believe that doing a *good* approximation (within 2 or 3 percent of the correct answer) rather quickly is a helpful skill that shows one's resourcefulness and understanding. One trick I often employ is to have a calculator handy, but use it only to verify the answer I have reached in my head or on paper. There are many anecdotes in this regard. This skill has generally come to be known as the "back of the envelop calculations" made famous by Enrico Fermi.

A recent collection of such *common sense* problems in mathematics alone is available in [9].

It is surprising how much one can do with the help of a relatively small stock of primary data. And such data is not only of academic interest. It helps us tremendously in matters of safety in particular and living life in general. For instance, knowing that 1 atmospheric pressure (1 atm) is equivalent to a 10-meter column of water and our deepest oceans are about 10-km deep, we can rather easily conclude that a marine field biologist needs to bear a pressure of 500 atm[6] only to reach half as deep as the deepest parts of the Pacific Ocean! I agree that when life is at stake and safety paramount, detailed calculations are a must, but a quickly done *reasoned estimate* often comes in handy.

Table 1: Useful Bits

What	How Much
Gravitational acceleration, g	$10\,\mathrm{m\,s^{-2}}$
Densities of solids and liquids	$10^3 - 10^4\,\mathrm{kg\,m^{-3}}$
Density of air at sea level	$\approx 1\,\mathrm{kg\,m^{-3}}$
Density of water at sea level	$1000\,\mathrm{kg\,m^{-3}}$ or $1\,\mathrm{g\,cm^{-3}}$
Number of drops in 1 cc ($= 1\,\mathrm{cm^{-3}} = 1\,\mathrm{ml}$) of water	≈ 25
Length of (Earth) day	$\approx 10^5$
Length of (Earth) year	$\approx 31 \times 10^6 \approx 10^{7.5}$ sec
Earth's radius	6400 km (= 4000 miles)
Angle subtended by a finger's thickness at arm's length	$\approx 1^\circ$

[6] This is enormous pressure.

Thickness of paper	$\approx$ 0.1 mm
Mass of a paperclip	$\approx$ 0.5 g
Highest mountains, deepest oceans on earth	$\approx$ 10 km
Earth-moon separation	3.84×10^5 km ($= 2.4 \times 10^5$ miles)
Earth-sun separation	1.5×10^8 km ($=$ 93 million miles)
Atmospheric pressure	$\equiv$ to a 10-meter column of water (Use dimensional analysis to get this) $=$ 14.7 psi (*Only in America!*)
Avogadro's Number	6.02×10^{23}
Atomic masses	$1.6 \times 10^{-27} - 4 \times 10^{-25}$kg
Linear atomic dimension	$\approx 10^{-10}$m $= 1$
Molecules per cc of a gas at STP	2.7×10^{19}
Atoms per cc in a solid	$\approx 10^{23}$ (they are much more tightly packed as compared to gas!)
Elementary charge (e)	1.6×10^{-19}C
Electron mass	$\approx 10^{-30}$kg
Speed of light	3×10^8m s^{-1}
Wavelengths of visible light	400nm (violet) – 700nm (red)

Binomial theorem is a remarkable mathematical tool. Newton generalized it for any exponent that is a real number [8] (or even a complex number):

$$(x+y)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^{r-k} y^k \tag{1}$$

$$= x^r + r x^{r-1} y + \frac{r(r-1)}{2} x^{r-2} y^2 + \dots \tag{2}$$

where $r \in \mathbb{R}$, $k \in \mathbb{N}$, and

$$\binom{r}{k} = \frac{r \cdot (r-1) \cdot (r-2) \cdot \dots \cdot (r-k+1)}{k!} = \frac{r^{\underline{k}}}{k!}$$

.

Note that

$$r^{\underline{n}} = \prod_{k=0}^{n-1} (r-k)$$

is called the "n^{th} falling factorial of r".

We can use this with any *real* exponent. This comes in handy in several *approximations*.

When the exponent is an integer n, we get a finite sequence (equation (3)) instead of the infinite series of equation (2):

$$(1+x)^n = 1 + n \cdot x + \frac{n(n-1)}{2}x^2 + \cdots + x^n \tag{3}$$

where $n \in \mathbb{N}$. This happens because when the exponent is a positive integer, the product

$$r \cdot (r-1) \cdot (r-2) \ldots (r-(k-1)) \cdot \boldsymbol{(r-k)} \cdot (r-(k+1)) \cdot (r-(k+2)) \ldots$$

$$= n \cdot (n-1) \cdot (n-2) \ldots (n-(k-1)) \cdot \boldsymbol{(n-k)} \cdot (n-(k+1)) \cdot (n-(k+2)) \ldots$$

must be zero for all the terms where $k > n$ (the bold term $\boldsymbol{(r-k) = (n-k)}$ which is a part of all binomial coefficients of the terms where $k > n$ is 0).

On the other hand, when the exponent is not an integer, no binomial coefficient is zero and we get an infinite series. This *dual* nature of binomial expansion is intriguing!

This leads to the approximations

$$(1+x)^n \approx 1 + nx \tag{4}$$

and

$$\sqrt{1-x} = (1-x)^{\frac{1}{2}} \approx (1+x)^{-\frac{1}{2}} \approx 1 - \frac{1}{2}x \tag{5}$$

when $\boldsymbol{x \ll 1}$.

For a right triangle with sides a, b, and hypotenuse c, we get

$$c = (a^2+b^2)^{\frac{1}{2}} = a(1+\frac{b^2}{a^2})^{\frac{1}{2}}$$

which leads to

$$c \approx a(1+\frac{b^2}{2a^2}) \tag{6}$$

when $\boldsymbol{\frac{b}{a} \ll 1}$.

Table 2: Useful Mathematical Approximations

What	How Much
π^2	≈ 10
e	≈ 2.71
$log_{10}e$	≈ 0.43 $(log_e 10 \approx 2.3)$

$log_{10}\pi$	≈ 0.5
$log_{10}2$	≈ 0.301
$log_{10}3$	≈ 0.48
$log_{10}5$	≈ 0.7
$log_{10}7$	≈ 0.85
πrad	180 deg
1rad	$\approx$ 57 deg
$\sin\theta$ where $\boldsymbol{\theta} \lll$ **1rad**	$\approx \theta$
$\cos\theta$ where $\boldsymbol{\theta} \lll$ **1rad**	≈ 1
e^x	$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ $\approx 1 + x \; \forall x \lll 1$ and then it follows that $log_e(1+x) = x \; \forall x \lll 1$

Estimation problems can be best solved with a pencil on paper[7]. I have attempted to *show my work* on paper whenever possible. The *finished* solution, neatly typeset in LaTeX, hardly shows the struggle that a paper shows. I have believed that the *rough work* is more enjoyable than the *fair, presentation material* although the latter might be more easily understood by others[8].

Problem **1** –

Solution **1** –

Problem **2** –

Solution **2** –

Problem **3** – Make reasoned estimates of

1. the total number of ancestors you would have (ignoring inbreeding) since the beginning of the human race, and

[7]Of course, some geniuses like Von Neumann can do it in their heads.
[8]In other words, although I like typesetting documents in LaTeX, I believe that content teaches more than the form.

2. the number of hairs on your head.

Solution **3** –

1. Population is a fascinating topic. I drew two pictures on paper that may explain my approach. Starting with "me" in the current generation, I drew an *isolated* family tree of my family. If I now add a second isolated family (say, of my friend), then its family tree would be more-or-less identical to mine. This will proliferate the number of the *First Gen* ancestors! We need to fix the number of the First Gen ancestors to some number[9] (I chose 2); the family trees have got to merge into each other.

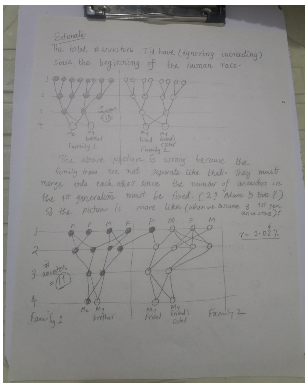

Figure 2: Estimate Ancestors: 1

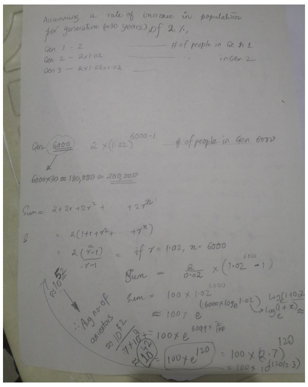

Figure 3: Estimate Ancestors: 2

[9]In Indian culture, some sages created families that cross-bred; who created those sages is left to speculation.

When I now think of these pictures, I believe they are wrong. But documenting this process is useful. Here are some simplifying assumptions that I am now making:

(a) There were 2 First Gen ancestors.

(b) On average, a generation is 25 years. Given that human race started about 200000 years ago, we have had about 8000 human generations.

(c) The population of the current (8000^{th}) generation is 7×10^9. Due to Thomas Robert Malthus [10], we assume that population grows in geometric progression.

(d) After they give birth to their last offspring, parents live for 25 years, i.e. one more generation.

(e) Total population up to and including the n^{th} generation are the ancestors of the entire population of the $(n+1)^{\text{th}}$ generation as a whole. Therefore, if we calculate the total number of humans who ever lived up to the previous generation and divide it by some number derived from the population of the current generation alone (we must exclude the population of the previous generation from that of the current generation (see 1d)), we can get the average number of ancestors of each member of the current generation.

Think of first four generations. *Assume* that the populations in those generations are 2, 8, 30, and 90 respectively. Then, the 2 people in the first generation have no human ancestors, $8 - 2 = 6$ people in the second generation together have 2 ancestors, $30 - 8 = 22$ people in the third generation together have $2 + 8 = 10$ ancestors, and $90 - 30 = 60$ people in the fourth generation together have $2 + 8 + 30 = 40$ ancestors.

I hope that this simplified model appears reasonable. Let's do some calculations:

Let

P_n be the population of the n^{th} generation; we assume $P_{8001} = 7 \times 10^9$,

r be the effective rate of per-generation increase of the population in a geometric progression,

C_n be the cumulative population up to and including (i.e. the total number of humans who ever lived on earth) the n^{th} generation,

A_n be the number of immediate ancestors (parents) of the n^{th} generation who are still alive. $A_n = P_{n-1}$,

M be the cumulative number of *my* ancetors (I live in the 8001$^{\text{st}}$ generation),

$n = 8000$.

We want to find M. Based on the above, for a geometric series,

$$\begin{aligned} a = 2, P_{8001} &= 7 \times 10^9 \\ 7 \times 10^9 &= 2 \cdot r^{8000} \\ \therefore r &= 1.0027525 \end{aligned} \tag{7}$$

$$\begin{aligned} M &= \frac{C_{8000}}{P_{8001} - A_{8001}} \\ &= \frac{C_{8000}}{P_{8001} - P_{8000}} \\ &= \frac{\frac{\cancel{a}\cdot(r^{7999})}{r-1}}{\cancel{a} \cdot (r^{8001} - r^{8000})} \\ &= \frac{1}{r \cdot (r-1)^2} \\ &\approx 131630 \end{aligned}$$

2. We assume that a human head has a surface area, S, of 100cm^2, only about 60% of which has hair and the average radius, r, of a human hair is about 40m giving a cross-sectional area of $\pi r^2 \approx 5000\text{m}^2$. We also assume that two hairs are about 9 hair diameters apart. The number of hairs on a human head, then, is about $120,000$:

$$H = \frac{1}{10} \cdot \frac{60}{5000 \times 10^{-8}} \frac{\text{cm}^2}{\text{cm}^2} = 12 \times 10^4 = 120000$$

Problem **4** –

Solution **4** –

Problem **5** –

Solution **5** –

Problem **6** –

Solution **6** –

Problem **7** –

Solution **7** –

Problem **8** –

Solution **8** –

Problem **9** –

Solution **9** –

Problem **10** –

Solution **10** –

Problem **11** –

Solution **11** –

Problem **12** –

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Problem **13** –

Solution **13** –

Problem **14** –

Solution **14** –

Problem **15** –

Solution **15** –

Problem **16** –

Solution **16** –

Problem **17** –

Solution **17** –

Problem **18** –

Solution **18** –

Problem **19** –

Solution **19** –

Problem **20** –

Solution **20** –

Problem **21** – How many inches per mile does a terrestrial (i.e. related to earth, land) great circle (e.g. a meridian or longitude) deviate from a straight line?

Solution **21** – A sphere and its great circle share their centers and a diameter.

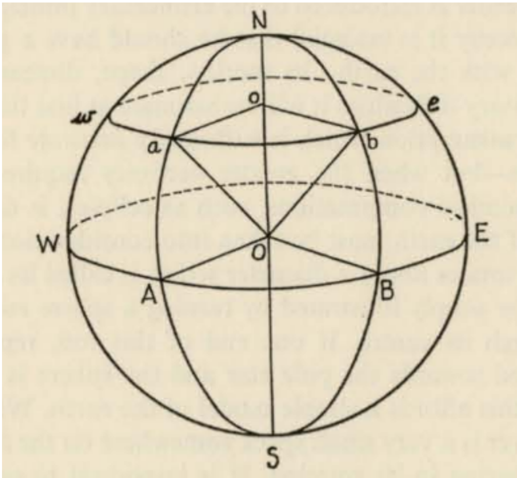

Figure 4: The terrestrial sphere. See [11]. N-a-A-S-N, N-b-B-S-N, W-A-B-E-W are all great circles and w-a-b-e-w is a small circle. The great circles with diameter NS make meridians (longitudes) whereas the circles with diameter parallel to WE make parallels (lattitudes).

In the Figure 5, as we start walking at B, if earth were flat, we would reach C after walking a mile. But since earth isn't flat, we would reach D. Thus, the deviation from the straight line $= CD$. From equation 6 we have:

$$c = r(1 + \frac{1}{2r^2})^{\frac{1}{2}}$$

$$CD = c - r = r(1 + \frac{1^2}{2r^2}) - r = \frac{1}{2r} = \frac{1}{8000}\text{mile}$$

$$CD \approx 0.66\text{feet}$$

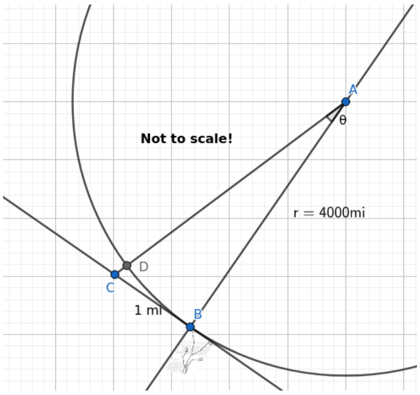

Figure 5: Walking on the Meridian.

Problem **22** –

Solution **22** –

Problem **23** –

Solution **23** –

Problem **24** –

Solution **24** –

Problem **25** –

Solution **25** –

Problem **26** –

Solution **26** –

Problem **27** –

Solution **27** –

Problem **28** –

Solution **28** –

Problem **29** –

Solution **29** –

Problem **30** – The table 3 lists the mean orbit radii of successive planets expressed in terms of the earth's orbit radius.

Table 3: Solar System Distances

n	Planet	$\frac{r}{r_E}$	$log_{10}\frac{r}{r_E}$
1	Mercury	0.39	$\approx 0.6015 - 1 \approx -0.4$
2	Venus	0.72	$\approx log_{10}72 - 2 \approx 3 * log_{10}2 + 2 * log_{10}3 - 2$ $\approx 3 * 0.3010 + 2 * 0.48 - 2$ ≈ -0.13
3	Earth	1	0
4	Mars	1.52	$\approx log_{10}19 + log_{10}8 - 2$ $\approx 1.28 + 0.903 - 2$ ≈ 0.183
5	Jupiter	5.2	0.72
6	Saturn	9.54	0.98
7	Uranus	19.2	1.28

Solution **30** –

1. Make a graph in which $log\frac{r}{r_E}$ is ordinate (Y-coordinate) and the number n abscissa (X-coordinate). On the same graph, replot the points for Jupiter, Saturn, and Uranus at values of n increased by unity each. A straight line can be fitted reasonably well through the new set of points. **Solution**: I made this plot (Figure 6) using the amazing tool, Desmos [13].

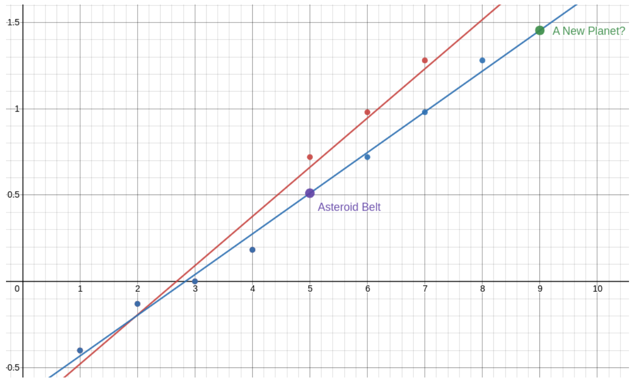

Figure 6: Semi-logarithmic Plot of Planet Numbers Versus Their Distances from the Sun

2. If $n = 5$ in the revised plot is taken to represent the asteroid belt between the orbits of Mars and Jupiter, what value of $log\frac{r}{r_E}$ would your new graph imply for this? Compare with the actual mean radius of the asteroid belt.

   Solution: The point on the revised semi-log scale plot is $(5, 0.511)$ for the later discovered asteroid belt. This gives $\frac{r}{r_E} = 10^{0.51} = 3.2$. *Universe Today* [12] reports this to be a little less (2.2 − 3.2 AU). It appears that the mass distributed in the asteroid belt was necessary to reason about the planetary motions.

3. If $n = 9$ is taken to suggest an orbit for the next planet beyond Uranus, what value would your graph imply? Compare it with observed value.

   Solution: The point is $(9, 1.454)$ which gives $10^{1.454} = 28.44$ AU for Neptune. Britannica [14] suggests a value of 30.05 AU. Our estimation is within an acceptable 5.66% of theirs.

4. Consider whether, in the light of (2) and (3), your graph can be regarded as the expression of a physical law with predictive value. (As a matter of history, it *was* so used. See the account of the discovery of Neptune near the end of Chapter 8.)

   Solution: The graph gives a glimpse of how the mass *might have* gotten distributed into various planets, if we believe the Big Bang. The distribution then gives rise to forces of attraction due to gravity and affects the periods of planets' revolutions around the sun. The line $log\frac{r}{r_E} = 0.255 \times n - 0.764$ (we found the slope and y-intercept of the line) shows some character of a physical law because it is a descriptive generalization of the planetary motion. If a planet is known to exist, then it can predict where it might be placed in the solar system.

Part I

The Approach to Newtonian Dynamics

It seems probable to me, that God in the beginning form'd Matter in solid, massy, hard, impenetrable, movable Particles ...

– Newton, Opticks(1730)

Chapter 1

A Universe of Particles

1. The *essence* of the Newtonian mechanics is that the motion of a given object is analyzed in terms of the *forces* to which it is subjected by its environment.
2. "Particles" are difficult to define. For the purpose of Newtonian Mechanics, however, we define a particle as an object that can be treated as a *point mass*.
3. A non-exhaustive list of *properties* of such particles consists of things like mass, size, shape, structure, magnetism, electric charge, and interaction with (similar or dissimilar) other particles.
4. We ask a question: "What is the smallest number of properties that suffices to characterize a particle?" The answer to it depends on whether the particle is *elementary*.
   (a) For an elementary particle, like an electron or proton, it appears that it is enough to specify its *charge* and *mass*.
   (b) Since larger objects which are made of many atoms are usually electrically neutral, it is enough to know their masses. Forces occur in larger objects as a result of *interactions* among them and therefore, we need to know the rules of these interactions as well. Size of an object is also useful mainly because it is the magnitude of its size that helps us determine if an object can be treated as a point mass.
5. We survey particles of interest starting with smallest and go up to some of the largest known.
6. We use SI units in this book.

7. **In keeping with the spirit of science, things change all the time. The book has a certain (the then current) view of *particles*. Particles, especially elementary particles, are researched aggressively. The 2006-view (35 years after this book was published) that is accessible to general public is captured in [15], and, of course, in Wikipedia's page on "The Standard Model" [16].**

8. Electrons and Nucleons:

   (a) The concept of size is not unambiguously defined for *any* object.

   (b) Electrons with a mass of about 9.1×10^{-31}kg are not yet analyzed as a collection of smaller particles and hence they are fundamental particles.

   (c) If we regard electron as a *sphere of electric charge*, its radius is estimated to be 10^{-15}m.

   (d) Nucleons, the inhabitants of the nucleus, have a mass of about 10^{-27}kg. The two principle nucleons are protons (charged) and neutrons (neutral). A free proton is stable whereas a free neutron decays, in less than 15 minutes, into a proton, an electron, and an antineutrino.

   (e) Nucleons have a more complicated internal structure as compared to electrons.

9. Atomic Nuclei:

   (a) The smallest and lightest nucleus is of course one proton (which is the same as a hydrogen *ion* (H^+)).

   (b) The heaviest naturally occurring nucleus (that of U_{92}^{238}) contains 238 nucleons. (**Before introducing a uranium nucleus, perhaps we should mention that there are 92 different, naturally occurring nuclei (along with electrons), each of which has a characteristic number of nucleons. This is a miracle in itself. Each such nucleus is that of an *element*.**)

   (c) If we assume that nucleons are equally densely packed in each element's nucleus, we get (denoting the nuclear mass by m and spherical nuclear volume by v):

$$\frac{m_1}{v_1} = \frac{m_2}{v_2}$$

$$\frac{m_n \times A_1}{\frac{4}{3}\pi r_1^3} = \frac{m_n \times A_2}{\frac{4}{3}\pi r_2^3}$$

where m_n denotes the mass of *a* nucleon (which is the same for each element), A the atomic mass number (that is a fancy name for the number of nucleons) of an element.

Upon simplification, we can see ($\frac{r_1^3}{r_2^3} = \frac{A_1}{A_2}$) that the diameter of a nucleus is roughly proportional to the cube root of its atomic mass number, A.

(d) The unit of distance, *Fermi, F* (named after the Italian physicist Enrico Fermi), is used to denote the extremely small nuclear distances: $1F = 10^{-15}$m. This can give us some sense of how dense nuclear material is, although it is difficult to comprehend such a dense matter:

$$d_U = \frac{4 \times 10^{-25}}{\frac{4}{3}\pi(10 \times 10^{-15})^3} \approx 10^{17} \mathrm{kg\,m^{-3}}$$

That makes the uranium nucleus 100 trillion times as dense as water.

10. Atoms:

(a) Chemists had established for elements a *relative mass scale* giving hydrogen a mass of 1. The idea of *mole* was introduced as **that amount of a pure substance whose mass in grams is numerically equal to its relative mass on this scale.**

(b) Avogadro's brilliant idea (of course, others contributed to it as well) of a *mole* (mol) said that equal moles of a pure substance (an element or compound) must contain equal numbers of their fundamental particles (atoms or molecules). Thus, 1mol of hydrogen contains as many H atoms as the number of H_2O molecules in 1mol of water or sucrose molecules in 1mol of sucrose. Although it was known that 2mol of hydrogen combine with 1mol of oxygen to produce 1mol water, the number of molecules or atoms in a mole (the so-called Avogadro's constant) itself was unknown.

(c) Existence of the *characteristic mass transfers* in electrolysis of ionic compounds (e.g. NaCl, $ZnCl_2$, AgCl) suggested a strong link between charge and atomic mass. Based on the amount of an ionic substance collected under a known electric potential at the electrodes, a correlation could be established. After precise measurement of electric charge by Millikan's experiments, atomic masses were established.

(d) The mass-to-charge ratio is measured by mass spectroscopy today.

(e) An *atomic mass unit, amu,* is defined as $\frac{1}{12}$ of the mass of the isotope ^{12}C (carbon 12).

$1\text{amu} = 1.67 \times 10^{-27}\text{kg}$

(f) Mass of an atom is roughly the same as the mass of its nucleus, but its diameter is about 10^4 times of that of the nucleus.

(g) The diameter of an atom is expressed in : $1 = 10^{-10}\text{m} = 10^5 F$.

(h) The heaviest atoms are *not* markedly bigger than the lightest ones. The general trend in the periodic table is that the atomic radius increases down a group and decreases across a period to the right. Alkali metals have the highest atomic radii in a given period.

(i) The atoms or molecules of a gas at normal atmospheric pressure are separated from one another, on average, by about 10 times their diameter.

11. Molecules and Living Cells:

(a) A convenient unit to represent sizes of things that are of interest to biologists is *micron*. 1 micron $= 10^{-6}$m.

(b) On average, a human cell is about 10 micron in diameter. Then, the estimated number of cells (assuming we have as many cells as there is non cellular matter) in a human body of volume 0.1m^3 is

$$n_c = \frac{0.1}{2 \times \frac{4}{3}\pi \times 125 \times 10^{-18}} \approx 10^{14}$$

A 70kg human who has about 10^{14} cells has, on average, a cell mass,

$$m_c = \frac{70}{10^{14}}\text{kg} = 7 \times 10^{-13}\text{kg}$$

Human body comprises 65%O, 18.5%C, 9%H, 3.2%N, and 1%P *by mass*. Then, on average, an atom of the human body has a mass of:

$$m_a = 0.65 \times 8 + 0.185 \times 12 + 0.09 + 0.032 \times 7 + 0.01 \times 15 \text{ amu}$$

$$m_a \approx 7.9 \text{ amu} \approx 13.19 \times 10^{-27}\text{kg}$$

Thus the number of atoms in a human cell on average = $\frac{m_c}{m_a} = 0.5 \times 10^{14}$.

12. Sand and Dust:

   (a) These particles, predominantly of quartz (crystalline SiO_2) are chemically inert.

   (b) The earth's surface is loaded with such particles (humans build houses that need sand as well).

   (c) Windblown particles have a diameter of about $0.01\text{mm} - 1\text{mm}$. Below that size the material is airborne. Smallest dust particles are about 0.1 micron in diameter.

13. Other Terrestrial (Belonging to Earth) Objects: These include objects in the range of masses $10\text{mg} - 1000\text{kg}$.

14. Planets and Satellites:

   (a) It does not occur to us to treat Earth as a particle while we are on it, but once we are able to leave it to go into the Space, we can think of it as a tiny speck.

   (b) Planets in the solar system can be regarded as point masses because of the vastness of the empty space. Of course, we need to take into account little distances and masses if the problem (e.g. tides on Earth) requires us to do so.

   (c) Mercury is the smallest planet and Jupiter is the largest. Earth is the densest. Together the planets **represent almost all of the mass around the sun in our solar system; Jupiter alone represents almost $\frac{2}{3}$**.

   (d) Planets have natural (and to a very small extent artificial) satellites. Examples of interesting artificial satellites (space probes) are the Hubble telescope, the International Space Station, Cassini that, while orbiting Saturn, crashed into Saturn (by design) after its mission was complete. In addition to these satellites, we have a belt of tens of thousands of tiny asteroids[1] between Mars and Jupiter that orbit the Sun.

15. Stars:

   (a) Stars are the most gigantic chemical plants we know. There is just so much going on on a star! And yet, when, with a naked eye, we look at it in the night sky and all it does is illuminate brightly or twinkle making us disbelieve any and all of the facts like, "Our Sun has space for 1 **million** Earths and that it is losing 4 million metric tonnes a second."

[1] For professional astronomers, these, along with some wealthy humans' artificial satellites being planned to be sent into the Space 2020-30, are the vermin of the skies.

(b) Astronomical distances are so large that a convenient unit of distance is 1 light-year which is the distance light travels in vacuum in a year = 9.46×10^{15}m. The nearest star the sun is about 4 light-years which is roughly 25 trillion = 25000000000000 miles away. Even when stars are massive, the interstellar separation is even more astounding. The ratio of diameters of large stars to the separation distance is of the order of 10^{-7} or 10^{-8}. It is very difficult to achieve the kind of vacuum that exists in the Space in a laboratory.

(c) Even though stars are huge, the interstellar distances allow us to use them as particles.

16. Galaxies:

(a) By 1924, Edwin Hubble produced a conclusive proof that our own Galaxy (which was mistaken, before 1920's, for the universe) was only one of innumerable systems of the same general kind.

(b) Galaxies are clusters of stars.

> Galaxies are the largest single aggregates of stars in the universe. They are to astronomy what atoms are to physics.
>
> *–Adam Sandage*

(c) A galaxy can contain anywhere between $10^6 - 10^{11}$ stars!

(d) And yet, the intergalactic distances make galaxies themselves as particles!

(e) Since galaxies are receding from each other at a speed proportional to the intergalactic distances[2], it may so happen that – because of "galactic red shift", the energy from them falls short of reaching us – a limit is put on the knowable universe.

(f) If we assume the density of matter throughout the Space as 10^{-26}kg m^{-3} and volume of the order of $(10^{26})^3$, then the total mass is about 10^{52}kg which will correspond to about 10^{11} galaxies.

(g) These masses and distances, that is, a *particulate* view of the matter is a strong candidate for the application of Newtonian mechanics.

Problem **1** –

[2] Another of Hubble's astounding findings.

Solution **1** –

Problem **2** –

Solution **2** –

Problem **3** – Calculate the approximate mass, in tons, of a teaspoonful of nuclear matter.

Solution **3** –

1 tsp = 4.92ml.

We have already established (see 9d) that nuclear matter is about 10^{14} times as dense as water. Therefore the mass of nuclear matter that only occupies 1 teaspoon,

$$m = 4.92 \times 10^{14}\text{g}$$
$$m = 4.92 \times 10^{14}\text{g} = 4.92 \times 10^{8}\text{t}$$
$$m \approx 500 \text{ million tonne}$$

(The book says it is 10^9t which is off by a factor of 2.)

Problem **4** – Sir James Jeans once suggested that each time any one of us draws a breath, there is a good chance that this lungful of air contains at least one molecule from the last breath of Julius Caesar. Make your own calculation to test this hypothesis.

Solution **4** – The volume of air in a breath, the so-called tidal volume, is estimated to be about $500\text{ml} = 500\text{cm}^3$. Assuming an air of density 1.2kg m^{-3}, the *amount* of air in a human breath = $1.2 \times 500 \times 10^{-6}\text{kg} = 6 \times 10^{-4}\text{kg} = 0.6\text{g}$. Assuming the molar mass of air to be 29g,

$$n_b = \frac{0.6 \times 6.022 \times 10^{23}}{29} \approx 1.23 \times 10^{22}$$

where, n_b is the number of *molecules* in a human breath. When Julius Caesar breathed his last, he exhaled n_b molecules into the atmosphere.

Atmosphere of Earth is extremely complex. Its mass is estimated to be about 5.1×10^{18}kg $= 5.1 \times 10^{21}$g.

$$n_a = \frac{5.1 \times 10^{21} \times 6.022 \times 10^{23}}{29} \approx 10^{44}$$

where n_a is the approximate number of molecules in the atmosphere. We can now see that *the fraction* of atmosphere that was Caesar's last breath was:

$$\frac{n_b}{n_a} = \frac{1.23 \times 10^{22}}{10^{44}} \approx 1.23 \times 10^{-22}$$

which is slightly higher than $\frac{1}{n_b} = 0.8 \times 10^{-22}$ the fraction of my breath that 1 molecule represents.

It appears that Sir James Jeans knew very well what he was talking about.

Chapter 2

Space, Time, and Motion

> Hitherto I have laid down the definitions of such words as are less known, and explained the sense in which I would have them to be understood in the following discourse. I do not define time, space, place, and motion, since they are well known to all. Only I must observe, that the vulgar (i.e. the common people) conceive those quantities under no other notions but from the relation they bear to sensible objects. And from there arise certain prejudices, for the removing of which, it will be convenient to distinguish them into absolute and relative, true and apparent, mathematical and common.
>
> *–NEWTON, Principia (1686) [17]*

1. What is Motion?
   (a) Our ability to give any precise account of motion depends upon the use of two separate concepts: *space* and *time*. **We say that an object is moving if it occupies different positions at different instants**.
   (b) Our ideas of motion align with those of Newton himself. Again, in keeping with the changing and falsifiable nature of science, these notions that Newton himself had are inadequate in the light of what we have learned since Newton's time. Here (in a pair of square brackets) are some of Newton's assertions which appear plausible, but should be *taken with a dose of healthy skepticism*:
      i. [Space is absolute.]
      ii. [Absolute Space, in its own nature, without relation to anything external, remains always similar and immoveable.]
      iii. [Each object in the universe exists at a particular point in space and time.]

iv. [Our physical measurements agree with the theorems of Euclidean geometry and Space is assumed to be Euclidean.]

v. [Time is also absolute and flows on without regard to any physical object or event. Principia [17] says, "Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called *duration*: relative, apparent, and common time is some sensible and external (whether accurate or equable[1]) measure of duration by means of motion, which is commonly used instead of *true time* ..."]

vi. [One can neither speed up nor slow down the *rate of the flow of time* and it exists uniformly throughout the universe. Identical and synchronized clocks placed anywhere in the universe can correctly mark off the flow of absolute time and remain synchronized forever.]

vii. [Space and Time are, although independent, in a sense interrelated.]

It was later found that although they sound plausible, many of the above ideas have consequences that are inconsistent with experience. This first became clearer with motions at very high ($\approx c$, the speed of light) speeds. This is not to say that Newton was not aware of relativistic ideas, but that it appears that he believed in an absolute space and time. It is true that we are unable to experience absolute motion: Right now, although the tree or the table that you are looking at is moving in an absolute sense, you are not able to experience that movement; you believe that it is stationary. A. I. A. Adewole carefully went through what Newton wrote in his Principia [17] at [18] and here is the summary of his commentary on Newton's views on *Absolute Space*:

i. Our freedom to relate motion to any designated body of our choice is *not* hindered by the existence of *absolute space*. In other words, some body does *not* need to be at *absolute rest* for us to relate motion of some other body to it. If the body we chose happens to be at absolute rest, then any motion *with respect to it* will also be absolute, otherwise, it will not be absolute.

ii. As long as we are only interested in *relative motions* of bodies, we can choose any body whatsoever and only consider those

[1] i.e. not varying

motions *with respect to it*.

iii. The fact that relative motion can be with respect to any body does not disprove the existence of an absolute space.

iv. We are unable to distinguish the "state of rest" from the "state of uniform motion" in both relative space and absolute space.

2. Frames of Reference:

   (a) "A car is moving" implies that "There is a relative motion between a car and earth (and all the things like trees, buildings etc. that are firmly attached to the earth)".

   (b) We accept the *local surroundings* – a collection of objects firmly attached to earth and therefore at rest relative to each other – as defining a *frame of reference* with respect to which the changes in position of other objects can be observed and measured.

   (c) The choice of a frame of reference is arbitrary, of course. However, it is convenient to choose a frame of reference in which *the description of motion is simplest*. Thus, while describing the motion of a car on earth or the movement of a roller-coaster, it is convenient to use a frame of reference that is attached to the earth rather than to the sun (which has a motion relative to earth and vice versa).

   For an excellent video introduction to frames of reference see [19]. There are theoretical reasons to choose some reference frames to others that we shall see later TODO: ref here.

3. Coordinate Systems:

   (a) A frame of reference is defined by some array of physical objects *that remain at rest relative to each other*.

   (b) *The Space of our experience* has three dimensions, we need to specify three independent quantities in order to uniquely fix the position of a point in Space.

   (c) Problems we consider are mainly two- and three-dimensional.

   (d) In a 2-D plane, we have the so-called Cartesian (x, y) and Polar coordinates(r, θ) of a point P that are equivalent:

$$\text{In 2-D} \begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \frac{y}{x} \\ x = r \cdot \cos\theta \qquad y = r \cdot \sin\theta \end{cases} \tag{2.1}$$

(e) In Cartesian coordinate system, if we denote $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ as *unit vector*s in the $x-$ and $y-$ directions respectively, we get

$$\text{(In 2-D)} \qquad \vec{\boldsymbol{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} \tag{2.2}$$

where $\vec{\boldsymbol{r}}$ is the *position vector* of $P(x, y)$.

(f) In Polar coordinate system, if $\hat{\boldsymbol{e}}_{\boldsymbol{r}}$ is the unit vector in the direction of the radius vector $\vec{\boldsymbol{r}}$ and $\hat{\boldsymbol{e}}_{\boldsymbol{\theta}}$ the unit vector in the direction perpendicular to the radius vector, then we get $\vec{\boldsymbol{r}} = r\hat{\boldsymbol{e}}_{\boldsymbol{r}}$. Note that $\hat{\boldsymbol{e}}_{\boldsymbol{\theta}}$ is not used in this expression, but it will be used often.

(g) For the 3-D space, the most generally useful coordinate systems are the 3-D Cartesian coordinates (x, y, z) and Spherical Polar coordinates (r, θ, φ).

$$\vec{\boldsymbol{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}} \tag{2.3}$$

where, like $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$, $\hat{\boldsymbol{k}}$ is the *unit vector* in the $+z$-axis. For a more detailed description of the Spherical Coordinate System, see [20].

(h) We use the right-handed Cartesian system. This means that if a *right-handed screw* is rotated from $+x$-axis toward $+y$-axis, it will move in the direction of $+z$-axis. Authors adopt different conventions, but we'll adopt the convention that is shown in Figure 2.1. According to this convention,

   i. The coordinates are (r, θ, φ).

   ii. The $+z$-axis is called the *zenith*.

   iii. The angle θ that the position vector of $P(x, y, z)$ makes with the $z+$-axis is called the *polar angle*. $0 \leq \theta \leq \pi$. The complementary angle $\lambda = \frac{\pi}{2} - \theta$ is called the *latitude* of P.

   iv. The angle φ that the *orthographic projection* of P in the XY-plane (the so-called plane of the meridian) makes with the $+x$-axis is called the *azimuthal angle*. $-\pi \leq \varphi \leq \pi$. φ is also called the *longitude* of P.

(i) Cartesian coordinates in terms of Spherical Polar coordinates:

$$\text{In 3-D} \begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases} \tag{2.4}$$

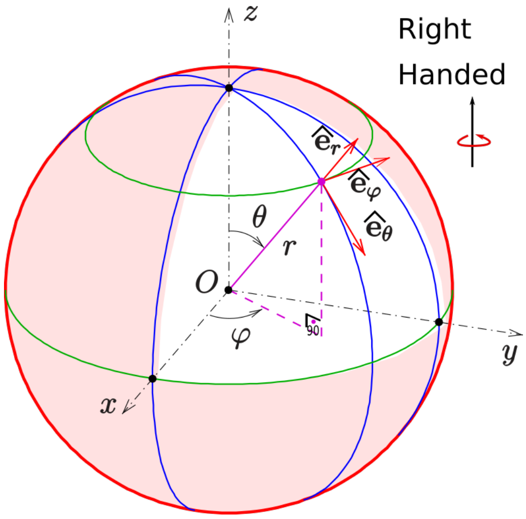

Figure 2.1: The Spherical Polar Coordinate System

(j) Spherical Polar coordinates in terms of Cartesian coordinates:

$$\text{In 3-D} \begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \arccos \frac{z}{r} = \arccos \frac{z}{\sqrt{x^2+y^2+z^2}} \\ \varphi = \text{atan2}\frac{y}{x} \end{cases}$$

where the function $atan2$ returns the angle in Euclidean plane, in radians, from the $+x$-axis to the ray joining $P(x, y)$ and the origin $O(0, 0)$; $-\pi \leq \text{atan2}\frac{y}{x} \leq \pi$.

(k) In the vector notation for Spherical Polar coordinates, we have three unit vectors (you can imagine standing on the surface of the sphere at P):

i. $\hat{\boldsymbol{e}}_{\boldsymbol{r}}$ – the unit vector along OP; this points vertically upward. As in the case of two dimensions (see: 3f), $\vec{\boldsymbol{r}} = r\hat{\boldsymbol{e}}_{\boldsymbol{r}}$.

ii. $\hat{\boldsymbol{e}}_{\boldsymbol{\theta}}$ – the unit vector perpendicular to OP; this points due south.

iii. $\hat{\boldsymbol{e}}_{\boldsymbol{\varphi}}$ – the unit vector also perpendicular to OP; this points due east.

4. Combination of Vector Displacements: This item is really a very short sampler on vectors.

(a) Vectors are directed entities with magnitude. This is not a thorough definition as not everything with a direction and magnitude is called a vector. On the other hand, certain vectors may not have both direction and magnitude.

(b) In this context, an entity that has magnitude but no direction is called a *scalar*. When the term "vector" is *not* in context, scalar just means number.

(c) Historically, vectors were defined before the formal definition of *vector spaces* and vector fields (see [21]). Therefore, in physics, we sometimes talk of vectors without referring to vector spaces to which they belong.

(d) If an entity is defined in mathematics, interesting (and perhaps useful) things start to occur when an instance of such entity combines with another. Interesting things also happen when an instance of one entity combines with an instance of another entity. In other words, *operations on entities* are perhaps more interesting than the entities in isolation. In essence, defining axioms of vector calculus is what vector spaces do. Few operations on vectors are more frequent than others (although all are useful):

i. *Adding* two vectors

ii. *Scaling* a vector, that is, growing or shrinking its magnitude by a number (or *scalar*)

iii. *Multiplying* two vectors

(e) Since the topic of vectors is developed by various mathematicians, physicists, and other scientists, there are various assumptions and interpretations. In general, in physics, the location of a vector does *not* matter. As long as two vectors have the same magnitude and direction, they are equivalent, wherever in the space they are located.

5. Resolution of Vectors:

(a) The choice of the *coordinate system* is arbitrary. Once we have decided a convenient coordinate system (it does *not* matter whether the axes are orthogonal), however, we can resolve a vector into components along those axes. The *vector sum* of components of a vector along the axes of all possible coordinate systems always yields the same vector.

(b) The $n-$dimensional vector space and a not-necessarily-orthogonal coordinate system are of interest from a generalization standpoint (See [22]), but we will stick to an orthogonal coordinate system **unless stated otherwise** and illustrate it using two dimensions.

(c) The following is then given:

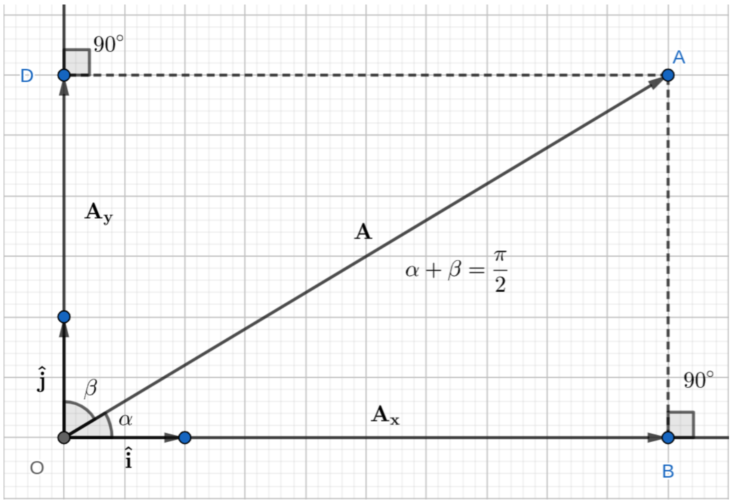

Figure 2.2: Basic 2-D Orthogonal Axes

i. The unit vectors (sometimes called the *Basis*) are denoted $\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}$. A vector is denoted by a bold capital letter with an arrow on top: $\vec{\boldsymbol{A}}, \vec{\boldsymbol{P}}$. The magitudes (which are numbers, or scalars) of vectors are denoted simply by a capital letter: A, B.

ii. The *scaling* of a vector $\vec{\boldsymbol{A}}$ by a number $n, n \in \mathbb{R}$ is denoted as a vector $n\vec{\boldsymbol{A}}$, which is simply a vector whose magnitude is $|n|$ times A, the magnitude of $\vec{\boldsymbol{A}}$. If n is negative, the scaled vector points in the exact opposite direction as $\vec{\boldsymbol{A}}$, otherwise it points in the same direction as $\vec{\boldsymbol{A}}$.

iii. A vector $\vec{\boldsymbol{A}}$ (in two dimensions) can be resolved as:

$$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{i}} + A_y\hat{\boldsymbol{j}} = (A\cos\alpha)\hat{\boldsymbol{i}} + (A\cos\beta)\hat{\boldsymbol{j}}$$

iv. The *scalar product* (also called the *dot product*) of two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ that are at an angle θ is defined as a number:

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = A \cdot B \cdot \cos\theta$$

Therefore, since $|\hat{\boldsymbol{i}}| = |\hat{\boldsymbol{j}}| = 1$, we get:

$$A_x = \vec{\boldsymbol{A}} \cdot \hat{\boldsymbol{i}}$$

$$A_y = \vec{\boldsymbol{A}} \cdot \hat{\boldsymbol{j}}$$

and

$$\vec{\boldsymbol{A}} = (\vec{\boldsymbol{A}} \cdot \hat{\boldsymbol{i}})\hat{\boldsymbol{i}} + (\vec{\boldsymbol{A}} \cdot \hat{\boldsymbol{j}})\hat{\boldsymbol{j}}$$

v. The above, although rather straightforward, may feel complicated. But it is useful in *relating* the components of a given vector in *different coordinate systems*. Consider, for instance, a coordinate system $x'y'$ obtained by rotating the xy coordinate system by an angle θ (See Figure 2.3). Let the unit vectors in the new system be $\hat{\boldsymbol{i}}', \hat{\boldsymbol{j}}'$ respectively.

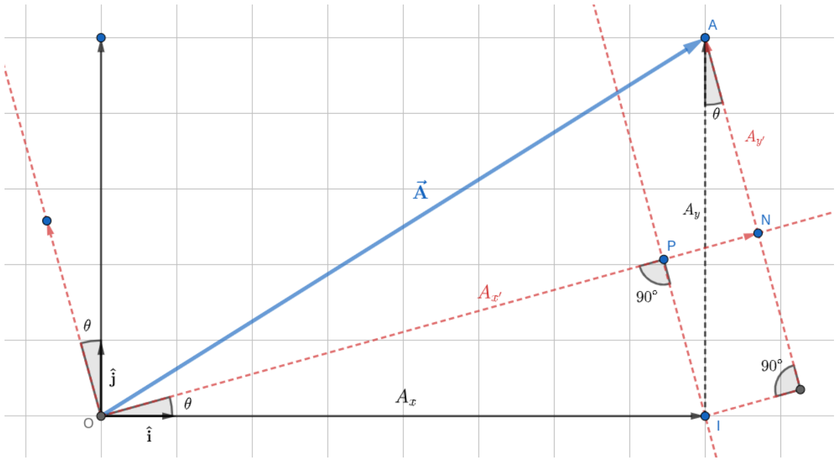

Figure 2.3: Basic 2-D Orthogonal Axes (rotated from Figure 2.2)

$$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{i}} + A_y\hat{\boldsymbol{j}} = A_{x'}\hat{\boldsymbol{i}}' + A_{y'}\hat{\boldsymbol{j}}'$$

We are interested in finding $A_{x'}$, $A_{y'}$ in terms of A_x, A_y. Taking the scalar product (which distributes over vector addition) of the above equation once with $\hat{\boldsymbol{i}}'$ and once with $\hat{\boldsymbol{j}}'$, we get:

$$A_{x'} = A_x(\hat{\boldsymbol{i}} \cdot \hat{\boldsymbol{i}}') + A_y(\hat{\boldsymbol{j}} \cdot \hat{\boldsymbol{i}}')$$

and

$$A_{y'} = A_x(\hat{\boldsymbol{i}} \cdot \hat{\boldsymbol{j}}') + A_y(\hat{\boldsymbol{j}} \cdot \hat{\boldsymbol{j}}')$$

because $\hat{\boldsymbol{i}} \cdot \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}} \cdot \hat{\boldsymbol{j}} = \hat{\boldsymbol{i}}' \cdot \hat{\boldsymbol{i}}' = \hat{\boldsymbol{j}}' \cdot \hat{\boldsymbol{j}}' = 1$ and $\hat{\boldsymbol{i}} \cdot \hat{\boldsymbol{j}} = \hat{\boldsymbol{i}}' \cdot \hat{\boldsymbol{j}}' = 0$. And,

$$A_{x'} = A_x \cos\theta + A_y \sin\theta$$

$$A_{y'} = -A_x \sin\theta + A_y \cos\theta$$

because $\hat{\boldsymbol{i}} \cdot \hat{\boldsymbol{i}}' = \cos\theta$, $\hat{\boldsymbol{j}} \cdot \hat{\boldsymbol{i}} = \cos(\frac{\pi}{2} + \theta) = -\sin\theta$, $\hat{\boldsymbol{j}} \cdot \hat{\boldsymbol{i}}' = \cos(\frac{\pi}{2} - \theta) = \sin\theta$, and $\hat{\boldsymbol{j}} \cdot \hat{\boldsymbol{j}}' = \cos\theta$.

The ease of algebraic manipulation (which can be generalized to more dimensions) is perhaps clear. In the Figure (2.3) one can see the geometric interpretation of the same result.

This can be generalized to a non-orthogonal or oblique coordinate system as well.

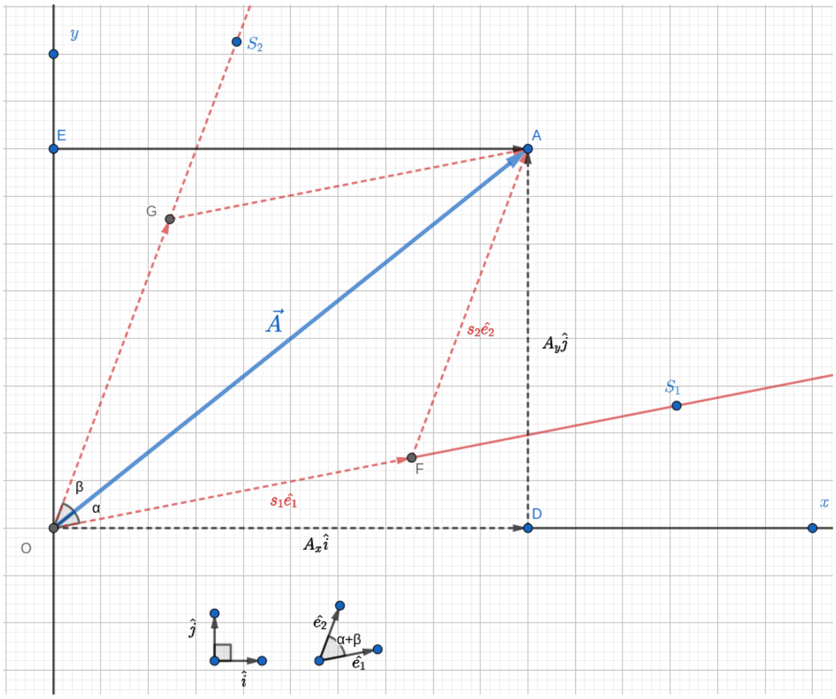

Figure 2.4: Finding Vector Components Along Non-orthogonal Axes

$$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{i}} + A_y\hat{\boldsymbol{j}} = s_1\hat{\boldsymbol{e}}_1 + s_2\hat{\boldsymbol{e}}_2$$

Taking the scalar product (which distributes over vector addition) of the above equation once with $\hat{\boldsymbol{e}}_1$ and once with $\hat{\boldsymbol{e}}_2$, we get:

$$A_x(\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{e}}_1) + A_y(\hat{\boldsymbol{j}}\cdot\hat{\boldsymbol{e}}_1) = s_1 + s_2(\hat{\boldsymbol{e}}_2\cdot\hat{\boldsymbol{e}}_1)$$

$$A_x(\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{e}}_2) + A_y(\hat{\boldsymbol{j}}\cdot\hat{\boldsymbol{e}}_2) = s_1(\hat{\boldsymbol{e}}_1\cdot\hat{\boldsymbol{e}}_2) + s_2$$

Since the axes are not orthogonal anymore, the scalar product $\hat{\boldsymbol{e}}_1\cdot\hat{\boldsymbol{e}}_2$ does *not* vanish. Nevertheless, the two above equations are linear in s_1, s_2 which can be solved since we know A_x, A_y and can find the scalar products $\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{e}}_1$, $\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{e}}_2$, $\hat{\boldsymbol{j}}\cdot\hat{\boldsymbol{e}}_1$, $\hat{\boldsymbol{j}}\cdot\hat{\boldsymbol{e}}_2$, and

$\hat{e}_1 \cdot \hat{e}_2$ knowing the various angles like α, β, and θ (for example, $\hat{i} \cdot \hat{e}_1 = \cos\theta$ if the angle between the $x-$ and S_1- axes is θ).

However, the use of oblique coordinate system like this is rather special.

6. Vector Addition and the Properties of Space: TODO: Revisit

   (a)

7. Time:

   (a) Although we *know* what time is, defining it is very hard.

   (b) Our concept of the passage of time is tied directly to the fact that things *change*. Would we have *felt* that the time passes had there been no perceptible change?

   (c) Although it may be valuable (as an intelligent construction perhaps) to have an abstract concept of time continuously flowing, our first-hand experience is only the observed behavior of a device called "clock".

Chapter 3

Accelerated Motion

Chapter 4

Force and Equilibrium

Chapter 5

Various Forces of Nature

Chapter 6

Force, Inertia, and Motion

Part II

Classical Mechanics at Work

Chapter 7

Using Newton's Law

Chapter 8

Universal Gravitation

Chapter 9

Collisions and Conservation Laws

Chapter 10

Energy Conservation in Dynamics; Vibrational Motions

Appendix A

History of Zeno's Paradoxes on Motion

1. Zeno's arguments have come to us via Plato, Aristotle, and Simplicius. And, of course, these arguments are understood from the translations of the Greek classics.

2. The four arguments of Zeno *against the notion of motion*:

   (a) **Dichotomy**[1]: You cannot traverse infinite *space* (thought of as a collection of an infinite number of *points*) in finite amount of *time*. This is akin to the reasoning that a rubber ball with a coefficient of restitution, say, 0.5 can never come to a standstill because it could not have made infinite number of bounces in a finite amount of time, if, indeed, it does come to a standstill.

   (b) **Achilles**: Achilles will never be able to beat the tortoise if tortoise has a head start even if Achilles is *faster* of the two.

   (c) **Arrow**:

   (d) **Stade**:

[1]Contrast or difference between two opposing things or ideas

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