Basic LaTeX Template

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Abstract

This paper computes the distance between two points and fits both linear and exponential functions through the two points.

1 Introduction

Consider the two points $(-1, 16)$ and $(3, 1)$. Section 2 computes the distance between these two points. Section 3 computes a linear equation $y = mx + b$ through the two points, and Section 4 fits a exponential equation $y = Ae^{kx}$ through the two points.

2 Distance

We can use the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{1}$$

to determine the distance between any two points (x_1, y_1) and (x_2, y_2) in $\mathbb{R}^2$. For our example, $(x_1, y_1) = (-1, 16)$ and $(x_2, y_2) = (3, 1)$, so plugging these values into the distance formula (1) tell us the distance between the two points is

$$d = \sqrt{(3 - (-1))^2 + (1 - 16)^2} = \sqrt{4^2 + (-15)^2} = \sqrt{241}.$$

3 Linear Fit

Consider a linear equation $y = mx + b$ through the two points. We will first determine the slope m of the line in Section 3.1, and we will then determine the y-intercept b of the line in Section 3.2.

3.1 Slope

The slope of the line passing through the two points is given by the forumula

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Plugging in our two points, we find the slope of the line between them is

$$m = \frac{1-16}{3-(-1)} = -\frac{15}{4}. \tag{2}$$

3.2 Intercept

To find the y-intercept of the line, we start with the point-slope form of the line of slope m through the point (x_0, y_0):

$$y - y_0 = m(x - x_0).$$

We plug in the point $(x_0, y_0) = (-1, 16)$ and the slope we found previously (2) to obtain the equation

$$y - 16 = -\frac{15}{4}(x+1).$$

Solving for y, we find the slope-intercept form of the line:

$$\begin{aligned} y &= -\frac{15}{4}x - \frac{15}{4} + 16 \\ &= -\frac{15}{4}x + \frac{49}{4}. \end{aligned}$$

Therefore, the y-intercept is $b = 49/4$, and the equation $y = -\frac{15}{4}x + \frac{49}{4}$ describes the line through the two points.

4 Exponential Fit

Let us consider the exponential function $y = Ae^{kx}$. For this function to pass through both points, we must find constants A and k that satisfy both equations $16 = Ae^{-k}$ and $1 = Ae^{3k}$. To solve these two simultaneous equations, we first take the ratio of the two equations, which gives us a single equation involving only k:

$$16 = \frac{Ae^{-k}}{Ae^{3k}} = e^{-4k}.$$

We can take the natural logarithm of this equation to solve for k:

$$-4k = \ln(16) = 4\ln(2),$$

which means $k = -\ln(2)$.

We can then use this value of k, along with either of the two points to solve for A. Let us consider the point $(-1, 16)$:

$$16 = Ae^{(-\ln(2))(-1)} = Ae^{\ln 2} = 2A.$$

Solving for A, we find $A = 8$, and the exponential equation through both points is

$$y = 8e^{-\ln(2)x} = 82^{-x} = 8\left(\frac{1}{2}\right)^x.$$

Here are examples of piecewise functions:

$$\chi_{\mathbb{Q}}(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q} \end{cases} \tag{3}$$

$$C_k = \begin{cases} 1 & \text{if } k = 1 \\ 1 & \text{if } k = 2 \\ C_{k-1} + C_{k-2} & \text{otherwise} \end{cases} \tag{4}$$