Calculus Made Easy - 02

Beautiful Ideas Presented Simply

Silvanus P. Thompson[1] Martin Gardner[2]

[1]Original Author

[2]Annotator

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Outline

First, on *Calculus Made Easy*

Thompson's Lively Treatise for Beginners

- These Slides Are Based on Silvanus Thompson's Good Little Book, Calculus Made Easy.
- After Quite Some Deliberation, [KM] Chose This Book To Introduce Calculus to Rujuta, A High Schooler.
  - Martin Gardner's[1] Thoughtful Recommendation Provided The Biggest Stimulus.
- We Also Go on 'Excursions' Outside The Book.
- Each Chapter Is A Separate Slide-deck to Keep Its Size Manageable.
- This Is Supposed to Be Your "Ready Reckoner".
  - Ideally, You Should Just Read The Book!
  - Use It as a Reference to Bring *It* All Back Later.
- New Words Are Introduced Slowly And Carefully, But Thought Experiments & Worthwhile Problems Are Often Introduced.
  - Let's Take the Leap of Guided Imagination Fearlessly!

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What Is A 'Function'?

An Everyday Word That Is Redefined ...

- In Math, We Precisely Define And Redefine Many Everyday Words!
- 'Function' Isn't An Exception.
- Different 'Views' to Look at Functions:
  - A Discrete (Relation, Table) View
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The *Discrete View* of A Function

The $\leq$ 1-Arrow Out Property

- One View: A Binary Relation with The "$\leq$ 1 Arrow Out" Property Between Elements of Sets, Say, A, B (on The *Relation Diagram*).

- This Is Discrete (Table) View of A Function: Each Discrete Point of Domain (A) Corresponds to A Discrete Point of Codomain[2] (B):$(a, 1), (b, 3), (e, 3), \ldots$.

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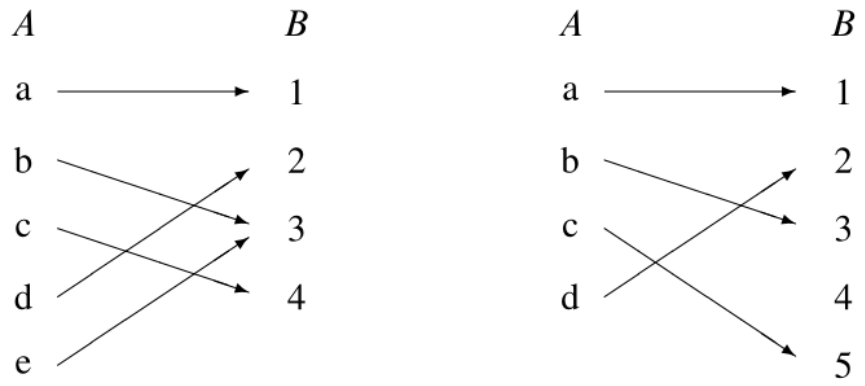

Figure: Examples of Function

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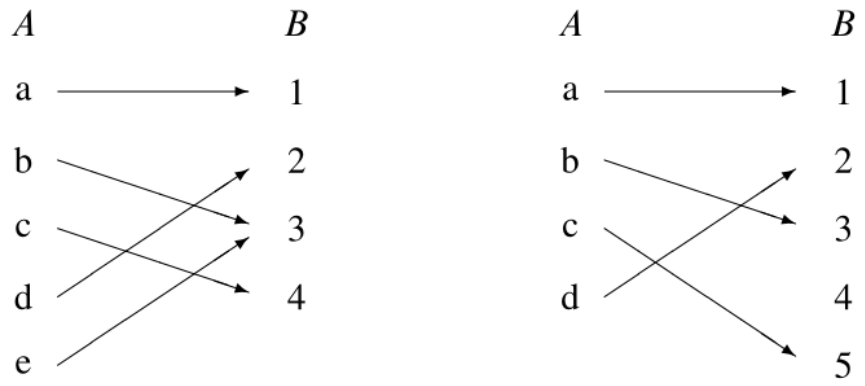

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- When No Point of Its Domain Has More Than 1 Arrow Out of Any Point, The Relation Becomes A Function.
  - IOW, One Domain Point Never Corresponds to More Than One Codomain Point.
    - It's Okay If A Domain Point Has No $\equiv$ Codomain Point. $\therefore \leq$ 1-Arrow Out (of Each Domain Point).
- Given A Relation Diagram, It's Straightforward to Prove If It Is A Function, Right?

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The *Discrete View* of A Function: Challenges

Tables Are Great, But ...

- Becomes Cumbersome When Domain and Codomain Become Too Big.
- Infinity Is Strange! What Happens When Domain and Codomain Are Infinite Sets?
- Although This View Is Self-Sufficient, It Deprives Us of A Connection between Sets.
- It's Well-suited for Computers. What about Humans :-)?

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Functions: Vocabulary

Common Words Redefined, Oh My!

- Domain, Codomain, Range
- Discrete, Continuous
- Function, Relation

What We Learned

A Summary of Sorts