

Learning Calculus from the Masters

How to Build on Elementary Ideas

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Aug 2025 / Free Learner's School Conversations

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WE LEARN FROM THE MASTERS.

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- Masters May Seem Austere And Sound Cryptic, But They Have Two Essential Traits
 - Kindness
 - Dependable and Thorough Knowledge of Something
- They Want You to Know What They Do (Know)
- They Understand Your Difficulties and Sympathize with You
- They Are Gentle, But *Don't* Dumb Down The Subject Matter

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THAT'S WHY.

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Figure: Augustus De Morgan Was A Master

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He Was A Great Mathematician Interested in Math
Education, And He Wrote Several Books to Help Students
Learn Math on Their Own!
Here Is a Tiny Story about Him:

Someone asked De Morgan, who lived in the 1800s, “Dear Mr. De Morgan, in which year were you born?”

De Morgan replied, “I was x years old in the year x^2 . In which year was I born?”

Which Year? :-)

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DON'T WE WANT TO LEARN FROM SUCH A
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- Reminder: In Mathematics, As in Everyday Speech, Words are Important. We Must Understand Them Precisely in The Given Context.
- Problem: Mathematics Uses Common Words and Gives Them Meanings That May Be Confusing at First.

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- Some Words of Mathematics are “Definitions”.
- Be Patient.
- Ask Sincere Questions Till They Are Clear.

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- Some Words of Mathematics are “Definitions”.
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Commensurable

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Here Is A Definition!

Definition (Commensurable)

Two Magnitudes or Lengths or Measures Are Commensurable If They Can Be Expressed in Whole Numbers of Some Common Unit.

To Recap [25] To Modern Language [30]

Note: The Common Unit Can Be Made As Small or As Big As We Please.

Note: Mensurable Means Measurable.

Commensurable

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But We Can Deconstruct it Slowly ...

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Consider An Example!

Example

‘Foot’ and ‘Yard’ Are Units of Length.

1 Foot is **Commensurable** with 1 Yard.

1 Foot = 12 Inches and 1 Yard = 36 Inches

- If the “Common Unit” is 3 (Inches), then 1 Foot Is 4 Common Units and 1 Yard Is 12 Common Units. Both 4 and 12 are Whole Numbers.
- If the “Common Unit” is 6 (Inches), then 1 Foot Is 2 Common Units and 1 Yard = 6 Common Units. Both 2 and 6 are Whole Numbers.
- If the “Common Unit” is 1 (Inch), then 1 Foot Is 12 Common Units and 1 Yard Is 36 Common Units. Both 12 and 36 are Whole Numbers.

Example: Commensurable

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We *Showed* That, with A Choice of 3 Common Units ¹,
(The Lengths) 1 Foot and 1 Yard Are Commensurable.

¹Although the Definition Requires Just One

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Consider Another Example!

Example

SFO \rightarrow NYC is a 6-Hour Flight.

SFO \rightarrow DTW is a $5\frac{1}{2}$ -Hour Flight.

Are These Two Periods, or Lengths of Time:

$T_1 = 6, T_2 = 5\frac{1}{2}$, Commensurable?

- What is Your Common Unit?
- How Many Whole Numbers of That Common Unit Are T_1, T_2 ?

Example: Commensurable

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Very Well Then!

To Show That Two Lengths Are Commensurable, We Find
A Common Unit and Demonstrate That Each Length
Equals Some Whole Number Times It.

Example: Commensurable

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You Are Convinced That $T_1 = 6, T_2 = 5\frac{1}{2}$ Are
Commensurable, Right?

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We Say ‘Lengths’ Because Humanity Measured Things Since Antiquity.

We Also Say ‘Magnitudes’.

Like In Everyday Language, We Use ‘Length’ and ‘Magnitude’ *Synonymously*.

Geometry Played A Big Role in Our Understanding of Mathematics ...

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We Saw 2 ‘Examples’ of Commensurable Magnitudes.
Sometimes, However, Counterexamples Are Helpful Too.
Are There Lengths That Are **Not** Commensurable?

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Commensurable Or Not?

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Figure: A Unit Square

In the Unit Square ABCD,
Are Its Side (e.g. AB) and Its Diagonal (e.g. BD)
Commensurable?

Commensurable Or Not?

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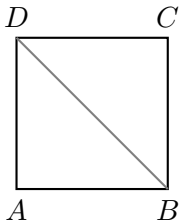


Figure: A Unit Square

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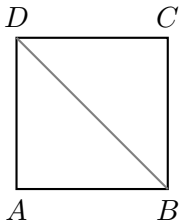


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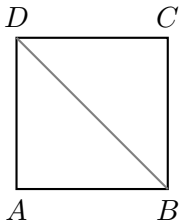


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In The Square ABCD [2],

$$l(AB) = 1, l(BD) = \sqrt{1^2 + 1^2} = \sqrt{2}$$

(From The Pythagorean Theorem).

Commensurable Or Not?

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Summary

- It Turns out That 1 and $\sqrt{2}$ Are Not Commensurable!
- There's No Common Unit, However Small, Of Which 1 and $\sqrt{2}$ Are Both Whole Numbers!

Commensurable Or Not?

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Summary

- It Turns out That **1 and $\sqrt{2}$ Are Not Commensurable!**
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Incommensurable?

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Summary

- We Repeat, **To Show That Commensurable Lengths**
(e.g. 2.1, 17.8) Are Indeed So ...
 - We *Creatively* Find A Common Unit (e.g. 0.1), And
 - Demonstrate That The Lengths Are Its Whole Numbers
(e.g. $21 \in \mathbb{Z}$ and $178 \in \mathbb{Z}$ respectively).
- Will That Approach Work **To Show That**
Incommensurable Lengths (e.g. $\sqrt[3]{3}$, $\sqrt[4]{5}$) Are Indeed So?
 - That Will Be An Unending Task! You See Why, Right?
 - And This Is Where We Need **Proofs!**

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Reread the Definition: Commensurable [1].

- 12 and 36 Are Commensurable. So Are 6 and $5\frac{1}{2}$. So Are Countless Other Pairs of Numbers.
- 1 and $\sqrt{2}$ Are **Not** Commensurable. So Are **Not** 2 and $\sqrt{5}$. So Are Countless Other Pairs of Numbers.
- Not Commensurable (Incommensurable) Pairs Are **Far Far More** Than Commensurable Pairs!
- When We Discovered Incommensurable Magnitudes Back in ~ 500 BC, A Few Decades Passed in A Confusion ...

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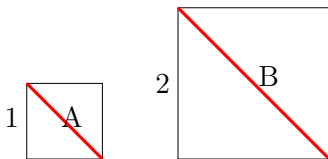


Figure: Diagonals of A 1x1 Square And A 2x2 Square

Are The Two **Diagonals: A, B** Commensurable?

- If Not, Why Not?
- If Yes, What Are The Common Unit And Whole Numbers for A, B?

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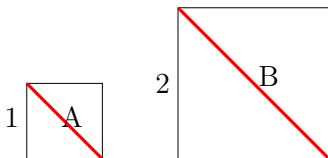


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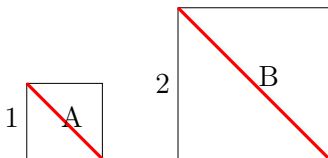


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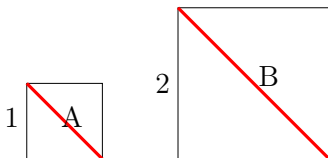


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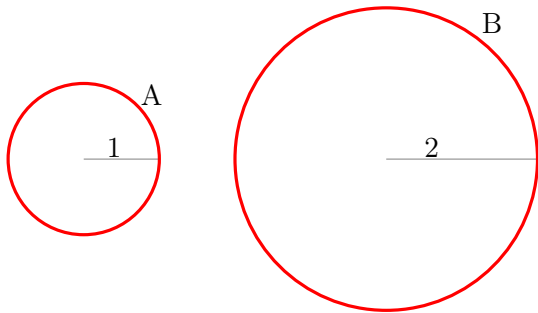


Figure: Circumferences of Circles of Radii 1 and 2

Are The Two Circumferences A, B Commensurable?

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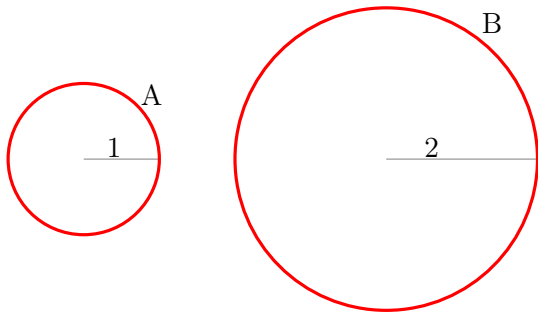


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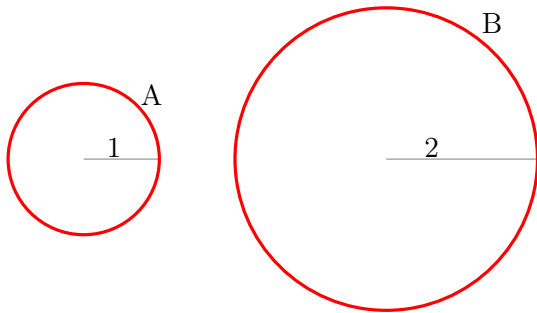


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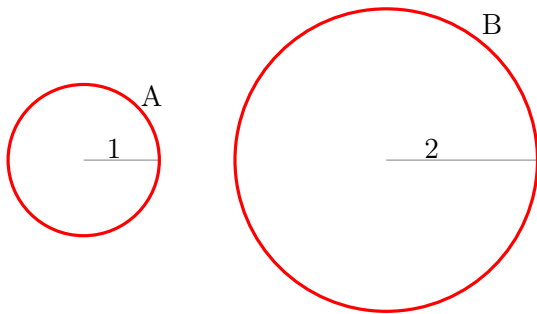


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They Are Special

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Integers Are Natural

God Made The Integers; All Else Is The Work of Man^a.

^aThe Mathematician Leopold Kronecker Said It First

- Do You Realize That Any Two Integers A, B Are Commensurable?
 - What Is the Common Unit?
 - What Are the Whole Numbers of A, B ?

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One Fact About Incommensurable Magnitudes

They Can Only Be Approximated

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Incommensurable Magnitudes Are Also Special

- For Incommensurable Magnitudes, A, B , There Exists No Common Unit!
- That Means, When Expressed as A Ratio, There's Nothing Common in Them.
- For Example, $\sqrt{2}$ Has No Exact Representation $\frac{A}{B}$, Where A, B Are Integers.
 - Otherwise, as We've Seen, 1 Would Be A Common Unit.
- That Means They Can Only Be Approximated as A Ratio of Two Integers: $\sqrt{2} \approx \frac{1414}{1000}$ Or $\sqrt{2} \approx \frac{14142}{10000}$.
 - That Funky Symbol \approx Means Approximately Equals

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Modern Language

We Build Our Math Vocabulary Using Previous Definitions

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Definition: Commensurable [1]

Definition (Rational Number)

If Magnitudes A, B Are Commensurable, Then The Ratio $\frac{A}{B}$ Is Called A *Rational Number*.

Definition (Irrational Number)

If A Number is Not Rational, Then It Is *Irrational*.

Note: This Is a *Negative* Definition. We Specify What Irrational Number Is Not.

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- Incommensurable.
- Rational Number.
- Irrational Number.
- Integer.
- Ratio.
- Fraction, Numerator, Denominator².
- \approx : Approximately Equals.

²We Didn't Learn These Here, But Assumed That You Know These Words ...

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It's A Matter of Perspective

Perhaps None of This Matters.

Commensurable Magnitudes, Irrational Numbers, ... Are All
Meaningless Without The Fun.

Having Fun Matters.

Let's Play Mindfully

This Playful Introduction Is Meant to Be ...

Well, Play!

Slowly, We Understand More, And Play More!

Does This Matter?

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The Ever-Changing Magnitudes

Things Change And That Changes Everything in Turn!

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- The Ever-Changing Universe Fascinates Us.
- The Study of Changing, Rather Than Fixed, Magnitudes Fascinates Us.
- Change Implies An Increase Or A Decrease.
- Let's Start with The Ratio of Magnitudes That Are Capable of Changing ...

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Ratio $r = \frac{A}{B}$

Both A and B Can Change ...

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- Let's Say You Have $A = \$1$ And I $B = \$2$.
- $B - A = \$1$, $r = \frac{A}{B} = \frac{1}{2}$.
- Which Is Right?
 - I Have Much More Money Than You.
 - I Have Almost as Much Money as You.
- Let's Say We Both Got \$98 (Things Changed!). You Now Have \$99 and I \$100.
- $B - A = \$1$, $r = \frac{A}{B} = \frac{99}{100}$.
- Now Which Is Right?
 - I Have Much More Money Than You.
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Their Difference Or Ratio?

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- Although in Both The Cases ...
- The Difference $B - A = \$1$ Is The Same ...
- It's **More Appropriate** To Say ...
- You And I Have *Almost* The Same Money **In the Second Case.**
- It's **More Appropriate** To Say ...
- You And I Have *Almost* The Same Money **When Their Ratio is Closer to 1.**
- Therefore,
- **The Ratio, Not The Difference** Indicates **The Equality of Changing Quantities.**
- **Do You Agree?**

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Observation, Reflection, Analysis ...

Hypothesis, Thinking and Experimentation, Generalization ...

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- Let's Recap!

- We Observe That

- Two Quantities³, Both Capable of Changing, *Approach Each Other ...*
- When Their *Ratio Approaches 1 Regardless* of Their *Initial Difference*.

- Is Our Observation Reliable Or Illusory?

- After All, We *Only Tried One Case*: Increasing A and B by 98.

- In Other Words, Does Our Observation Hold *In General?*

³Or Numbers, Magnitudes, Lengths, ... 

Observation, Reflection, Analysis ...

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Equality of Changing Quantities: Ratio or Difference?

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Informally ...

We Want to Show That, In General, Two Different Magnitudes Approach Equality When Both Are Increased While Preserving The Difference.

- We Need to Be Precise.
- But We Have Got to Start Somewhere and Proceed Logically.
- We Use Symbols Or Notation for This Purpose.

Equality of Changing Quantities: Ratio or Difference?

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Informally ...

We Want to Show That, In General, Two Different Magnitudes Approach Equality When Both Are Increased While Preserving The Difference.

- We Need to Be Precise.
- But We Have Got to Start Somewhere and Proceed Logically.
- We Use Symbols Or Notation for This Purpose.

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- Let A, B *Stand for* Initial Values of The Magnitudes.
- Let $A < B$. In Other Words, A *Stands for* The Lesser of The Two Magnitudes.
- Let d *Stand for* The Difference between A, B .

$$B = A + d \quad (1)$$

- Let r *Stand for* The Ratio of A, B .

$$r = \frac{A}{B} \therefore r = \frac{A}{A + d} \quad (2)$$

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- Where, m Stands for The Increase in Their Amount.

- What Does r Now Become?

$$r \rightarrow \frac{A_{new}}{B_{new}} = \frac{A + m}{A + m + d} \quad (3)$$

- What Is The Question Now?
- Right, Is $\frac{A+m}{A+m+d}$ Closer to 1 Than $\frac{A}{A+d}$?
Given: m Is A Positive Number (Because? Right, We're Increasing Magnitudes).

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Mathematically

Which **Ratio** Is Closer to 1?

$$r = \frac{A+m}{A+m+d} \text{ (Eq. [3]) Or}$$

$$r = \frac{A}{A+d} \text{ (Eq. [2])?}$$

Given: $A < B, m > 0$

- Notice That The Translation **Isn't Exact**.
- “Mathematically Speaking,” We Clearly Specified Our Assumptions (e.g., $A < B$).
- Without Clarifying Assumptions, The Translation Fails!
- Correct Translation Is An Art. Fortunately, Practice Makes Us Better!

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Will You Analyze This Question Then?

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The Result of Your Analysis Will Be a Proof.

I Trust You to Prove That $\frac{A+m}{A+m+d}$ is Closer to 1.

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- **Proofs** Are The Way of Mathematics to Settle Questions Once And for All.
- Once You Provide That Proof, The Truth Will Be Established (Under The Given Assumptions) Forever!

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- Commensurable Magnitudes (e.g. 2.5, 3.7) Can Be Expressed As **Exact Ratios** (e.g. *Exactly* $\frac{25}{10}$).
- Incommensurable Magnitudes (e.g. $\sqrt{3}$, 2) Can Only Be Approximated (e.g. *Approximately* $\frac{866}{1000}$, or $\frac{86602}{100000}$).
- Changing Magnitudes (**Commensurable Or Incommensurable**) A, B ($A < B$) Approach Equality When Increased While Keeping Their Initial Difference.

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- Commensurable Magnitudes (e.g. 2.5, 3.7) Can Be Expressed As **Exact Ratios** (e.g. *Exactly* $\frac{25}{37}$).
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- Changing Magnitudes (**Commensurable Or Incommensurable**) A, B ($A < B$) Approach Equality When Increased While Keeping Their Initial Difference.

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