Sta 601/360: Lab 4

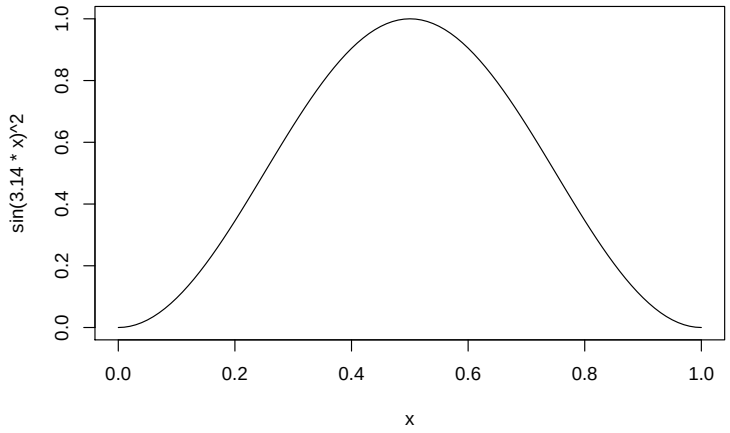
Moving on to the next form of sampling!

As you have already seen, we often end up with a form for the posterior which we know only up to a proportionality constant. Today we're going to look at a method for sampling from such a distribution directly, known as *rejection sampling*.

The idea of rejection sampling is to create an *envelope function* which is greater than or equal to the distribution we wish to sample from at all points, and which we can sample from easily. For example, suppose we end up with the following form for our posterior:

$$f(\theta) \propto \sin^2(\pi\theta), \quad \theta \in [0, 1].$$

The density looks like this:



We could use a Uniform(0,1) density as your envelope, since it is greater than or equal to the given density (up to proportionality) at all points and is easy to sample from. Rejection sampling works as follows:

1. Sample a point x from the envelope function
2. Calculate the ratio r of the *target* density to the *envelope* density at the point x you sampled.
3. With probability r accept the point x and add it to your list of sampled points; otherwise, throw it out.
4. Go to Step 1 and repeat until you have the desired sample size from your target density.

Your assignment is:

1. Implement this algorithm in R (or the programming language of your choice).
2. Propose and implement a different envelope function.
3. How does the efficiency of your envelope function compare to the Uniform envelope function?
4. Provide plots (histograms or density estimates) of the samples from (1) and (2) above.