STA 601 – Homework 16

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→ In class we discussed the <u>Unordered Categorical Data Model</u>:

$$y_i \in \{1, 2, 3 \ldots d\}$$

$X_i \in (n \times p)$ Matrix of features/predictors

$\beta_j \in (p \times 1)$ Vector of co-efficients

Z_{ij} – Latent Utility

$$\therefore \; Z_{ij} \sim N(x_i \beta, 1)$$

$$y_i = j \text{ if } Z_{ij} = \max\{Z_{i1}, Z_{i2} \ldots Z_{id}\}$$

→ <u>Full Posterior</u>:

$$(\beta_1, \beta_2 \ldots \beta_d, Z^n \mid y^n, x^n) \propto \left[\prod_{i=1}^{d} N_p(\beta_j; \beta_{0j}, \Sigma_{\beta_j})\right] \times$$

$$\left[\prod_{i=1}^{n} \prod_{j=1}^{d} N(Z_{ij}; x_i \beta_j, 1)\right] \times$$

$$\left[\prod_{i=1}^{n} \mathbb{1}(Z_{iy_i} = \max(Z_i))\right]\}$$

→ We also computed Full Conditionals for Gibbs Sampler:

- $(\beta_j \mid \beta_{(\sim j)}, Z^n, x^n, y^n) \sim N(\beta^*, \Sigma_{\beta^*})$

- $(Z_{ij} \mid Z_{i(\sim j)} \beta, x^n, y^n) \sim \underbrace{N(x_i \beta_j, 1)}$ constrained on $Z_{ij} = \max(Z_i)$

$\Downarrow$

This is a multivariate normal with covariance matrix as identity

- This means each Z_i is independent of others conditioned on β, x^n, y^n

- Because Z_i's are independent we will not have an ~~identiatibt~~ Identifiability problem with the model.

(I am not confident about this answer).