Lab 6

(Adapted from Doug Vanderwerken)

Let

$$\pi(\theta) \propto \exp\{-\frac{1}{2}\theta^2\} + \frac{1}{2}\exp\{-\frac{1}{2}(\theta-3)^2\}$$

be a normal mixture with modes near 0 and 3. Find the normalizing constant analytically (this need not be a complicated derivation).

Now, implement a Metropolis-Hastings sampler with proposal density

$$p(\theta'|\theta) = N(\theta, \sigma_{cand}).$$

Choose $\sigma_{cand} > 0$ such that the acceptance probability is very close to 45%, and report your choice. Generate at least 10,000 draws (after burnin–you determine how much burnin is needed) from your Metropolis-Hastings sampler. Provide a plot of the analytic and sampled densities of $\pi(\theta)$ on the same axes.

Whats wrong with using $\sigma_{cand} = .05$, in theory or in practice? What about $\sigma_{cand} = 8$? $\sigma_{cand} = 100$? Be specific.