STA 601 - Homework 6

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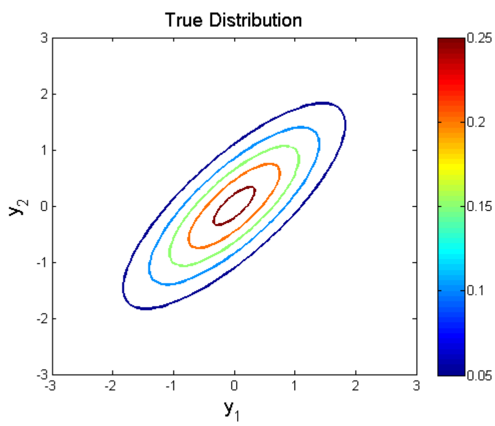
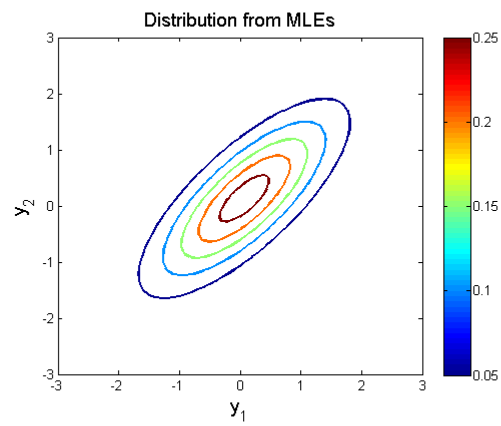
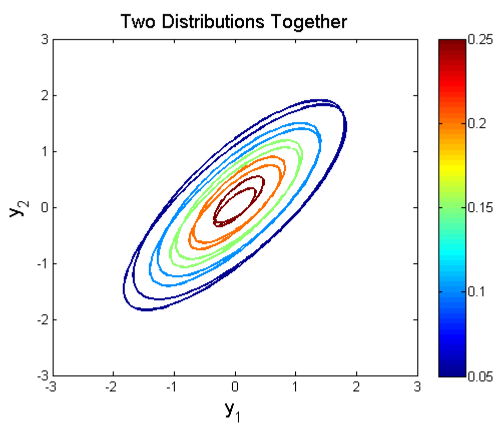
1. <u>Data:</u>
   $y_i \sim \mathcal{N}_2(\mu, \Sigma)$, $i = 1, 2, \ldots, 100$.
   $\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$, $\rho = 0.8$.

2. <u>Maximum Likelihood Estimates:</u>
   $\mu_{MLE} = \begin{bmatrix} 0.0715 \\ 0.0969 \end{bmatrix}$
   $\Sigma_{MLE} = \begin{bmatrix} 1.0789 & 0.9091 \\ 0.9091 & 1.1541 \end{bmatrix}$







3. Gibbs Sampling:
   I used the following priors:
   $\mu \sim \mathcal{N}_2 \left[\begin{pmatrix} 0.2 \\ 0.2 \end{pmatrix}, \begin{pmatrix} 1.25 & 0.6 \\ 0.6 & 1.25 \end{pmatrix} \right]$. $\Sigma \sim$ Inverse-Wishart $\left[4, \begin{pmatrix} 1.2 & 0.4 \\ 0.4 & 1.2 \end{pmatrix}^{-1} \right]$.
   Also, I used 10,000 samples and burn-in of 1000.

4. Estimates from Posterior:
   $\mu_{posterior} = \begin{bmatrix} 0.0735 \\ 0.0994 \end{bmatrix}$
   $\Sigma_{posterior} = \begin{bmatrix} 1.0791 & 0.9031 \\ 0.9031 & 1.1551 \end{bmatrix}$

5. Comparison:
   If we compare the Bayes estimates and MLEs, they are very close to the true values of (μ, Σ)

Appendix:

```
%% STA 601 - Homework 6
% Author: Kedar Prabhudesai
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close all;
clear all;

%% Setup Data and Distributions
% Create Bivariate Distribution
nSamples = 100;
rho = 0.8;
mu = [0 0];
SIGMA = [1 rho; rho 1];
y1 = -3:0.1:3;y2 = -3:0.1:3;
[Y1,Y2] = meshgrid(y1,y2);
yPDF = mvnpdf([Y1(:) Y2(:)],mu,SIGMA);
yPDF = reshape(yPDF,length(Y2),length(Y1));

% Get Data
rng('shuffle');
rSamples = mvnrnd(mu,SIGMA,nSamples);

% Get MLE
muMLE = mean(rSamples)
sigmaMLE = cov(rSamples)

% Plot Contours
% figure;contour(y1,y2,yPDF,'Linewidth',2);
% colorbar;
% xlabel('y_1','Fontsize',14);
% ylabel('y_2','Fontsize',14);
% title('True Distribution','Fontsize',14);
%
% figure;contour(y1,y2,reshape(mvnpdf([Y1(:) Y2(:)],muMLE,sigmaMLE),length(Y2),length(Y1)),'Linewidth',2);
% colorbar;
% xlabel('y_1','Fontsize',14);
% ylabel('y_2','Fontsize',14);
% title('Distribution from MLEs','Fontsize',14);
%
% figure;contour(y1,y2,yPDF,'Linewidth',2);hold on;
% contour(y1,y2,reshape(mvnpdf([Y1(:) Y2(:)],muMLE,sigmaMLE),length(Y2),length(Y1)),'Linewidth',2);hold off;
% colorbar;
% xlabel('y_1','Fontsize',14);
% ylabel('y_2','Fontsize',14);
% title('Two Distributions Together','Fontsize',14);

%% Gibbs Sampler
nGibbs = 10000;
ySamples = rSamples';
ybar = muMLE';
mu0 = [0.2 0.2]';
L0 = [1.25 0.6;0.6 1.25];

nu0 = 4;
S0 = [1.2 0.4;0.4 1.3];
% S0 = [625 312.5;312.5 625];

thetaSamples = zeros(2,nGibbs);
sigmaSamples = zeros(2,2,nGibbs);
sigmaSamples(:,:,1) = S0;
```

```
for iSample = 2:nGibbs
    % Update theta
    Ln = inv(inv(L0) + nSamples.*inv(sigmaSamples(:,:,iSample-1)));
    mun = Ln*(inv(L0)*mu0 + nSamples.*inv(sigmaSamples(:,:,iSample-1))*ybar);
    thetaSamples(:,iSample) = mvnrnd(mun,Ln);

    % Update Sigma
    Sn = S0 + (bsxfun(@minus,ySamples,thetaSamples(:,iSample)))*(bsxfun(@minus,ySamples,thetaSamples(:,iSample)))
    Z = mvnrnd([0 0],inv(Sn),nu0+nSamples);
    sigmaSamples(:,:,iSample) = inv(Z'*Z);
end

% Burn-In
thetaSamples(:,1:1000) = [];
sigmaSamples(:,:,1:1000) = [];

muPost = mean(thetaSamples,2)';
sigmaPost = mean(sigmaSamples,3);
```