STA 601 - Lab 1

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Beta-Binomial Model:

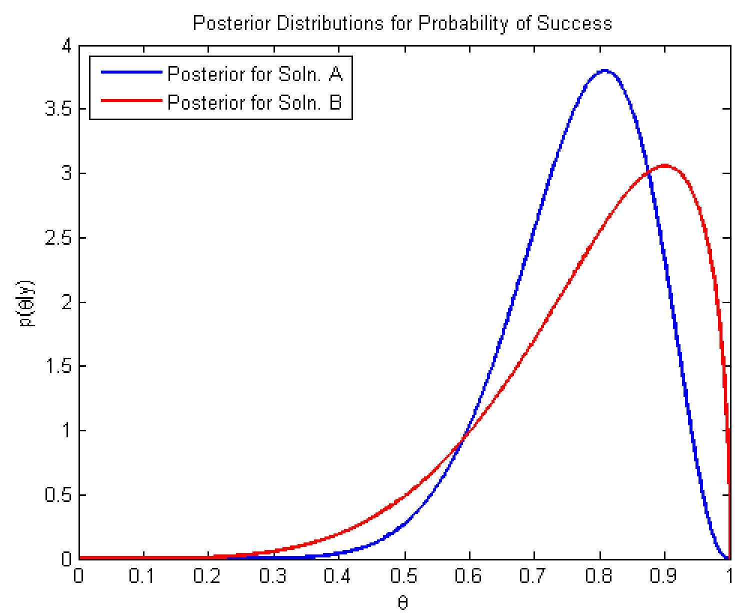
Likelihood function: $y|\theta \sim \binom{n}{y}\theta^y(1-\theta)^{n-y}$

Prior: $\theta \sim Beta(a, b)$.

Posterior: $\theta|y \sim Beta(a + y, b + n - y)$

1. Using *Jeffrey's Prior* $[Beta(\frac{1}{2}, \frac{1}{2})]$ as default prior, the posterior distributions for probabilities of success are:

$$p_A \sim Beta(11.5, 3.5)$$
$$p_B \sim Beta(5.5, 1.5)$$



2. To find that the probability of success is at least 80%, we can find the area under the posterior density to the right of 0.8. This is equivalent finding the $1 - cdf(0.8)$ for each distribution. Hence we get,

$$P(p_A \geq 0.8) = 1 - P(p_A < 0.8) = 42.043790\%$$
$$P(p_B \geq 0.8) = 1 - P(p_B < 0.8) = 53.546303\%$$

3. To truly determine if Solution B has higher success rate then Solution A, we can do a Monte-Carlo simulation by drawing a large number $(100,000)$ of random samples from the two posterior distributions. Then we can compare the proportion of samples from B greater than the ones from A. Doing so we get,

$$P(p_B > p_A) = 57.422000\%$$

4. From these results we do see that Solution B is indeed performing better than Solution A.

Appendix:

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%% STA 601: Lab 1
% Author: Kedar Prabhudesai
% Created on: 9/6/2013
%%
% Support of pdf
x = 0:0.001:1;
%% Part 1: Find Posterior Distributions and Plot them
% Prior parameters - Using Jeffrey's Prior
a = 0.5; b = 0.5;

% Likelihood Parameters - Assuming Binomial Distribution
% n - Total number of trials
% y - Total number of successes
SolnA.n = 14;
SolnA.y = 11;

SolnB.n = 6;
SolnB.y = 5;

% Beta Distribution Object
SolnA.posterior = makedist('Beta','a',(a + SolnA.y),'b',(b + SolnA.n - SolnA.y));
SolnB.posterior = makedist('Beta','a',(a + SolnB.y),'b',(b + SolnB.n - SolnB.y));

% Plot Posterior
plot(x,SolnA.posterior.pdf(x),'b','LineWidth',2);hold on;
plot(x,SolnB.posterior.pdf(x),'r','LineWidth',2);hold off;
xlabel('\theta');
ylabel('p(\theta|y)');
title('Posterior Distributions for Probability of Success');
legend('Posterior for Soln. A','Posterior for Soln. B','location','NW');

%% Part 2:
% Find Probability that Soln will be successful at least 80%
% Pr(SolnA => 0.8) = 1 - Pr(SolnA < 0.8).
%                  = 1 - Value of cdf at 0.8
ProbASuccess = 1 - SolnA.posterior.cdf(0.8);
ProbBSuccess = 1 - SolnB.posterior.cdf(0.8);
disp('Probabilities of being successful at least 80% of the time:');
fprintf('\tSoln. A = %f %%\n\tSoln. B = %f %%\n',ProbASuccess*100,ProbBSuccess*100);

%% Part 3:
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% Find the true success rate of Soln.B being more successful than Soln.A.
% We can do this by drawing a large number of random numbers from the
% posterior beta distributions and finding the success rate
nTrials = 100000;
pARndVals = SolnA.posterior.random(1,nTrials);
pBRndVals = SolnB.posterior.random(1,nTrials);
TrueSuccessRate = sum(pBRndVals > pARndVals)/nTrials;
fprintf('Probability of Soln. B being more successful than Soln. A = %f %%\n',TrueSuccessRate*100);
```