STA 601 - Homework 10

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Logistic Regression Model:

Likelihood,

$L(y \mid x, \beta) = \prod_{i=1}^{n} \left(\frac{1}{1+e^{-x_i'\beta}}\right)^{y_i} \left(\frac{1}{1+e^{x_i'\beta}}\right)^{1-y_i}$

$\therefore log[L(y \mid x, \beta)] = \sum_{i=1}^{n} \left[y_i log\left(\frac{1}{1+e^{-x_i'\beta}}\right) + (1-y_i) log\left(\frac{1}{1+e^{x_i'\beta}}\right)\right]$

Priors,

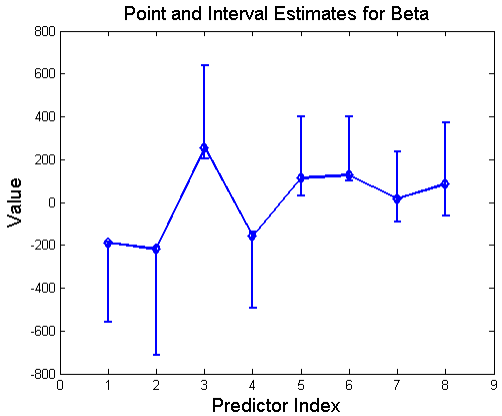
$(\beta_0, \beta_1, \ldots, \beta_p) \sim \mathcal{N}_p(\mu_p, \Sigma_p)$. p-Variate Normal Distribution.

Metropolis-Hastings Algorithm:

We update each β_i as follows:

- Draw, $\beta_i^* \sim \mathcal{N}(\beta_i^{(s)}, 5)$.
- Compute, $r = log[L(y \mid x, \beta_i^*, \beta_j^{(s)})] + log[\pi(\beta_i^*, \beta_j^{(s)})] - log[L(y \mid x, \beta_j^{(s)})] - log[\pi(\beta_j^{(s)})]$, where, j represents all coefficients other than i.
- Accept, $\beta_i^{(s+1)} = \beta_i^*$ with probability$(1, min(r))$.

Results:



Appendix:

```
%% STA 601 - Homework 10
% Author: Kedar Prabhudesai
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close all;
clear all;

% Get data
tmp = importdata('china500.dat');
data = tmp.data;
data(:,[1 6]) = [];

% Outcome variable
y = data(:,1);
% Predictors
x = data(:,2:end);
x = [ones(size(x,1),1) x];
p = size(x,2);


%% Metropolis Hastings Algorithm
nSamples = 10000;
Δ2 = 10;
betaSamples = zeros(p,nSamples);
% Predictor Weights. Initialize with random values
beta = mvnrnd(zeros(p,1),Δ2*eye(p));
beta = beta';
betaSamples(:,1) = beta;

for iSample = 2:nSamples
    betaStar = mvnrnd(betaSamples(:,iSample-1),Δ2*eye(p));
    betaStar = betaStar';

    home;
    disp(iSample);
    for iPred = 1:p
        betaZeroHere = betaSamples(:,iSample-1);
        betaStarHere = betaZeroHere;
        betaStarHere(iPred) = betaStar(iPred);

        LStarHere = sum(y.*log((1./(1+exp(-x*betaStarHere))) + (1-y).*log((1./(1+exp(x*...
            betaStarHere))))));
        LZeroHere = sum(y.*log((1./(1+exp(-x*betaZeroHere))) + (1-y).*log((1./(1+exp(x*...
            betaZeroHere))))));

        piStarHere = log(mvnpdf(betaStarHere,betaZeroHere,Δ2*eye(p)));
        piZeroHere = log(mvnpdf(betaZeroHere,betaZeroHere,Δ2*eye(p)));

        lhr = exp(LStarHere + piStarHere - (LZeroHere + piZeroHere));
        AcceptRejectFlag = binornd(1,min(1,lhr));

        if AcceptRejectFlag
            betaSamples(iPred,iSample) = betaStar(iPred);
        else
            betaSamples(iPred,iSample) = betaSamples(iPred,iSample-1);
        end
    end
end

% Manage Plotting
```

```
figure;
MeanBi = zeros(p,1);
ConfInts = zeros(p,2);
for iPred = 1:p
    MeanBi(iPred) = mean(betaSamples(iPred,:));
    ConfInts(iPred,:) = quantile(betaSamples(iPred,:),[0.025 0.975]);
end
figure;
errorbar(1:p,MeanBi,ConfInts(:,1),ConfInts(:,2),'Marker','diamond','Linewidth',2);
title('Point and Interval Estimates for Beta','FontSize',14);
xlabel('Predictor Index','FontSize',14);
ylabel('Value','FontSize',14);
```