STA 601/360: Lab 3
(adapted from Doug Vanderwerken)

Your friend, "Sloppy" Jo(e) is an aspiring pollster, and she (he) agrees to conduct a poll for you, free of charge. You give the following instructions: "Please ask about 25 people whether they are in favor of more gun control, and report back to me the number who are in favor." After a few days Sloppy returns with the poll results: there were $y = 20$ in favor. "And how many people did you ask?" you inquire. "Ummm, I dunno. You didnt ask me to record that. All I know is that it was about 25."

Assume $y|N, \beta \sim \text{Binomial}(N, \beta)$. Furthermore, assume a uniform prior on β and a (shifted) Poisson prior on N. Do the following:

1. Derive the joint posterior distribution of N and β given y.

2. Derive the posterior complete conditionals for N and β.

3. Use these to sample (using Gibbs sampling) from the joint posterior of β and N, using a starting value of $\beta = .05, N = 50$.

4. Show the 2D trace plot for the first 10 draws of the Gibbs sampler. I want to see both the points and the connecting lines.

5. Give the central 90% posterior credible interval for β, rounded to the nearest 1% for both upper and lower limits.

6. What is the probability that exactly 20 people were polled? Base answer on at least 10,000 draws (post-burn-in), and round to nearest one-tenth of 1% (answer need not be accurate to the nearest one tenth of 1%).

Hint: When finding full conditional for N, you may want to find the distribution of N-y and add this to the given value for y.