STA 360/601: Homework 2 Answers

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1 Regular Beta Prior

Let θ be the probability of a bad reaction associated with a specific agent. Suppose we have a $Beta(a, b)$ prior for θ, and we observe n trials $x_1, ..., x_n$ with probability θ of there being a bad reaction. If we observe no bad reactions, the posterior of θ is $Beta(a, b + n)$.

$$\begin{aligned} f(\theta \mid x_1, ..., x_n) &\propto \theta^{a-1}(1-\theta)^{b-1} \times (1-\theta)^n \\ &\propto \theta^{a-1}(1-\theta)^{b+n-1} \\ &= Beta(a, b+n) \end{aligned}$$

Sample code to calculate the number of trials can be found in Sakai.

2 Restricted Uniform Prior

If we use a restricted prior, $(U(0, 0.01)$, the posterior will be:

$$f(\theta \mid x_1, ..., x_n) \propto (1-\theta)^n, \quad 0 \leq \theta \leq 0.1$$

We can recognize this kernel as that of a $Beta(1, 1 + n)$ distribution. However, because the prior puts zero weight on values outside of $[0, 0.1]$, the posterior is only defined in $[0, 0.1]$ as well. Therefore, the posterior is actually a truncated $Beta(1, 1 + n)$ distribution. Sample code to calculate the number of trials can be found in Sakai.

Table 1: Number of Trials Needed

Prior	Beta Prior 1	Beta Prior 2	Beta Prior 3	Beta Prior 4	Uniform Prior
Trials	1330	145	2310	1563	1995