# STA 360/601: Homework 3 Answers

Daniel Li

## 1 Sampling from the Posterior

From the lecture slides, we know the posterior of $\theta_2$ is $Ga(68, 45)$. We sample from the posterior distribution with $S = 10, 100, 1000$, and calculate the mean, $95\%$ central credible interval, and $\Pr(\theta_2 < 1.5)$ for each of the three cases. Results are shown in the figure below and in Table 1.

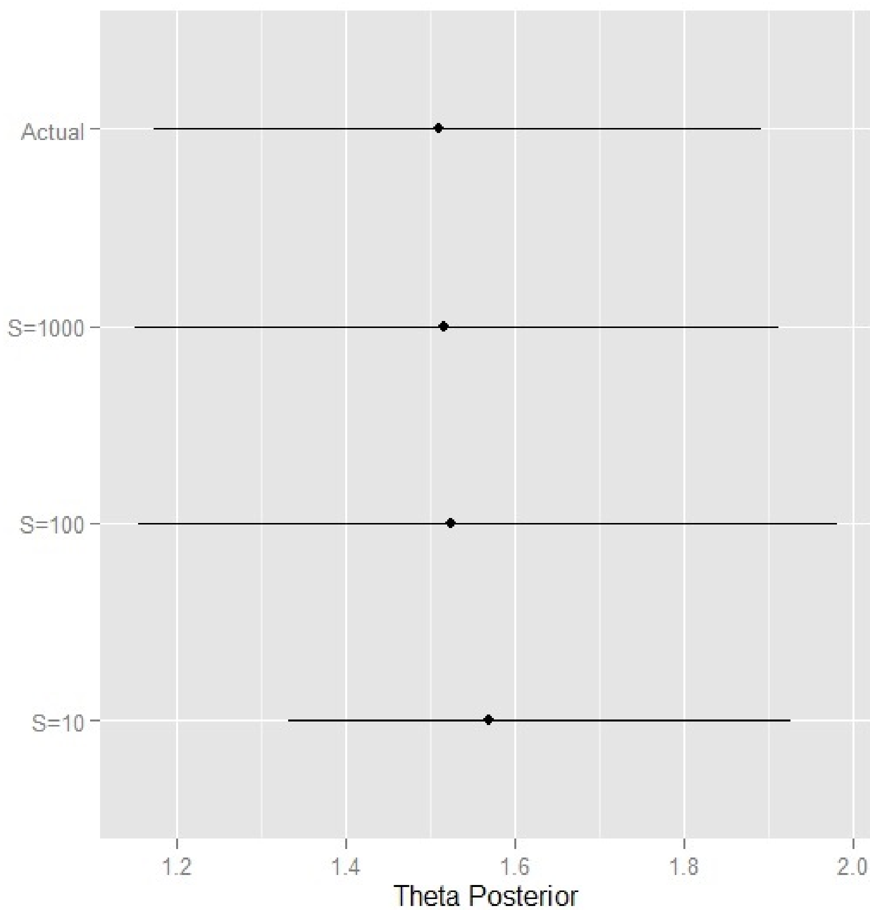

Table 1: Sample Summaries

| S | Mean | 95% Credible Interval | $\Pr(\theta_2 < 1.5)$ |
|---|---|---|---|
| 10 | 1.57 | $[1.33, 1.93]$ | 0.60 |
| 100 | 1.53 | $[1.16, 1.98]$ | 0.47 |
| 1000 | 1.52 | $[1.15, 1.91]$ | 0.48 |
| Actual | 1.51 | $[1.18, 1.89]$ | 0.49 |

# 2 Minimum Sample Size

## 2.1 Method 1

For the Monte Carlo estimate of the posterior mean, $\bar{\theta}$, we sample $S$ independent draws from the posterior and take the mean of these $S$ draws. One way to go about this problem is to using the following properties of the gamma distribution. For $X_i \sim Ga(a_i, b)$ for $i = 1, .., n$:

$$\sum_{i=1}^{n} X_i \sim Ga\left(\sum_{i=1}^{n} a_i, b\right)$$
$$cX_i \sim Ga\left(a, \frac{b}{c}\right)$$

This means that for $S$ draws from our posterior $Ga(68, 45)$, the distribution of Monte Carlo estimate of the posterior mean should be $\bar{\theta} \sim Ga(68S, 45S)$. We can increase S until at least $95\%$ of the mass is between $\frac{68}{45} \pm 0.001$. This method suggests we need $128, 998$ draws.

## 2.2 Method 2

A second method would be to use the central limit theorem, which tells us that $\bar{\theta} \approx N\left(E(\theta \mid y), \frac{Var(\theta|y)}{S}\right)$. Note that in this case, we know $E(\theta \mid y) = \frac{68}{45}$ and $Var(\theta \mid y) = \frac{68}{45^2}$. Using the normal approximation, we can increase S until at least $95\%$ of the mass is between $\frac{68}{45} \pm 0.001$. This method also suggests we need $128, 998$ draws.

## 2.3 Method 3

We can also draw $S$ samples, calculate the sample mean, and repeat this for a total of 10,000 (or more) times. We can then calculate the proportion of sample means that are within $\frac{68}{45} \pm 0.001$. We can increase $S$ until this proportion is at least $0.95$. We find that for $S = 128, 998$, this proportion is $\approx 0.9511$.