STA 601/360: Lab 2
(adapted from Doug Vanderwerken)

1. Suppose $X|(\tau^2) \sim \text{Normal}(0, 1/\tau^2)$ and $\tau^2 \sim \text{Gamma}(shape = \nu/2, rate = \nu/2)$. Derive the marginal distribution of X.

2. Let $\nu = 1$. Draw a sample of 10,000 from the marginal distribution of X by drawing 10,000 τ^2's and then 10,000 X's given the τ^2's. Plot the sample (either histogram or density is fine). Give two names for the actual marginal distribution $f(X)$ when $\nu = 1$.

3. Use the Kolmogorov-Smirnov test (`ks.test` in R) to test whether your observed distribution is equal to a t distribution with 1 degree of freedom. Report the p-value and your conclusion.

4. Now, repeat the above sampling and `ks.test` 1000 times, using 50 draws from $f(X)$ each time (instead of 10,000 draws as above). Record the p-value at each iteration. (Do not report, but this will be used for the next step. The p-value can be grabbed using this R code: `ks.test(x,'pt',1)$p`.) Plot a histogram of the 1000 p-values and include this in report. What distribution should this be? Hint: it's a beta(a,b) for some a,b in $\{1, 2, 3, ..., \}$.

5. Does the Central Limit Theorem hold for the mean of a sample from $f(X)$ when $\nu = 1$? What about $\nu = 2$? $\nu = 3$? Why or why not? A quick explanation will do; an involved proof is NOT required.