STA 601 - Homework 5

Kedar Prabhudesai

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In the first Two Parts of the Assignment, we got the following expressions,

Model for Data:

$$y_i \sim Poisson(\lambda\gamma^{x_i})$$

Priors:

$$\lambda \sim Gamma(1,1)$$
$$\gamma \sim Gamma(1,1)$$
$$p(\lambda,\gamma) = p(\lambda)p(\gamma).$$

Likelihood:

$$L(\mathbf{y};\lambda,\gamma) = C(\mathbf{y})\lambda^{\sum_{i=1}^{n} y_i}\gamma^{\sum_{i=1}^{n} y_i x_i}\prod_{i=1}^{n} exp(-\lambda\gamma^{x_i})$$

Full Conditionals:

$$\lambda|\gamma,\mathbf{y} \sim Gamma\left(\sum_{i=1}^{n} y_i + 1, \sum_{i=1}^{n}\gamma^{x_i} + n\right).$$
$$\gamma|\lambda,\mathbf{y} \sim Gamma\left(\sum_{i=1}^{n} y_i x_i + 1, \lambda m + 1\right).$$

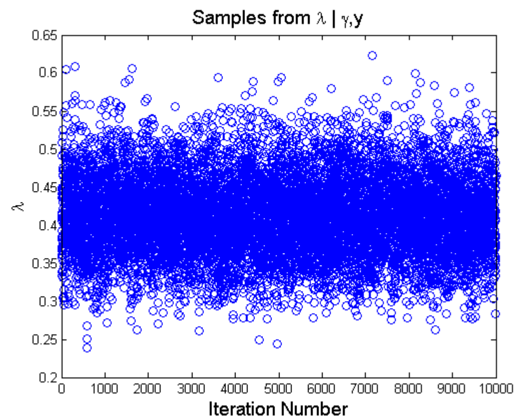
where, m is the number of treated subjects.

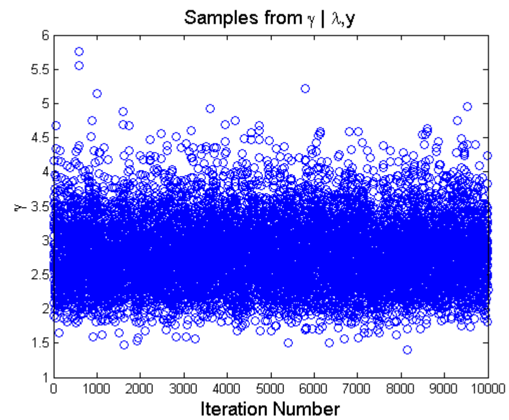
To sample from the Joint Posterior we can do Gibbs Sampling:

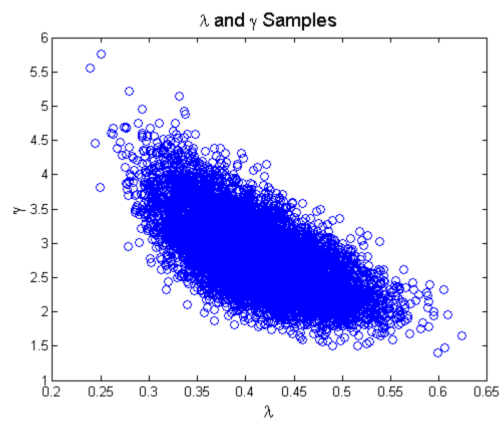
- Select, $\gamma^{(0)}$,
- Draw, $\lambda^{(1)} \sim p(\lambda|\gamma^{(0)},\mathbf{y})$
- Draw, $\gamma^{(1)} \sim p(\gamma|\lambda^{(1)},\mathbf{y})$
- Hence, we get $\{\lambda^{(1)},\gamma^{(1)}\}$.
- Repeat.

3. Simulation:

We can get sample data from $y_i \sim Poisson(\lambda\gamma^{x_i})$, using $\lambda = \gamma = 1$. For the both $x_i = 0$ and $x_i = 1$, the Poisson parameter, $\lambda\gamma^{x_i} = 1$. The following are trace plots of the samples from the Full Conditional Distributions.

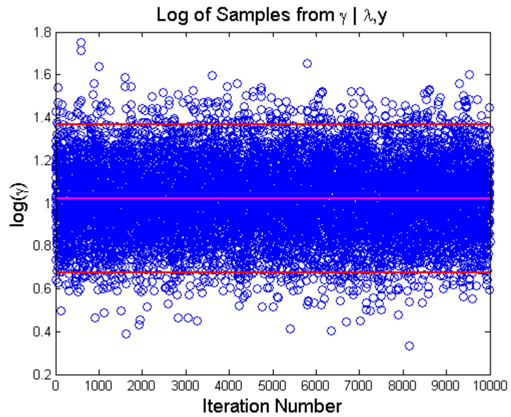






4.(i) Estimation of $log(\gamma)$:

Using the samples of γ from previous step, I get Posterior Mean of $log(\gamma) = 1.0198$. 95% Credible Intervals=[0.6758 1.3684]. The mean is indicated in magenta in the following figure, and the credible intervals in red.



4.(ii) Posterior Predictive Distribution $p(\tilde{y}|\mathbf{y})$:

This can be written as,

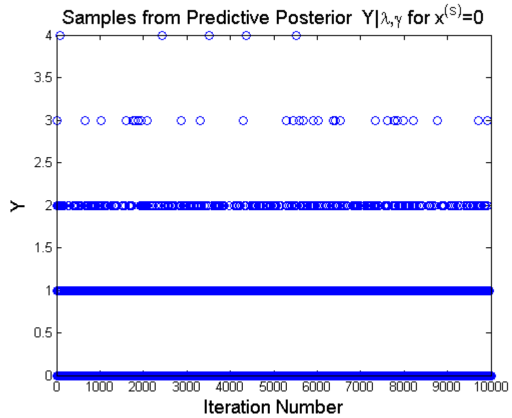
$$\begin{aligned} p(\tilde{y}|\mathbf{y}) &= \int\int p(\tilde{y}|\lambda,\gamma,\mathbf{y})p(\lambda|\gamma,\mathbf{y})p(\gamma|\lambda,\mathbf{y})d\lambda d\gamma \\ &= \int\int p(\tilde{y}|\lambda,\gamma)p(\lambda|\gamma,\mathbf{y})p(\gamma|\lambda,\mathbf{y})d\lambda d\gamma \end{aligned}$$

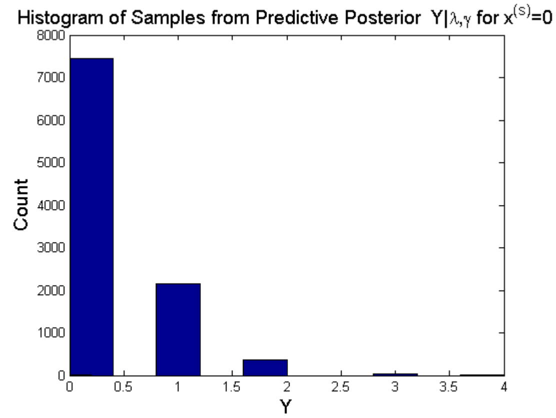
We can estimate the predictive posterior, as follows:

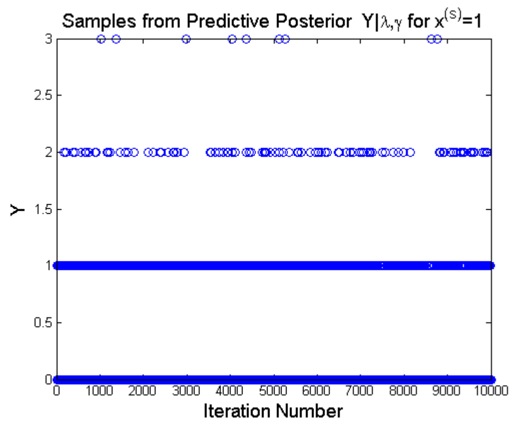
- Select, $\gamma^{(0)}$,
- Draw, $\lambda^{(1)} \sim p(\lambda|\gamma^{(0)},\mathbf{y})$
- Draw, $\gamma^{(1)} \sim p(\gamma|\lambda^{(1)},\mathbf{y})$
- Using, $\{\lambda^{(1)},\gamma^{(1)}\}$, draw $\tilde{y}^{(1)} \sim Poisson(\lambda\gamma^{x_i})$
- Repeat.

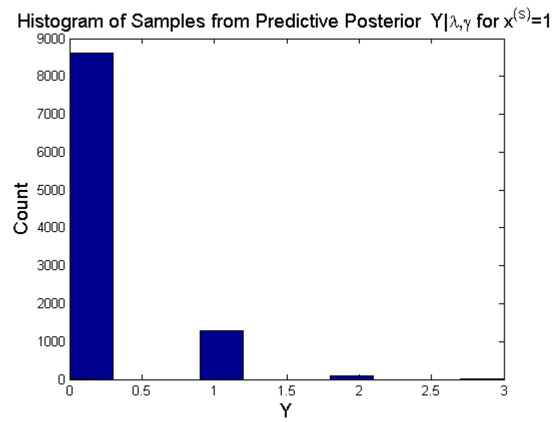
Depending on the value of $x^{(s)}$, we have different Posterior Predictors,
$x^{(s)} = 0$, $\tilde{y}^{(s)} \sim Poisson(\lambda)$.
$x^{(s)} = 1$, $\tilde{y}^{(s)} \sim Poisson(\lambda\gamma)$.

The following are trace plots and histograms, with sampling.

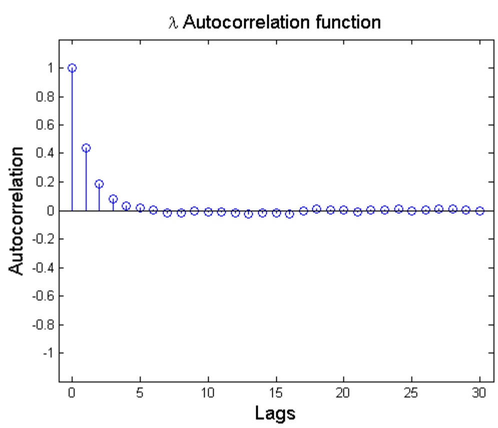


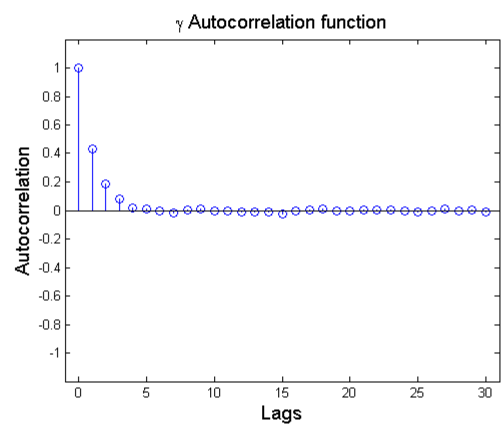






5. Convergence Diagnostics: Given below are Autocorrelation function plots for λ and γ samples. We can see that it drops as number of lags increase. This does indicate that we have good mixing in the chain. Also, the mixing in λ and γ is quite similar.





Sample Size:

The effective sample size can be computed by using the formula, $S_{eff} = Var[\phi]/Var_{MCMC}[\tilde{\phi}]$.