**Linear quadrilateral element formulation**

A general approximation for  in terms of and  can be expressed as



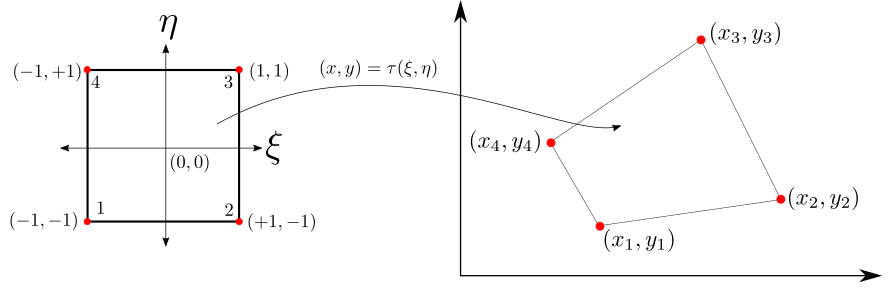


Figure 1: Geometrical Transformation

We have four nodes where we can evaluate the above equation as follows. In other words, takes four values of (-1, -1), (+1, -1), (+1, +1) and (-1, +1). Substituting these values, we arrive at four equations as shown below.









Representing the above equations in matrix form



or in a more compact form as



Thus, the coefficients can be calculated by inverting A as shown below



We find that



Substituting this we get





Further simplifying,



where









**8-node hexahedron element formulation**



Evaluating this equation at eight nodes of the reference element,

















Expressing the above equation in matrix form, we get







We can express the above equation as





















The derivatives of the shape functions are

1. With respect to 

















1. With respect to 

















1. With respect to 

















The infinitesimal strain is written as,



Now,



The infinitesimal strain displacement relations are













Which can be written in matrix form as



The element is isoparametric, therefore the shape functions also define the geometrical transformation between the reference and the parent element. The coordinates x and y of any point of the parent element are given by







The Jacobian of this transformation is give by



which can be expressed as



which when simplified yields,



The derivatives of the shape functions can be written as follows using the chain rule,







which can be written as



which is



Thus the derivatives of the shape functions in the (x,y,z) system are obtained by





