Statistical Signal Processing
Exercise 2

Sagi Kedmi

Nov. 21, 2014

1 KL Divergence

Wiki: In probability theory and information theory, the **Kullback-Leibler divergence** is a non-symmetric measure of the difference between two probability distribution P and Q. Specifically, the KL-divergence of Q from P is a measure of the information lost when Q is used to approximate P: The KL divergence measure the expected number of extra bits required to code [Huffman code] samples from P when using a code based on Q, rather than using the the true code based on P. Typically P represents the "true" distribution of data, observations, or a precisely calculated theoretical distribution. The measure Q typically represents a theory, mode, description or approximation of P.

In this question we're finding the KL divergence of f_2 from f_1.

Using the Law of the Unconscious Statistician:

$$\mathcal{D}_{KL}\left(f_1||f_2\right)=\mathbb{E}_{f_1}\left[\log\left(\frac{f_1(y)}{f_2(y)}\right)\right]=\int_{-\infty}^{\infty}\log\left(\frac{f_1(y)}{f_2(y)}\right)f_1(y)\,\mathrm{d}y=\int_{-\infty}^{\infty}-\log\left(\frac{f_2(y)}{f_1(y)}\right)f_1(y)\,\mathrm{d}y$$

Known inequality: $\forall x\geq 0:\log(x)\leq x-1$