A Comprehensive Analysis of the Triangle Inequality

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Intro

This semester, we studied *Euclid's Elements* to develop our own understanding of the Triangle Inequality and other propositions used in its proof. Then, we outlined its proof and other related topics in a way that is accessible for high school mathematics students. Finally, we created a number of GeoGebra Applets to accompany these ideas in order for students to interact with the material in a hands-on way.

Motivation

The Triangle Inequality is a fundamental concept in the Common Core State Standard Curriculum. As such, our efforts reflect what we believe to be the most effective method for teaching: structured, discovery-based learning through applications, extensions, and a thorough review of the foundations of the Triangle Inequality.

Definition

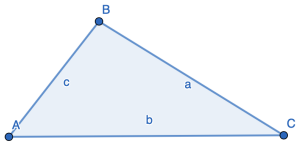
Definition 0.1 (Metric Space) *A metric space is a set X with a distance function $d : X \times X \to \mathbf{R}$. The function d is called the distance function and obeys the following properties for all points a, b, c in X:*

1. $d(a, b) = 0 \leftrightarrow a = b$
2. $d(a, b) = d(b, a)$
3. $d(a, b) + d(b, c) \geq d(a, c)$

Main finding goes here, translated into **plain English**. **Emphasize** the important words.

The Triangle Inequality

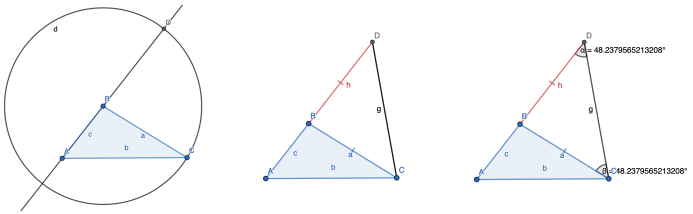
Theorem 1 ([1]) *In any triangle, (the sum of) two sides taken together in any possible way is greater than the remaining side.*



In the diagram on the left, the Triangle Inequality tells us that

1. $a + b > c$
2. $a + c > b$
3. $b + c > a$

Euclid's Proof of the Triangle Inequality



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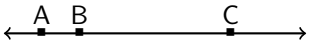
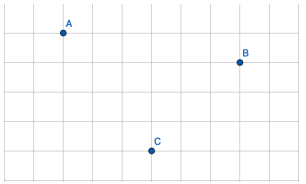
Other Metric Spaces

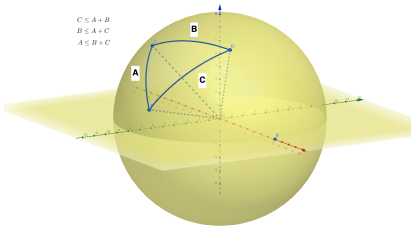
The first metric space that the Triangle Inequality holds in is the Number Line! Can you see how this is true?





The next metric space in which the Triangle Inequality holds is Gridland. Be careful not to leave the road!

We can also consider how the Triangle Inequality works on a sphere. We use arc lengths instead of line segments when working in this metric space.



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