LOG(M) PROJECT ZERO

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1. Introduction

In general, orthogonal polynomials are polynomials that are perpendicular to other polynomials as defined by a specific inner product. Our research will focus on the specific inner product on polynomials P and Q_n defined as following.

$$\langle P, Q_n \rangle = \int_{-1}^{1} P(x)Q_n(x)w(x)dx$$

Where $w(x)$ is a predefined weight function around which we are studying, and P is any polynomial of degree less than or equal to $n-1$.

In order to find and calculate orthogonal polynomials, we must find Q_n such that the above integral evaluates to 0 for any polynomial P of degree less than n. Linearity of integrals implies that it suffices to find Q_n that satisfy the following.

$$\int_{-1}^{1} x^k Q_n(x)w(x)dx = 0 \quad (k = 0, 1, ..., n-1)$$

Below are some examples of the Orthogonal Polynomials

1.1. **Legendre Polynomials** ($w(x) \equiv 1$).

$$\int_{-1}^{1} x^k L_n(x)dx = 0 \quad (k = 0, 1, ..., n-1)$$

1.2. **Jacobi Polynomials** ($w(x) = (1-x)^\alpha(1+x)^\beta$).

$$\int_{-1}^{1} x^k J_n(x)w_j(x)dx = 0 \quad (k = 0, 1, ..., n-1)$$

1.3. **Chebyshev Polynomials** ($w(x) = 1/\sqrt{(1-x^2)}$).

$$\int_{-1}^{1} x^k C_n(x)w_c(x)dx = 0 \quad (k = 0, 1, ..., n-1)$$

References