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30th International Mathematical Olympiad (1989)

Day I

1. Prove that the set $\{1, 2, \ldots, 1989\}$ can be expressed as the disjoint union of subsets A_i $(i = 1, 2, \ldots, 117)$ such that each A_i contains 17 elements and the sum of the elements of each A_i is the same.

2. In an acute-angled triangle ABC, the internal bisector of angle A meets the circumcircle again at A_1. Points B_1 and C_1 are defined similarly. Let A_0 be the intersection of AA_1 with the external bisectors of angles B and C. Points B_0 and C_0 are defined similarly. Prove that:

(i) The area of triangle $A_0B_0C_0$ is twice the area of the hexagon $AC_1BA_1CB_1$.

(ii) The area of triangle $A_0B_0C_0$ is at least four times the area of triangle ABC.

3. Let n and k be positive integers and let S be a set of n points in the plane such that no three points are collinear. If for any point P of S there are at least k points of S equidistant from P, prove that

$$k < \frac{1}{2} + \sqrt{2n}.$$

Day II

4. Let $ABCD$ be a convex quadrilateral such that $AB = AD + BC$. There exists a point P inside the quadrilateral at a distance h from the line CD such that $AP = h + AD$ and $BP = h + BC$. Show that

$$\frac{1}{\sqrt{h}} \geq \frac{1}{\sqrt{AD}} + \frac{1}{\sqrt{BC}}.$$

5. Prove that for each positive integer n there exist n consecutive positive integers none of which is an integral power of a prime number.

6. A permutation $(x_1, x_2, \ldots, x_{2n})$ of the set $\{1, 2, \ldots, 2n\}$ has property P if

$$|x_i - x_{i+1}| = n$$

for at least one i. Show that for each n there are more permutations with property P than without.

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31st International Mathematical Olympiad (1990)

Day I

1. Chords AB and CD of a circle intersect at a point E inside the circle. Let M be an interior point of segment EB. The tangent at E to the circle through D, E, M meets the lines BC and AC at F and G respectively. If

$$\frac{AM}{AB} = t,$$

find $\frac{EG}{EF}$ in terms of t.

2. Let $n \geq 3$ and let E be a set of $2n-1$ distinct points on a circle. Suppose exactly k points are colored black. Such a coloring is called good if there exists a pair of black points such that the interior of one arc between them contains exactly n points of E. Find the smallest value of k such that every coloring is good.

3. Determine all integers $n > 1$ such that

$$\frac{2^n + 1}{n^2}$$

is an integer.

Day II

4. Let $\mathbb{Q}^+$ be the set of positive rational numbers. Construct a function $f : \mathbb{Q}^+ \to \mathbb{Q}^+$ such that

$$f(xf(y)) = \frac{f(x)}{y}$$

for all $x, y \in \mathbb{Q}^+$.

5. Starting from an integer $n_0 > 1$, two players A and B choose integers alternately. Player A chooses n_{2k+1} with

$$n_{2k} \leq n_{2k+1} \leq n_{2k}^2,$$

and player B chooses n_{2k+2} such that

$$\frac{n_{2k+1}}{n_{2k+2}}$$

is a prime power. Player A wins with 1990, player B wins with 1. Determine winning strategies for n_0.

6. Prove that there exists a convex 1990-gon with equal angles whose side lengths are

$$1^2, 2^2, 3^2, \ldots, 1990^2$$

in some order.

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32nd International Mathematical Olympiad (1991)

Day I

1. Given a triangle ABC, let I be the center of its incircle. The internal bisectors meet the opposite sides at A', B', C' respectively. Prove that

$$\frac{1}{4} < \frac{AI \cdot BI \cdot CI}{AA' \cdot BB' \cdot CC'} \leq \frac{8}{27}.$$

2. Let $n > 6$ be an integer and let $a_1, a_2, \ldots, a_k$ be all natural numbers less than n and relatively prime to n. If

$$a_2 - a_1 = a_3 - a_2 = \cdots = a_k - a_{k-1} > 0,$$

prove that n is either a prime or a power of 2.

3. Let $S = \{1, 2, 3, \ldots, 280\}$. Find the smallest integer n such that every n-element subset of S contains five pairwise relatively prime numbers.

Day II

4. Suppose G is a connected graph with k edges. Prove that the edges can be labeled $1, 2, \ldots, k$ such that at each vertex incident with two or more edges, the greatest common divisor of the labels is 1.

5. Let ABC be a triangle and P an interior point. Show that at least one of the angles

$$\angle PAB,\ \angle PBC,\ \angle PCA$$

is less than or equal to $30°$.

6. For any real number $a > 1$, construct a bounded infinite sequence $x_0, x_1, x_2, \ldots$ such that

$$|x_i - x_j|\,|i - j|^a \geq 1$$

for all distinct nonnegative integers i, j.