

EXERCISE 7.1

Find an anti derivative (or integral) of the following functions by the method of inspection.

1. $\sin 2x$
2. $\cos 3x$
3. e^{2x}
4. $(ax + b)^2$
5. $\sin 2x - 4e^{3x}$

Find the following integrals in Exercises 6 to 20:

6. $\int (4e^{3x} + 1) dx$
7. $\int x^2 \left(1 - \frac{1}{x^2}\right) dx$
8. $\int (ax^2 + bx + c) dx$
9. $\int (2x^2 + e^x) dx$
10. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$
11. $\int \frac{x^3 + 5x^2 - 4}{x^2} dx$
12. $\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$
13. $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$
14. $\int (1 - x)\sqrt{x} dx$
15. $\int \sqrt{x}(3x^2 + 2x + 3) dx$

16. $\int (2x - 3 \cos x + e^x) dx$
17. $\int (2x^2 - 3 \sin x + 5\sqrt{x}) dx$
18. $\int \sec x (\sec x + \tan x) dx$
19. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$
20. $\int \frac{2-3 \sin x}{\cos^2 x} dx$

Choose the correct answer in Exercises 21 and 22.

21. The anti derivative of $(\sqrt{x} + \frac{1}{\sqrt{x}})$ equals:
 (A) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ (B) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^2 + C$
 (C) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ (D) $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$
22. If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$ is:
 (A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$
 (C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

1. $\frac{2x}{1+x^2}$
2. $\frac{(\log x)^2}{x}$
3. $\frac{1}{x+x \log x}$
4. $\sin x \sin(\cos x)$
5. $\sin(ax+b) \cos(ax+b)$
6. $\sqrt{ax+b}$
7. $x\sqrt{x+2}$
8. $x\sqrt{1+2x^2}$
9. $(4x+2)\sqrt{x^2+x+1}$
10. $\frac{1}{x-\sqrt{x}}$
11. $\frac{x}{\sqrt{x+4}}, x > 0$
12. $(x^3-1)^{\frac{1}{3}}x^5$
13. $\frac{x^2}{(2+3x^3)^3}$
14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$
15. $\frac{x}{9-4x^2}$
16. e^{2x+3}

17. $\frac{x}{e^{x^2}}$

18. $\frac{e^{\tan^{-1} x}}{1+x^2}$

19. $\frac{e^{2x}-1}{e^{2x}+1}$

20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

21. $\tan^2(2x-3)$

22. $\sec^2(7-4x)$

23. $\frac{\sin^{-1} x}{\sqrt{1-x^2}}$

24. $\frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x}$

25. $\frac{1}{\cos^2 x (1 - \tan x)^2}$

26. $\frac{\cos \sqrt{x}}{\sqrt{x}}$

27. $\sqrt{\sin 2x} \cos 2x$

28. $\frac{\cos x}{\sqrt{1+\sin x}}$

29. $\cot x \log \sin x$

30. $\frac{\sin x}{1+\cos x}$

31. $\frac{\sin x}{(1+\cos x)^2}$

32. $\frac{1}{1+\cot x}$

33. $\frac{1}{1-\tan x}$

34. $\frac{\sqrt{\tan x}}{\sin x \cos x}$

35. $\frac{(1+\log x)^2}{x}$

36. $\frac{(x+1)(x+\log x)^2}{x}$

37. $\frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$

Summary

Integration is the inverse process of differentiation. In the differential calculus, we are given a function and we have to find the derivative or differential of this function, but in the integral calculus, we are to find a function whose differential is given.

Let $\frac{d}{dx}F(x) = f(x)$. Then we write $\int f(x) dx = F(x) + C$. These integrals are called indefinite integrals or general integrals, C is called constant of integration. All these antiderivatives differ by a constant.

Properties of Indefinite Integrals:

1. $\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$
2. For any real number k , $\int kf(x) dx = k \int f(x) dx$
3. $\int [k_1f_1(x) + k_2f_2(x) + \cdots + k_nf_n(x)] dx = k_1 \int f_1(x) dx + \cdots + k_n \int f_n(x) dx$

Standard Integrals:

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, (n \neq -1)$
2. $\int \frac{1}{x} dx = \log |x| + C$
3. $\int \cos x dx = \sin x + C$
4. $\int \sin x dx = -\cos x + C$
5. $\int \sec^2 x dx = \tan x + C$
6. $\int \operatorname{cosec}^2 x dx = -\cot x + C$
7. $\int \sec x \tan x dx = \sec x + C$
8. $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$
9. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
10. $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$
11. $\int e^x dx = e^x + C$
12. $\int a^x dx = \frac{a^x}{\log a} + C$

Integration by Partial Fractions:

1. $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}, a \neq b$
2. $\frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$
3. $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
4. $\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
5. $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

Integrals of Special Functions:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$
2. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$
3. $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
4. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log \left| x + \sqrt{x^2-a^2} \right| + C$
5. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C$
6. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log \left| x + \sqrt{x^2+a^2} \right| + C$

Fundamental Theorem of Integral Calculus:

1. **First Fundamental Theorem:** Let $A(x) = \int_a^x f(t) dt$, then $A'(x) = f(x)$.
2. **Second Fundamental Theorem:** $\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$.

Integration by parts:

$$\int f_1(x) \cdot f_2(x) dx = f_1(x) \int f_2(x) dx - \int [f_1'(x) \cdot \int f_2(x) dx] dx$$

Also, $\int e^x [f(x) + f'(x)] dx = e^x f(x) + C$