Distributed Algorithms

Nancy Lynch

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차 례

제 1 장

필사

1.1 공부하기

분산 처리는 게임 서버에서 가장 중요한 기반 기술 중 하나이지만 제대로 이해하고 활용해서 단단한 토대를 구성하는 노력이 부족했다.

이제 Nancy Linch의 고전적인 대학원 강의 자료를 필사하면서 중요한 개념들을 이해하고 개발에 활용하고자 한다.

1.1.1 LaTeX으로 필사하기

인터넷에서 구한 원본이 TeX으로 작성되었고 언젠가 한번은 마스터 해야겠기에 이번 필사를 LaTeX으로 진행한다.

문서 작성에 이만한 도구가 없기도 해서 이번에 제대로 같이 배우고자 한다.

LaTeX익히기

한글 article 템플릿 문서에서 시작한다. HCR 기본 폰트가 가독성도 높고 기본 구성도 괜찮은 편이다. 예쁜 문서가 되려면 할 일이 많겠으나 별첨에 LaTeX공부한 내용을 정리하면서 진행한다.

1.1.2 끝까지 하기

대학원 강의라 생각보다 어려운 경우가 많고 활용을 바로 하기도 쉽지 않을 터라 중간 중간 좌절할 수 있겠지만 끝까지 진행한다.

기본은 필사이다.

제 2 장

Lecture 1

2.1 Introduction to the Course

2.1.1 The Subject Matter

”Distributed Algorithms” including a wide range of parallel algorithms, which can be classified by a variety of attributes:

- Interprocess Communication (IPC) method: shared memory, message passing, dataflow.
- Timing Model: synchronous, asynchronous, partially synchronous
- Failure Model: reliable system, faulty links, faulty processors.
- Problems addressed: resource allocation, communication, agreement, database concurrency control, deadlock detection, and many more.

Some of the major intended application areas of distributed algorithms are:

- communication systems
- shared-memory multiprocessor computation

- distributed operating systems
- distributed database systems,
- digital circuits, and
- real-time process-control systems.

The algorithms to be studied in this course are distinguished by having a higher degree of uncertainty, and more independence of activities: Some of the types of uncertainty that will consider are:

- unknown number of processors,
- unknown shape of network,
- independent inputs at different locations,
- serveral programs executing at once, starting at different times, going at different speeds,
- nondeterministic processors
- ucertain message delivery times,
- unknown message ordering
- failures: procesor (stopping, transient omission, Byzantine); link (message loss, duplication, reodering)

Because of all this uncertainty, no component of a distributed system ”knows” the entire system state.

Distributed algorithms can be extremely complex, at least in their details, and can be quite difficult to understand. Even though the actual code may be short, the fact that many processors are executing the code in parallel, with steps interleaved in some undetermined way, implies that ther can be prohibitively many different executions, even for the same inputs. This implies that it is nearly impossible to understand everything about the executions of distributed algorithms.

Therefore, instead of trying to understand all the details of the execution, one tends to assert certain properties of the execution, and just understand and prove these properties.

Style

The general flavor of the work to be studied is as follows:

- Identify problems of major significance in (proactical) distributed computing and define abstract versions of the problems for mathematical study.
- Give precise problem statements.
- Describe algorithms precisely.
- Prove rigorously that the algorithms solve the problems.
- Analyze the complexity of the algorithms.
- Prove corresponding impossibility results.

Note the emphasis on right; A rigorous approach seems necessary to be sure that the problems are meaningful, the algorithms are correct, the impossibility results are true and meaningful, and the interfaces are sufficiently well-defined to allow system building.

...

So, rigor is a goal to be striven for, rather than one that we will achiee entirely in this course.

Overview of the Course

Timing models:

- synchronous:
- asynchronous:
- partially synchronous (timing based):

IPC mechanism: shared memory vs. message-passing. And finally, each model and problem can be considered with various failure assumptions.

Models and Proof Methods. The basic models used are *automata-theoretic*, starting with a basic state-machine model with little structure. *Invariant assertions* are often proved about automaton states, by induction. Another model is the I/O automaton model for *reactive systems*, i.e., systems that interact with an external environment in an ongoing fashion. This model can model systems based on shared variables, but is more appropriate for message-passing systems. One of the key features of this model is that it has good *compositionality* properties, e.g., that the correctness of a compound automaton can be proved using the correctness of its components. *Temportal logic* is an example of a special set of methods (language, logic) mainly designed for proving *liveness* properties (e.g., something eventually happens). *Timed models* are mainly newer research work. Typically , there are specially-tailored models for talking about timing-based systems - e.g., those whose components have access to system clocks, can use timeouts, etc. *Algebraic methods* are an important research subarea (but we will not have time for this). The algebraic methods describe concurent processes and systems using algebraic expressions, the use equations involving these expressions to prove equialences and implementation relationships amonth the processes.

Synchronous Message-Passing. We have the processors at the nodes of a graph G, communicating with their neighbors via messages in the edges. We start with a simple toy example, involving *ring computation*. ...... For example, we shall show upper and lower bounds for the time and the amount of communication (i.e., number of messages) required. Then we turn to the problem of *reaching consensus*. The uncertainty here stems from not only from different initial opinions, but also from *processor failures*. We consider failures of different types: stopping, ommision, where messages may be lost en route; and byzantine, where a faulty processor is completely unrestricted.

Asynchronous Shared Memory. After ”warming up” with synchronous algorithms (in which there is only a little uncertainty), we move into the more characteristic (and possibly more interesting) part of the course, on *asynchronous* algorithms.

The first problem we deal with is *mutual exclusion*. This is one of the fundamental (and historically first) problems in this area, and consequently, much work has been dedicated to exploring it. ... Many important concepts for this field will be illustrated in this context, including progress, fairness, fault-tolerance, and time analysis for asynchronous algorithms. We shall see upper bounds on the amount of shared memory, corresponding lower bounds, and impossibility results. We shall also discuss generalizations of mutual exclusion to more general resource allocation problems. For example, we will consider the *Dining Philosophers* problem - a prototypical resource allocation problem.

Next, we shall study the concept of *atomic registers*: so far, we have been assuming indivisible access to shared memory. But how can one implement this on simpler architectures? An interesting new property that appears here is *wait-freeness*, which means that any operation on the register must complete regardless of the failure of other concurrent operations.

An *atomic snapshot* is a convenient primitive for share read-write memory. Roughly speaking, the objective is to take an instantaneous snapshot of all memory locations at once.

A *concurrent timestamp system* is another nice primitive. This is a system that issues, upon request, timestamps that can be used by programs to establish a consistent order among their operations. The twist here is how to implement such systems with bounded memory. A concurrent timestamp system can be used to build more powerful forms of shared memory, such as multi-writer multi-reader memory.

We shall also reconsider the *concensus* problem in the asynchronous Shared memory model, and prove the interesting fact it is impossible to solve in this setting.

Asynchronous Message-Passing Systems. This section deals with algorithms that operate in async networks. Again, the system is modeled as a graph with processors at nodes, and communication links are represented by the edges, but now the system does not operate in rounds. In particular, messages can arrive at arbitrary times and the processors can take steps at arbitrary speeds. One might say that we now have ”looser coupling” of the components of the system: we have more independence and uncertainty.

Computing in static graphs. Graph가 고정되고 시작 시 입력이 전달되고 하나의 결과만 출력하는 고정된 그래프 구성.

Network Synchronization. At this point, we could plunge into a study the many special-purpose algorithms designed expressly for asynchronous distributed networks. But instead, we shall first try to impose some structure on such algorithms by considering ”algorithm transformations” that can be used to run algorithms designed for a simpler computation model on a complex asynchronous network.

The first example here arises in the very important paper by Lamport, where he shows a simple method of assigning consistent *logical times* to events in a distributed network. This can be used to allow an asynchronous network to simulate one in which the nodes have access to perfectly synchronized real-time clocks. The second example is Awerbuch's *synchronizer*, which allows an asynchronous network to simulate the lock-step synchronous networks discussed in the first part of the course, and to do so efficiently.

Detection of stable properties refers to a class of problems with a similar flavor and a common solution. Suppose that there is a separate algorithm running, and we want to design another algorithm to ”monitor” the first. To monitors here mean, for instance, to detect when it terminates or deadlocks, or to take ”consistent snapshot” of its state.

Dtalink protocols involve the implementation of a reliable communication link in terms of unrealible underlying channles. We shall see the basic Alternating Bit Protocol (the standard case study for concurrent algorithm verification papers).

Special-Purpse Network Building Blocks. Major examples are the protocols of broadcast-convergecast, reset, end-to-end.

Self-stabilization. Informally, a protocol is said to be self-stabilizing if its specification does not require a certain ”initial configuratin” to be imposed on the system to ensure correct behavior of the protocol.

Timing-based Systesm. These systesm lie between synchronous and asynchronous, so they have somewhat less uncertainty than the latter. In these systems, processors have some knowledge of time, for example, access to real time, or approximate real time, or some timeout facility.

2.1.2 Synchronous Network Algorithms

Nodes organized into a directed graph
$G = (V, E)$: directed graph. size is $n = |V|$.
M : Message alphabet. $null$ denote the absence of a message.

Each node $i \subset V$ has:

$states(i)$: a set of states

$start(i)$: a subset of $states(i)$

$msgs(i)$: mapping $states(i) \times out - nbrs(i)$ to elements of M or $null$

$trans(i)$: mapping $states(i)$ and a vector of messages in $M \cap null$, one per in-neighbor of i, to $states(i)$

노트

$states, start, msgs, trans$ are all denoted with a function of a node index i. $msgs(i)$ is a function $msg : M \cup null \times out - nbrs(i) \rightarrow states(i)$. 방향을 갖는 그래프로 $in - nbrs(i)$ 와 $out - nbrs(i)$ 는 단방향, 양방향에 모두 적용될 수 있다. $|V|$는 그냥 n으로 많이 표시되므로 문맥에서 잘 찾아야 한다.

Execution begins with all the nodes in some start states, and all channels empty. Then the nodes, in lock step, repeatedly execute the following procedure called round.

1. Apply message generation function to generate messages for out neighbors
2. Put them in the channels.

3. Apply transition function to the incoming messages and the state to get the new state.

> **노트**
>
> channel이 노드의 edge에 M 의 알파벳을 보내고 받는 기능을 부여했다. lock step을 어떻게 달성할 지에 대한 언급 없이 그냥 가능하다고 가정했다. 그 자체로 매우 중요한 가정이다.

Problem Example: Leader Election in a Ring

Graph that is a ring, plus an extra dummy node. The ring is unidirectional (messages can be sent only clockwise). The ring is of arbitrary size, unknown to the processors (i.e., size information is not built into their states).

The requirement is that eventually, exactly one process outputs a *leader* message on its dummy outgoing channel.

Proposition 2.1.1 (Impossibility under Symmetry)**.** *If all the nodes in the ring have identical state sets, start states, message functions and transition functions, then they do not solve the leader election problem for* $n > 1$*.*

증명. It is straightforward to verify, by **induction on the number of rounds**, that all the processors are in identical states, after any number of rounds.

□

제 3 장

Appendix

3.1 Appendix 1. LaTeX익히기

3.1.1 Drawing Diagrams

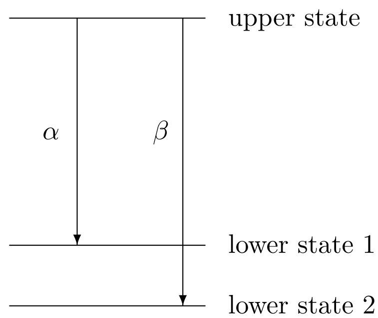
빠르게 그릴 수 있다면 직관적인 이해를 높일 수 있다.

기본 기능

```
\setlength{\unitlength}{0.20mm}
\begin{picture}(400,250)
\put(75,10){\line(1,0){130}}
\put(75,50){\line(1,0){130}}
\put(75,200){\line(1,0){130}}
\put(120,200){\vector(0,-1){150}}
\put(190,200){\vector(0,-1){190}}
\put(97,120){$\alpha$}
\put(170,120){$\beta$}
\put(220,195){upper state}
\put(220,45){lower state 1}
\put(220,5){lower state 2}
```

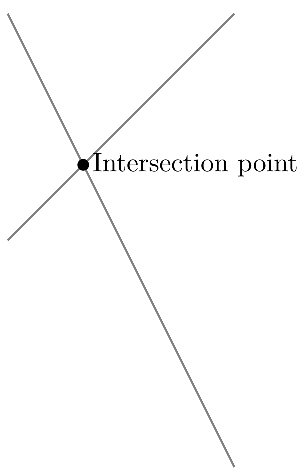
```
\end{picture}
```

아래 그림을 만든다. 좌표계와 위치를 지정하고 line / vector / circle / oval을 사용한다. 추가 오브젝트들도 있을 것이다.



TikZ package

tikz와 pgf는 위대한 라이브러리이다. 다른 위대함이 텍 안에 많지만 놀랍다.



완전한 매뉴얼이 상세한 튜토리얼을 포함해서 아래에 있다.

http://mirror.utexas.edu/ctan/graphics/pgf/base/doc/pgfmanual.pdf