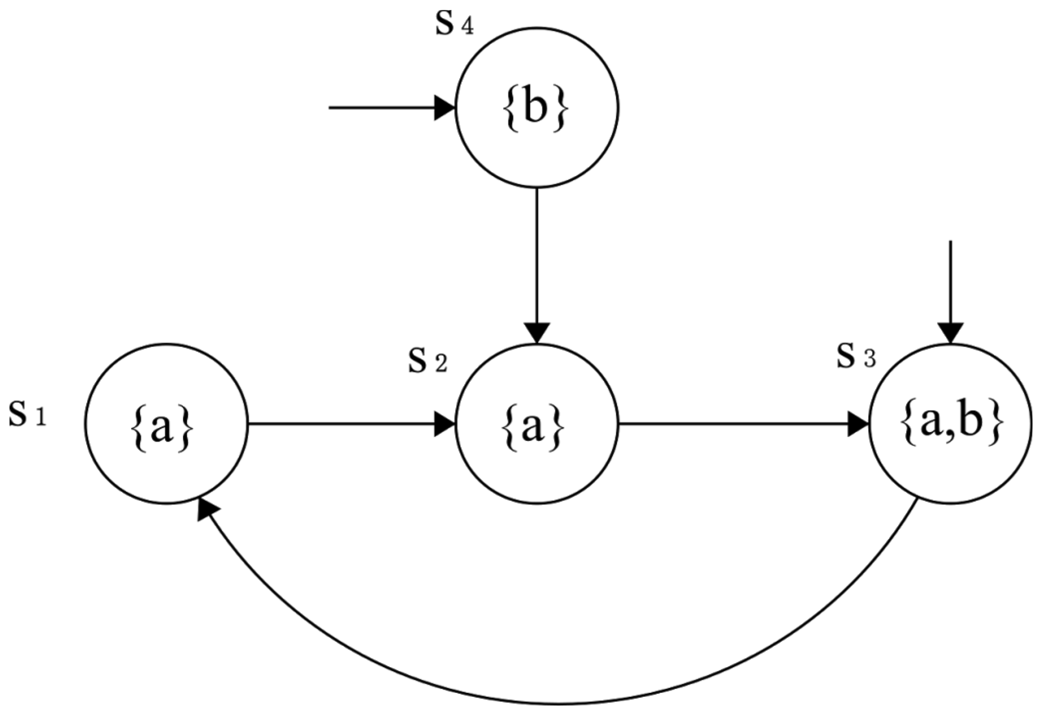
CA2: LTL & CTL Model Checking

Complete the following exercises, ensuring that you explain your solution on each case.

Q1. LTL (6 marks)

Consider the following transition system over the set of atomic propositions {a,b}



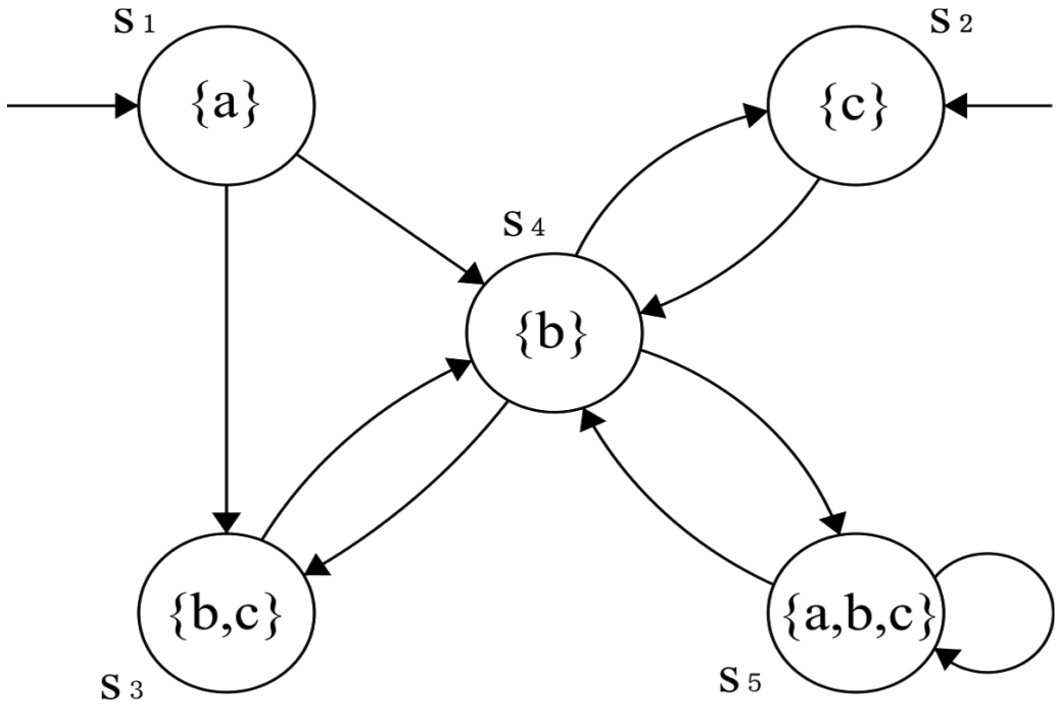
Indicate for each of the following LTL formulae the set of stages for which these formulae are fulfilled:

(a) X a
(b) X X X a
(c) G b
(d) G F a
(e) G(b U a)
(f) F(a U b)

Explain your answers.

Q2. LTL paths (6 marks)

Consider the transition system TS over the set of atomic propositions AP = {a,b,c}:



Decide for each LTL formulae φ_i below if TS $\models \varphi_i$ holds. Justify your answer. If TS $\not\models \varphi_i$, provide a path π such that $\pi \not\models \varphi_i$.
(a) φ_1 = F G c
(b) φ_2 = G F c
(c) φ_3 = X ¬c → X X c
(d) φ_4 = G a
(e) φ_5 = a U G (b ∨ c)
(f) φ_6 = (X X b) U (b ∨ c)

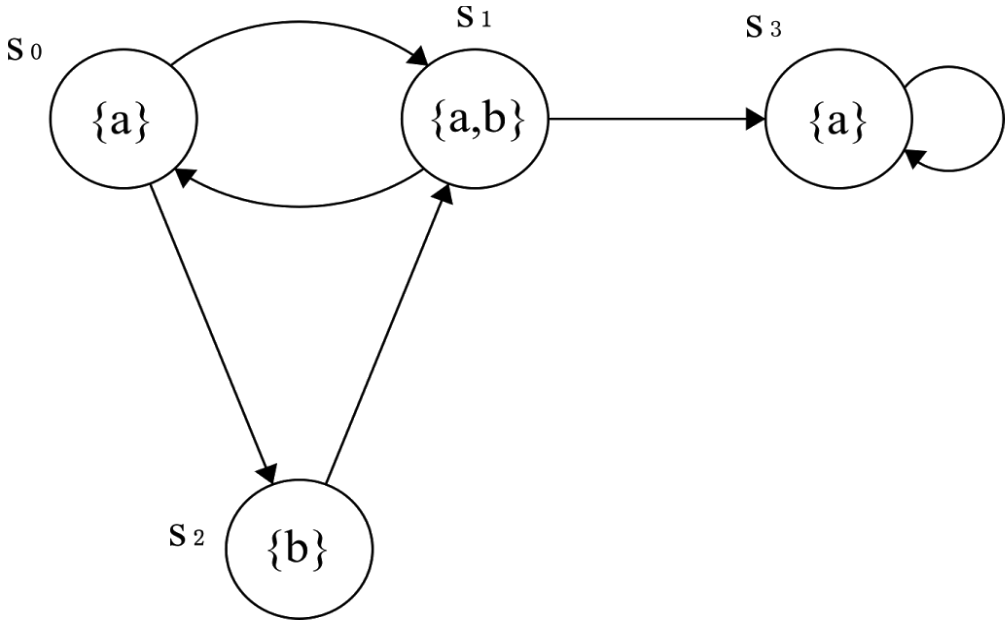
Q3. Printer (10 marks)
Suppose we have two users, *Peter* and *Jane*, and a single printer device *Printer*. Both users perform several tasks, and every now and then they want to print their results on the *Printer*. Since there is only one printer, only one user can print a job at a time. Suppose we have the following atomic propositions for *Peter* at our disposal:

- *Peter.request* ::= indicates that *Peter* request usage of the printer.
- *Peter.use* ::= indicates that *Peter* uses the printer.
- *Peter.release* ::= indicates that *Pete*r releases the printer.

For *Jane*, similar predicates are defined. Specify in LTL the following properties:

(a) Mutual Exclusion, i.e., only one user at a time can use the printer
(b) Finite time of usage, i.e., a user can print only for a finite amount of time.
(c) Absence of individual starvation, i.e., if a user wants to print something, he/she eventually is able to do so.
(d) Absence of blocking, i.e., a user can always request to use the printer.
(e) Alternating access, i.e., users must strictly alternate in printing.

Q4. CTL (7 marks)
Consider the transition system below:



In what states do the following formulae hold? Explain.
(a) E X a
(b) A X a
(c) E G a
(d) A G a
(e) E F (E G a)
(f) A(a U b)
(g) E (a U (¬ a ∧ A(¬ a U b)))

Q5. Elevator (10 marks)
Consider an elevator that services N >0 floors numbered 0 through N-1. There is an elevator door at each floor with a call button and an indicator light that signals whether or not the elevator has been called. In the elevator cabin there are N send buttons (one per floor) and N indicator lights that inform to which floor the elevator is going to be sent. For simplicity consider N = 4. Present a set of atomic propositions that are needed to describe the following properties of the elevator system and give the corresponding CTL formulae:

(a) The doors are safe i.e. a floor door is never open if the cabin is not present at the given floor
(b) The indicator lights correctly reflect the current requests. That is, each time a button is pressed, there is a corresponding request that needs to be memorised until fulfilment, if ever.
(c) The elevator only services the requested floors and does not move when there is no request.
(d) All request are eventually satisfied.

Q6. BDDS (6 marks)
Draw a Binary Decision Trees to represent the following formulae. Show how to translate these trees into Binary Decision Diagrams (BDDs). Expplain why each BDD provides a more efficient representation of the formulae:
(a) (x1 ∧ x3) ∨ (x1 ∧ x3)
(b) x1 ∧ (¬x2 ∨ x3)