30400 ELECTROMAGNETICS

Fall 2024 - Lecture 21

(David K. Cheng, pp. 390-397, 406-417)

Plane waves IV

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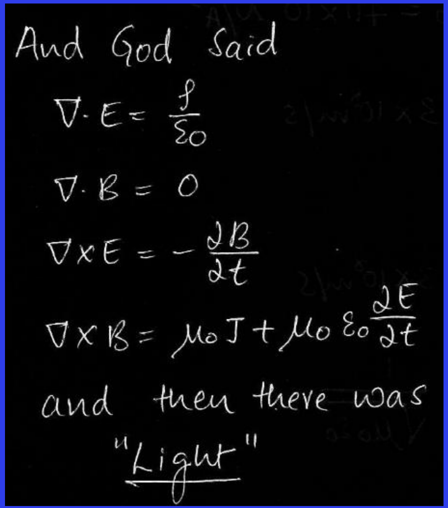
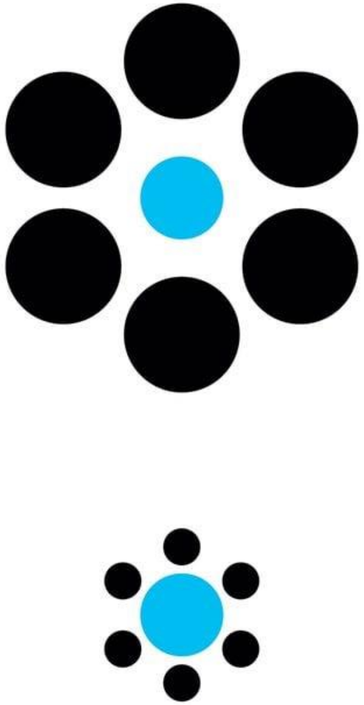
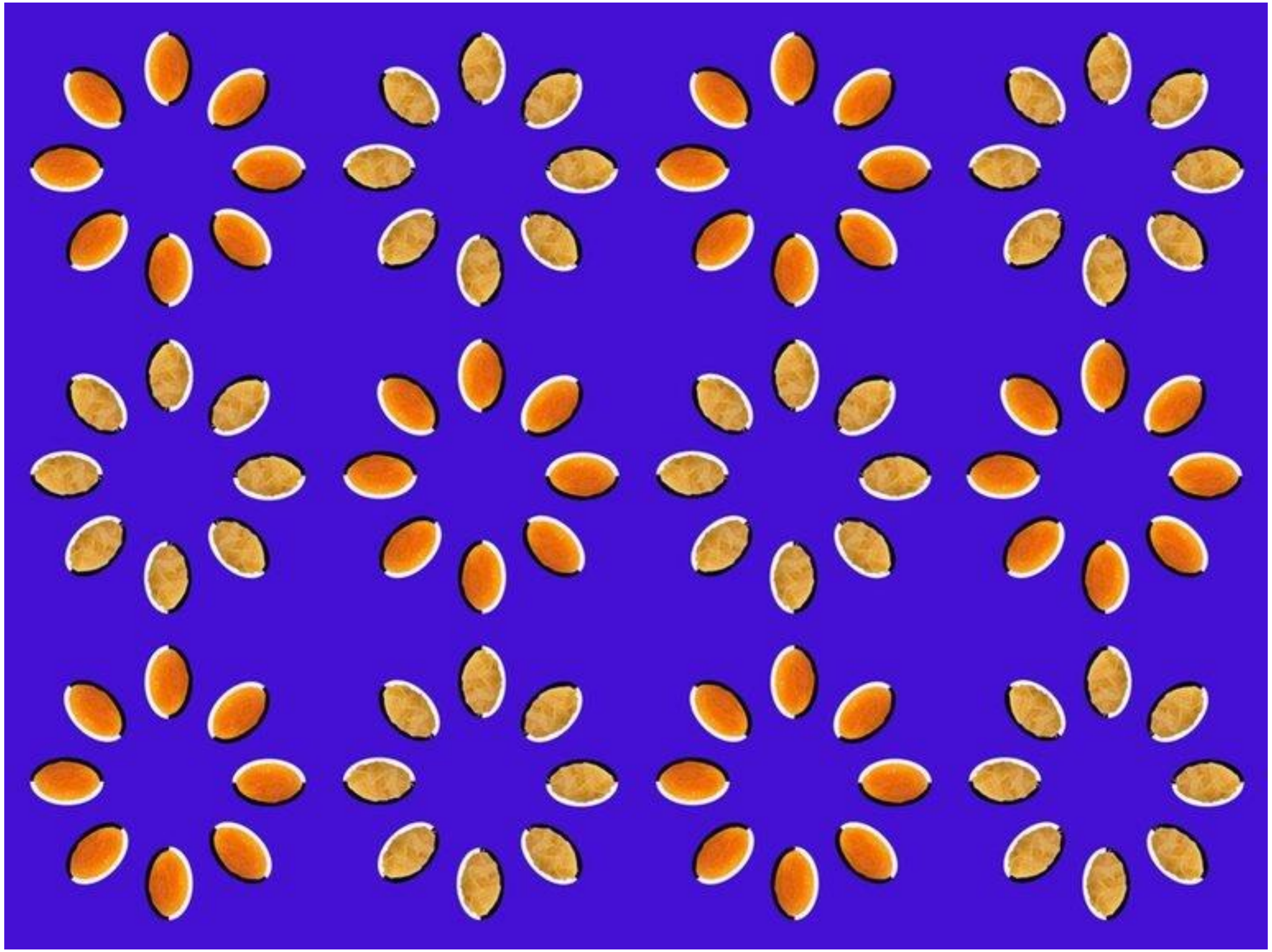






LECTURE NO. 21

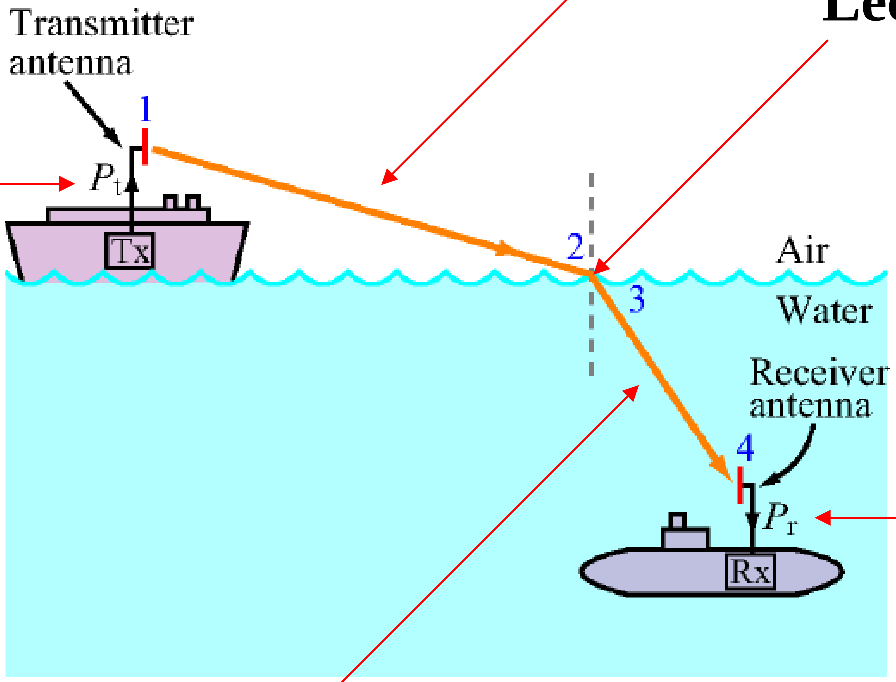
- **Summary** on Lecture 20
- **Oblique** incidence of plane wave on **PEC plane boundary**
- **Oblique** incidence of plane wave on **dielectric plane boundary**

WHAT ARE WE GOING TO DO?

Lecture 18

Lecture 20 and 21

Lecture 22-26

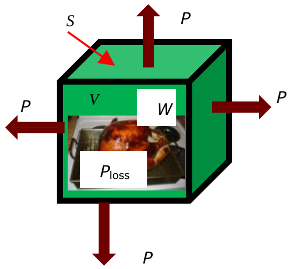


Lecture 22-26

Lecture 19

Summary on Lecture 20

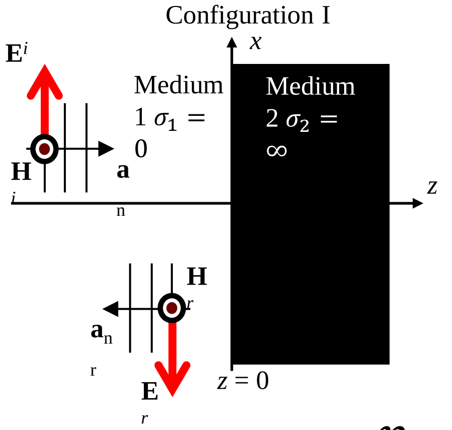
- Energy balance (the Law of conservation of energy) $$P + P_{loss} = -\frac{dW}{dt}$$



- **Poynting vector $\boldsymbol{\mathcal{P}}(t,z) = \mathbf{E}(t,z) \times \mathbf{H}(t,z)$:** The instantaneous power flow density – direction and magnitude at time *t* in point *z*.
- **Poynting complex vector** $\boldsymbol{\mathcal{P}}(z) = \frac{1}{2}\{\mathbf{E} \times \mathbf{H}^*\}$ $\quad (e^{j\omega t})$
- **Time-average** power flow density $\boldsymbol{\mathcal{P}}_{av}(z) = \frac{1}{2}\mathrm{Re}\{\mathbf{E} \times \mathbf{H}^*\} = \mathrm{Re}\boldsymbol{\mathcal{P}}$
- For a uniform plane wave in a lossless medium $\boldsymbol{\mathcal{P}}_{av} = \mathbf{a}_n \frac{|\mathbf{E}_0|^2}{2\eta}$ $\quad (e^{j\omega t})$
- **For a uniform plane wave in a lossy medium** $\boldsymbol{\mathcal{P}}_{av} = \mathbf{a}_n \frac{|\mathbf{E}_0|^2}{2|\eta_c|} e^{-2\alpha z} \cos\varphi, \eta_c = |\eta_c| e^{j\varphi}$ $\quad (e^{j\omega t})$

Summary on Lecture 20

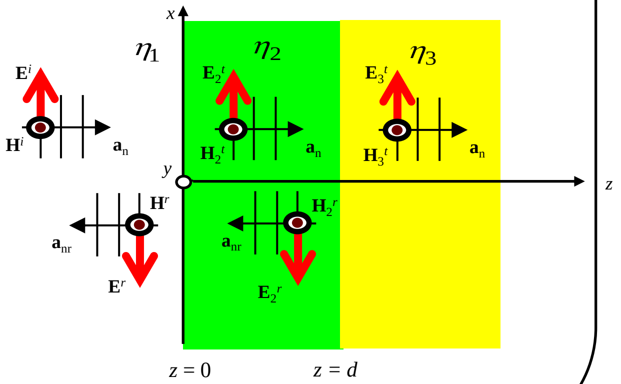
- Interface between two media, normal incidence: $\mathbf{E}^i = \mathbf{a}_x E_0^i e^{-j\beta_1 z} \qquad (e^{j\omega t})$
- Reflection from a perfect conductor: $\mathbf{E}^r = \mathbf{a}_x E_0^r e^{j\beta_1 z},\ E_0^i = -E_0^r \qquad (e^{j\omega t})$



- Total field is a **standing wave** $\mathbf{E}(z,t) = 2\mathbf{a}_x E_0^i \sin 2\beta z\ \sin\omega t\,;\ \boldsymbol{\mathcal{P}}_{av} = 0$
- **Reflection and transmission coefficients** $\Gamma = \frac{E_0^r}{E_0^i} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}\ \tau = \frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_2+\eta_1}$ $\quad 1+\Gamma=\tau \quad 1-\Gamma^2 = \frac{\eta_1}{\eta_2}\tau^2$
- **Wave impedance** $\quad Z(z) = \frac{\text{Total } E_x(z)}{\text{Total } H_y(z)} \qquad \Gamma = \frac{E_0^r}{E_0^i} = \frac{Z_2(0)-\eta_1}{Z_2(0)+\eta_1}$
- **Reflectionless layer**: $\quad Z_2(0) = \eta_1$

Case $\eta_1 = \eta_3 : d = \frac{n\lambda_2}{2}, n = 0, 1, 2$ **Half-wave dielectric window**

Case $\eta_1 \neq \eta_3 : \eta_2 = \sqrt{\eta_1\eta_3}, d = \frac{(2n+1)\lambda_2}{4}$ **Quarter-wave transformer**



One more time about polarization

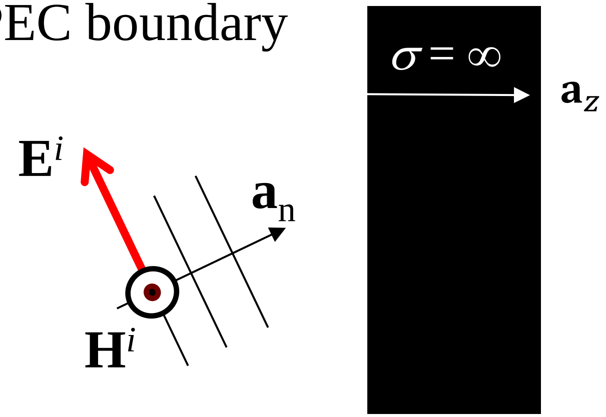
$\mathbf{E}(z) = \mathbf{E}_0\, e^{-j\beta z}, \qquad \Longrightarrow \mathbf{E}(t,z) = \mathbf{E}_0 \cos(\omega t - \beta z)$ - propagating wave

$\mathbf{E}(z) = -2j\mathbf{E}_0 \sin \beta_1 z \Longrightarrow \mathbf{E}(t,z) = 2\mathbf{E}_0 \sin \beta_1 z \sin \omega t$ – standing wave

- If vector $\mathbf{E}_0$ ($\mathbf{H}_0$) is real – we have a **linear** polarization
- If vector $\mathbf{E}_0$ ($\mathbf{H}_0$) is pure imaginary – we have a **linear** polarization
- If vector $\mathbf{E}_0$ ($\mathbf{H}_0$) = $\mathbf{a}E_0$ with real vector $\mathbf{a}$ – we have a **linear** polarization
- If vector $\mathbf{E}_0$ ($\mathbf{H}_0$) is complex – we have a general **elliptic** polarization
- If vector $\mathbf{E}_0$ ($\mathbf{H}_0$) is complex and $|\mathrm{Re}\,\mathbf{E}_0| = |\mathrm{Im}\,\mathbf{E}_0|$ – we have a **circular** polarization

- In a TEM wave vectors $\mathbf{E}_0$ and $\mathbf{H}_0$ are in the plane orthogonal to vector $\mathbf{k}$
- Any vector $\mathbf{E}_0$ ($\mathbf{H}_0$) in a plane can be presented as a sum of two orthogonal basis vectors, for example $\mathbf{a}_x$and $\mathbf{a}_y$. Therefore, a wave with any polarization can be represented as a sum of two **linearly** polarized waves or a sum of two **circularly** polarized waves.

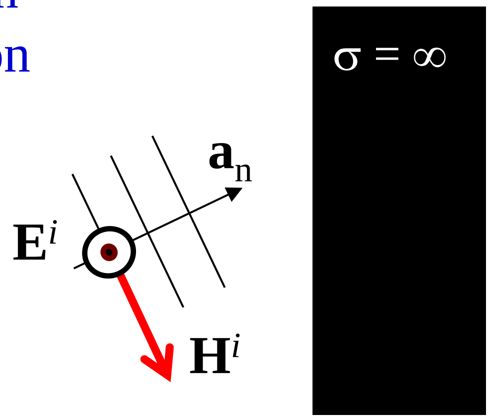
Configuration IV: PEC boundary



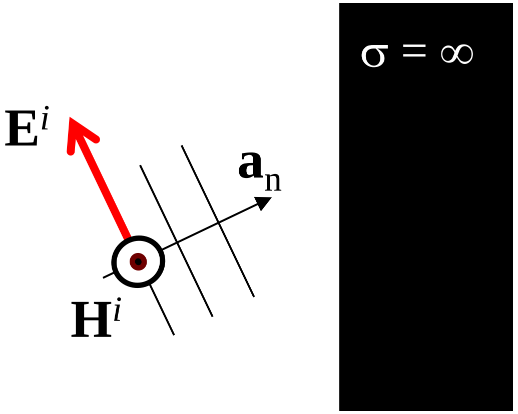
$$\mathbf{a}_n \neq \mathbf{a}_z \neq -\mathbf{a}_{nr}$$

”**Plane of incidence**” is defined by $\mathbf{a}_n$ and $\mathbf{a}_z$

Perpendicular polarization /
Transverse electric (TE) polarization
s-polarization
E-polarization

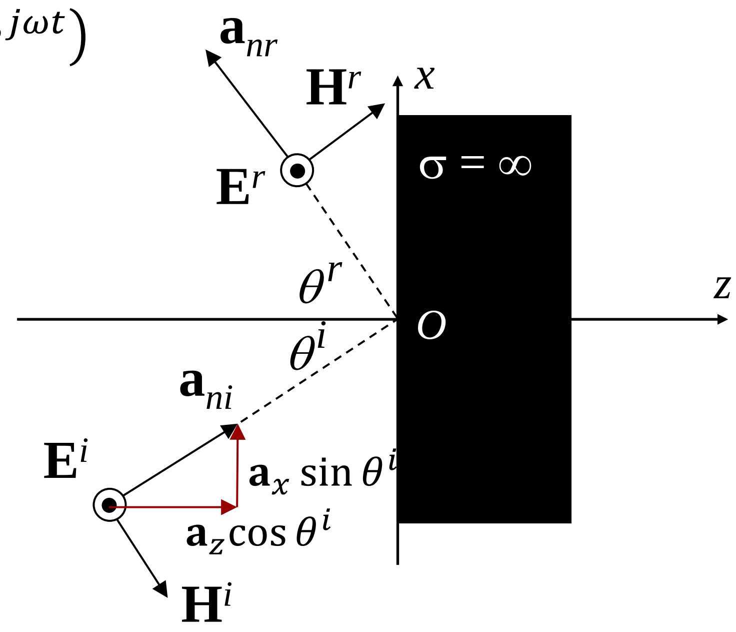


Parallel polarization /
Transverse magnetic (TM) polarization
p-polarization
H-polarization



IV. PEC Boundary - Oblique TE Incidence - I

$(e^{j\omega t})$



xOz : plane of incidence

θ^i: angle of incidence

θ^r: angle of reflection

$$\mathbf{a}_{ni} = \mathbf{a}_x \sin\theta^i + \mathbf{a}_z \cos\theta^i$$

$$\mathbf{E}^i = \mathbf{E}_0^i e^{-j\mathbf{k}^i\cdot\mathbf{R}} = \mathbf{a}_y E_0^i e^{-j\beta_1(x\sin\theta^i + z\cos\theta^i)}$$

$$\mathbf{H}^i = \frac{1}{\eta_1}\mathbf{a}_{ni} \times \mathbf{E}_0^i e^{-j\mathbf{k}^i\cdot\mathbf{R}} =$$

$$(-\mathbf{a}_x \cos\theta^i + \mathbf{a}_z \sin\theta^i)\frac{E_0^i}{\eta_1}\, e^{-j\beta_1(x\sin\theta^i + z\cos\theta^i)}$$

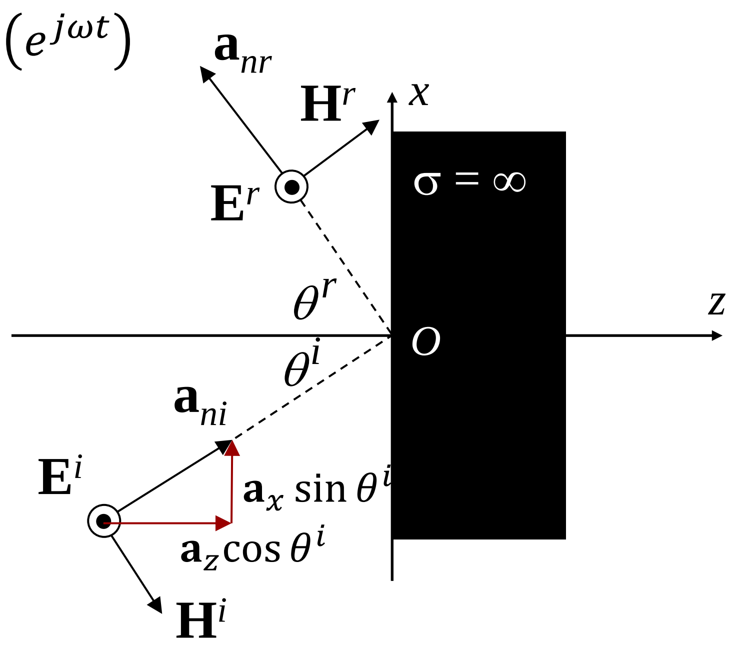
$$\mathbf{a}_{nr} = \mathbf{a}_x \sin\theta^r - \mathbf{a}_z \cos\theta^r$$

$$\mathbf{E}^r = \mathbf{E}_0^r e^{-j\mathbf{k}^r\cdot\mathbf{R}} = \mathbf{a}_y E_0^r e^{-j\beta_1(x\sin\theta^r - z\cos\theta^r)}$$

$$\mathbf{H}^r = \frac{1}{\eta_1}\mathbf{a}_{nr} \times \mathbf{E}_0^r e^{-j\mathbf{k}^r\cdot\mathbf{R}} =$$

$$(\mathbf{a}_x \cos\theta^r + \mathbf{a}_z \sin\theta^r)\frac{E_0^r}{\eta_1}\, e^{-j\beta_1(x\sin\theta^r - z\cos\theta^r)}$$

IV. PEC Boundary - Oblique TE Incidence - II



$$\mathbf{E}^i = \mathbf{a}_y E_0^i e^{-j\beta_1(x\sin\theta^i + z\cos\theta^i)} \qquad \mathbf{E}^r = \mathbf{a}_y E_0^r e^{-j\beta_1(x\sin\theta^r - z\cos\theta^r)}$$

Boundary condition @ z = 0: $\mathbf{E}_{tan} = 0 \Longrightarrow E_y^i + E_y^r = 0$

$$E_0^i\, e^{-j\beta_1 x\sin\theta^i} + E_0^r e^{-j\beta_1 x\sin\theta^r} = 0$$

i) The "phase matching" argument gives $\theta^r = \theta^i$

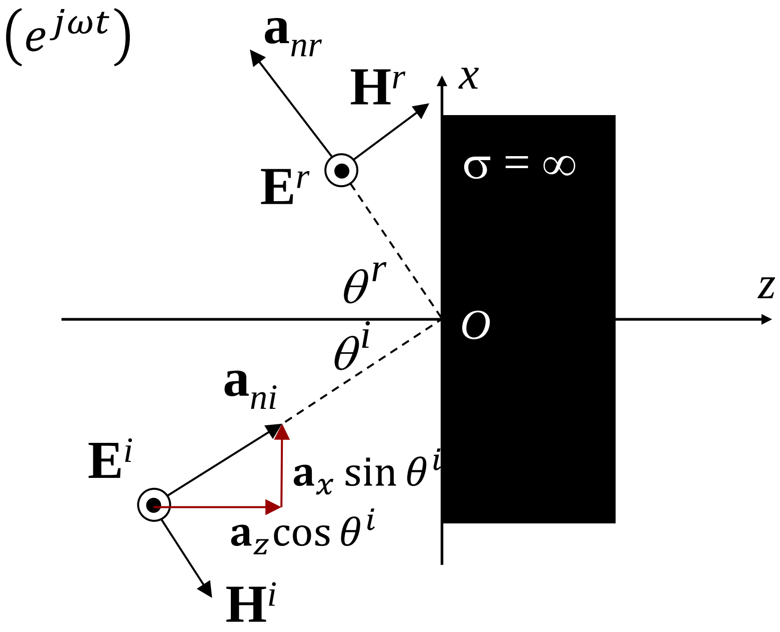
Snell's law of reflection

ii) Amplitudes' expression gives: $E_0^r = -E_0^i$

Total field $\mathbf{E}(x,z) = \mathbf{a}_y E_0^i (e^{-j\beta_1 z\cos\theta^i - j\beta_1 x\sin\theta^i} - e^{j\beta_1 z\cos\theta^i - j\beta_1 x\sin\theta^i}) =$

$$= \mathbf{a}_y\, E_0^i e^{-j\beta_1 x\sin\theta^i}\left(e^{-j\beta_1 z\cos\theta^i} - e^{j\beta_1 z\cos\theta^i}\right) = -2j\mathbf{a}_y E_0^i e^{-j\beta_1 x\sin\theta^i}\sin(\beta_1 z\cos\theta^i)$$

IV. PEC Boundary - Oblique TE Incidence - III



1. Propagating wave in the x-direction:

$$\frac{\partial}{\partial t}(\omega t - \beta_1 x \sin\theta^i) = 0 \Leftrightarrow u_{1x} = \frac{\partial x}{\partial t} = \frac{\omega}{\beta_1 \sin\theta^i} = \frac{u_1}{\sin\theta^i}$$

$$\beta_1 \lambda_x \sin\theta^i = 2\pi \Leftrightarrow \lambda_x = \frac{2\pi}{\beta_1 \sin\theta^i} = \frac{\lambda_1}{\sin\theta^i}$$

2. Standing wave in the z-direction:

$$\boldsymbol{\mathcal{P}}_{av} = \frac{1}{2}\mathrm{Re}\{\mathbf{E}\times\mathbf{H}^*\} = \mathbf{a}_x 2\frac{{E_0^i}^2}{\eta_1}\sin\theta^i \sin^2(\beta_1 z\cos\theta^i), \mathcal{P}_{av,z} = 0$$

3. Non-uniform plane wave

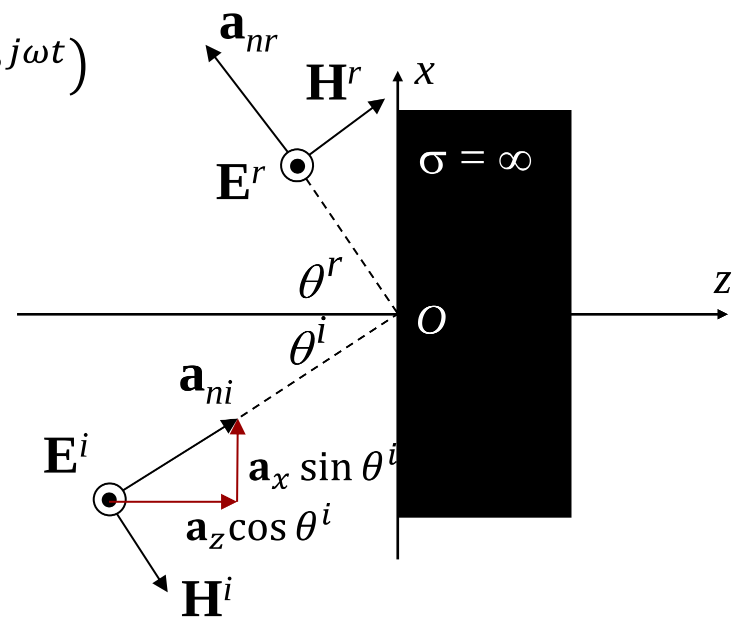
- phase propagation in x-direction
- power propagation in x-direction
- not a TEM wave

Properties of the **total TE -field**:

$$\mathbf{E}(x,z) = -2j\mathbf{a}_y E_0^i e^{-j\beta_1 x\sin\theta^i}\sin(\beta_1 z\cos\theta^i),$$

$$\mathbf{H}(x,z) = -2\frac{E_0^i}{\eta_1}\left[\mathbf{a}_x \cos\theta^i \cos(\beta_1 z\cos\theta^i) + \mathbf{a}_z j\sin\theta^i \sin(\beta_1 z\cos\theta^i)\right]e^{-j\beta_1 x\sin\theta^i}$$

$\left(e^{j\omega t}\right)$



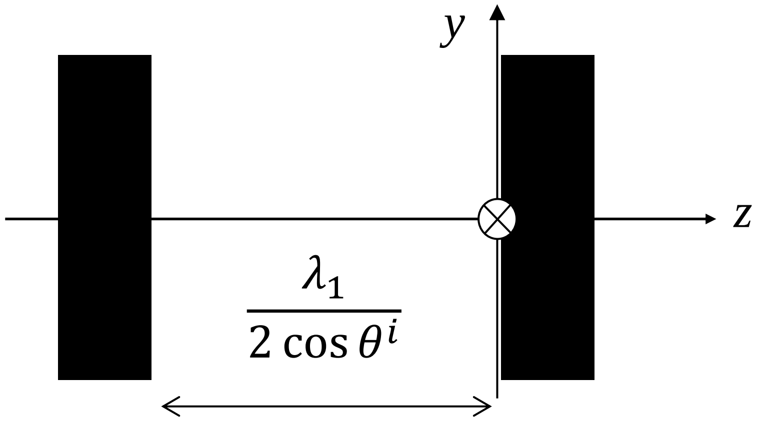
$$\mathbf{E}(x,z) = -2j\mathbf{a}_y E_0^i e^{-j\beta_1 x \sin\theta^i} \sin(\beta_1 z \cos\theta^i)$$

$$\mathbf{E}(x,z) = 0 \;\Rightarrow \beta_1 z \cos\theta^i = -n\pi$$

$$n = 0 \Leftrightarrow z = 0$$

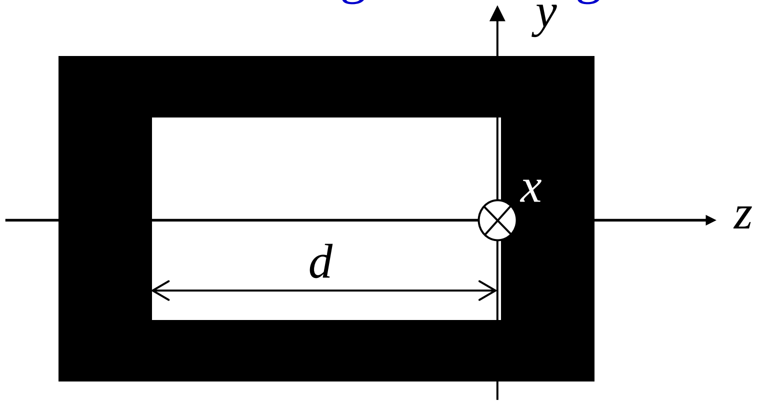
$$n = 1 \Leftrightarrow z = -\frac{\pi}{\beta_1 \cos\theta^i} = -\frac{\lambda}{2\cos\theta^i}$$

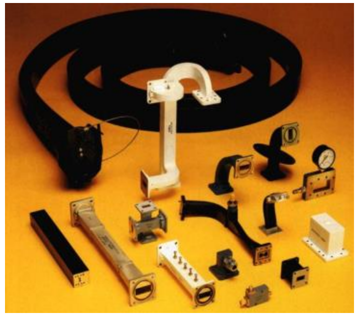
The metallic parallel plate waveguide



Waveguide ”cut-off” $> \dfrac{\lambda}{2}$

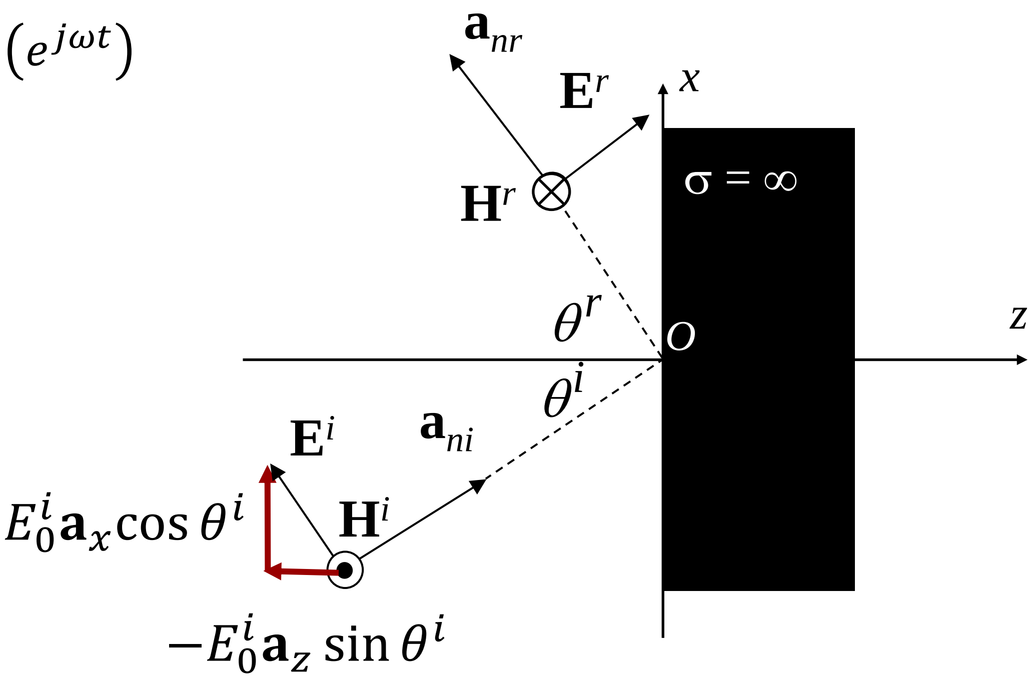
metallic rectangular waveguide





IV. PEC Boundary - Oblique TM Incidence - I

$\left(e^{j\omega t}\right)$



Incident TM field:

$$\mathbf{E}^i = E_0^i\left(\mathbf{a}_x \cos\theta^i - \mathbf{a}_z \sin\theta^i\right)e^{-j\beta_1 x \sin\theta^i - j\beta_1 z \cos\theta^i}$$

$$\mathbf{H}^i = \mathbf{a}_y \frac{E_0^i}{\eta_1} e^{-j\beta_1 x \sin\theta^i - j\beta_1 z \cos\theta^i}$$

Reflected TM field:

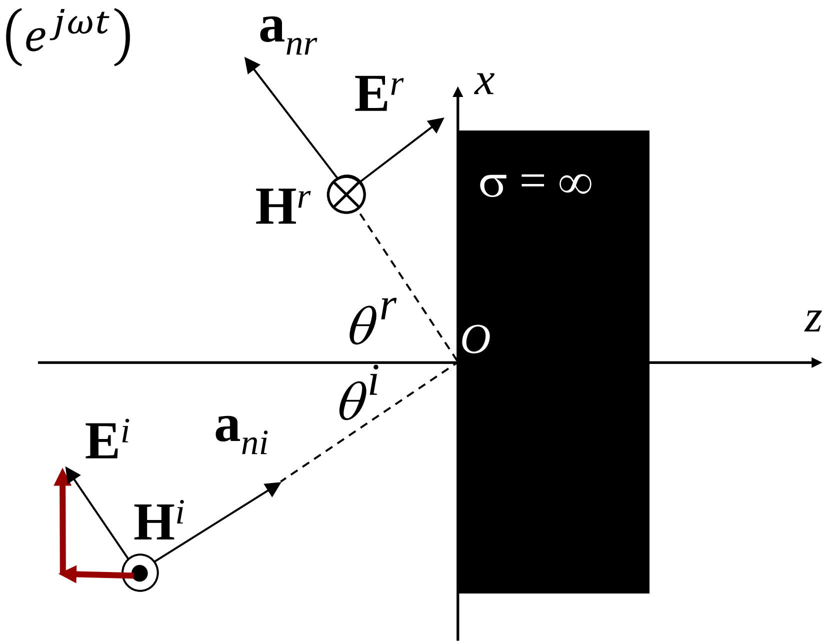
$$\mathbf{E}^r = E_0^r\left(\mathbf{a}_x \cos\theta^r + \mathbf{a}_z \sin\theta^r\right)e^{-j\beta_1 x \sin\theta^r + j\beta_1 z \cos\theta^r}$$

$$\mathbf{H}^r = -\mathbf{a}_y \frac{E_0^r}{\eta_1} e^{-j\beta_1 x \sin\theta^r + j\beta_1 z \cos\theta^r}$$

Total TM field: $\mathbf{E}^i + \mathbf{E}^r, \mathbf{H}^i + \mathbf{H}^r$

Boundary conditions @ z = 0: $\mathbf{E}_{tan} = 0 \Longrightarrow E_0^r = -E_0^i, \theta^r = \theta^i$

IV. PEC BOUNDARY - OBLIQUE TM INCIDENCE - II



1. Propagating wave in the x-direction:

$$u_x = \frac{\omega}{\beta_1 \sin\theta^i} = \frac{u_1}{\sin\theta^i} \quad \lambda_x = \frac{2\pi}{\beta_1 \sin\theta^i} = \frac{\lambda_1}{\sin\theta^i}$$

2. Standing wave in the z-direction:

$$\boldsymbol{\mathcal{P}}_{av} = \frac{1}{2}\mathrm{Re}\{\mathbf{E}\times\mathbf{H}^*\} = \mathbf{a}_x 2\frac{{E_0^i}^2}{\eta_1}\sin\theta^i\cos^2(\beta_1 z\cos\theta^i)\,, \mathcal{P}_{av,z} = 0$$

3. Non-uniform plane wave

- phase propagation in *x*-direction
- power propagation in *x*-direction
- not a TEM wave

Total TM field:

$$\mathbf{E}(x,z) = -2E_0^i\left[\mathbf{a}_x j\cos\theta^i\sin(\beta_1 z\cos\theta^i) + \mathbf{a}_z\sin\theta^i\cos(\beta_1 z\cos\theta^i)\right]e^{-j\beta_1 x\sin\theta^i}$$

$$\mathbf{H}(x,z) = 2\mathbf{a}_y\frac{E_0^i}{\eta_1}e^{-j\beta_1 x\sin\theta^i}\cos(\beta_1 z\cos\theta^i)$$

IV. PEC Boundary - Oblique TM Incidence - III

$$\mathbf{E}(x,z) = -2E_0^i\left[\mathbf{a}_x\, j\cos\theta^i \sin(\beta_1 z\cos\theta^i) + \mathbf{a}_z \sin\theta^i \cos(\beta_1 z\cos\theta^i)\right]e^{-j\beta_1 x\sin\theta^i}$$

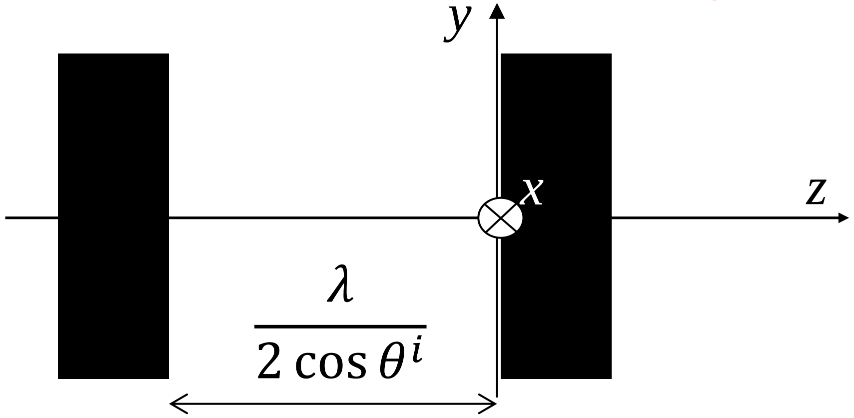
$(e^{j\omega t})$

$$E_x(x,z) = 0, \beta_1 z\cos\theta^i = -n\pi$$

$$n = 0 \Leftrightarrow z = 0$$

$$n = 1 \Leftrightarrow z = -\frac{\pi}{\beta_1\cos\theta^i} = -\frac{\lambda}{2\cos\theta^i}$$

The metallic parallel plate waveguide



$$\lim_{\theta^i\to\pi/2}\mathbf{E}(x,z) = -\,2\mathbf{a}_z E_0^i e^{-j\beta_1 x}$$

$E_x(x,z) = 0$ for all z

Waveguide "cut-off"

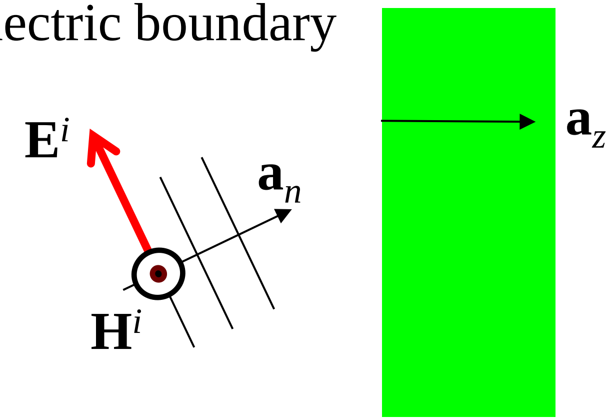
$> \frac{\lambda}{2}$

For this field there is no minimum requirement to the parallel plate distance!

The total field does not represent a metallic rectangular waveguide field

V. DIELECTRIC BOUNDARY - OBLIQUE INCIDENCE - I

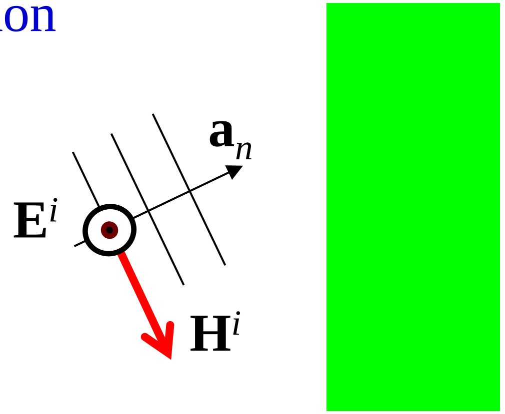
Configuration V: Dielectric boundary



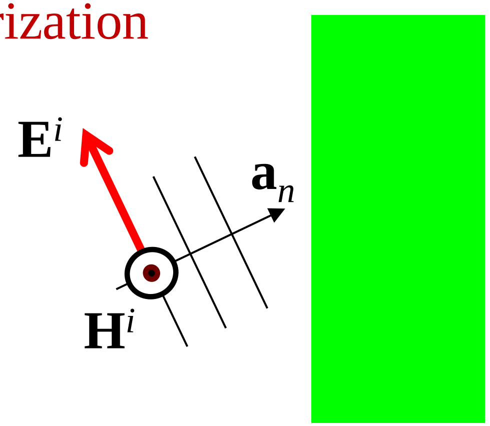
$$\mathbf{a}_n \neq \mathbf{a}_z \neq -\mathbf{a}_{nr}$$

”**Plane of incidence**” is defined by $\mathbf{a}_n$ and $\mathbf{a}_z$

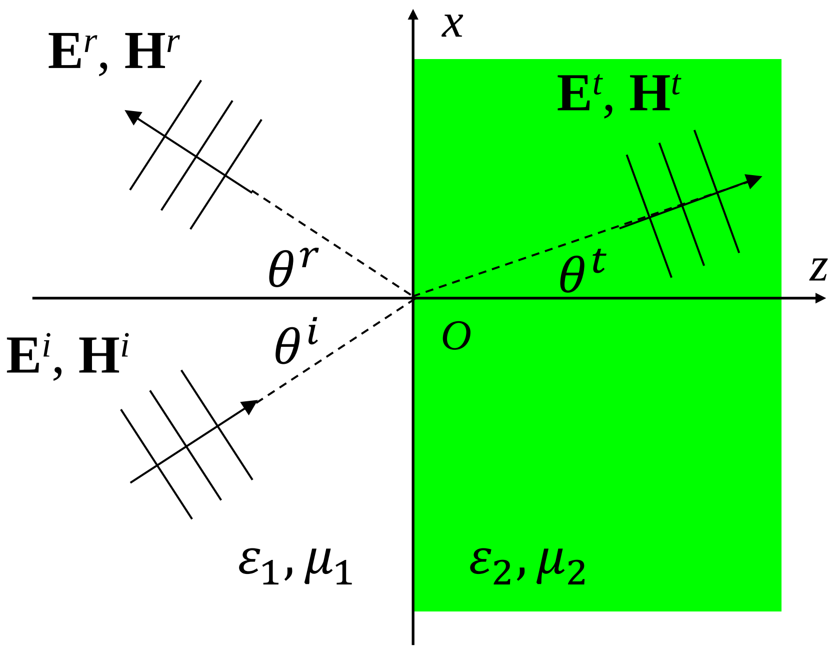
Perpendicular polarization /
Transverse electric (**TE**) polarization
s-polarization
E-polarization



Parallel polarization /
Transverse magnetic (**TM**) polarization
p-polarization
H-polarization



V. Dielectric Boundary - Oblique Incidence - II



Snell's law of reflection: $\theta^r = \theta^i$

Snell's law of refraction: $\dfrac{\sin\theta^t}{\sin\theta^i} = \dfrac{u_2}{u_1} = \dfrac{n_1}{n_2}$

Speed of propagation $u = \dfrac{1}{\sqrt{\varepsilon\mu}}$

Index of refraction $n = \dfrac{c}{u} = \dfrac{\frac{1}{\sqrt{\varepsilon_0\mu_0}}}{\frac{1}{\sqrt{\varepsilon\mu}}} = \sqrt{\varepsilon_r\mu_r}$

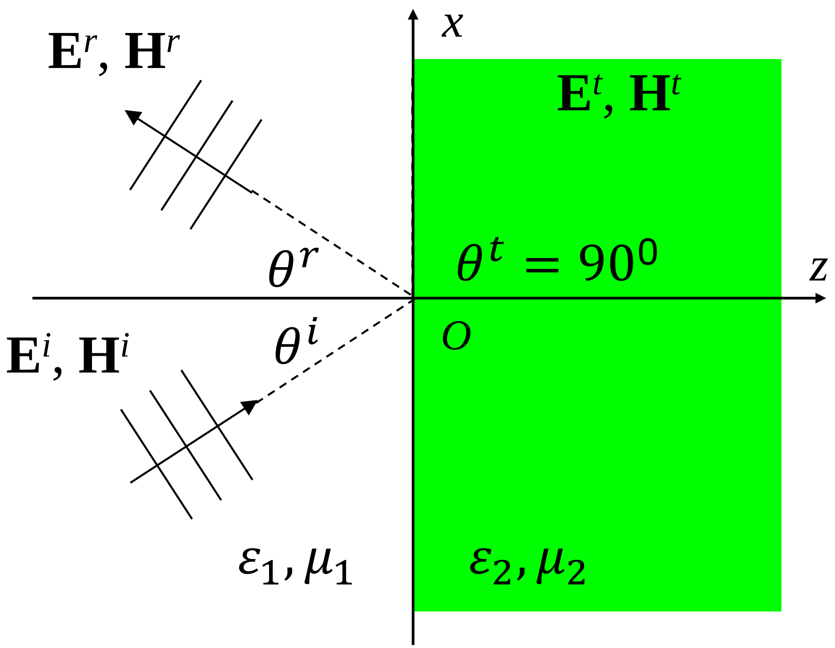
Willebrord van Roijen Snell (1580-1626)

Total reflection: No propagation away from the boundary in medium 2: $\theta^t = 90°$

Total reflection requires: $n_1 > n_2$

$$\sin\theta^t = \frac{n_1}{n_2}\sin\theta^i = 1 \Leftrightarrow \sin\theta^i = \frac{n_2}{n_1} \Leftrightarrow \theta^i = \sin^{-1}\frac{n_2}{n_1} \equiv \theta_c^i$$ Critical angle of incidence

V. Dielectric Boundary - Oblique Incidence - III



Field at critical angle of incidence: $\theta_c^i = \sin^{-1}\frac{n_2}{n_1}$

$$\mathbf{a}_{ni} = \mathbf{a}_x \sin\theta^i + \mathbf{a}_z \cos\theta^i$$

$$\mathbf{E}^t = \mathbf{E}_0^t e^{-j\beta_2 \mathbf{a}_k \cdot \mathbf{R}} = \mathbf{E}_0^t e^{-j\beta_2(x\sin\theta^t + z\cos\theta^t)} = \mathbf{E}_0^t e^{-j\beta_2 x}$$

Plane wave

What happens if $\theta^i > \theta_c^i$?

$$\sin\theta^t = \frac{n_1}{n_2}\sin\theta^i = \frac{\sin\theta^i}{\sin\theta_c^i} > 1$$

$$\cos\theta^t = \sqrt{1 - \sin^2\theta^t} = \pm j\sqrt{\sin^2\theta^t - 1}$$

$$\mathbf{E}^t = \mathbf{E}_0^t e^{-\beta_2 z\sqrt{\sin^2\theta^t - 1}} e^{-j\beta_2 x\sin\theta^t}$$

Nonuniform plane wave $\mathbf{E}_0(z)$

Surface/evanescent wave

$u_{phase} < u_2$

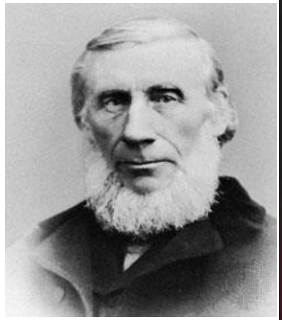
EXAM 2017

Question 2.9

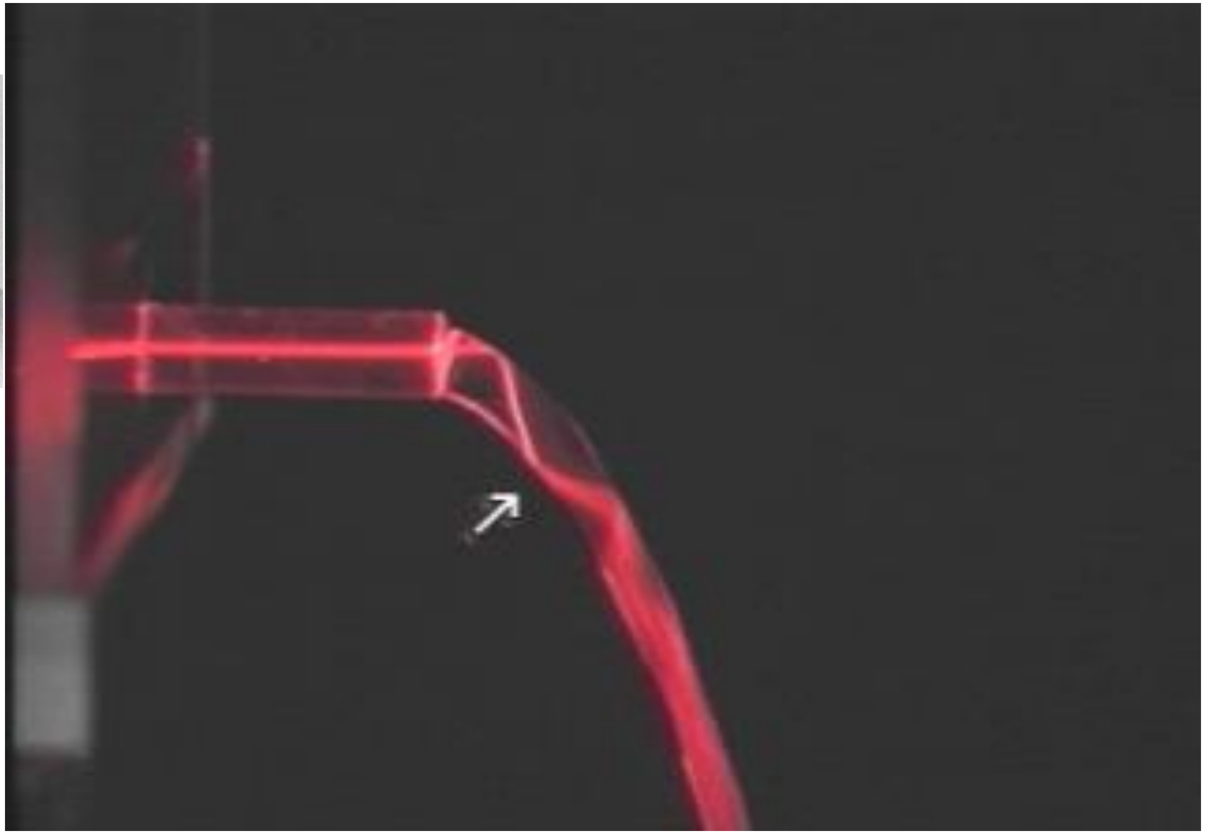
How does the amplitude of the transmitted field vary with distance from a plane dielectric boundary when a uniform plane wave is incident at an angle of incidence equal to the critical angle?

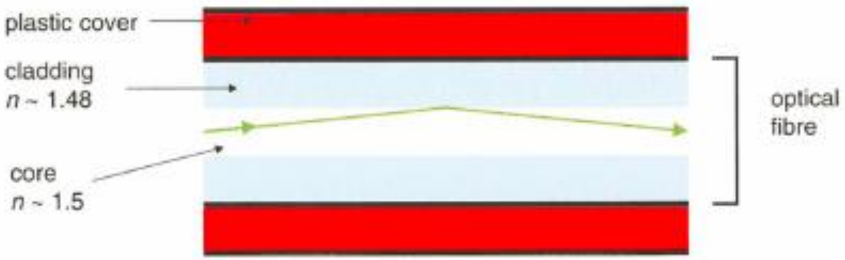
a) The amplitude decreases exponentially with distance
b) The amplitude decreases linearly with distance
c) The amplitude is always zero
d) The amplitude is constant

Total Internal Reflection (TIR)



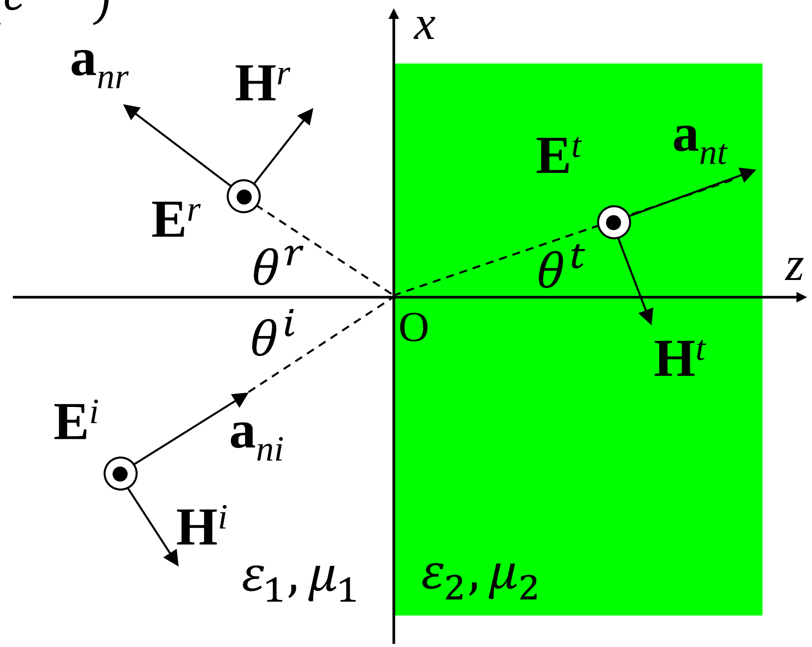
John Tyndall
(1820-1893)





V. DIELECTRIC BOUNDARY - OBLIQUE TE INCIDENCE - I

$(e^{j\omega t})$



$$\mathbf{E}^i = \mathbf{a}_y E_0^i e^{-j\beta_1(x\sin\theta^i + z\cos\theta^i)}$$

$$\mathbf{E}^r = \mathbf{a}_y \boxed{E_0^r} e^{-j\beta_1(x\sin\theta^r - z\cos\theta^r)}$$

$$\mathbf{E}^t = \mathbf{a}_y \boxed{E_0^t} e^{-j\beta_2(x\sin\theta^t + z\cos\theta^t)}$$

$$\mathbf{H}^i = \frac{1}{\eta_1}\mathbf{a}_{ni} \times \mathbf{E}_0^i e^{-j\mathbf{k}^i\cdot\mathbf{R}} =$$

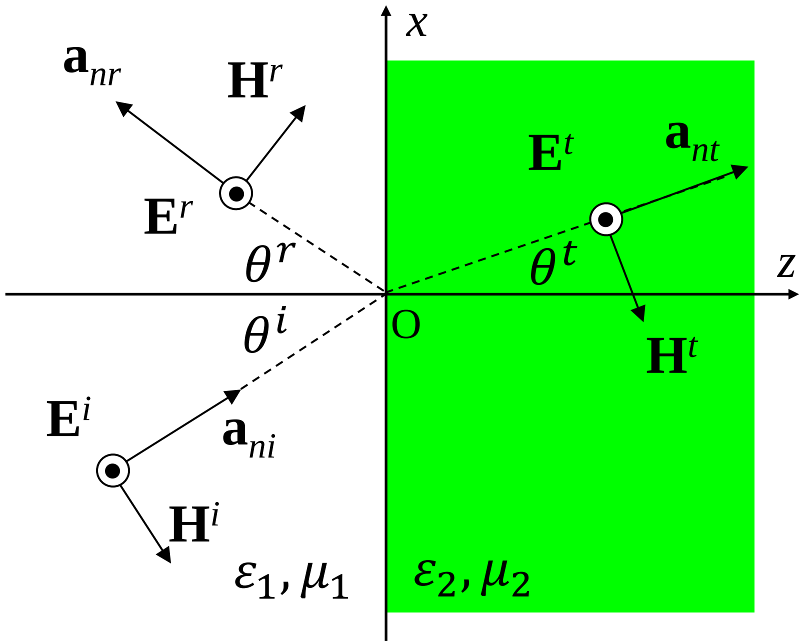
$$(-\mathbf{a}_x\cos\theta^i + \mathbf{a}_z\sin\theta^i)\frac{\boxed{E_0^i}}{\eta_1} e^{-j\beta_1(x\sin\theta^i + z\cos\theta^i)}$$

$$\mathbf{H}^r = \frac{1}{\eta_1}\mathbf{a}_{nr} \times \mathbf{E}_0^r e^{-j\mathbf{k}^r\cdot\mathbf{R}} =$$

$$(\mathbf{a}_x\cos\theta^r + \mathbf{a}_z\sin\theta^r)\frac{\boxed{E_0^r}}{\eta_1} e^{-j\beta_1(x\sin\theta^r - z\cos\theta^r)}$$

The interface is the xOy plane. The TE electric field is tangential, but the TE magnetic field should be projected having only the x-component as the tangential field

V. DIELECTRIC OUNDARY - OBLIQUE TE INCIDENCE - II



Boundary Conditions @ z = 0

$$\mathbf{E}^i + \mathbf{E}^r = \mathbf{E}^t \Leftrightarrow {E_0}^i + {E_0}^r = {E_0}^t$$

$$ {H_x}^i + {H_x}^r = {H_x}^t \Leftrightarrow \frac{\cos\theta^i}{\eta_1}\left({E_0}^i - {E_0}^r\right) = \frac{\cos\theta^t}{\eta_2}{E_0}^t$$

Solution

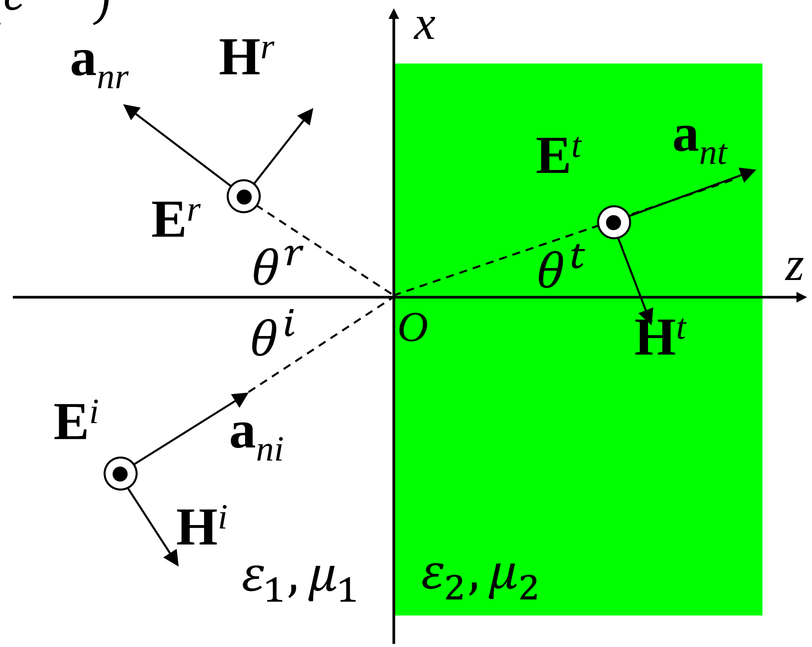
$$E_0^r = \frac{\eta_2/\cos\theta^t - \eta_1/\cos\theta^i}{\eta_2/\cos\theta^t + \eta_1/\cos\theta^i} E_0^i \equiv \Gamma_\perp E_0^i$$

$$E_0^t = \frac{2\eta_2/\cos\theta^t}{\eta_2/\cos\theta^t + \eta_1/\cos\theta^i} E_0^i \equiv \tau_\perp E_0^i$$

$$1 + \Gamma_\perp = \tau_\perp$$

V. Dielectric Boundary - Oblique TE Incidence - III

$(e^{j\omega t})$



$$E_0^r = \frac{\eta_2/\cos\theta^t - \eta_1/\cos\theta^i}{\eta_2/\cos\theta^t + \eta_1/\cos\theta^i} E_0^i \equiv \Gamma_\perp E_0^i$$

Is there a condition for no reflection?

$$\Gamma_\perp = 0 \Rightarrow \eta_2/\cos\theta^t = \eta_1/\cos\theta^i$$

$$\Rightarrow \quad \cos\theta^i = \frac{\eta_1}{\eta_2}\cos\theta^t$$

$$\sqrt{1-\sin^2\theta^i} = \frac{\eta_1}{\eta_2}\sqrt{1-\sin^2\theta^t} = \frac{\eta_1}{\eta_2}\sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta^i}$$

$$\sin^2\theta^i = \frac{1-\mu_1\varepsilon_2/\mu_2\varepsilon_1}{1-(\mu_1/\mu_2)^2}$$

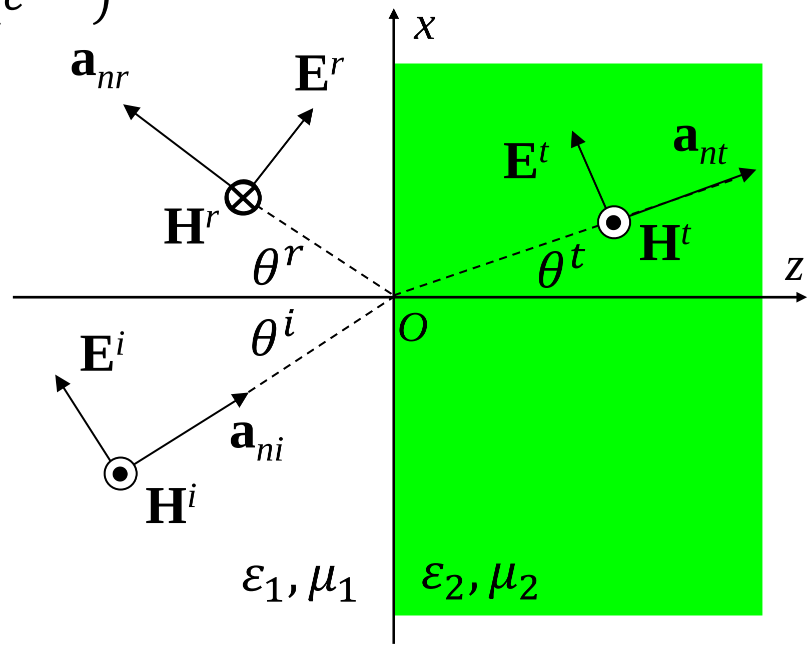
If $\mu_1 = \mu_2, \sin^2\theta_i = \infty$

If $\varepsilon_1 = \varepsilon_2, \sin^2\theta_i = \frac{1}{1+\mu_1/\mu_2}$

In theory yes – in practice no (or very rarely)!

V. Dielectric Boundary - Oblique TM Incidence - I

$\left(e^{j\omega t}\right)$



Boundary Conditions @ z = 0

$$E_x^i + E_x^r = E_x^t \Leftrightarrow \left(E_0^i + E_0^r\right) \cos\theta^i = E_0^i \cos\theta^t$$

$$H_y^i + H_y^r = H_y^t \Leftrightarrow \frac{E_0^i - E_0^r}{\eta_1} = \frac{E_0^t}{\eta_2}$$

Solution

$$E_0^r = \frac{\eta_2 \cos\theta^t - \eta_1 \cos\theta^i}{\eta_2 \cos\theta^t + \eta_1 \cos\theta^i} E_0^i \equiv \Gamma_\parallel E_0^i$$

$$E_0^t = \frac{2\eta_2 \cos\theta^i}{\eta_2 \cos\theta^t + \eta_1 \cos\theta^i} E_0^i \equiv \tau_\parallel E_0^i$$

$$\mathbf{E}^i = E_0^i\left(\mathbf{a}_x \cos\theta^i - \mathbf{a}_z \sin\theta^i\right) e^{-j\beta_1 x \sin\theta^i - j\beta_1 z \cos\theta^i}$$

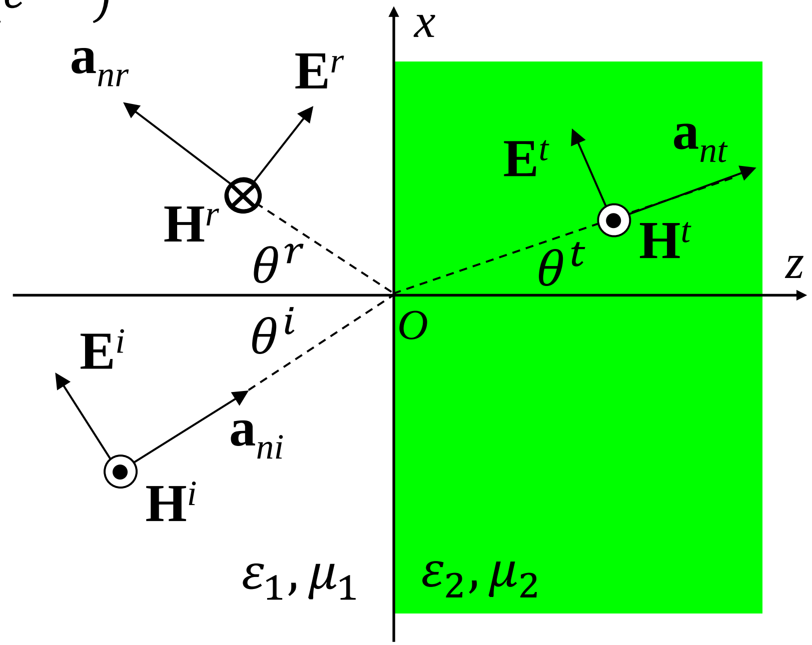
$$1 + \Gamma_\parallel = \tau_\parallel \frac{\cos\theta^t}{\cos\theta^i}$$

$$\mathbf{E}^r = \boxed{E_0^r}(\mathbf{a}_x \cos\theta^r + \mathbf{a}_z \sin\theta^r) e^{-j\beta_1 x \sin\theta^r + j\beta_1 z \cos\theta^r}$$

$$\mathbf{E}^t = \boxed{E_0^t}(\mathbf{a}_x \cos\theta^t - \mathbf{a}_z \sin\theta^t) e^{-j\beta_2 x \sin\theta^t - j\beta_2 z \cos\theta^t}$$

V. DIELECTRIC BOUNDARY - OBLIQUE TM INCIDENCE - II

$\left(e^{j\omega t}\right)$



$$E_0^r = \frac{\eta_2 \cos\theta^t - \eta_1 \cos\theta^i}{\eta_2 \cos\theta^t + \eta_1 \cos\theta^i} E_0^i \equiv \Gamma_{\parallel} E_0^i$$

Is there a condition for no reflection?

$$\Gamma_{\parallel} = 0 \Rightarrow \ \eta_2 \cos\theta^t = \eta_1 \cos\theta^i$$

$$\Rightarrow \ \cos\theta^i = \frac{\eta_2}{\eta_1} \cos\theta^t$$

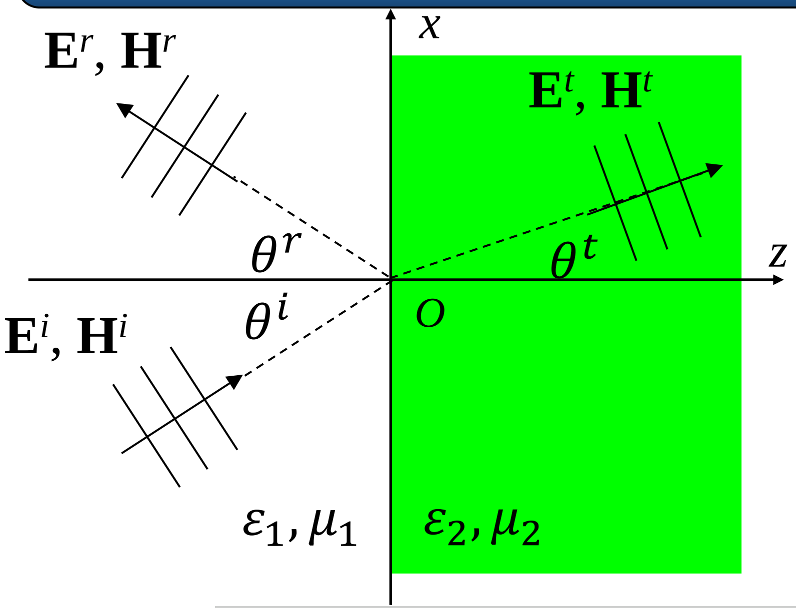
$$\sqrt{1 - \sin^2\theta^i} = \frac{\eta_2}{\eta_1}\sqrt{1 - \sin^2\theta^t} = \frac{\eta_2}{\eta_1}\sqrt{1 - \frac{n_1^2}{n_2^2}\sin^2\theta^i}$$

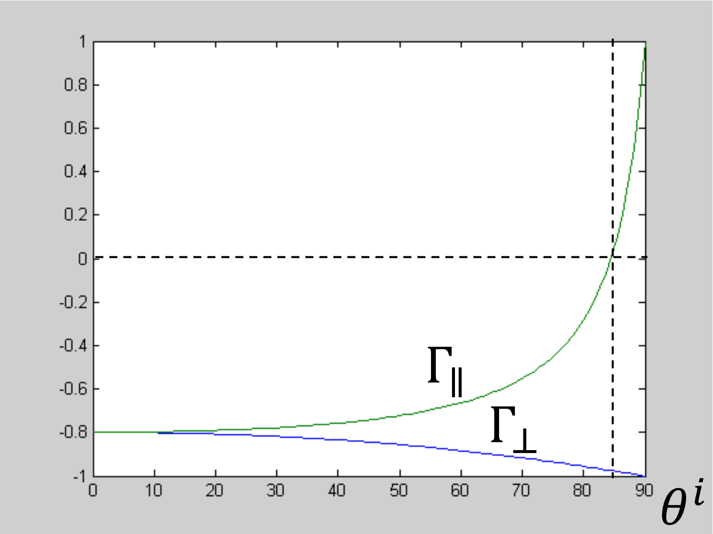
$$\sin^2\theta^i = \frac{1 - \mu_2\varepsilon_1/\mu_1\varepsilon_2}{1 - (\varepsilon_1/\varepsilon_2)^2} \equiv \sin^2\theta_B \quad \text{YES!}$$

Brewster angle

$$\mu_1 = \mu_2 \colon \sin^2\theta_{B\parallel} = \frac{1}{1 + \varepsilon_1/\varepsilon_2} \text{ or } \tan\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1}$$

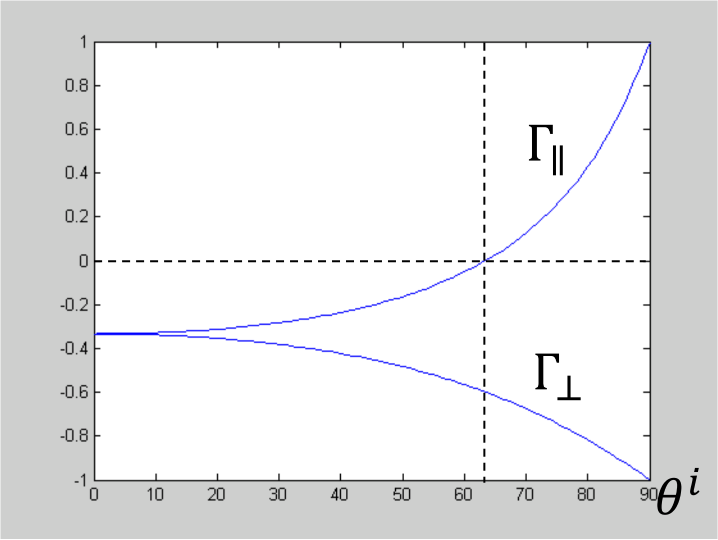
V. DIELECTRIC BOUNDARY - OBLIQUE INCIDENCE

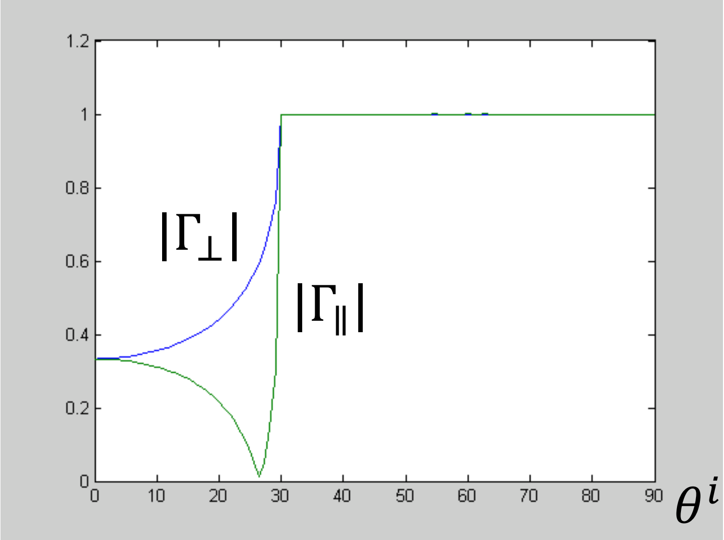




$\mu_2 = \mu_1$
$\varepsilon_2 = 80\varepsilon_1$

$\mu_2 = \mu_1$
$\varepsilon_2 = 4\varepsilon_1$





$\mu_2 = \mu_1$
$\varepsilon_2 = 0.25\varepsilon_1$

Power balance

$$\mathcal{P}_{av1} = \mathcal{P}_{av2}$$

Normal incidence

$$1 - \Gamma^2 = \frac{\eta_1}{\eta_2}\tau^2$$

See Example 8-11, p.400-401

$$R \equiv \left(\frac{E_0^r}{E_0^i}\right)^2 = \Gamma^2 \qquad T \equiv \frac{\eta_1}{\eta_2}\left(\frac{E_0^t}{E_0^i}\right)^2 = \frac{\eta_1}{\eta_2}\tau^2$$

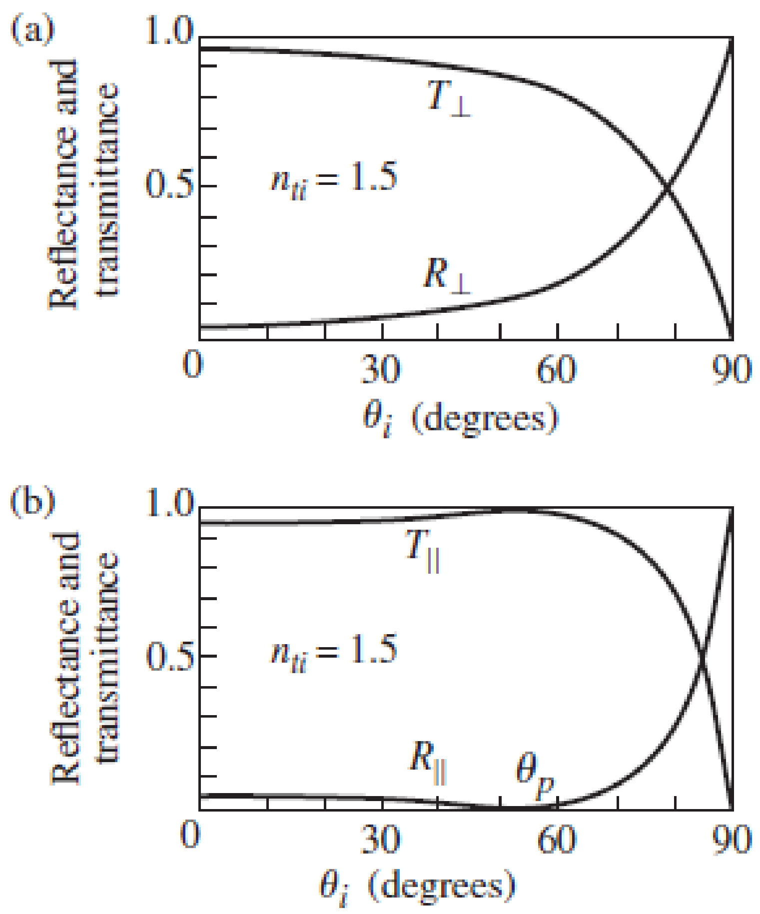
Reflectance Transmittance

$$1 = R + T$$

Oblique incidence

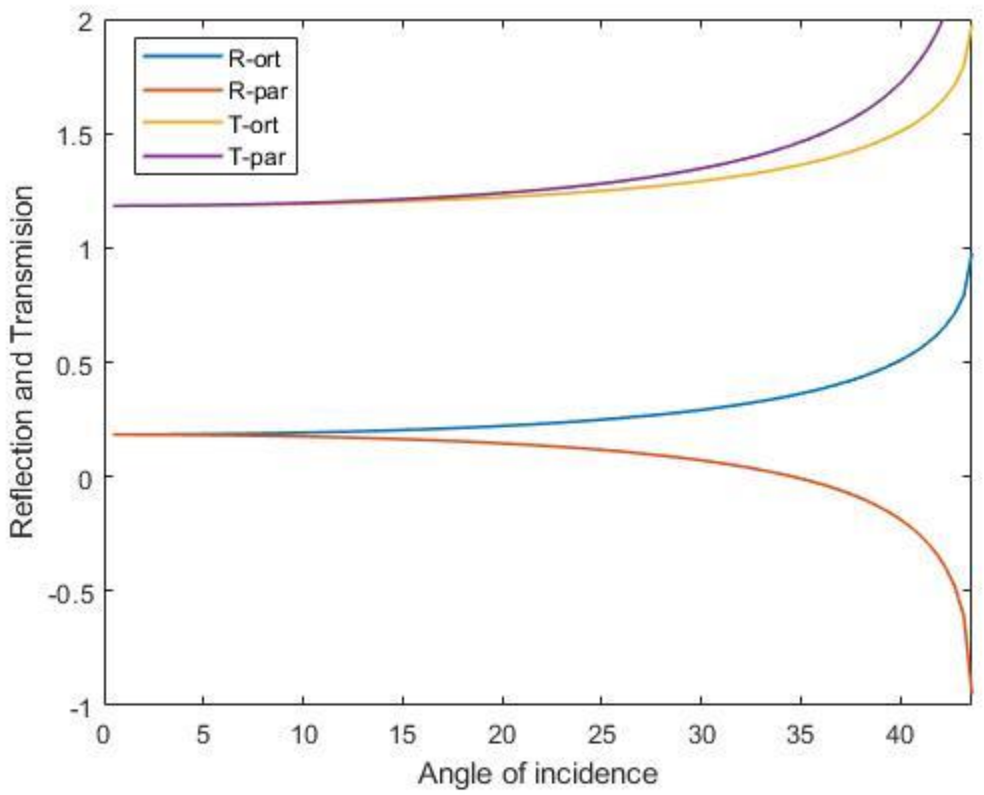
$$T \equiv \frac{\eta_1 \cos\theta^t}{\eta_2 \cos\theta^i}\left(\frac{E_0^t}{E_0^i}\right)^2 = \frac{\eta_1 \cos\theta^t}{\eta_2 \cos\theta^i}\tau^2$$

$$1 = R_{\parallel} + T_{\parallel} \qquad 1 = R_{\perp} + T_{\perp}$$

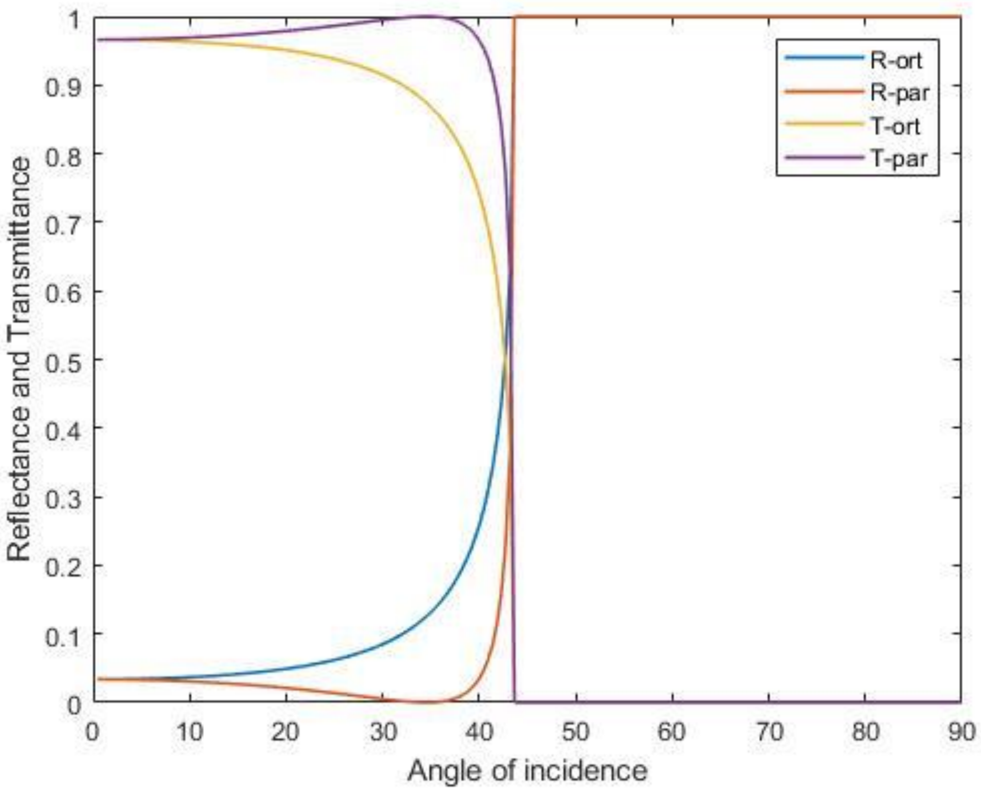


Reflection/transmission at glass/air interface

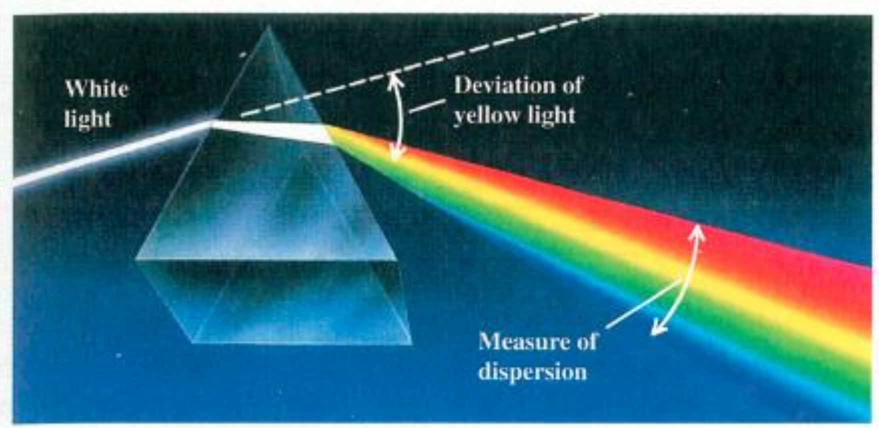
Reflection and transmission coefficients $\Gamma_{\perp,\parallel}, \tau_{\perp,\parallel}$



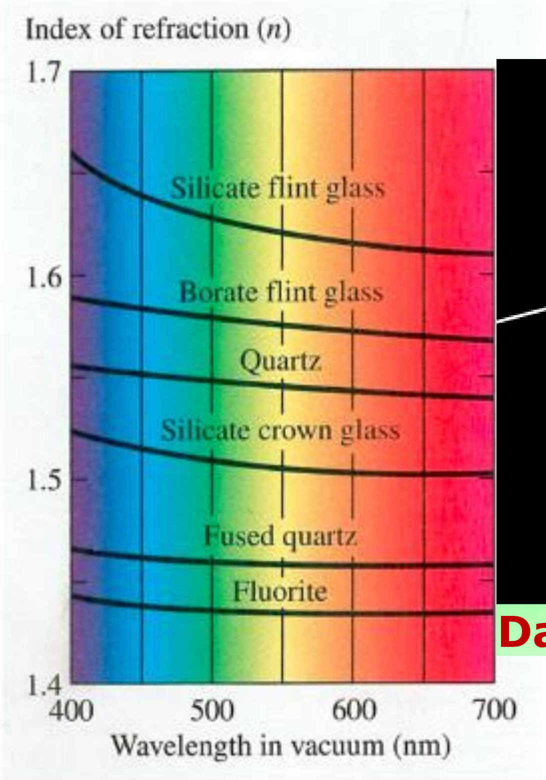
Reflectance and transmittance $R_{\perp,\parallel}, T_{\perp,\parallel}$



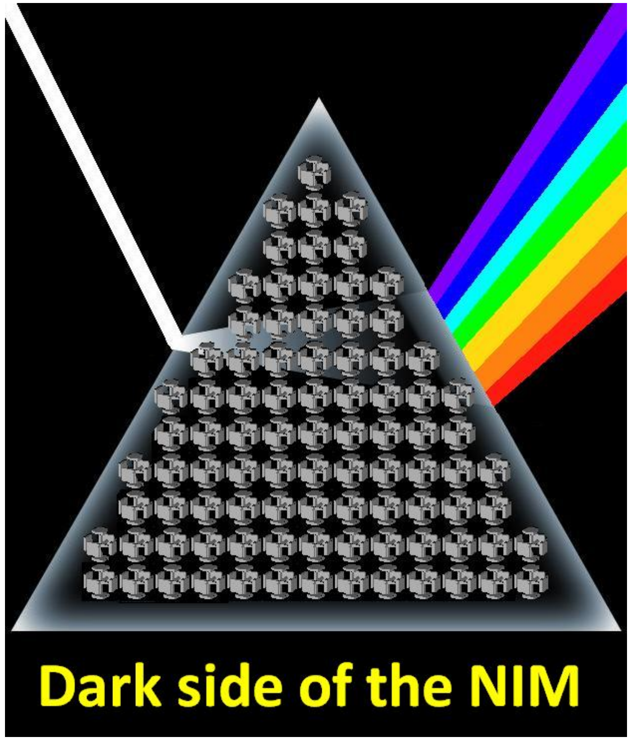
Something more …



$$\frac{\sin\theta_t}{\sin\theta_i}=\frac{n_1}{n_2}$$



Dark Side of the Moon



Summary of Lecture 21

- For oblique incidence, distinction must be made between the **TE** and **TM** polarizations
- **Snell's law of reflection** follows from the electromagnetic boundary conditions
- The total field in front of a PEC boundary is a **non-uniform** plane wave
- The total field in front of a PEC boundary illustrates the fields in **waveguides**
- The effect of **total reflection** is common to the **TE**- and **TM**-polarizations
- Angles of incidence larger than the **critical angle** give rise to surface waves
- Reflection and transmission coefficients for the **TE**- and **TM**-polarizations are different
- **Brewster angle** occurs in practice only for the **TM**-polarization
- The power balance is provided by the reflectance and transmittance