MATH 410 502
Homework 2

Keegan Smith

September 11, 2024

Problem 1

Initially our memory space looks like:
16 byte free list: [0, ..., 15]
8 byte free list: []
4 byte free list: []
2 byte free list: []
1 byte free list: []
after 1 byte is requested:
16 byte free list: []
8 byte free list: [8,..., 15]
4 byte free list: [4,..., 7]
2 byte free list: [2, 3]
1 byte free list: [1]
after 2 byte requested (so now 3 bytes in total requested):
16 byte free list: []
8 byte free list: [8,..,15]
4 byte free list: [4,..., 7]
2 byte free list: []
1 byte free list: [1]
after 4 byte requested (so now 7 bytes in total requested):
16 byte free list: []
8 byte free list: [8,..,15]
4 byte free list: []
2 byte free list: []
1 byte free list: [1]
after 2 byte requested (so now 9 bytes in total requested):
16 byte free list: []
8 byte free list: []
4 byte free list: [12,...,15]
2 byte free list: [10,11]
1 byte free list: [1]

Problem 2

a) There are $\frac{2^{36}}{2^{13}} = 2^{23}$ pages in the virtual address space.

b) If each page table entry is 4 bytes $= 2^5 = 32$ bits, then there must be 2^{32} physical pages. Thus the total addressable memory should be that times the size of a page: $2^{32} \cdot 2^{13} = 2^{45}$ bytes.

c) I would use a 1 level page table because an 8GB process would be using most of the memory anyways. A 2 or 3 level page table would only serve to obfuscate things as you would still have to allocate memory for pretty much all of the 2nd and 3rd level pages.

d) For a single page table, the number of page table entries would be $\frac{2^33}{2^13} = 2^{20}$.

Problem 3

a) The page size would be 2^8 bytes

b) A process that has 2^{18} bytes would be using $\frac{2^{18}}{2^8} = 2^{10}$ pages. This means that the number of allocated page table entries in the third level table must be 2^{10}.
Each third level page table has 2^6 entries, so there must have been $\frac{2^{10}}{2^6} = 2^4$ 2nd level page entries allocated (because 2^4 third level pages were allocated).
The size of a level 2 page table is 2^8 entries, since only 2^4 entries were needed, we only allocated one level 2 page table.
This means we only needed one page entry in the level 1 page table, and thus we only allocated one level one page table. So in total, we allocated 3 level 3 tables, one level 2 table, and one level one table.
The total number of pages is $\frac{2^{32}}{2^8} = 2^{24}$, so the size of a page entry is 24 bits or 3 bytes.
So in total, the page table size was $2^6 \cdot 2^4 \cdot 3 + 2^8 \cdot 3 + 2^{10} \cdot 3$ bytes, and wasted $(2^8 - 3 + 2^{10} - 1) \cdot 3$ bytes due to internal fragmentation

c) For the code segment, we are allocating a total of $2^{14} \cdot 3$ bytes which is $\frac{2^{14} \cdot 3}{2^8} = 2^6 \cdot 3$ pages. Thus we need $2^6 \cdot 3$ entries in our L3 page table, L3 has 2^6 entries per table, so we allocated 3 tables. Since we allocated 3 tables in L3, we must've used 3 entries in L2 page table, the number of entries in an L2 table is 2^8 so we only needed to allocate 1 L2 page table. So of course we only allocated 1 L1 page table. So for the code segment's

page tables we consumed: $(2^6 \cdot 3 + 2^8 + 2^10) \cdot 3$ bytes. Memory which was not used (in the L1 and L2 page tables) is $(2^8 - 3 + 2^{10} - 1) \cdot 3$ (we only used 3 entries in the L2 table and only one entry in L1).
For the data segment: $600K = 75 \cdot 2^{15}$ bytes $= \frac{75 \cdot 2^{15}}{2^8}$ pages $= 75 \cdot 2^7$ pages. So we will need $\frac{75 \cdot 2^7}{2^6} = 75 \cdot 2$ L3 page tables. So we will need $\lceil \frac{75 \cdot 2}{2^8} \rceil = 1$ L2 page (which is wasting $(2^8 - 150) \cdot 3 = 318$ bytes). We are only using 1 L1 entry so 1 L1 page table, and so we are wasting $(2^{10} - 1) \cdot 3$ bytes.
In total for the data segment, we allocated: $(75 \cdot 2 \cdot 2^6 + 2^8 + 2^{10}) \cdot 3$ bytes. We wasted $318 + (2^{10} - 1) \cdot 3$ bytes.
For the stack segment, I already did this in b). We allocated a total of $2^6 \cdot 2^4 \cdot 3 + 2^8 \cdot 3 + 2^{10} \cdot 3$ and wasted $(2^8 - 3 + 2^{10} - 1) \cdot 3$ bytes.
So in total we allocated $(2^6 \cdot 3 + 2^8 + 2^{10}) \cdot 3 + (75 \cdot 2 \cdot 2^6 + 2^8 + 2^{10}) \cdot 3 + 2^6 \cdot 2^4 \cdot 3 + 2^8 \cdot 3 + 2^{10} \cdot 3$ for page tables and wasted $(2^8 - 3 + 2^{10} - 1) \cdot 3 + 318 + (2^{10} - 1) \cdot 3 + (2^8 - 3 + 2^{10} - 1) \cdot 3$ bytes due to internal fragmentation.